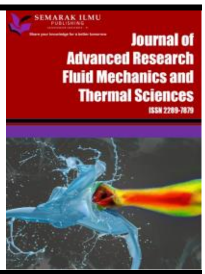
Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage:
https://semarakilmu.com.my/journals/index.php/fluid_mechanics_thermal_sciences/index
ISSN: 2289-7879



Unsteady MHD Free Convection Flow of a Second Grade Casson Fluid with Ramped Wall Temperature

K. Venkata Chandra Sekhar[1,*]

[1] Department of Mathematics, College of Engineering, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur, Andhra Pradesh, India

ARTICLE INFO

Article history:
Received 9 August 2023
Received in revised form 20 October 2023
Accepted 29 October 2023
Available online 15 November 2023

Keywords:
Unsteady; MHD; second grade; Casson fluid; Laplace transform

ABSTRACT

The present study delves into the analysis of unsteady free convection flow involving a non-Newtonian Casson fluid adjacent to an infinitely tall vertical plate. The primary objective of this article is to examine the flow behaviour of a second-grade Casson fluid under varying critical parameters. The plate exhibits a gradual increase in wall temperature and is situated within a porous medium. The governing equations are transformed into a dimensionless form, using non dimensionless variables. The method of Laplace transform (MOLT) is employed to derive accurate solutions for the temperature and velocity within the flow field. To enhance comprehension of the issue, The impact of different physical parameters on the flow field has been examined through the utilization of graphs and tables. Outcome of the problem was compared with existing one which was in good agreement. The study revealed that the velocity magnitude was lower in the scenario with ramped temperature compared to the scenario with isothermal temperature. Finally, skin friction and Nusselt numbers of the present work are tabulated. The significant outcome of the present work is that the flow field for the present phenomena is same as that of the existing one in absence of radiation parameter and heat source parameter when $u(0,t)=0$, $\beta=\infty$.

1. Introduction

Natural convection heat transfer in between a vertical plate and a fluid is a well-studied topic as it has many engineering applications. Researchers have investigated diverse thermal boundary conditions at bounding plates, encompassing continuous and well-defined conditions, non-uniform and arbitrary wall conditions, as well as the impact of radiation and porosity. Some of the most notable studies include those by Raptis and Singh [1,2], Sacheti *et al.,* [3] Chandran *et al.,* [4,5], Ganesan and Palani [6], Fetecau *et al.,* [7,8], Chandran *et al.,* [9], Seth and Ansari [10], Seth *et al.,* [11], Narahari and Bég [12] and Samiulhaq *et al.,* [13]. These studies have led to the development of Closed-form solutions for the velocity and temperature fields under a variety of conditions. Although all of the aforementioned results refer to incompressible viscous fluids, there is still a need to study natural convection heat transfer in other types of fluids, such as non-Newtonian fluids. The

* *Corresponding author.*
E-mail address: kasulanati22@rediffmail.com

https://doi.org/10.37934/arfmts.111.1.8095

importance of non-Newtonian fluids has been increased significantly in recent years, and numerous researchers have made significant contributions to the study of flow characteristics. Non-Newtonian fluids are essential to understand because they are used in a wide variety of applications, including plastics manufacturing, lubricants, food processing, and biological fluid dynamics.

Non-Newtonian fluids are fluids whose viscosity varies with the shear rate. This has led to the development of various models of non-Newtonian fluids. Within this category, analysing second-grade fluids is straightforward, allowing for effective modelling of various types of fluids like dilute polymer solutions, slurry flows, and industrial oils. Second-grade fluids offer a versatile approach for examining a diverse array of problems encompassing varying geometries and mechanical as well as thermal boundary conditions. For instance, an examination by Tan and Masuoka [14] delved into the flow of a second-grade fluid in a porous half-space near a heated flat plate. In a separate study, Hayat and Abbas [15] utilized the homotopy analysis method to scrutinize heat transfer in the magneto hydrodynamic (MHD) flow of second-grade fluids within porous media. Khan *et al.,* [16] derived a closed-form solution, Srinivasa *et al.,* [17] investigated thermal effects in Stoke's problem, and Mustafa *et al.,* [18] investigated the natural convection flow of second-grade fluids along a vertical plate, taking into account their viscoelastic properties. Within the realm of engineering, magneto-hydrodynamics (MHD) has been employed to produce electricity from ionized gas through the application of a magnetic field. The investigation of heat and mass transfer within magneto hydrodynamic (MHD) flows holds significance due to its numerous applications in both scientific research and technology. The utilization of heat and mass transfer principles in magneto hydrodynamic (MHD) flows extends to buoyancy-induced flows occurring in the atmosphere, bodies of water, and quasi-solid structures like the Earth. As a result, several researchers Katagiri [19], Jana *et al.,* [20], Mandal and Mandal [21], Ghosh [22], Jha and Apere [23], Krishna *et al.,* [24] have conducted research on heat and mass transfer in the context of magneto hydrodynamic (MHD) flow. Kataria and Mittal [25,26] explored the heat and mass transfer effects of various types of magnetohydrodynamic (MHD) nanofluids from different perspectives, Sheikholeslami *et al.,* [27], Krishna, and Chamkha [28] also investigated similar phenomena. Teh and Ashgar [29], along with Patel *et al.,* [30] conducted studies in this domain. Furthermore, Krishna and Chamkha [31] made a significant contribution to comprehending the impact of Hall and ion slip effects on the unsteady magnetohydrodynamic (MHD) convective rotating flow of nanofluids. Additionally, Krishna *et al.,* [32] delved into the Hall effects on the MHD peristaltic flow of Jeffrey fluid. Magnetic fields have a significant influence on a wide range of natural and technological phenomena. Magnetic fields are also used in a variety of technological applications, such as fast-breeder reactors, controlled thermonuclear fusion, and MHD power generation. To explore intriguing findings on second-grade fluids, one can refer to the citations [33-37]. The purpose of this analysis is to extend the work done by Samiulhaq *et al.,* [33] for Casson fluid flow with variable velocity, radiation and heat source included with ramped wall temperature. Moreover, exact solutions are derived for velocity and temperature for second grade Casson fluid flow by using powerful Laplace transformation method. The solutions obtained meet all the specified initial and boundary conditions.

2. Mathematical Representation of the Phenomenon

An investigation was conducted on the unsteady flow of a non-Newtonian fluid exhibiting Casson behaviour in the vicinity of an infinitely tall vertical plate under the influence of magnetohydrodynamics (MHD), with the added condition of the plate's temperature increasing linearly with time. The electrically conducting fluid flows through a porous medium. We define the x-axis as the upward direction along the plate, and the y-axis as the normal direction to the plane of the plate. A uniform magnetic field with strength B_0 is applied perpendicular to the flow, as illustrated in Figure 1. At the initial time (t=0), both the fluid and the plate are stationary and maintained at a constant temperature. At a later time $t=t_0$, the temperature of the plate is either increased or decreased to $T_0+\left(T_w-T_0\right)\frac{t}{t_0}$ for $t>t_0$, and it is maintained at this constant temperature T_w. The goal of this study is to investigate the natural convection flow of a Casson fluid, incorporating factors such as a heat source with variable velocity, radiation effects, and a temperature that varies with a ramped profile on the bounding plate.

