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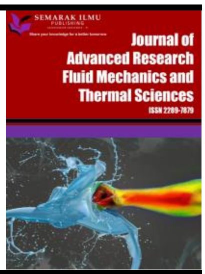

Numerical Solution of Falkner-Skan Equation for a Moving Wedge in Hybrid Nanofluids

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ABSTRACT

The flow and heat transfer past a moving wedge in a hybrid nanofluid is studied. A set of governing equations is transformed into ordinary differential equations using the similarity transformation. The resulting equations are then solved using bvp4c solver in MATLAB. The effects of the wedge angle parameter m and nanoparticle volume fraction parameter of $\mathrm{Al_2O_3-Cu}$/water on the skin friction coefficient and heat transfer characteristics are investigated. It is found that increasing the wedge angle parameter m and Cu nanoparticles volume fraction gives rise to the skin friction coefficient and heat transfer on the surface. Further, dual solutions exist when the wedge moves in the opposite direction with the free stream.

1. Introduction

Nanotechnology is a technology that has captured the attention of an abundant number of researchers since the last century. In 1959, nanotechnology was first introduced by Nobel Laureate Richard P. Feynman, an American physicist during his lecture *"There's Plenty of Room at the Bottom"*. Since then, nanotechnology has been widely used in the industry as it provides cleaner and more efficient energy supplies, which may help decrease construction, maintenance, repair, and decommissioning activities [1,2].

Nanofluids are a technology that falls under nanotechnology. Nanofluids were first proposed by Choi and Eastman [3]. A nanofluid can be categorized as a group of base fluids that consists of nanometer-sized particles (1-100 nm) [4]. The nanoparticles were usually made of metals, oxides, carbides, nitrides, or nonmetals. Nanofluids are produced by dispersing the nano-sized particles into

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conventional heat transfer fluids such as water, ethylene glycol, toluene, and engine oil [5]. The advantages of using nanofluids are that they boost thermophysical properties such as thermal conductivity, thermal diffusivity, viscosity, convective heat transfer, and have great stability. However, the thermal conductivity of nanofluid varies according to the size, shape, and materials of nanoparticles. The smaller the size of the nanoparticles, the higher the thermal conductivity of nanofluids [6,7]. As the size of nanoparticles is diminutive, they can easily flow through a microscopic channel without clogging. Furthermore, several studies stated that the usage of nanoparticles in their studies had increased by 15% - 40% thermal conductivity [5]. Yacob *et al.,* [8] numerically analyzed the steady two-dimensional boundary layer flow passing a static or moving wedge immersed into nanofluids. Three nanoparticles considered are Cu, TiO_2, Al_2O_3 with water as the base fluid. Furthermore, the Keller-box method and the NAG routine DO2HAF were used to generate the numerical solutions. The findings show that Cu-water has the highest skin friction coefficient and heat transfer rate at the surface. Aziz *et al.,* [9] scrutinized the flow and heat transfer of nanofluid in a thin film over an unsteady stretching sheet. Three different nanoparticles were utilized in the study which are Cu, TiO_2, Al_2O_3, and water as base fluid. The nonlinear differential equation was solved by using the Homotopy Analysis Method (HAM) and resulted that when the volume fraction of nanoparticles increases, the rate of heat transfer decreases. Furthermore, Sreedevi *et al.,* [10] conducted a comparative study to analyze the chemical reaction and thermal radiation on mixed convection flow and heat and mass transfer characteristics of water-TiO_2 and water-Al_2O_3 over a wedge. Velocity, temperature, and concentration slip boundary condition were taken into consideration and was solved numerically by using the Galarkin finite element method. The result shows that, as the magnetic field parameter increases, the temperature and concentration fields decrease. Currently, most of the industry that uses the heat transfer method changed to nanofluids for energy conversion, heat exchangers, and cooling systems [11].

Nanofluids were said to be widely used in the industry due to their high thermal properties and recently, a new generation of nanofluids was developed, called hybrid nanofluids. Hybrid nanofluids are considered strong because they have better conductivity at a lesser nanoparticle concentration. A hybrid nanofluid is a fluid made up of two or more different types of nanoparticles in a based fluid [12]. The innovation of this nanotechnology helps in enhancing the performance of heat transformation because it is more effective in chemical and thermal conductivity compared to nanofluids which also increase the area of nanoparticles, energy efficiency, and show better performance [13]. According to most research conducted, hybrid nanofluid is very helpful and plays an important role in most technological innovation industries such as the biochemical industry, refrigeration, aircraft, spacecraft, thermal storage, solar heating, power systems, etc. There are two methods to be used in utilizing hybrid nanofluids which are the one/single-step and two-step methods. Moreover, the best result can be executed with the usage of a hybrid nanofluid by adjusting the nano-powder concentration of each nanoparticle [14,15].

