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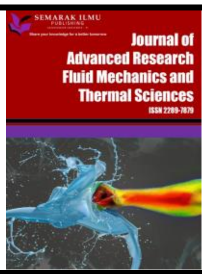

Mathematical Model of Reiner-Philippoff Embedded with Al_2O_3 and Cu Particles over a Shrinking Sheet with Mixed Convection and Mass Flux Effect

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ABSTRACT

The investigation on the Reiner–Philippoff fluid model embedded with two different nanoparticles (Al_2O_3 and Cu) over a shrinking sheet is carried out. The Tiwari and Das model are applied in the study covering the continuity, momentum, energy equations, and Reiner-Philippoff relation. The flow studied also considers the mixed convection and mass flux influences. The respective equations are first transformed into ordinary differential equation form using the similarity transformation before performing the computation work using the bvp4c function in MATLAB. The present model is identical to the established model in special cases, and then a direct comparative study is executed to verify the current model. The results for respective problems are presented in tabular and graphical form. It is perceived that the presence of nanoparticles affects the fluid characteristic significantly.

1. Introduction

An efficient working fluid is required for industrial and technological applications to regulate processes and produce superior final products. Although many processes use non-Newtonian fluid types to accelerate advancements, pure water (Newtonian) is still used as a cooling agent. There are various non-Newtonian fluids, and each has distinctive properties. In contrast to Newtonian fluid,

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whose strain is by the stress tensor, the non-Newtonian fluid type is classified by either shear thinning, which displays pseudo-plasticity, or shear thickening, which describes the dilatant. Shear-thinning fluids show the behaviour of Newtonian fluids at extreme shear rates, whereas shear-thickening fluids show the development in viscosity proportional to shear rate. Krishnan *et al.,* [1] indicates that the fluid in models reflecting shear thickening and shear thinning characteristics includes the Powell-Eyring, Sisko, Carreau-Yasuda, and Carreau viscosity models and Reiner-Philippoff.

The Reiner-Philippoff model, which belongs to the non-Newtonian group, is the most fascinating to study because it exhibits Newtonian fluid behaviour at low or high values (shear stress) and non-Newtonian behaviour at other values. Furthermore, the Reiner-Philippoff model is essential and unique in representing natural fluid in industrial applications. In certain situations, it can exhibit three fluid characteristics where it can behave like Newtonian, dilatant, and pseudo-plastics. This is significant in manufacturing procedures since the applied fluid might vary in a specific process to achieve the best production. In addition, numerous researches examined the flow's movement over various geometries and its effects on the flow field [2-11].

The study of fluid flow has advanced and become more fascinating since the development of nanofluids, which can control heat and mass transfer and flow behaviour. In the industrial engineering and manufacturing processes, the boundary layer flow triggered by the stretching or shrinking surface is widely employed, such as in wire drawing, continuous glass casting, and polymer or metal extrusions. The flow across a linearly stretched surface appears to have been studied for the first time historically by Crane [12]. Flows over shrinking surfaces have recently drawn attention, in contrast to flows over stretched surfaces. The flow that the shrinking surface causes is effectively a reverse flow, claims Goldstein [13]. In addition, numerous researches have examined the impact of various physical parameters on stretching and shrinking surfaces [14-23].

Convection heat transfer is a process caused by differences in temperature and density, through which heat is transferred from one part of the fluid to another. There are two types of convection: forced convection and natural convection. Forced convection is the process of fluid motion imparted by an external source, while natural convection is caused by natural means such as buoyancy effects. Mixed convection is formed when forced and natural convection systems combine. Due to its significance in industrial systems, including nuclear reactors, solar collectors, and electronic devices, mixed convection flow is a topic of great interest to researchers. Merkin [24] examined the mixed convection flow toward a vertical plate in a porous material. Ingham [25] researched the mixed convection flow over a moving vertical flat plate. Ramachandran *et al.,* [26] then applied this work to the stagnation flow problem and discovered that the opposing flow area was where the solution's non-uniqueness occurred. In addition, previous researches have also considered the work on the mixed convection flow over various geometry [27-31].

In recent years, hybrid nanofluids have replaced nanofluids in several technologies to enhance thermal performance. Turcu *et al.,* [32] and Jana *et al.,* [33] are among the first researchers to integrate hybrid nano-composite particles in their experimental work. Due to the synergistic effects of its various nanoparticles, a hybrid nanofluid is an innovative fluid that can accelerate the heat transfer rate [34]. The appropriate nanoparticles can also be combined or hybridized to achieve the optimum heat transfer [35]. Suresh *et al.,* [36] created a nanocrystalline Cu- Al_2O_3 hybrid nanocomposite using a thermochemical process. Then, the produced nanocomposite powder was dissolved in deionized water to form the hybrid Cu-Al_2O_3/water nanofluid. The experimental findings show that nanoparticles' volume concentration increases with the hybrid nanofluid's thermal conductivity and viscosity. When the viscosity and thermal conductivity of the nanofluids were examined, it was found that the viscosity increase was considerably more than the thermal conductivity increase. Moreover, they claimed that even while alumina has a low heat conductivity, it has a good level of chemical inertness, which could help to keep the hybrid nanofluid stable. Suresh *et al.,* [37] investigated the pressure drop characteristics of the Cu-Al_2O_3/water hybrid nanofluid and laminar convection heat transfer in uniformly heated circular tubes. Compared to the Nusselt number of waters, the testing results show a maximum rise in the Nusselt number of 13.56%. 0.1% hybrid Cu- Al_2O_3/water nanofluid has a slightly higher friction factor than 0.1% Al_2O_3/water nanofluid. Because of Suresh's discoveries, Cu- Al_2O_3/water is an incredibly efficient hybrid nanofluid in transferring heat. Additionally, Singh and Sarkar [38] and Farhana *et al.,* [39] also commented on the relevance of the combination of alumina and other nanoparticles. In this regard, Devi and Devi [40] investigated the Al_2O_3-Cu hybrid nanofluid boundary layer flow problem across a stretching surface using new correlations of the thermophysical properties that matched the findings of Suresh *et al.,* [37]. They found that the higher nanoparticle volume fractions enhanced the heat transfer rate in those studies. In recent years, a hybrid nanofluid's boundary layer flow past a stretching or shrinking surface has been thoroughly studied. The significance of this field's uses in manufacturing processes, such as synthetic fiber synthesis, paper manufacture, and polymer extraction, has led to a tremendous rise in studies in this area. Waini *et al.,* [41] reported a temporal stability analysis on the flow via a stretching and shrinking surface in a hybrid nanofluid. They found that one of the solutions was unstable over time, whilst the other was stable and physically trustworthy.

Motivated by the above literature survey, this study focuses on the mixed convection of Reiner-Philippoff hybrid nanofluid. The flow is expected to pass across a shrinking sheet. Also, as water is regarded as a base fluid, Al_2O_3-Cu nanohybrid particles are added to the based fluid to expedite the heat transfer rate. Influences from mass flux are also considered in the flow. The governing equations are derived into ordinary differential equation form using the similarity transformation before it is solved computationally using the bvp4c function in MATLAB. The findings are presented graphically and briefly discuss how various physical factors were influenced. This problem has not been studied before, so the reported results are new.