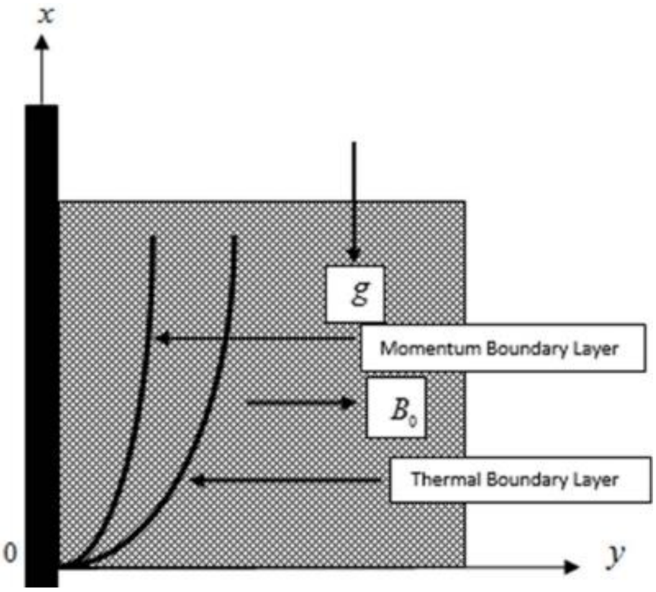


Fig. 1. Physical model of the phenomena

In the course of the analysis, the following assumptions were taken into consideration.

(i) The impact of viscous dissipation on the energy equation is deemed insignificant.
(ii) Lorentz force, which is given by *J* × B, is the body force. B represents the strength of the magnetic field everywhere in space, while J represents the amount of current flowing through a given area.
(iii) The current density can be calculated using Ohm's law, which states that $J=\sigma(E+V\times B)$, here, σ represents the electrical conductivity of the fluid, and E denotes the electric field.
(iv) V is the velocity vector field, total magnetic field $B=B_0+b_1$.
(v) The magnetic field applied is denoted as B_0, while the resulting induced magnetic field is represented as b1.
(vi) The current density, J, is expressed under specific assumptions $E=0$, $b_1=0$ and $B=B_0=(0,B_0,0)$. B_0 modified as $J=\sigma(V\times B_0)$.
(vii) At last, the Lorentz force becomes $J\times B=-\sigma B_0{}^2V$.

Under the Boussinesq approximation the equations governing the flow with reference to Krishna *et al.,* [24] are as follows:

$$\frac{\partial u}{\partial t}=\left[\nu\left(1+\frac{1}{\beta}\right)+\frac{\alpha_1}{\rho}\frac{\partial}{\partial t}\right]\frac{\partial^2 u}{\partial y^2}+g\beta\left(T-T_\infty\right)-\frac{\sigma B_0{}^2 u}{\rho}-\frac{\varphi}{k_1}\left(\nu\left(1+\frac{1}{\beta}\right)+\frac{\alpha_1}{\rho}\frac{\partial}{\partial t}\right)u(y,t) \tag{1}$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}-\frac{Q}{\left(T\text{-}T_\infty\right)\rho c_p} \tag{2}$$

Here u is the velocity of the flow in x direction, T is its temperature, ρ is the density of the fluid, g is gravitational force, β is Casson parameter, ν is kinematic viscosity, φ is the porosity of the medium, k is permeability, κ is the thermal conductivity, c_p is specific heat at the constant pressure, and α_1 is the material module of second grade fluid. In reference to Samiulhaq *et al.,* [33] the pertinent initial and boundary conditions are outlined below:

$$u^*[y,0]=0, T^*[y,0]=T_\infty, y\geq 0, u^*[0,t]=\frac{t^* t_0\sqrt{t_0}}{\nu}, t>0$$

$$T^*[0,t]=T_\infty+\left(T_w\text{-}T_\infty\right)\frac{t}{t_0}, \text{for } 0<t<t_0 \tag{3}$$

$$T^*[0,t]=T_w{}^* \text{ for } t>t_0$$

$$u^*[y,t]\to 0 \text{ as } y\to\infty, T^*[y,t]\to 0 \text{ as } y^*\to\infty \text{ and } t\geq 0$$

Introducing the subsequent dimensionless parameters

$$y^*=\frac{y}{\sqrt{\nu t_0}}, t^*=\frac{t}{t_0}, u^*=u\sqrt{\frac{t_0}{\nu}}, \theta=\frac{T^*\text{-}T_\infty}{T_w\text{-}T_\infty}, \Pr=\frac{\mu c_p}{\kappa}=\frac{\nu\rho c_p}{\kappa}$$

$$\alpha=\frac{\alpha_1}{\mu t_0}, M^2=\frac{\sigma B_0{}^2 t_0}{\rho}, \frac{1}{k}=\frac{\nu f}{\kappa_1}t_0 \tag{4}$$

$t_0=\left[\frac{\sqrt{\nu}}{g\beta\left(T_w{}^*\text{-}T_\infty\right)}\right]^{\frac{2}{3}}, q_r=\frac{-4\sigma}{3\kappa_1}\frac{\partial T^{*4}}{\partial y}, Q^*=\frac{Q}{\left(T\text{-}T_\infty\right)\rho c_p}$, into Eq. (1) and Eq. (2) and dropping * transformed equations are

$$\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial t\partial y^2}-\left(1+\frac{\alpha}{k}\right)\frac{\partial u}{\partial t}-\left(M^2+\frac{1+\frac{1}{\beta}}{k}\right)u+\theta=0 \tag{5}$$

$$\frac{\partial\theta}{\partial t}=\left(\frac{1+R}{\Pr}\right)\frac{\partial^2\theta}{\partial y^2}-Q\theta \tag{6}$$

The transmuted boundary conditions are

$$
\begin{aligned}
&u[y,0]=0,\theta[y,0]=0, y \geq 0 \\
&u[0,t]=t \quad t>0 \\
&\theta[0,t]=t \quad 0<t<0 \\
&\qquad =1 \quad t \geq 1 \\
&u[y,t] \to 0, \theta[y,t] \to 0 \text{ as } y \to \infty \; t \geq 0
\end{aligned}
\tag{7}
$$

3. Methodology (Laplace Transform Method)

By applying the Laplace transform to coupled partial differential equations (PDEs) (5) and (6) with respect to the boundary conditions (7), analytical expressions have been derived. These expressions yield the Laplace velocity and temperature equations.

$$
\left(A+\alpha s\right)\frac{d^2u}{dy^2}-\left(Bs+C\right)u+\theta=0 \tag{8}
$$

$$
\frac{d^2\theta}{dy^2}-E\left(s+Q\right)\theta=0 \tag{9}
$$

The corresponding Laplace initial and boundary conditions are determined by

$$
\begin{aligned}
&u[y,0]=0,\theta[y,0]=0, \; y \geq 0 \\
&u[0,s]=s^{-2}, \; \theta[0,s]=\frac{1-e^{-s}}{s^2} \\
&u[y,s] \to 0, \theta[y,s] \to 0 \text{ as } y \to \infty
\end{aligned}
\tag{10}
$$