Izady *et al.,* [16] investigated two-dimensional boundary layer flow over a porous stretching/shrinking wedge with radiation and magnetohydrodynamic (MHD) effects and Fe_2O_3-CuO/water was used as the hybrid nanofluids. Later, Hussain *et al.,* [11] conducted a numerical study to clarify the dynamics of water conveying of hybrid nanofluid over an exponentially stretchable sheet with the presence of Navier's partial slip and thermal jump condition using a finite difference scheme based, Lobatto IIIa-bvp4c in MATLAB. In contrast to Izady *et al.,* [16], Hussain *et al.,* [11] considered three types of hybrids nanofluids which are Al_2O_3-TiO_2, Cu-Al_2O_3 and Cu-TiO_2, while H_2O is used as the base fluid. Kakar *et al.,* [17] extended the work of Yacob *et al.,* [8] by including magnetic and melting effects with velocity slip condition on the stagnation point flow over a stretching/shrinking wedge in hybrid nanofluids. Two different nanoparticles namely Al_2O_3 and Cu, and water as base fluid are taken into consideration and the numerical result were obtained by using bvp4c technique in MATLAB. Futhermore, Anuar *et al.,* [18] analysed the flow of stagnation point over an exponentially shrinking sheet in a hybrid nanofluid with suction/injection effects. The authors stated that in some ranges of shrinking parameters, non-unique solutions are observable. Thus, stability analysis is conducted and it is found that the first solution is stable while the second solution is unstable. Khashi'ie *et al.,* [19] numerically studied the stagnation point flow of Cu-$\mathrm{Al_2O_3}$/water hybrid nanofluid over a permeable stretching or shrinking cylinder. The result shows, that in a shrinking cylinder, the suction parameter is essential in providing dual similarity; otherwise, no solution is found if the surface is impermeable. Alumina-water nanofluid, as opposed to Cu-water and Cu-$\mathrm{Al_2O_3}$/water hybrid nanofluids, has the lowest heat transfer rate for shrinking cylinder and is also found that by conducting the stability analysis, first solution is more stable and realistic than the second solution.

In this study, we will extend the work of Yacob *et al.,* [8] and Kakar *et al.,* [17] by investigating the numerical solution of the Falkner-Skan equation for a moving wedge in a hybrid nanofluid. There are different coefficients in the momentum and energy equations of Yacob *et al.,* [8] and Kakar *et al.,* [17], where the numerical results obtained are different when the wedge angle does not equal 1. We are to consider similar nanoparticles which are copper (Cu) and alumina $(\mathrm{Al_2O_3})$ as hybrid nanoparticles and water as base fluid. The numerical results will be generated using bvp4c function in MATLAB software. The influence of the wedge angle parameter and hybrid nanoparticle volume fraction on heat transfer properties will be explored.

2. Methodology

The steady two-dimensional boundary layer flow is considered to pass over a static or moving wedge. Two distinct types of nanoparticles are utilized, which are copper (Cu) and alumina $(\mathrm{Al_2O_3})$ and water is used as the base fluid. It is assumed that the velocity of the free stream is $u_e(x) = U_\infty x^m$ and the velocity of moving wedge is $u_w(x) = U_w x^m$ where U_∞ and U_w are constant. Here m is within the range $0 \leq m \leq 1$ and $m = \beta/(2-\beta)$ where m is the wedge angle, $\beta = \Omega/\pi$ is the Hartree pressure gradient parameter and Ω is the total wedge angle. We consider the Cartesian coordinate system (x, y) where x and y are the coordinates measured along the wedge's surface and normal to it, respectively as presented in Figure 1. Based on the above assumptions, the boundary layer approximations and the nanofluid model introduced by Tiwari and Das [20], Yacob *et al.,* [8] and Waini *et al.,* [21], the governing equations of mass conversation, momentum, and energy are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho Cp)_{hnf}}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

subject to the boundary conditions

$$\begin{gathered} v = 0, u = u_w(x), T = T_w \; at \; y = 0 \\ u_e(x), T = T_\infty \; as \; y \to \infty \end{gathered} \tag{4}$$