2. Methodology

Figure 1 depicts the physical configuration of Reiner-Philippoff nanofluid across a shrinking surface where the velocity's surface is $u = ax^{1/3}f'(\eta)$ with $a > 0$. The mass flux velocity $v_w(x)$ represents the surface permeability, while given $T_w = T_\infty + T_0 x^{-1/3}$ is the surface temperature where the constant ambient temperature is T_∞ and T_0 is the reference temperature. Thus, the comprehensive equations for the suggested model are as follows [10,31]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial y} = \frac{\tau}{\mu_\infty + \dfrac{\mu_{hnf} - \mu_\infty}{1 + \left(\dfrac{\tau}{\tau_s}\right)^2}} \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_{hnf}}\frac{\partial \tau}{\partial y} + \frac{(\rho\beta_T)_{hnf}}{\rho_{hnf}} g\left(T - T_\infty\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{hnf}}{\left(\rho C_p\right)_{hnf}}\frac{\partial^2 T}{\partial^2 y} \tag{4}$$

subject to:

$$\begin{aligned} &u=\varepsilon u_w(x),\ v=v_w(x),\ T=T_w \ \text{ at } \ y=0 \\ &u\to 0,\ \ T\to T_\infty \ \text{ as } \ y\to\infty \end{aligned} \tag{5}$$

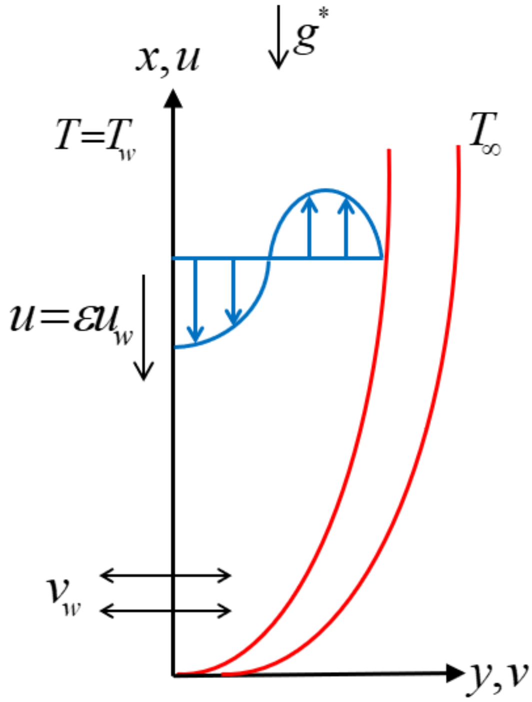

Fig. 1. The physical model

where (u,v) are the velocity components in the (x,y) directions, respectively. Further, ρ_{hnf} is fluid density, $(\rho\beta)_{hnf}$ is thermal expansion, $\left(\rho C_p\right)_{hnf}$ is heat capacity, k_{hnf} is thermal conductivity, μ_{hnf} is dynamic viscosity, μ_∞ is limiting dynamic viscosity, *T* is temperature, *g* is acceleration due to gravity, τ is shear stress of Reiner-Philippoff fluid, τ_s is references shear stress and ε is stretching/shrinking parameter. The subscripts of *hnf* and *f* stand for hybrid nanofluid and fluid, respectively. The similarity transformation is as follows [42]:

$$\psi=\sqrt{a\nu x^{1/3}}\,f(\eta),\qquad \tau=\rho\sqrt{a^3\nu}\,g(\eta),\qquad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\qquad \eta=\frac{y}{x^{1/3}}\sqrt{\frac{a}{\nu}} \tag{6}$$

The term ψ is express by $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$ yield:

$$u=ax^{1/3}f'(\eta),\qquad v=-\sqrt{a\nu_f}\,x^{-1/3}\left(\frac{2}{3}f(\eta)-\frac{1}{3}\eta f'(\eta)\right) \tag{7}$$

At $\eta = 0$, the wall mass flux velocity obtained as:

$$v_w(x) = -\frac{2}{3}\sqrt{a\nu_{hnf}}\, x^{-1/3} S \tag{8}$$

in which $f(0) = S$ indicates the parameter of constant mass flux. There are three different situations of the value of S, where S = 0 denote the impermeable surface, S < 0 for injection and S > 0 is for suction, while $\nu_f = \mu_\infty/\rho_f$ is the fluid kinematic viscosity. The similarity between Eq. (9) to Eq. (12) are obtained after employing Eq. (6) and Eq. (7):

$$g = f''\left(\frac{g^2 + \left(\dfrac{\mu_{hnf}}{\mu_f}\right)\lambda\gamma^2}{g^2+\gamma^2}\right) \tag{9}$$

$$g' - \frac{\rho_{hnf}}{\rho_f}\left(\frac{1}{3}f'^2 - \frac{2}{3}ff''\right) + \left[\frac{(\rho\beta_T)_{hnf}}{(\rho\beta_T)_f}\right] Z\theta = 0 \tag{10}$$

$$\frac{1}{\Pr}\left[\frac{k_{hnf}}{k_f}\right]\theta'' + \left[\frac{(\rho C_p)_{hnf}}{(\rho C_p)_f}\right]\left(\frac{1}{3}f'\theta + \frac{2}{3}f\theta'\right) = 0 \tag{11}$$

subject to:

$$\begin{aligned} &f(0) = S,\; f'(0) = \varepsilon,\; \theta(0) = 1 \\ &f'(\eta) \to 0,\;\; \theta(\eta) \to 0 \;\text{ as }\; \eta \to \infty \end{aligned} \tag{12}$$

The dimensionless parameter Bingham number γ, Reiner–Philippoff fluid λ, mixed convection Z, and Prandtl number Pr, are defined by:

$$\gamma = \frac{\tau_s}{\rho_f\sqrt{a^3\nu_f}}, \qquad \lambda = \frac{\mu_f}{\mu_\infty}, \qquad Z = \frac{Gr}{\mathrm{Re}_x^2} = \frac{g(\beta_T)_f T_0}{a^2}, \qquad \Pr = \frac{(\mu C_p)_f}{k_f} \tag{13}$$

Note that λ = 1 presents viscous Newtonian type, whereas λ > 1 signifies the shear-thinning and λ < 1 is the shear-thickening fluid type, respectively. Further, ε = 0 signifies the static sheet, ε > 0 indicates the stretching sheet, and ε < 0 is the shrinking sheet. The quantity of physical in terms of skin friction and local Nusselt number is given by:

$$C_f = \frac{\tau_w}{\rho_f u_w^2}, \quad Nu_x = \frac{xq_w}{k_f(T_w - T_\infty)} \tag{14}$$

where

$$\tau_w = \rho_f \sqrt{a^3 \nu_f} \left(g(\eta) \right)_{y=0}, \qquad q_w = -k_{hnf} \left(\frac{\partial T}{\partial y} \right) \Bigg|_{y=0} \tag{15}$$

The τ_w symbolizes the quantity of τ on *y* = 0, and q_w presenting the surface heat flux. Then, one gets:

$$\mathrm{Re}_x^{1/2} C_f = g(0), \qquad \mathrm{Re}_x^{-1/2} Nu_x = -\frac{k_{hnf}}{k_f} \theta'(0) \tag{16}$$

where $Re_x = u_w(x)x/v_f$ is the local Reynolds number and $Gr_x = \left(g^*(\beta_T)_f \left(T_0 x^{-1/3}\right) x^3\right)/v_f^2$ is the Grashof number. The physical properties of water, Al_2O_3, and Cu are listed in Table 1, whereas the thermophysical characteristics of nanofluid and hybrid nanofluid are listed in Table 2. Meanwhile, the nanoparticle volume fractions of Al_2O_3 and Cu are symbolized by ϕ_1 and ϕ_2, respectively. Also, the subscripts of *hnf, nf,* and *f*, stand for hybrid nanofluid, nanofluid, and fluid, respectively.