The formulations for Laplace velocity and temperature are presented as follows:

$$
u(y,s)=\left[\frac{1}{s^2}-\frac{1-\frac{1}{e^s}}{s^2(s-I_1)(s-I_2)}\right]e^{-y\sqrt{\frac{Bs+c}{\alpha s+A}}}+\frac{1-e^{-s}}{s^2(s-I_1)(s-I_2)}e^{-y\sqrt{E}\sqrt{s+Q}} \tag{11}
$$

$$
u(y,s)=\left(1-\frac{1}{e^s}\right)s^{-2}e^{-y\sqrt{E}\sqrt{s+Q}} \tag{12}
$$

Utilizing the inverse Laplace transform to solve the Eq. (11) and Eq. (12) the corresponding velocity and temperature expressions describing the flow are as follows.

$$
u(y,t)=\frac{1}{2}\left[\left(t-\frac{yk_3}{2\sqrt{k1}}\right)e^{-yk_3\sqrt{k_1}}\operatorname{erfc}\left(\frac{yk_3}{2\sqrt{t}}-\sqrt{k_1t}\right)+\left(t+\frac{yk_3}{2\sqrt{k1}}\right)e^{-yk_3\sqrt{k_1}}\operatorname{erfc}\left(\frac{yk_3}{2\sqrt{t}}+\sqrt{k_1t}\right)\right]
$$

$$-J\Bigg[\frac{1}{2}\Bigg\{e^{-yk_3\sqrt{k_1}}\operatorname{erfc}\Bigg(\frac{yk_3}{2\sqrt{t}}-\sqrt{k_1 t}\Bigg)+e^{-yk_3\sqrt{k_1}}\operatorname{erfc}\Bigg(\frac{yk_3}{2\sqrt{t}}+\sqrt{k_1 t}\Bigg)\Bigg\}$$

$$-\frac{1}{2}\Bigg\{e^{-yk_3\sqrt{k_1}}\operatorname{erfc}\Bigg(\frac{yk_3}{2\sqrt{t-1}}-\sqrt{k_1(t-1)}\Bigg)+e^{-yk_3\sqrt{k_1}}\operatorname{erfc}\Bigg(\frac{yk_3}{2\sqrt{t}}+\sqrt{-k_1(1-t)}\Bigg)\Bigg\}H(t-1)\Bigg]$$

$$-K\Bigg[\frac{1}{2}\Bigg\{\Bigg(t-\frac{yk_3}{2\sqrt{k_1}}\Bigg)e^{-yk_3\sqrt{k_1}}\operatorname{erfc}\Bigg(\frac{yk_3}{2\sqrt{t}}-\sqrt{k_1 t}\Bigg)+\Bigg(t+\frac{yk_3}{2\sqrt{k_1}}\Bigg)e^{yk_3\sqrt{k_1}}\operatorname{erfc}\Bigg(\frac{yk_3}{2\sqrt{t}}+\sqrt{k_1 t}\Bigg)\Bigg\}$$

$$-\frac{1}{2}\Bigg\{\Bigg(t-1-\frac{yk_3}{2\sqrt{k_1}}\Bigg)e^{-yk_3\sqrt{k_1}}\operatorname{erfc}\Bigg(\frac{yk_3}{2\sqrt{t-1}}-\sqrt{k_1(t-1)}\Bigg)+\Bigg(t-1+\frac{yk_3}{2\sqrt{k_1}}\Bigg)e^{yk_3\sqrt{k_1}}\operatorname{erfc}\Bigg(\frac{yk_3}{2\sqrt{t-1}}+\sqrt{k_1(t-1)}\Bigg)\Bigg\}H(t-1)\Bigg]$$

$$-L\Bigg[\frac{1}{2}\Bigg\{e^{I_1 t-yk_3\sqrt{k_4}}\operatorname{erfc}\Bigg(\frac{yk_3}{2\sqrt{t}}-\sqrt{k_4 t}\Bigg)+e^{I_1 t+yk_3\sqrt{k_4}}\operatorname{erfc}\Bigg(\frac{yk_3}{2\sqrt{t}}+\sqrt{k_4 t}\Bigg)\Bigg\}$$

$$-\frac{1}{2}\Bigg\{e^{I_1(t-1)-yk_3\sqrt{k_4}}\operatorname{erfc}\Bigg(\frac{yk_3}{2\sqrt{t-1}}-\sqrt{-k_4(1-t)}\Bigg)+e^{I_1(t-1)+yk_3\sqrt{k_4}}\operatorname{erfc}\Bigg(\frac{yk_3}{2\sqrt{t-1}}+\sqrt{-k_4(1-t)}\Bigg)\Bigg\}\Bigg]$$

$$+L\Bigg[\frac{1}{2}\Bigg\{e^{I_2 t-yk_3\sqrt{k_5}}\operatorname{erfc}\Bigg(\frac{yk_3}{2\sqrt{t}}-\sqrt{k_5 t}\Bigg)+e^{I_1 t+yk_3\sqrt{k_5}}\operatorname{erfc}\Bigg(\frac{yk_3}{2\sqrt{t}}+\sqrt{k_5 t}\Bigg)\Bigg\} \tag{13}$$

$$-\frac{1}{2}\Bigg\{e^{I_2(t-1)-yk_3\sqrt{k_5}}\operatorname{erfc}\Bigg(\frac{yk_3}{2\sqrt{t-1}}-\sqrt{k_5(t-1)}\Bigg)+e^{I_1(t-1)+yk_3\sqrt{k_5}}\operatorname{erfc}\Bigg(\frac{yk_3}{2\sqrt{t-1}}+\sqrt{-k_5(1-t)}\Bigg)\Bigg\}H(t-1)\Bigg]$$

$$+J\Bigg[\frac{1}{2}\Bigg\{e^{-y\sqrt{EQ}}\operatorname{erfc}\Bigg(\frac{y\sqrt{E}}{2\sqrt{t}}-\sqrt{Qt}\Bigg)+e^{y\sqrt{EQ}}\operatorname{erfc}\Bigg(\frac{y\sqrt{E}}{2\sqrt{t}}+\sqrt{Qt}\Bigg)\Bigg\}$$

$$-\frac{1}{2}\Bigg\{e^{-y\sqrt{EQ}}\operatorname{erfc}\Bigg(\frac{y\sqrt{E}}{2\sqrt{t-1}}-\sqrt{Q(t-1)}\Bigg)+e^{y\sqrt{EQ}}\operatorname{erfc}\Bigg(\frac{y\sqrt{E}}{2\sqrt{t-1}}+\sqrt{-Q(1-t)}\Bigg)\Bigg\}H(t-1)\Bigg]$$

$$+K\Bigg[\frac{1}{2}\Bigg\{\Bigg(t-\frac{y\sqrt{E}}{2\sqrt{Q}}\Bigg)e^{-y\sqrt{EQ}}\operatorname{erfc}\Bigg(\frac{y\sqrt{E}}{2\sqrt{t}}-\sqrt{Qt}\Bigg)+\Bigg(t+\frac{y\sqrt{E}}{2\sqrt{Q}}\Bigg)e^{-y\sqrt{EQ}}\operatorname{erfc}\Bigg(\frac{y\sqrt{E}}{2\sqrt{t}}+\sqrt{Qt}\Bigg)\Bigg\}$$