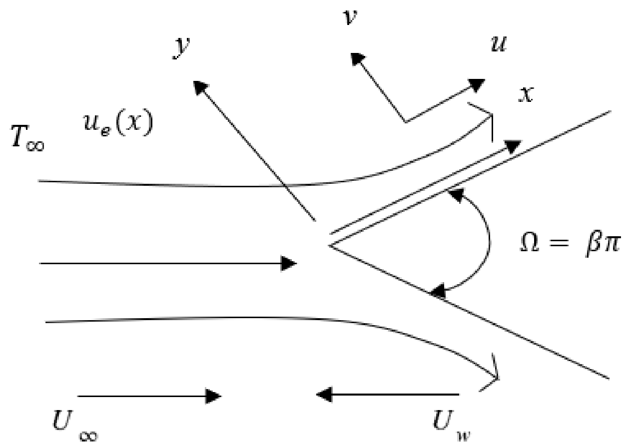

Fig. 1. Physical model

where u and v are velocity components along x *and* y axis, respectively. Further, μ_{hnf} is the dynamic viscosity of the hybrid nanofluid, ρ_{hnf} is the density of the hybrid nanofluid, k_{hnf} is the thermal conductivity of the hybrid nanofluid, $(\rho Cp)_{hnf}$ is the specific heat capacity of hybrid nanofluid, T is the fluid temperature while T_w is the wedge temperature and T_∞ is the ambient temperature. The formula of thermophysical properties of nanofluid, hybrid nanofluid and thermophysical properties of fluid and nanoparticle are presented in Table 1 and Table 2, respectively.

Table 1
Thermophysical properties of nanofluid and hybrid nanofluid [21]

Properties	Nanofluid	Hybrid Nanofluid
Density	ρ_{nf} = (1-$\varphi_1)\rho_f + \varphi_1\rho_{s1}$	$\rho_{hnf} = (1-\varphi_2)\left[(1-\varphi_1)\rho_f + \varphi_1\rho_{s1}\right] + \varphi_2\rho_{s2}$
Heat capacity	$(\rho C_p)_{nf} = (1-\varphi_1)\left(\rho C_p\right)_f + \varphi_1\left(\rho C_p\right)_{s1}$	$(\rho C_p)_{hnf} = (1-\varphi_2)[(1-\varphi_1)(\left(\rho C_p\right)_f + \varphi_1\left(\rho C_p\right)_{s1} + \varphi_2\left(\rho C_p\right)_{s2}$
Dynamic viscosity	$\mu_{nf} = \dfrac{\mu_f}{(1-\varphi_1)^{2.5}}$	$\mu_{hnf} = \dfrac{\mu_f}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}$
Thermal conductivity	$k_{nf} = \dfrac{k_{s1}+2k_f-2\varphi_1(k_f-k_{s1})}{k_{s1}+2k_f+\varphi_1(k_f-k_{s1})} \times (k_f)$	$k_{hnf} = \dfrac{k_{s2}+2k_{nf}-2\varphi_2(k_{nf}-k_{s2})}{k_{s2}+2k_{nf}+\varphi_2(k_{nf}-k_{s2})} \times (k_{nf})$ where $k_{nf} = \dfrac{k_{s1}+2k_f-2\varphi_1(k_f-k_{s1})}{k_{s1}+2k_f+\varphi_1(k_f-k_{s1})} \times (k_f)$

Table 2
Thermophysical properties of fluid and nanoparticles [22]

Physical properties	Fluid phase (water)	Al_2O_3	Cu
ρ (kg/m^3)	9932	3970	8933
C_p (J/kgK)	4179	765	385
K (W/mK)	0.613	40	400
α (m^2/s)	1.47	131.7	1163.1

The method of similarity transformation is applied to reduce the governing equations to the ordinary differential equations by introducing the following similarity variables,

$$\psi = \left[\frac{2v_f x u_e(x)}{m+1}\right]^{1/2} f(\eta), \eta = \left[\frac{(m+1)u_e(x)}{2v_f x}\right]^{1/2} y, \theta(\eta) = \frac{T-T_\infty}{T_w - T_\infty} \tag{5}$$

where ψ, η and θ are stream function, similarity variable and dimensionless temperature, respectively. Let ψ denotes as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ and these definitions satisfy Eq. (1). Using Eq. (5) and the appropriate terms that are derived from Eq. (5) and Eq. (2) to Eq. (4) are reduced to