Table 1
Thermophysical properties of water, Al_2O_3, and Cu [43]

Thermophysical properties	Water	Al_2O_3	Cu
$\rho\left(kg/m^3\right)$	997.1	3970	8933
$C_p\left(J/kgK\right)$	4179	765	385
$k\left(W/mK\right)$	0.613	40	400
$\beta \times 10^{-5}\left(1/K\right)$	0.85	1.67	21

Table 2
Thermophysical properties of *nf* and *hnf* [44]

Thermophysical properties	Nanofluid	Hybrid nanofluid
Density	$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$	$\rho_{hnf} = \left(1-\phi_{hnf}\right)\rho_f + \phi_1\rho_{s1} + \phi_2\rho_{s2}$
Heat capacity	$\left(\rho C_p\right)_{nf} = (1-\phi)\left(\rho C_p\right)_f + \phi\left(\rho C_p\right)_s$	$\left(\rho C_p\right)_{hnf} = (1-\phi)_{hnf}\left(\rho C_p\right)_f + \phi_1\left(\rho C_p\right)_{s1} + \phi_2\left(\rho C_p\right)_{s2}$
Dynamic viscosity	$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\phi)^{2.5}}$	$\frac{\mu_{hnf}}{\mu_f} = \frac{1}{(1-\phi_1-\phi_2)^{2.5}}$
Thermal conductivity	$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi\left(k_f - k_s\right)}{k_s + 2k_f + 2\phi\left(k_f - k_s\right)}$	$\frac{k_{hnf}}{k_f} = \frac{\frac{\phi_1 k_1 + \phi_2 k_2}{\phi_{hnf}} + 2k_f + 2\left(\phi_1 k_1 + \phi_2 k_2\right) - 2\phi_{hnf} k_f}{\frac{\phi_1 k_1 + \phi_2 k_2}{\phi_{hnf}} + 2k_f - 2\left(\phi_1 k_1 + \phi_2 k_2\right) - 2\phi_{hnf} k_f}$
Thermal expansion coefficient	$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s$	$(\rho\beta)_{hnf} = \left(1-\phi_{hnf}\right)(\rho\beta)_f + \phi_1(\rho\beta)_{s1} + \phi_2(\rho\beta)_{s2}$ where $\phi_{hnf} = \phi_1 + \phi_2$

3. Results and Discussion

The numerical solutions of Eq. (9) to Eq. (12) are obtained using the boundary value problem solver, bvp4c, a feature of the MATLAB software. It uses the three-stage Lobatta IIIa formula and is a finite difference approach. To acquire the necessary solutions, the selection of initial guess and boundary layer thickness η_∞ will depend on the parameters used. This solver is also being utilized by various researchers to solve boundary layer flow problems [45-49].

A direct comparison analysis is conducted on the existing value of $f''(0)$ provided by Cortell [50], Ferdows *et al.,* [51], and Waini *et al.,* [18] to vouch for the dependability of the current model. It should be noted that the equations on the current model were the same for the limiting case, making a comparison between the present findings and the current output appropriate. In Table 3 and Table 4, respectively, the validation data on the values of $f''(0)$ are presented. The comparison reveals excellent agreement, which supports the current mathematical formulation and the provided numerical results.

Table 3
Comparative model in terms of momentum equations

Author	Model (momentum)	Limiting cases
Current	$g' - \frac{\rho_{hnf}}{\rho_f}\left(\frac{1}{3}f'^2 - \frac{2}{3}ff''\right) + \left[\frac{(\rho\beta_T)_{hnf}}{(\rho\beta_T)_f}\right]Z\theta = 0$	*Z = 0*
Cortell [50]	$3f''' + 2ff'' - (f')^2 = 0$	-
Ferdows *et al.,* [51]	$f''' + \frac{2}{3}ff'' - \frac{1}{3}\left(f'^2 + Mf'^2\right) + Gr\theta + Gc\phi = 0$	*M = Gr = Gc = 0*
Waini *et al.,* [18]	$3\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}f''' + 2ff'' - f'^2 = 0$	-

Table 4
Comparative value of $f''(0)$ at $\varepsilon = \lambda = \gamma = 1$, Pr = 2 and *Z* = 0 for different value of *S*

S	Cortell [50]	Ferdows *et al.,* [51]	Waini *et al.,* [18]	Current
-0.75	-0.453521	-0.453523	-0.453523	-0.453523325
-0.5	-0.518869	-0.518869	-0.518869	-0.518869429
0	-0.677647	-0.677648	-0.677648	-0.677647983
0.5	-0.873627	-0.873643	-0.873643	-0.873642863
0.75	-0.984417	-0.984439	-0.984439	-0.984439388

To strengthen the current formulation and the current output, the values of $g(0)$ are also compared with the result obtained by Sajid *et al.,* [10] and Waini *et al.,* [52] for various values of the Reiner-Philippoff fluid parameter λ, the Bingham number γ and Pr = 2. A strong agreement can be seen in the comparison; hence, Table 5 and Table 6 show the corresponding numerical values. For higher values of γ, the values of $g(0)$ significantly increase. However, with the rise of λ, there is a slight decrease in the values of $\mathrm{Re}_x^{1/2} C_f$.

Table 5

Comparative model in terms of momentum equations

Author	Model (momentum)	Limiting cases
Current	$g' - \frac{\rho_{hnf}}{\rho_f}\left(\frac{1}{3}f'^2 - \frac{2}{3}ff''\right) + \left[\frac{(\rho\beta_T)_{hnf}}{(\rho\beta_T)_f}\right]Z\theta = 0$	*Z = 0*
Sajid *et al.,* [10]	$g' + \frac{2}{3}ff'' - \frac{1}{3}f'^2 = 0$	-
Waini *et al.,* [52]	$g' + \frac{2}{3}ff'' - \frac{1}{3}f'^2 - M\sin^2(\beta)f' = 0$	*M = 0*

Table 6
Comparative value of $g(0)$ for λ and γ when S = Z = 0 and $\varepsilon = 1$

γ	λ	Sajid *et al.,* [10]	Waini *et al.,* [52]	Current
0.1	0.1	-0.660273	-0.660275	-0.660275189
0.5		-0.380604	-0.380604	-0.380603983
1		-0.246415	-0.246415	-0.246414994
0.1	0.3	-0.664497	-0.664498	-0.664497827
	0.5	-0.668484	-0.668486	-0.668486422
	0.7	-0.672282	-0.672277	-0.672276682