$$-\frac{1}{2}\left\{\left(t-1-\frac{y\sqrt{E}}{2\sqrt{Q}}\right)e^{-yk_3\sqrt{k_1}}\mathrm{erfc}\left(\frac{y\sqrt{E}}{2(t-1)^{\frac{1}{2}}}-\sqrt{-k_1(1-t)}\right)+\left(t-1+\frac{y\sqrt{E}}{2\sqrt{Q}}\right)e^{-yk_3\sqrt{k_1}}\mathrm{erfc}\left(\frac{yk_3}{2}(t-1)^{-\frac{1}{2}}+\left(k_1(t-1)\right)^{\frac{1}{2}}\right)\right\}H(t-1)\Bigg]$$

$$+L\left[\frac{1}{2}\left\{e^{-y\sqrt{E}\sqrt{k_6}}\mathrm{erfc}\left(\frac{y\sqrt{E}}{2\sqrt{t}}-\sqrt{k_6t}\right)+e^{y\sqrt{E}\sqrt{k_6}}\mathrm{erfc}\left(\frac{y\sqrt{E}}{2\sqrt{t}}+\sqrt{k_6t}\right)\right\}\right.$$

$$\left.-\frac{1}{2}\left\{e^{-y\sqrt{E}\sqrt{k_6}}\mathrm{erfc}\left(\frac{y\sqrt{E}}{2\sqrt{t-1}}-\sqrt{k_6(t-1)}\right)+e^{y\sqrt{E}\sqrt{k_6}}\mathrm{erfc}\left(\frac{y\sqrt{E}}{2\sqrt{t-1}}+\sqrt{-k_6(1-t)}\right)\right\}H(t-1)\right]$$

$$-L\left[\frac{1}{2}\left\{e^{-y\sqrt{E}\sqrt{k_7}}\mathrm{erfc}\left(\frac{y\sqrt{E}}{2\sqrt{t}}-\sqrt{k_7t}\right)+e^{y\sqrt{E}\sqrt{k_7}}\mathrm{erfc}\left(\frac{y\sqrt{E}}{2\sqrt{t}}+\sqrt{k_7t}\right)\right\}\right.$$

$$\left.-\frac{1}{2}\left\{e^{-y\sqrt{E}\sqrt{k_7}}\mathrm{erfc}\left(\frac{y\sqrt{E}}{2\sqrt{t}}-\sqrt{k_6t}\right)+e^{y\sqrt{E}\sqrt{k_6}}\mathrm{erfc}\left(\frac{y\sqrt{E}}{2\sqrt{t}}+\sqrt{k_6t}\right)\right\}\right]\Bigg]$$

$$-\frac{1}{2}\left\{e^{-y\sqrt{E}\sqrt{k_7}}\mathrm{erfc}\left(\frac{y\sqrt{E}}{2\sqrt{t-1}}-\sqrt{k_7(t-1)}\right)+e^{y\sqrt{E}\sqrt{k_7}}\mathrm{erfc}\left(\frac{y\sqrt{E}}{2\sqrt{t-1}}+\sqrt{k_7(t-1)}\right)\right\}H(t-1)\Bigg]$$

$$\theta(y,t)=\frac{1}{2}\left[\left(t-\frac{y\sqrt{E}}{2\sqrt{Q}}\right)\mathrm{erfc}\left(\frac{y\sqrt{E}}{2\sqrt{t}}-\sqrt{Qt}\right)+\left(t+\frac{y\sqrt{E}}{2\sqrt{Q}}\right)\mathrm{erfc}\left(\frac{y\sqrt{E}}{2\sqrt{t}}+\sqrt{Qt}\right)\right] \quad (14)$$

$$-\frac{1}{2}\left[\left(t-1-\frac{y\sqrt{E}}{2\sqrt{Q}}\right)\mathrm{erfc}\left(\frac{y\sqrt{E}}{2\sqrt{t-1}}-\sqrt{Q(t-1)}\right)+\left(t-1+\frac{y\sqrt{E}}{2\sqrt{Q}}\right)\mathrm{erfc}\left(\frac{y\sqrt{E}}{2\sqrt{t-1}}+\sqrt{Q(t-1)}\right)\right]$$

4. Discussion on Findings

To gain a better understanding of the current problem The velocity, denoted as u [y, t], and temperature, denoted as T [y, t], are graphically represented for various y values. The impact of various critical parameters on the velocity and temperature fields of the flow phenomenon is described and visually presented. Elevating the Casson parameter results in an augmented viscosity. High viscosity lowers the fluid velocity. This was depicted in Figure 2. In Figure 3, velocity profiles corresponding to varying permeability parameters are displayed. As the values of k increase, the boundary layer density also rises. Due to this the opposing nature of penetrable medium is negligible which causes rise in velocity of the fluid. In Figure 4, the influence of the second-grade parameter on the flow velocity is depicted. Increase in second grade parameter decreases the boundary layer thickness that results in decrease of the fluid flow. The interaction between induced currents and the magnetic field produces a resistive force that opposes the flow of the fluid. This was shown in Figure 5. A heat source outside of a flow field adds heat to the fluid, which increases the temperature and velocity of the flow. Both the facts are displayed in Figure 6 and Figure 10. When thermal radiation increases in a fluid medium, the kinetic energy of the fluid particles decreases. This leads to a corresponding decrease in fluid velocity, as shown in Figure 7. The influence of the Prandtl number is illustrated in Figure 8. The Prandtl number is the ratio of kinematic viscosity to thermal diffusivity. With an increase in the Prandtl number, heat transfer efficiency decreases, and the viscosity of the fluid also rises. This results in an increase in the viscosity of the fluid. An increase in fluid viscosity results in a decrease in flow velocity. Figure 9 shows temperature profiles for different values of Prandtl number. As the Prandtl number increases, the thermal conductivity of the fluid decreases, resulting in a decrease in fluid temperature. Figure 11 shows how the radiation parameter affects the temperature field of the flow. Elevating the radiation parameter results in a higher temperature of the fluid. This is likely due to the increase in the radiation variable, leading to a thicker thermal boundary layer. From Figure 12, it is evident that the flow velocity is greater in the case of an isothermal temperature compared to a ramped temperature. The boundary layer is consistently thinner under a ramped temperature condition compared to an isothermal temperature scenario. This is attributed to the steeper temperature gradient induced by the ramped temperature profile, promoting enhanced heat transfer and thereby minimizing the boundary layer thickness.