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f} f''' + ff'' + \frac{2m}{m+1} f'^2 = 0 \tag{6}$$

$$\frac{1}{Pr} \frac{k_{hnf}/k_f}{(\rho C_p)_{hnf}/(\rho C_p)_f} \theta'' + f\theta' = 0 \tag{7}$$

subject to the boundary conditions

$$\begin{gathered} f(0) = 0, f'(0) = \lambda, \theta(0) = 1 \text{ at } \eta = 0 \\ f'(\eta) = 1, \theta(\eta) = 0 \text{ as } \eta \to \infty \end{gathered} \tag{8}$$

where φ_1 and φ_2 are the nanoparticle volume fraction of $\mathrm{Al_2O_3}$ and Cu, respectively. Meanwhile, ρ is the density, k is the thermal conductivity, $Pr = \nu/\alpha$ is Prandtl number where ν is the kinematic viscosity and α is thermal diffusivity, $\lambda = u_w/u_\infty$ is constant moving wedge parameter ($\lambda = 0$ represents static wedge and $\lambda > 0$ represents the wedge moves in the same direction with the free stream and $\lambda < 0$, represents the opposite direction), while the subscripts n_1, n_2, f and nf represent $\mathrm{Al_2O_3}$ solid component, Cu solid component, fluid and nanofluid.

The skin friction coefficient C_f and Nusselt number Nu_x are defined by

$$C_f = \frac{\tau_w}{\rho_f u_e^2} \quad \text{and} \quad Nu_x = \frac{x q_w}{k_f(T_w - T_\infty)} \tag{9}$$

where τ_w represents the surface shear stress and q_w is the surface heat flux from the wedge which are given by

$$\tau_w = \mu_{hnf}\left(\frac{\partial u}{\partial y}\right)_{y=0} \text{ and } q_w = -k_{hnf}\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{10}$$

Substituting (5) into Eq. (9) and Eq. (10), we obtain

$$C_f Re_x^{\frac{1}{2}} = \frac{\mu_{hnf}}{\mu_f}\sqrt{\frac{m+1}{2}} f''(0) \text{ and } Nu_x Re_x^{-\frac{1}{2}} = -\frac{k_{hnf}}{k_f}\sqrt{\frac{m+1}{2}}\theta'(0) \tag{11}$$

where $Re_x = \frac{x u_e}{v_f}$ is the local Reynolds number.

3. Results

The set of ordinary differential Eq. (6) to (8) are solved numerically using bvp4c function in MATLAB where the generated numerical results are represented graphically and in tabular form. Two different types of nanoparticles, which are Copper (Cu) and Alumina ($\mathrm{Al_2O_3}$) are taken into consideration with water as a based fluid and Prandtl number is fixed at 6.2. It is worth mentioning that the results of the current study are reduced to those of a regular or viscous fluid when $\varphi_1 = \varphi_2 = 0$. Table 3 presents the numerical values of $f''(0)$ for some values of m when λ = 0 (static wedge) and φ = 0 (viscous fluid), which show a favourable agreement with those obtained by Van Dyke [23], Watanabe [24], Yih [25], Yacob *et al.,* [8] and Kakar *et al.,* [17]. Table 4 shows the results of first solution for the value of $f''(0)$ and $-\theta'(0)$ for $\mathrm{Al_2O_3}$– Cu/water hybrid nanofluid when φ_1 = 0.1, λ = −1, Pr = 6.2 and m = 0.5 with different values of φ_2. It shows that increasing φ_2 tends to increase the values of $f''(0)$ and $-\theta'(0)$.

Table 3
Comparisons of the values of $f''(0)$ for various values of m when λ = 0 and $\varphi_1 = \varphi_2 = 0$

m	Van Dyke [23]	Watanabe [24]	Yih [25]	Yacob *et al.,* [8]	Kakar *et al.,* [17]	Present results
0		0.46960	0.469600	0.4696	0.469600	0.469599985
0.5				1.0389		1.038903481
1	1.232588		1.232588	1.2326		1.232587654

Table 4
Values of $f''(0)$ and $-\theta'(0)$ for $\mathrm{Al_2O_3}$ – Cu/water hybrid nanofluid when $\varphi_1 = 0.1, \lambda = -1$, Pr = 6.2, and m = 0.5 with different values of φ_2

	φ_2	First solution
$f''(0)$	0	0.899425810
	0.02	0.927307928
	0.04	0.950217233
$-\theta'(0)$	0	0.008372007
	0.02	0.013487628
	0.04	0.019565131