The analysis conducted for this study is shown graphically in terms of physical quantities and profile data. The values of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ for various values of dimensionless physical parameters are shown in Table 7. At a fixed value for the measured parameters (ε = -1, S = 2.4, λ = 1.5, γ = 0.1, Pr = 10 and Z = -0.5) where $\phi_1 = \phi_2 = 0$ (pure fluid), it appears that the values of $\mathrm{Re}_x^{1/2} C_f$ increase when S, Z, and Pr rise, whereas they decrease when ε, λ and γ are added. The values of $\mathrm{Re}_x^{-1/2} Nu_x$ (thermal rate) increase when ε, S, Z, and Pr are increased while it trends downward with the accession of λ and γ. Stretching flow, suction, mixed convection parameters, and a high Prandtl number are dominant phenomena that tend to release energy to the flow. On the other hand, the flow energy is slowed by the presence of the Bingham number and the Reiner-Philippoff fluid parameter. The fluctuation of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ under the influence of S and λ at constant values of ε = -1, γ = 0.1, Pr = 10 and Z = -0.5 is depicted in Figure 2 and Figure 3. The values of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ are decreased due to the rise of λ in a shrinking sheet. Larger values of λ, from a physical perspective, establish barriers to the shear-thinning effect, which lessens the fluid's interaction with surfaces and produces less drag. Meanwhile, the increase in S substantially raised the values of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$.

Table 7

Values of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ for various values of physical parameters

ε	S	λ	γ	Z	Pr	$\mathrm{Re}_x^{1/2} C_f$	$\mathrm{Re}_x^{-1/2} Nu_x$
-1	2.4	1.5	0.1	-0.5	10	1.071559759	15.797838638
-0.5						0.673644536	15.902427543
1						-2.671121502	16.173117605
-1	2.35					0.983959247	15.458950581
	2.38					1.040255984	15.662372870
	2.42					1.100157249	15.933222541
	2.4	0.5				1.123427917	15.798679715
		1.2				1.089582739	15.798125718
		1.8				1.051917467	15.797528114
		1.5	0.09			1.076613256	15.797920603
			0.11			1.066023793	15.797748671
			0.15			1.037822480	15.797291518
			0.1	-0.7		1.057149539	15.797731777
				0		1.107545857	15.798105180
				0.8		1.165010666	15.798529902
				-0.5	3	0.698234629	4.597082725
					5	0.728543011	7.795010391
					12	0.765218974	18.993257535

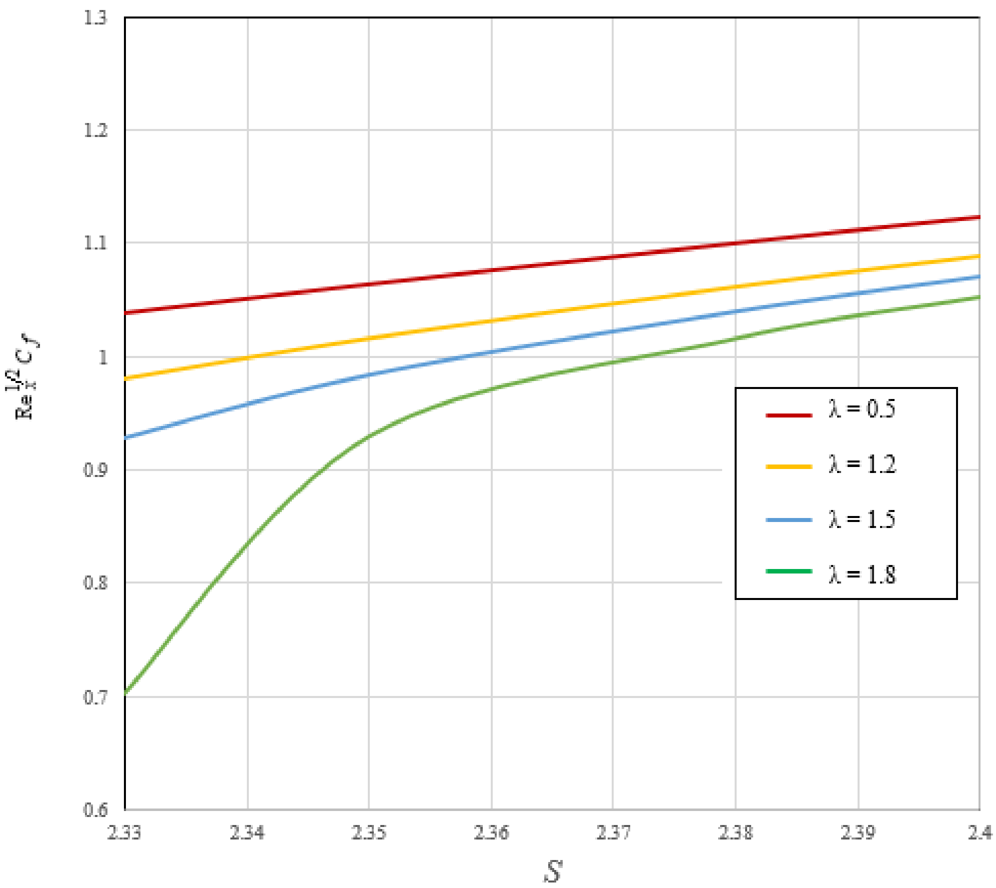

Fig. 2. $\mathrm{Re}_x^{1/2} C_f$ vs S for various values of λ where ε = -1, γ = 0.1, Pr = 10 and Z = -0.5

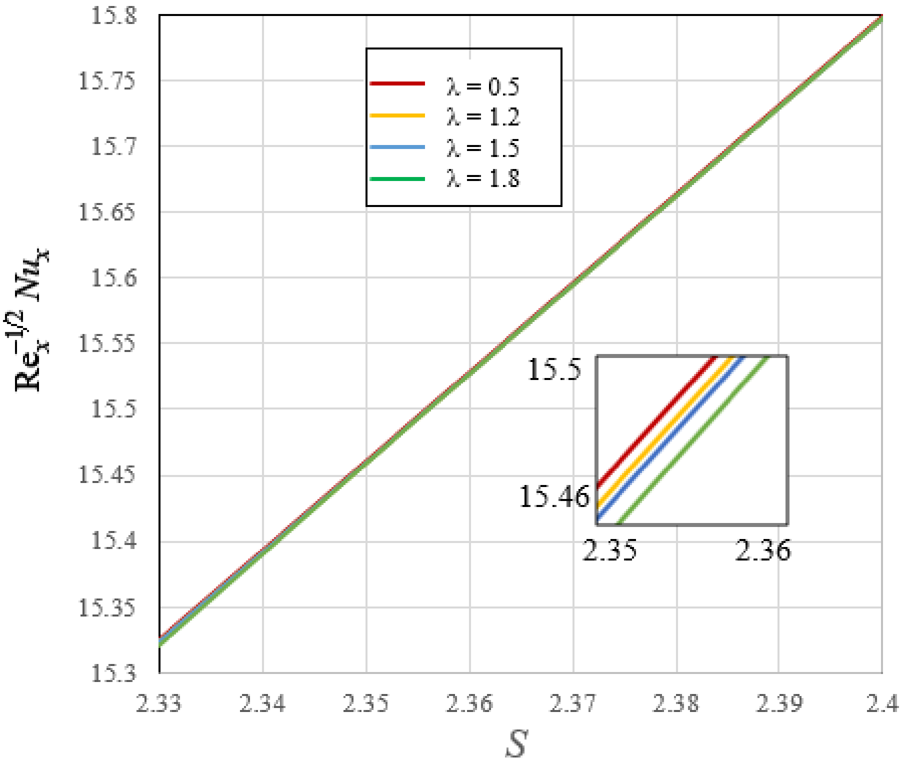

Fig. 3. $\mathrm{Re}_x^{-1/2} Nu_x$ vs *S* for various values of *λ* where *ε* = -1, *γ* = 0.1, Pr = 10 and *Z* = -0.5