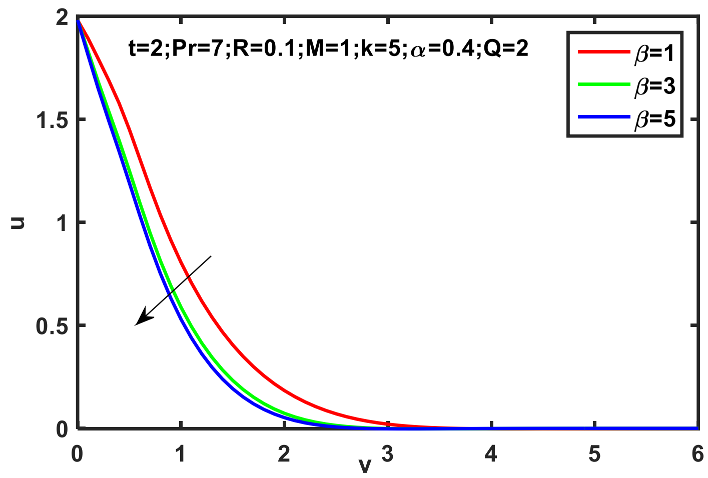


Fig. 2. Influence of Casson parameter on velocity field

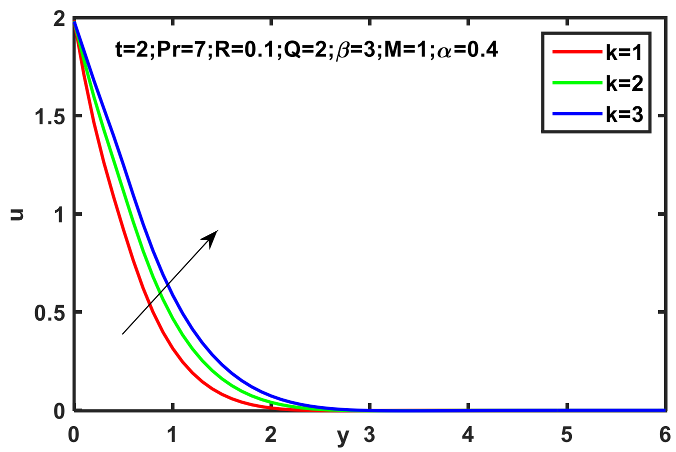


Fig. 3. Velocity Field Variation with Respect to Permeability Parameter

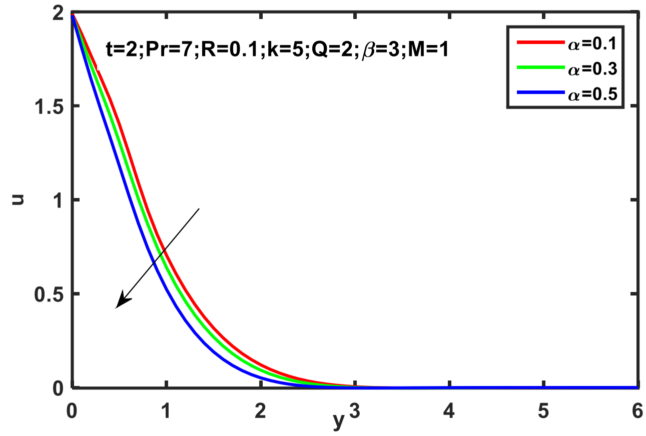


Fig. 4. Impact of second grade parameter on velocity field

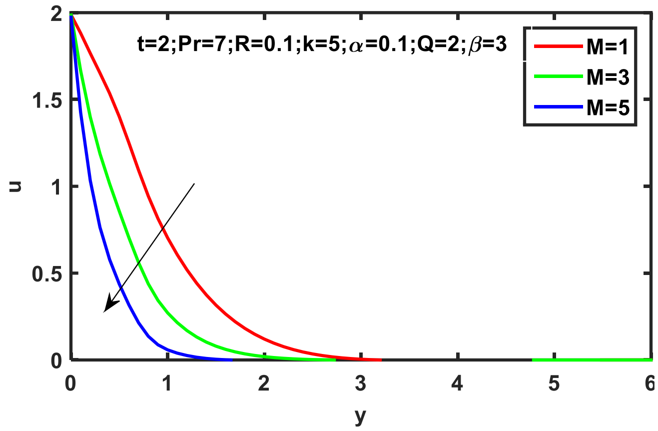


Fig. 5. Variations in velocity field under the Influence of Magnetic parameter

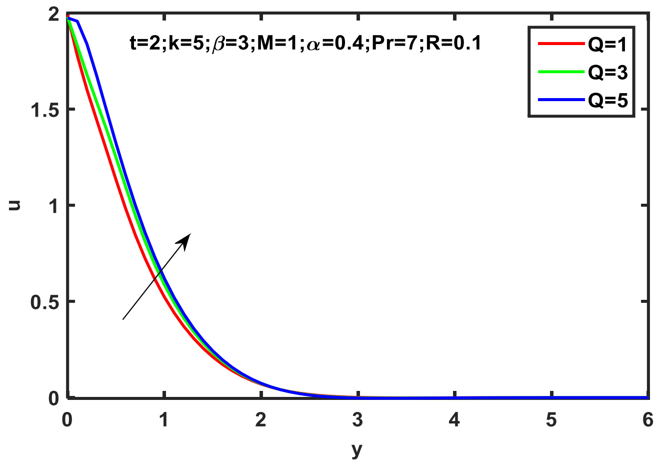


Fig. 6. Result of velocity field with reference to Heat source parameter

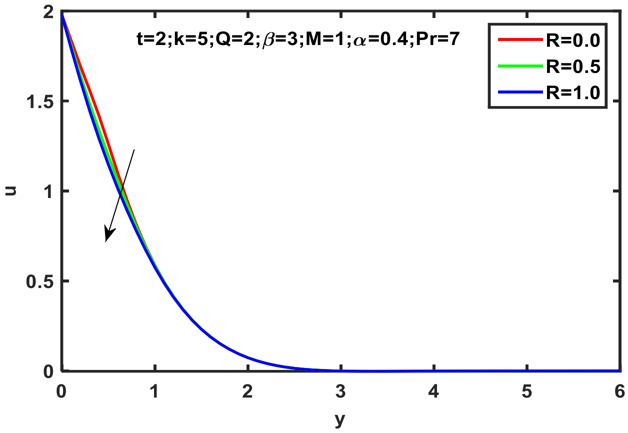


Fig. 7. Impact of the Radiation Parameter on Velocity Distribution

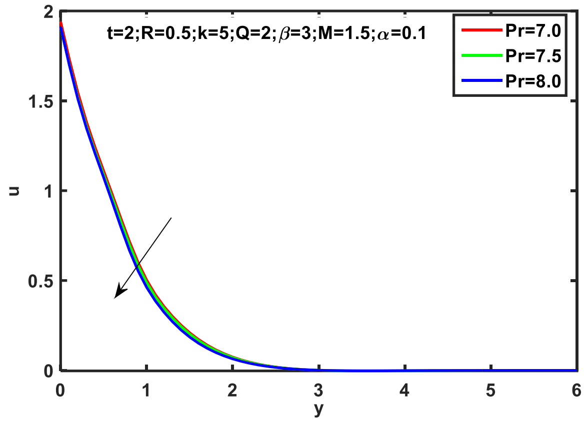


Fig. 8. Influence of the Prandtl parameter on the velocity distribution

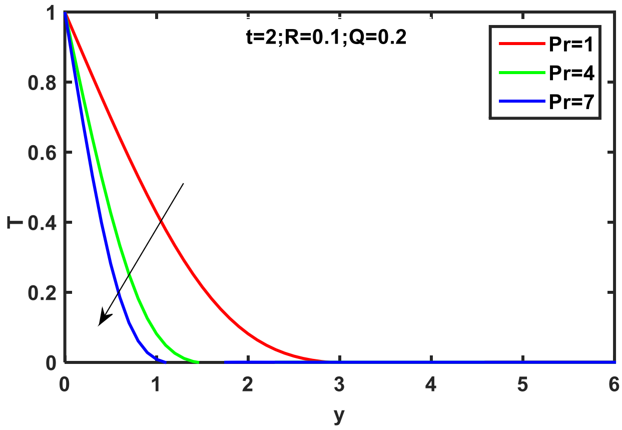


Fig. 9. Effect of the Prandtl parameter on the temperature distribution

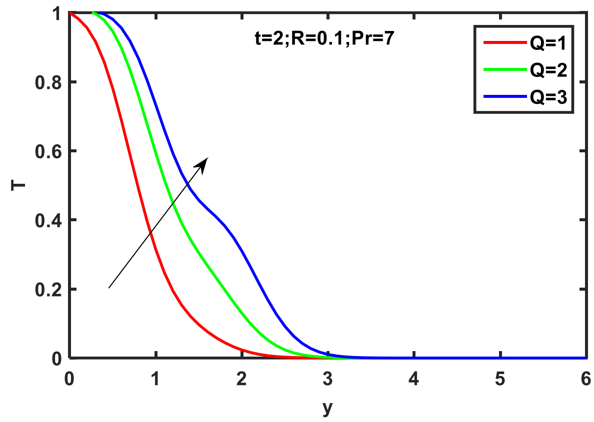


Fig. 10. Impact of the heat source parameter on the temperature distribution

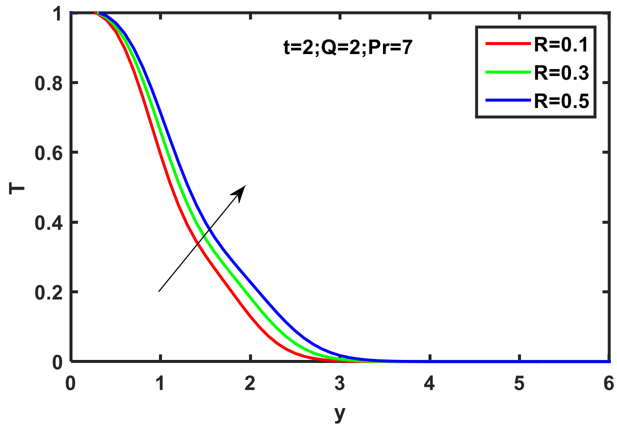


Fig. 11. Influence of Radiation parameter on temperature field

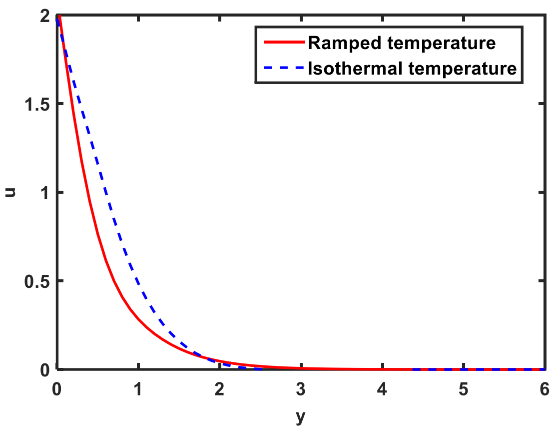


Fig. 12. The impact of ramped and isothermal temperatures on the velocity field

5. Skin Friction and Nusselt Number

Skin friction pertains to the resistance generated by friction between a fluid and the surrounding surface. It holds significance in the context of water-bed boundary conditions, because frictional forces can cause sediment particles to erode from the bed.

Skin friction (s_f) = $\mu \left.\frac{\partial u}{\partial y}\right|_{y=0} + \alpha \left.\frac{\partial^2 u}{\partial t \partial y}\right|_{y=0}$.

The Nusselt number is a nondimensional parameter that quantifies the rate of heat transfer by convection relative to the rate of heat transfer by conduction under the same conditions.

Nusselt number (Nu) = $-\left.\frac{\partial T}{\partial y}\right|_{y=0}$

Table 1 and Table 2 display the changes in skin friction and Nusselt number concerning distinct physical parameters.

Table 1
Skin friction for different values of M, β, α, k and Q when $Pr=7$, $R=0.1$, $t=1.2$

M	β	α	k	Q	sf