Figure 2 and Figure 3 demonstrate the variation of the skin friction coefficient and the local Nusselt number with λ for different values of m where $\varphi_1 = \varphi_2 = 0.1$ for $\mathrm{Al_2O_3}$– Cu/water hybrid nanofluid. As seen in Figure 2, the curves intersect at point (1,0) where the skin friction coefficient is zero when λ = 1, regardless of other parameters. This is expected as there is no shear stress at the surface when the wedge and fluid move at the same velocity. Additionally, the skin friction coefficient has negative value when λ > 1, indicating that the moving wedge exerts a drag force on the fluid, while it has a positive value when λ < 1, indicating the opposite. Figure 3 and Figure 4 show that there is an increase in both skin friction coefficient and local Nusselt number when the wedge angle increases. However, the reverse trend occurs when λ > 1 for the skin friction coefficient as display in Figure 3. The corresponding velocity and temperature profiles with a specific value of λ = −1.09 and φ_1 = φ_2 = 0.1 of the Figure 2 and Figure 3 are shown in Figure 4 and Figure 5. As m increases, both the velocity and thermal boundary layer thicknesses decrease, resulting in an increase in the velocity and temperature gradients at the surface. This is consistent with the data presented in Figure 2 and Figure 3.

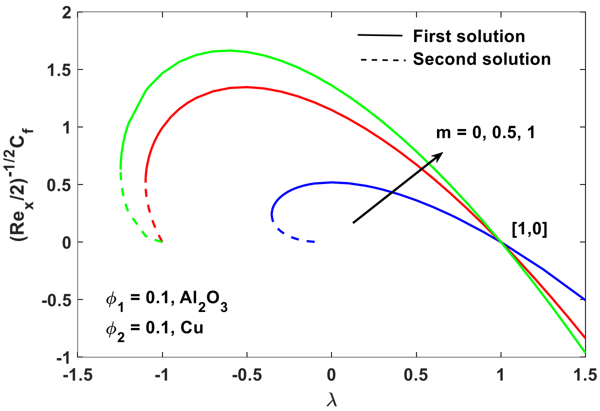

Fig. 2. Variation of skin friction coefficient with λ for different values of m when $\varphi_1 = \varphi_2 = 0.1$ for Al_2O_3– Cu/water hybrid nanofluid

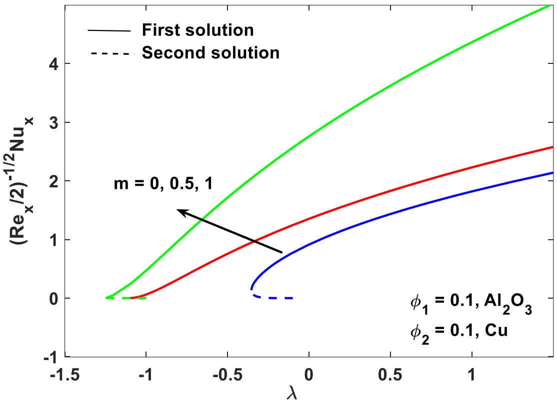

Fig. 3. Variation of local Nusselt number with λ for different values of m when $\varphi_1 = \varphi_2 = 0.1$ for Al_2O_3– Cu/water hybrid nanofluid

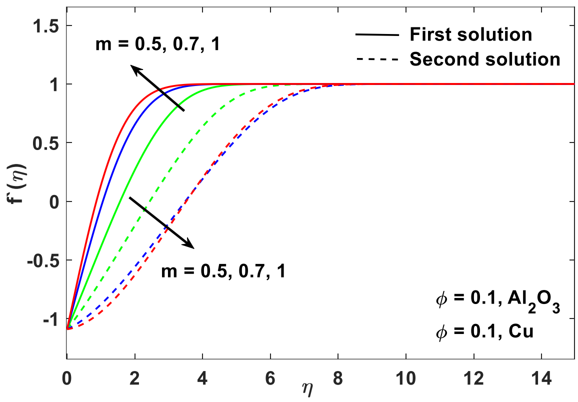

Fig. 4. Velocity profiles $f'(\eta)$ for different values of m when $\varphi_1 = \varphi_2 = 0.1$ and λ = –1.09 Al_2O_3– Cu/water hybrid nanofluid