Figure 4 and 5 illustrate how *λ* and *γ* affect variations in $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ when *ε* = 1, Pr = 10 and *S* = *Z* = 0. For this model to be a non-Newtonian Reiner-Philippoff fluid model, ϕ_1 and ϕ_2 are both set to zero. The increase in $\mathrm{Re}_x^{-1/2} Nu_x$ and the decrease in $\mathrm{Re}_x^{1/2} C_f$ were both influenced by the rise in *λ*. When *γ* = 0.3, 0.5, 2 and *λ* = 1 (Newtonian fluid), the values of $\mathrm{Re}_x^{1/2} C_f = -0.67764793$ and $\mathrm{Re}_x^{-1/2} Nu_x = 1.117224634$ remain unchanged (Table 8). Further, as *γ* increases, it becomes clear that the quantity of $\mathrm{Re}_x^{1/2} C_f$ rises when *λ* < 1 (dilatant fluid) and decreases when *λ* > 1 (pseudoplastic fluid), while the thermal rate yields opposite outcomes. Additionally, for future reference, Table 8 tabulates the computed values of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ with various values of *λ* and *γ*.

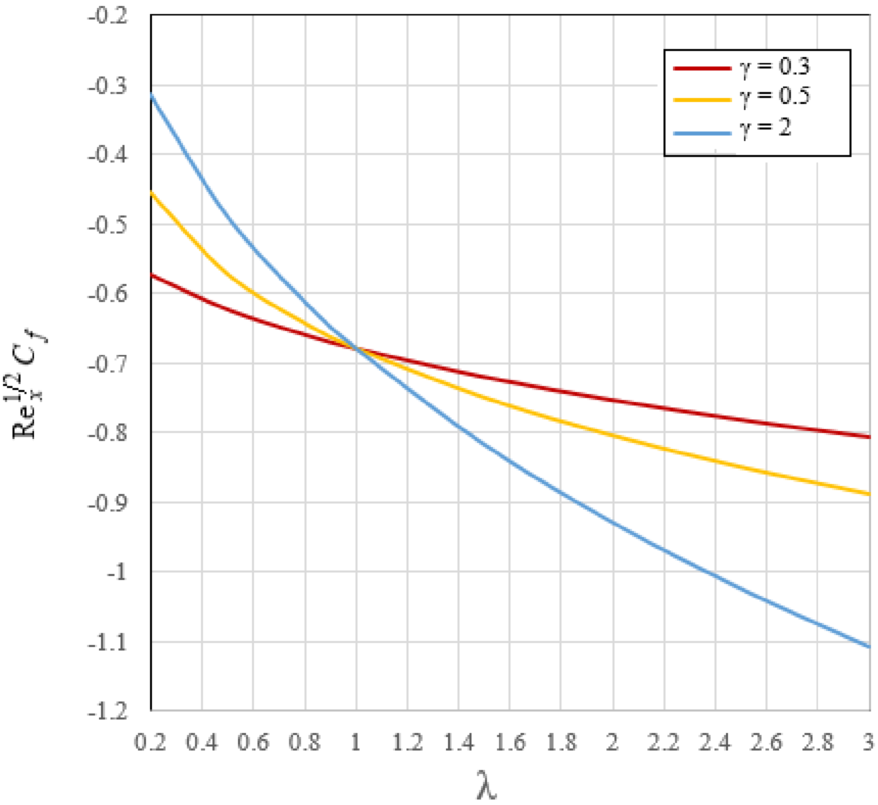

Fig. 4. $\mathrm{Re}_x^{1/2} C_f$ vs λ for various values of γ where ε = 1, Pr = 10 and S = Z = 0

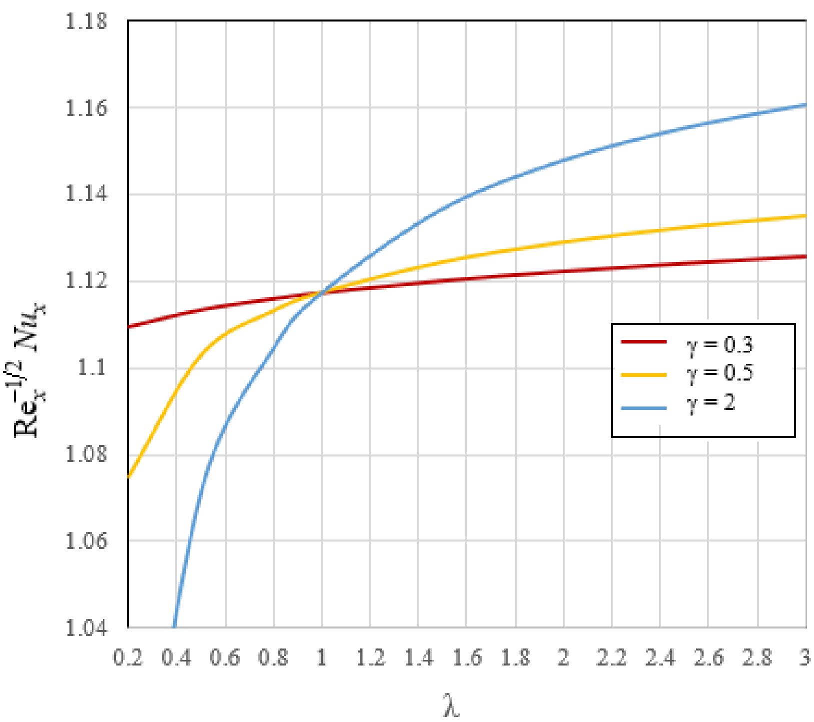

Fig. 5. $\mathrm{Re}_x^{-1/2} Nu_x$ vs λ for various values of γ where ε = 1, Pr = 10 and S = Z = 0

Table 8
Values of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ for *λ* and *γ* when *ε* = 1, Pr = 10 and $S = Z = \phi_1 = \phi_2 = 0$

λ	*γ* = 0.3		*γ* = 0.5		*γ* = 2	
	$\mathrm{Re}_x^{1/2} C_f$	$\mathrm{Re}_x^{-1/2} Nu_x$	$\mathrm{Re}_x^{1/2} C_f$	$\mathrm{Re}_x^{-1/2} Nu_x$	$\mathrm{Re}_x^{1/2} C_f$	$\mathrm{Re}_x^{-1/2} Nu_x$
0.2	-0.571405913	1.109494135	-0.454413417	1.074919188	-0.312096465	0.976476966
0.5	-0.621589532	1.113354604	-0.571307799	1.102878900	-0.487759442	1.071410812
0.8	-0.657721255	1.115877294	-0.641511772	1.113010824	-0.610294917	1.104426708
1.0	-0.677647983	1.117224634	-0.677647983	1.117224634	-0.677647983	1.117224634
1.5	-0.718769953	1.119945712	-0.748419875	1.124193035	-0.816690687	1.136574988
2.0	-0.752029995	1.122104847	-0.802981734	1.128741894	-0.929233695	1.147725714
2.5	-0.780309880	1.123916366	-0.848005566	1.132107848	-1.024871047	1.155141945
3.0	-0.805104451	1.125486397	-0.886660341	1.134776439	-1.108575709	1.160506653