2.1					1.4503
2.2					1.8225
2.3					2.2012
	1				1.4503
	2				2.1701
	3				2.3399
		0.1			1.4503
		0.2			3.2178
		0.3			4.4246
			5		1.4503
			6		1.3884
			7		1.3441
				1	3.9937
				2	1.8225
				3	0.3447

Table 2
Nusselt number for different values t=2, Pr=7, Q=1, R=0.1

Pr	Q	R	t	Nu
7				0.1023
8				0.1169
9				0.1315
	1			0.1023
	2			0.0410
	3			0.0336
		0.1		0.1023
		0.2		0.0937
		0.3		0.0865

6. Conclusions

A second grade Casson fluid flow phenomena was studied under the influence of different physical parameters. The governing partial differential equations are made dimensionless. Exact solutions are derived for Flow velocity and temperature by applying the method of Laplace transforms (MOLT). Graphical analysis was performed to examine the non-uniform distribution of velocity and temperature within the flow influenced by varying critical parameters. The disparity in tables displaying skin friction and Nusselt number corresponding to various physical parameters are presented. During the analysis the following conclusions were drawn

(i) Increase of Permeability parameter and heat source parameter increases the speed of the fluid flow, whereas increase of Casson parameter, second grade fluid parameter, Magnetic parameter, Prandtl number, Radiation parameter decreases the same.

(ii) An increase in Prandtl number decreases the temperature of the flow field, while an increase in heat source and radiation parameter increases the temperature of the flow field. It is noticed that the velocity of the flow field with ramped temperature lower than that of isothermal temperature.

(iii) In the present problem boundary condition at wall was taken as $u(0,t)=t$, where as Samiulhaq *et al.,* [33] taken as $u(0,t)=0$.

(iv) To verify the accuracy of the flow field solution presented in this article, results are compared to existing solutions using graphs. Figure 13 depicts the disparity in velocity fields of existing problem with that of Samiulhaq *et al.,* [33]. Figure 14 depicts that velocity profiles of Present work and Samiulhaq *et al.,* [33] are coincided when the boundary condition $u(0,t)=t$ replaced with $u(0,t)=0$ taking $R=0, \beta=\infty$ and $Q=0$.