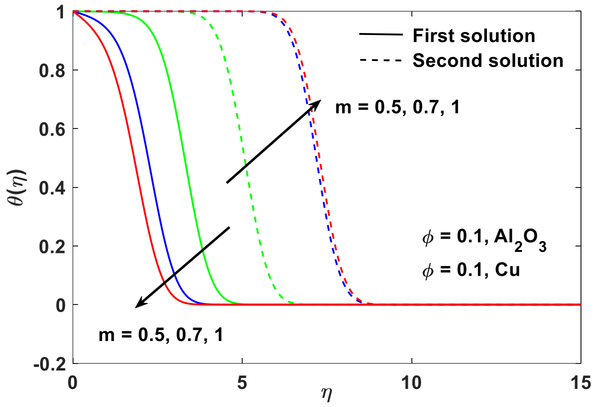

Fig. 5. Temperature profiles $\theta(\eta)$ for different values of m when $\varphi_1 = \varphi_2 = 0.1$ and λ = –1.09 for Al_2O_3– Cu/water hybrid nanofluid

Figure 6 and Figure 7 show the velocity and temperature profiles for different values of m when $\varphi_1 = 0.1$, $\varphi_2 = 0.5$ and $\lambda = -1.09$ for Al_2O_3– Cu/water hybrid nanofluid. For the first solution in Figure 6, m increases then the velocity gradient will increase and in consequence decrease the velocity boundary layer thickness. Additionally, the velocity and temperature profiles in Figure 4 to Figure 7 asymptotically meet the boundary conditions, validating the accuracy of the numerical results obtained. Further, dual solutions are only found to exist for $\lambda < 0$. For m = 0, dual solutions exist from −0.3541 to −0.1. For m = 0.5, dual solutions exist from -1.09926 to -1 and for m = 1, dual solutions exist from -1.24658 to -1.

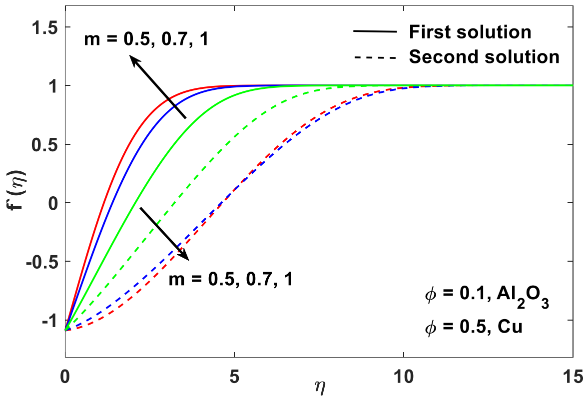

Fig. 6. Velocity profiles $f'(\eta)$ for different values of m when $\varphi_1 = 0.1$, $\varphi_2 = 0.5$ and $\lambda = -1.09$ for Al_2O_3- Cu/water hybrid nanofluid

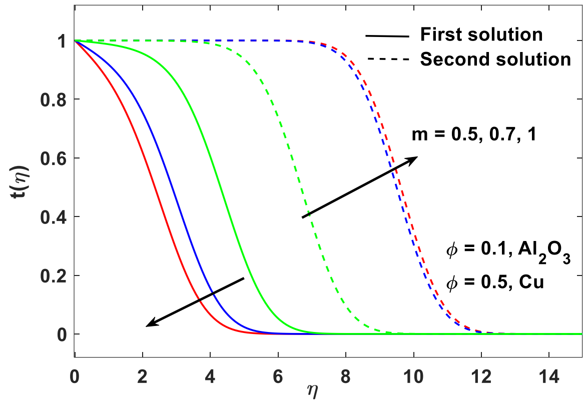

Fig. 7. Temperature profiles $\theta(\eta)$ for different values of m when $\varphi_1 = 0.1$, $\varphi_2 = 0.5$ and $\lambda = -1.09$ for 1, Al_2O_3- Cu/water hybrid nanofluid

4. Conclusions

In this study, we numerically examined the flow and heat transfer over a stretching or shrinking wedge in hybrid nanofluids using the Tiwari-Das model. The governing equations were transformed into non-linear ordinary differential equations using a similarity transformation, which were then solved using the bvp4c function in MATLAB. Two types of nanoparticles, copper (Cu) and alumina (Al_2O_3) were considered in the water-based hybrid nanofluid. The results showed that increasing the wedge angle (m) and Cu nanoparticles volume fraction lead to an increase in the skin friction coefficient and heat transfer rate at the surface. Additionally, dual solutions were only found to exist for $\lambda < 0$. As the value of wedge parameter increases, the range of boundary layer separation also increases.

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