This study also looks at the impact of volumetric concentration on a related topic. According to Table 9, it was shown that the $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ improve higher when the concentration of Cu (ϕ_2) nanoparticles increases, while for Z = -1, the reverse behaviour is seen. A higher concentration of Cu nanoparticles produces more kinematic energy, which boosts the fluid particles' ability to transfer heat (Figure 6 and Figure 7). Additionally, it is seen that as Z and ϕ_2 increases, so do the values of $\mathrm{Re}_x^{-1/2} Nu_x$. This finding demonstrates that the synergistic effects mentioned by Sarkar *et al.,* [34] can increase the heat transfer rate when hybrid nanoparticles are added.

Figure 8 to 13 show several samples of the velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles for particular parameters. The boundary condition is asymptotically satisfied by these profiles, confirming the accuracy of the numerical findings. Figure 8 and 9 show the analysis with Reiner-Philippoff parameters considered for the velocity and temperature profiles. The profiles demonstrate that the fluid's velocity decreases as λ increases. The temperature profiles, however, exhibit the opposite behaviour. The analysis of the velocity and temperature profiles with variations in S and Z was shown in Figure 10 to 13, respectively (with fixed values of ϕ_1 = 0.01 and ϕ_2 = 0.02). Figure 10 and Figure 12 show that the increasing behaviour is seen as S and Z increase. Nevertheless, the temperature profiles shown in Figure 11 and 13 exhibit conflicting behaviour.

Table 9
Values of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ for selected values of ϕ_1, ϕ_2 and *Z* when *ε* = -1, Pr = 10 and *S* = 2.4 where ϕ_1 = Al_2O_3 and ϕ_2 = Cu

Z	$\phi_1 = 1.5\%$, $\phi_2 = 0.5\%$		$\phi_1 = 1\%$, $\phi_2 = 1\%$		$\phi_1 = 0.5\%$, $\phi_2 = 1.5\%$	
	$\mathrm{Re}_x^{1/2} C_f$	$\mathrm{Re}_x^{-1/2} Nu_x$	$\mathrm{Re}_x^{1/2} C_f$	$\mathrm{Re}_x^{-1/2} Nu_x$	$\mathrm{Re}_x^{1/2} C_f$	$\mathrm{Re}_x^{-1/2} Nu_x$
-1	0.646807266	15.696747986	0.575950470	15.703827628	0.505812081	15.710987918
0	1.281574759	15.705673614	1.329781786	15.714050900	1.376873461	15.722440787
1	1.451175837	15.707110093	1.580218334	15.716154487	1.707536315	15.725196750

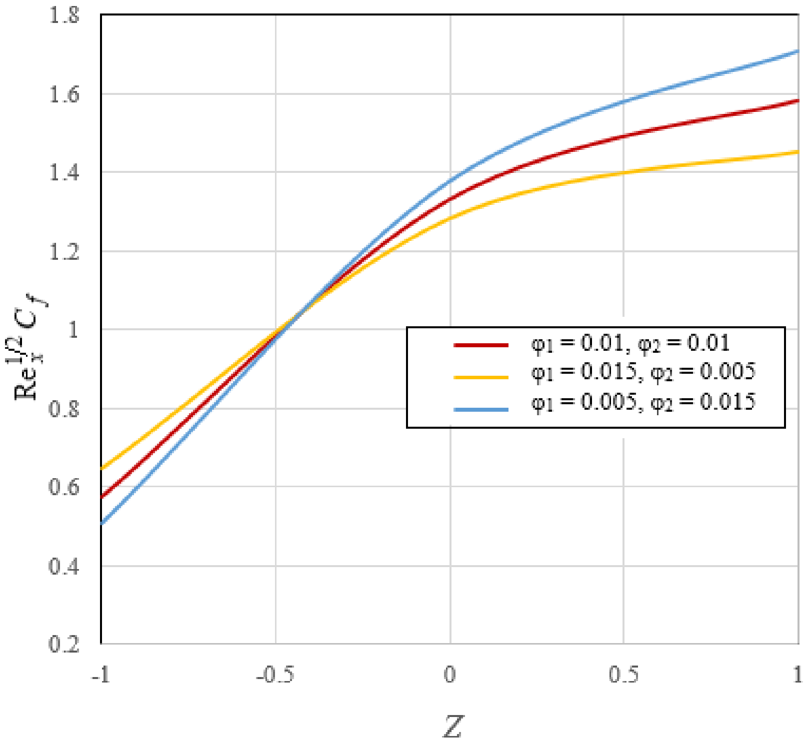

Fig. 6. $Re_x^{1/2} C_f$ vs Z for various values of ϕ_1 and ϕ_2 where ε = -1, Pr = 10 and S = 2.4

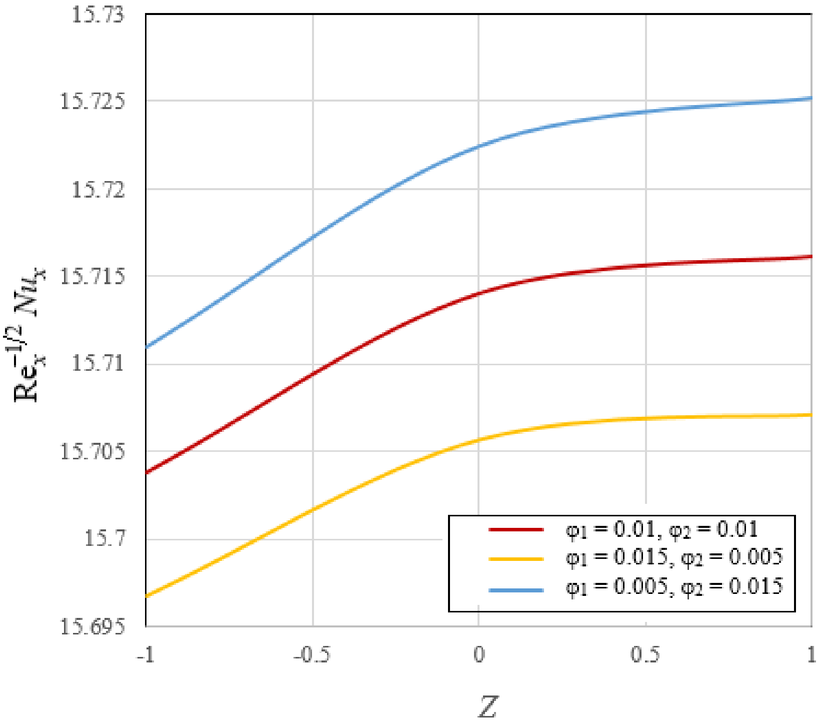

Fig. 7. $Re_x^{-1/2} Nu_x$ vs Z for various values of ϕ_1 and ϕ_2 where ε = -1, Pr = 10 and S = 2.4