(v) Table 3 describes the nature of the flow phenomena for different values of y and t. It can be noticed that velocity of the flow converges to zero far away from the plate

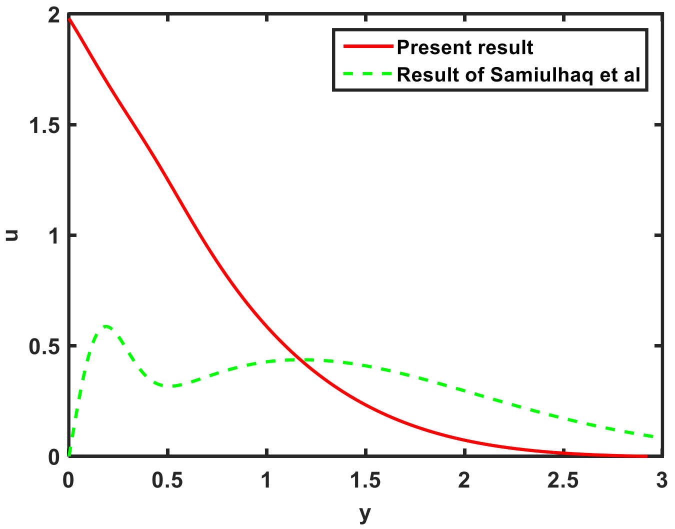


Fig. 13. Comparison of existing results of velocity field with Samiulhaq *et al.,* [33]

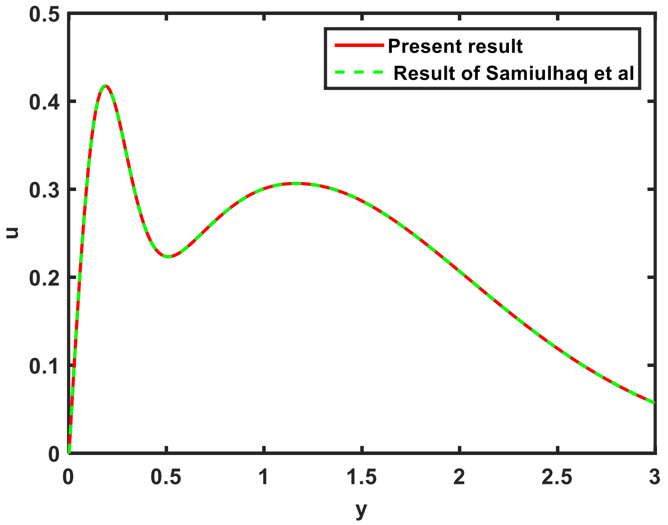


Fig. 14. Comparison of existing results of velocity field with Samiulhaq *et al.,* [33]

Table 3
Velocity distribution for various values of y and t

y	t=1.5	t=2.0	t=2.5	t=3.5
0.0	1.5	2.0	2.5	3.5
0.5	1.0	1.4	1.7	2.3
1.0	0.3	0.7	0.9	1.3
1.5	0.0	0.3	0.4	0.7
2.0	0.0	0.1	0.2	0.3
2.5	0.0	0.0	0.0	0.1
3.0	0.0	0.0	0.0	0.0

Acknowledgement
This research work was not funded by any grant.

References
[1] Raptis, A., and A. K. Singh. "MHD free convection flow past an accelerated vertical plate." *International Communications in Heat and Mass Transfer* 10, no. 4 (1983): 313-321. https://doi.org/10.1016/0735-1933(83)90016-7

[2] Raptist, A., and A. K. Singh. "Rotation effects on MHD free-convection flow past an accelerated vertical plate." *Mechanics Research Communications* 12, no. 1 (1985): 31-40. https://doi.org/10.1016/0093-6413(85)90032-1

[3] Sacheti, Nirmal C., Pallath Chandran, and A. K. Singh. "An exact solution for unsteady magnetohydrodynamic free convection flow with constant heat flux." *International Communications in Heat and Mass Transfer* 21, no. 1 (1994): 131-142. https://doi.org/10.1016/0735-1933(94)90090-6

[4] Chandran, Pallath, Nirmal C. Sacheti, and Ashok K. Singh. "Unsteady hydromagnetic free convection flow with heat flux and accelerated boundary motion." *Journal of the Physical Society of Japan* 67, no. 1 (1998): 124-129. https://doi.org/10.1143/JPSJ.67.124

[5] Chandran, P., N. C. Sacheti, and A. K. Singh. "Exact solutions for the convective flow of fluids of different Prandtl numbers near an infinite vertical plate in a rotating system." *International Journal of Applied Mechanics and Engineering* 6, no. 3 (2001): 573-590.

[6] Ganesan, P., and G. Palani. "Natural convection effects on impulsively started inclined plate with heat and mass transfer." *Heat and Mass Transfer* 39, no. 4 (2003): 277-283. https://doi.org/10.1007/s00231-002-0380-1

[7] Fetecau, Constantin, Samiulhaq, Ilyas Khan, Farhad Ali, and Sharidan Shafie. "Radiation and porosity effects on the magnetohydrodynamic flow past an oscillating vertical plate with uniform heat flux." *Zeitschrift für Naturforschung A* 67, no. 10-11 (2012): 572-580. https://doi.org/10.5560/zna.2012-0070

[8] Fetecau, Corina, Mehwish Rana, and Constantin Fetecau. "Radiative and porous effects on free convection flow near a vertical plate that applies shear stress to the fluid." *Zeitschrift für Naturforschung A* 68, no. 1-2 (2013): 130-138. https://doi.org/10.5560/zna.2012-0083

[9] Chandran, Pallath, Nirmal C. Sacheti, and Ashok K. Singh. "Natural convection near a vertical plate with ramped wall temperature." *Heat and Mass Transfer* 41 (2005): 459-464. https://doi.org/10.1007/s00231-004-0568-7

[10] Seth, G. S., and Md S. Ansari. "MHD natural convection flow past an impulsively moving vertical plate with ramped wall temperature in the presence of thermal diffusion with heat absorption." *International Journal of Applied Mechanics and Engineering* 15, no. 1 (2010): 199-215.

[11] Seth, G. S., Md S. Ansari, and R. Nandkeolyar. "MHD natural convection flow with radiative heat transfer past an impulsively moving plate with ramped wall temperature." *Heat and Mass Transfer* 47 (2011): 551-561. https://doi.org/10.1007/s00231-010-0740-1

[12] Narahari, M., and O. Anwar Bég. "Radiation effects on free convection flow past an impulsively started infinite vertical plate with ramped wall temperature and constant mass diffusion." In *AIP Conference Proceedings*, vol. 1225, no. 1, pp. 743-750. American Institute of Physics, 2010. https://doi.org/10.1063/1.3464923

[13] Samiulhaq, Samiulhaq, Ilyas Khan, Farhad Ali, and Sharidan Shafie. "MHD free convection flow in a porous medium with thermal diffusion and ramped wall temperature." *Journal of the Physical Society of Japan* 81, no. 4 (2012): 044401. https://doi.org/10.1143/JPSJ.81.044401

[14] Tan, Wenchang, and Takashi Masuoka. "Stokes' first problem for a second grade fluid in a porous half-space with heated boundary." *International Journal of Non-Linear Mechanics* 40, no. 4 (2005): 515-522. https://doi.org/10.1016/j.ijnonlinmec.2004.07.016