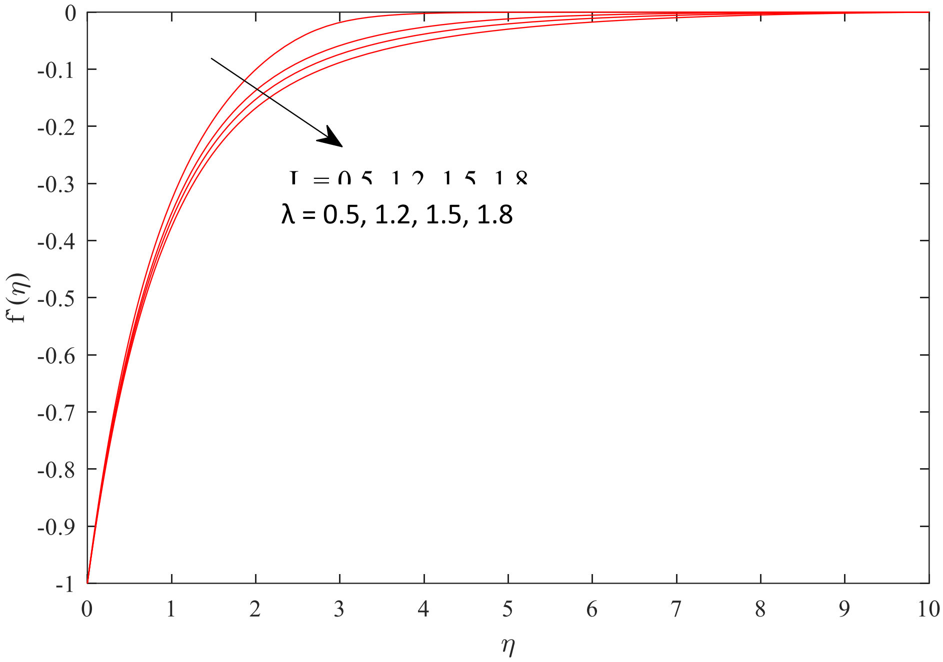

Fig. 8. Velocity profiles $f'(\eta)$ vs η for various values of λ where ε = -1, γ = 0.1, Pr = 10, S = 2.4 and Z = -0.5

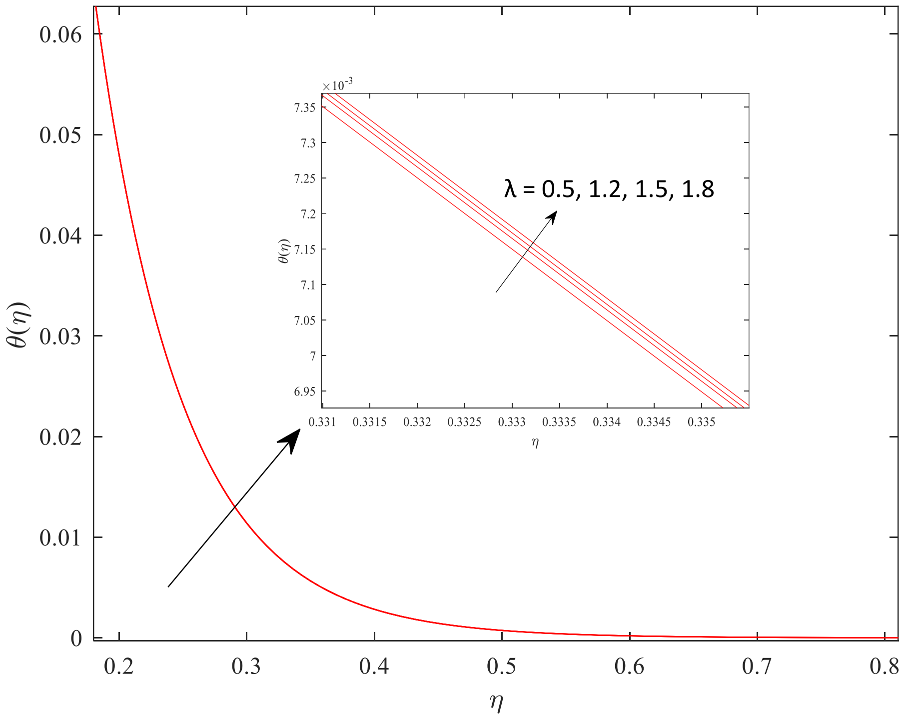

Fig. 9. Temperature profiles $\theta(\eta)$ vs η for various values of λ where ε = -1, γ = 0.1, Pr = 10, S = 2.4 and Z = -0.5

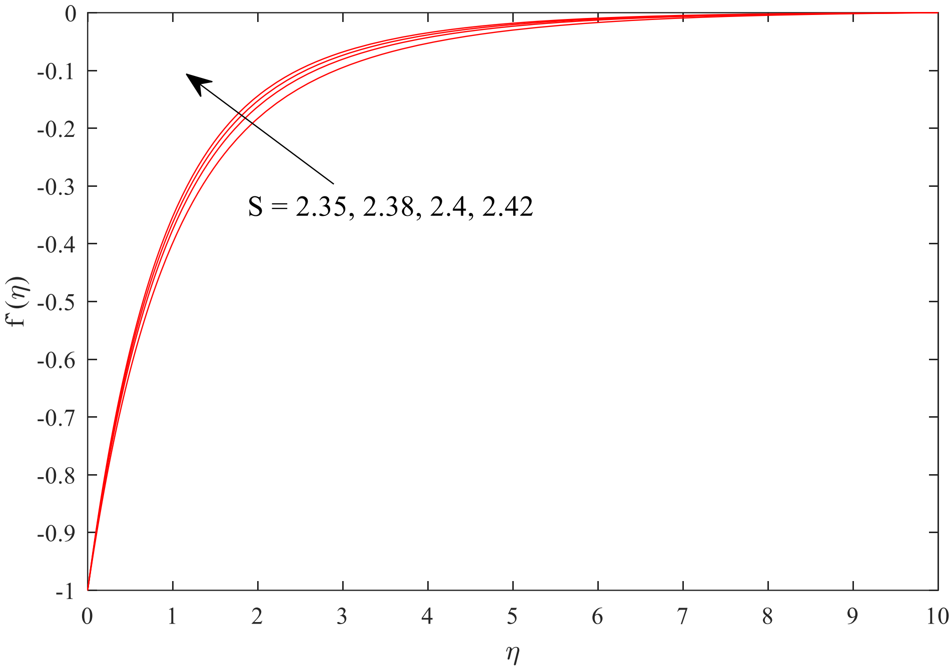

Fig. 10. Velocity profiles $f'(\eta)$ vs η for various values of S where ε = -1, λ = 1.5, γ = 0.1, Pr = 10 and Z = -0.5

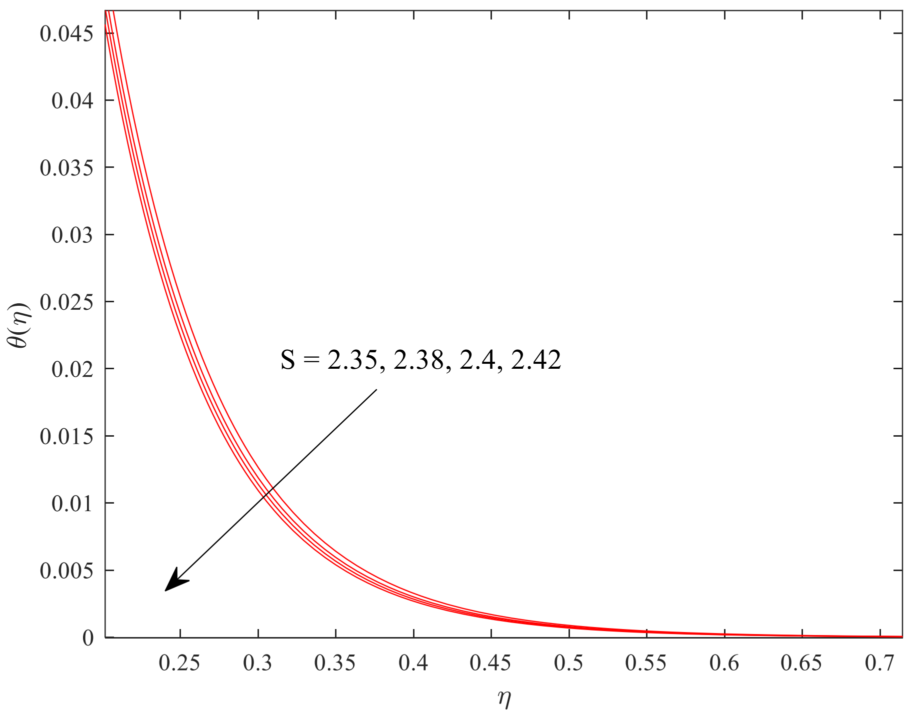

Fig. 11. Temperature profiles $\theta(\eta)$ vs η for various values of S where ε = -1, λ = 1.5, γ = 0.1, Pr = 10 and Z = -0.5

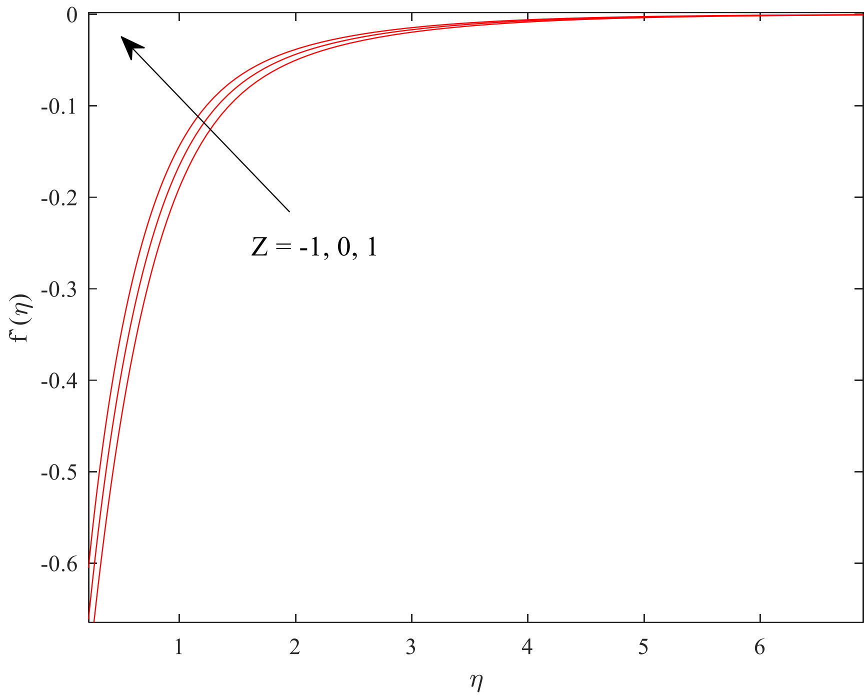

Fig. 12. Velocity profiles $f'(\eta)$ vs η for various values of Z where ε = -1, S = 2.4, λ = 1.5, γ = 0.1, Pr = 10, ϕ_1 = 0.01 and ϕ_2 = 0.02

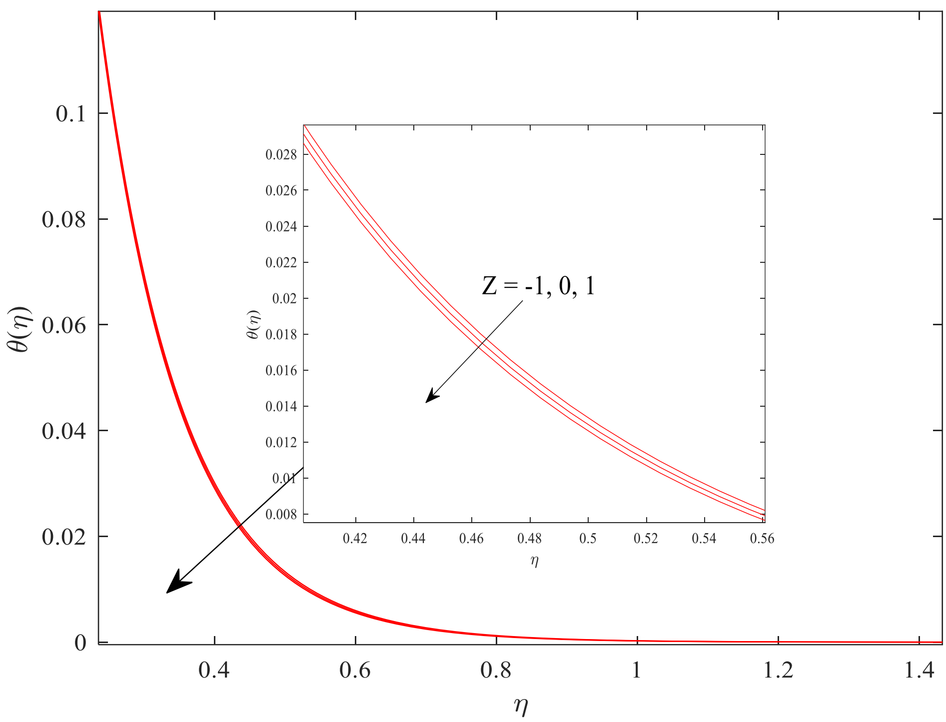

Fig. 13. Temperature profiles $\theta(\eta)$ vs η for various values of Z where ε = -1, S = 2.4, λ = 1.5, γ = 0.1, Pr = 10, ϕ_1 = 0.01 and ϕ_2 = 0.02

4. Conclusions

A mathematical analysis of mixed convection of the Reiner-Philippoff fluid flow past a shrinking sheet is established. The Reiner-Philippoff parameter shows that the increase in λ reduces the velocity of the fluid while the reverse behaviour is attained for the temperature profiles. On the other hand, the increasing behaviour of the velocity profile is observed with the increasing of *S* and *Z* (where $\phi_1 = 0.01$ and $\phi_2 = 0.02$). Still, it shows a contradictory behaviour for the temperature profiles. The increase in the injection/suction parameter, mixed parameter and Prandtl number increase the local Nusselt number, while the increases in the Reiner-Philippoff parameter and Bingham number decrease the skin friction coefficient.

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