[15] Hayat, T., and Z. Abbas. "Heat transfer analysis on the MHD flow of a second grade fluid in a channel with porous medium." *Chaos, Solitons & Fractals* 38, no. 2 (2008): 556-567. https://doi.org/10.1016/j.chaos.2006.12.004

[16] Khan, Masood, Kaleem Iqbal, and M. Azram. "Closed-form solutions for MHD flow of a second-grade fluid through porous space." *Special Topics & Reviews in Porous Media: An International Journal* 2, no. 2 (2011). https://doi.org/10.1615/SpecialTopicsRevPorousMedia.v2.i2.60

[17] Srinivasa, R. K., B. R. B. Reddy, and P. K. Rao. "Thermal effects in Stokes' second problem for unsteady second grade fluid flow through a porous medium under effect of a magnetic field." *Journal of Mathematics (IOSR-JM)* 7, no. 2 (2013): 5-14. https://doi.org/10.9790/5728-0720514

[18] Mustafa, Naeem, S. Asghar, and M. A. Hossain. "Natural convection flow of second-grade fluid along a vertical heated surface with power-law temperature." *Chemical Engineering Communications* 195, no. 3 (2007): 209-228. https://doi.org/10.1080/00986440701569127

[19] Katagiri, Masakazu. "Flow formation in Couette motion in magnetohydrodynamics." *Journal of the Physical Society of Japan* 17, no. 2 (1962): 393-396. https://doi.org/10.1143/JPSJ.17.393

[20] Jana, R. N., Nanigopal Datta, and B. S. Mazumder. "Magnetohydrodynamic Couette flow and heat transfer in a rotating system." *Journal of the Physical Society of Japan* 42, no. 3 (1977): 1034-1039. https://doi.org/10.1143/JPSJ.42.1034

[21] Mandal, G., and K. K. Mandal. "Effect of Hall current on MHD Couette flow between thick arbitrarily conducting plate in a rotating system." *Journal of the Physical Society of Japan* 52, no. 2 (1983): 470-477. https://doi.org/10.1143/JPSJ.52.470

[22] Ghosh, S. K. "Unsteady hydromagnetic flow in a rotating channel with oscillating pressure gradient." *Journal of the Physical Society of Japan* 62, no. 11 (1993): 3893-3903. https://doi.org/10.1143/JPSJ.62.3893

[23] Jha, Basant K., and Clement A. Apere. "Combined effect of hall and ion-slip currents on unsteady mhd couette flows in a rotating system." *Journal of the Physical Society of Japan* 79, no. 10 (2010): 104401. https://doi.org/10.1143/JPSJ.79.104401

[24] Krishna, M. Veera, B. V. Swarnalathamma, and Ali J. Chamkha. "Investigations of Soret, Joule and Hall effects on MHD rotating mixed convective flow past an infinite vertical porous plate." *Journal of Ocean Engineering and Science* 4, no. 3 (2019): 263-275. https://doi.org/10.1016/j.joes.2019.05.002

[25] Kataria, Hari R., and Akhil S. Mittal. "Mathematical model for velocity and temperature of gravity-driven convective optically thick nanofluid flow past an oscillating vertical plate in presence of magnetic field and radiation." *Journal of the Nigerian Mathematical Society* 34, no. 3 (2015): 303-317. https://doi.org/10.1016/j.jnnms.2015.08.005

[26] Kataria, Hari R., and Akhil S. Mittal. "Velocity, mass and temperature analysis of gravity-driven convection nanofluid flow past an oscillating vertical plate in the presence of magnetic field in a porous medium." *Applied Thermal Engineering* 110 (2017): 864-874. https://doi.org/10.1016/j.applthermaleng.2016.08.129

[27] Sheikholeslami, M., Hari R. Kataria, and Akhil S. Mittal. "Effect of thermal diffusion and heat-generation on MHD nanofluid flow past an oscillating vertical plate through porous medium." *Journal of Molecular Liquids* 257 (2018): 12-25. https://doi.org/10.1016/j.molliq.2018.02.079

[28] Krishna, M. Veera, and Ali J. Chamkha. "Hall effects on MHD squeezing flow of a water-based nanofluid between two parallel disks." *Journal of Porous Media* 22, no. 2 (2019). https://doi.org/10.1615/JPorMedia.2018028721

[29] Teh, Yuan Ying, and Adnan Ashgar. "Three dimensional MHD hybrid nanofluid Flow with rotating stretching/shrinking sheet and Joule heating." *CFD Letters* 13, no. 8 (2021): 1-19. https://doi.org/10.37934/cfdl.13.8.119

[30] Patel, Harshad R., Akhil S. Mittal, and Rakesh R. Darji. "MHD flow of micropolar nanofluid over a stretching/shrinking sheet considering radiation." *International Communications in Heat and Mass Transfer* 108 (2019): 104322. https://doi.org/10.1016/j.icheatmasstransfer.2019.104322

[31] Krishna, M. Veera, and Ali J. Chamkha. "Hall and ion slip effects on Unsteady MHD Convective Rotating flow of Nanofluids-Application in Biomedical Engineering." *Journal of the Egyptian Mathematical Society* 28, no. 1 (2020): 1. https://doi.org/10.1186/s42787-019-0065-2

[32] Krishna, M. Veera, K. Bharathi, and Ali J. Chamkha. "Hall effects on MHD peristaltic flow of Jeffrey fluid through porous medium in a vertical stratum." *Interfacial Phenomena and Heat Transfer* 6, no. 3 (2018). https://doi.org/10.1615/InterfacPhenomHeatTransfer.2019030215

[33] Samiulhaq, Sohail Ahmad, Dumitru Vieru, Ilyas Khan, and Sharidan Shafie. "Unsteady magnetohydrodynamic free convection flow of a second grade fluid in a porous medium with ramped wall temperature." *PLoS One* 9, no. 5 (2014): e88766. https://doi.org/10.1371/journal.pone.0088766

[34] Krishna, M. Veera, Kamboji Jyothi, and Ali J. Chamkha. "Heat and mass transfer on MHD flow of second-grade fluid through porous medium over a semi-infinite vertical stretching sheet." *Journal of Porous Media* 23, no. 8 (2020). https://doi.org/10.1615/JPorMedia.2020023817

[35] Rehman, Aziz Ur, Muhammad Bilal Riaz, and Adam Wojciechowski. "Thermo diffusion impacts on the flow of second grade fluid with application of (ABC),(CF) and (CPC) subject to exponential heating." *Scientific Reports* 12, no. 1 (2022): 18437. https://doi.org/10.1038/s41598-022-21773-5

[36] Shah, Nehad Ali, Se-Jin Yook, and Oreyeni Tosin. "Analytic simulation of thermophoretic second grade fluid flow past a vertical surface with variable fluid characteristics and convective heating." *Scientific Reports* 12, no. 1 (2022): 5445. https://doi.org/10.1038/s41598-022-09301-x

[37] Sadia, Haleema, and M. Mustafa. "Numerical exploration of slip effects on second-grade fluid motion over a porous revolving disk with heat and mass transfer." *Heliyon* 9, no. 8 (2023). https://doi.org/10.1016/j.heliyon.2023.e18683