Soret and MHD Effects of Parabolic Flow Past through an Accelerated Vertical Plate with Constant Heat and Mass Diffusion in the Presence of Rotation, Chemical Reaction and Thermal Radiation

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1. Introduction

Due to the vast range of fields in which it may be used, the research of magnetic field on viscous, electrically conducting fluid incompressible flow has piqued the attention of several researchers. In the fields of agriculture, the petroleum industry, geophysics, and astrophysics, MHD is crucial. The research of the electrically conducting fluids’ motion under a magnetic field is known as MHD (magnetic hydrodynamics). This research focuses on the examination of interactions among magnetic fields and electrically conducting fluids, including liquid metals, electrolytes, plasma, etc.

Thermal diffusion, sometimes referred to as the Soret effect, is the transferring of mass process that results from the combined impacts of concentration and temperature gradients. Charles Soret conducted the first experimental research on thermal diffusion impact on the transferring of mass. Connected concerns in 1879. Research conducted by Reddy et al., [1] focused on a vertical porous plate that was unlimited in size and included a suction that could be adjusted. Maran et al., [2] worked on the impact of the transfer of mass as well as the heat on the unstable flow of convection

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that is MHD free over a vertical plate containing a thermal diffusion or chemical reaction. The researchers Devi and Raj [3] investigated the various effects of thermal diffusion on an irregular hydromagnetic free convection flow. This flow involved the transmission of mass and heat in a slip flow regime through a sliding vertical plate. In current study of MHD variable suction free convective flow across an infinite vertical porous plate that has been conducted by Mythreye and Balamurugan [4], both the Soret effect along with the Chemical reaction were taken into consideration. The effect of Soret on MHD free convective flow was discussed by Bhavana et al., [5] in their research, using a vertical plate through a heat source as the test subject. Aruna et al., [6] worked on magnetic fields effect and hall effects on flow through a parabolically accelerated vertical plate when heat radiation was present. Selvaraj and Jothi [7] investigated the heat source’s effect on MHD as well as the radiation absorption fluid across an increasingly vertical plate that was surrounded by a porous medium.

Soundalgekar [8] and Vidhya et al., [9] examined the impacts of MHD on spontaneously initiated vertical infinite plate having a change of temperature when there is the existence of transverse magnetic field. The researchers Soundalgekar [10] also investigated how a fluid’s flow which was electrically conducting was impacted during transverse magnetic field that is given to fluid via a spontaneously started infinite isothermal vertical plate. Habibishandiz and Saghir [11] and Jose et al., [12] explained the method of Laplace transform was utilised in order to find solutions to the dimensionless governing equations. The research conducted by Amar et al., [13] looked at the optically thin grey gas radiative free convection flow that occurred across a semi-infinite vertical plate. Jose and Selvaraj [14] performed research on the various consequences of rotating effects of parabolically flow past on a vertical plate that was undergoing a chemical reaction at the convective transfer of mass and heat. Lakshmikaanth et al., [15] finds an escalation in radiation 'R' leads to a reduction in temperature, whereas a higher heat source 'Q' results in an elevation in temperature. Lakshmikaanth et al., [16] reported that increasing radiation 'R', rotation 'w', and MHD parameter ‘M’ leads to a reduction in velocity, while rising Grashof 'Gc', 'Gr', Hall Current, and heat source 'Q' result in increased velocity effects of Rotation on MHD stream accelerated isothermal perpendicular plate with warmth and mass dispersion had been focused by Muthucumaraswamy et al., [17]. Muthucumaraswamy et al., [18] emphasized on the impact of rotation on the MHD stream flowing through an accelerating perpendicular plate in their research. Selvaraj et al., [19] investigated MHD Parabolic flow, including mass and heat diffusion and rotation, over an accelerating isothermal vertical plate. In their study, Gowri and Selvaraj [20] and Bafakeeh et al., [21] looked into the rotational impacts of an unsteady MHD-parabolic flow on a vertical plate in a porous medium with temperature and mass diffusions that were constant throughout. A. Selvaraj and Constance Angela conducted research into the effect that Dufour and Hall have on MHD flow by using an exponentially accelerated vertical plate [22]. Amar et al., [13] and Goud and Reddy [23] explored the effect of the Dufour principle on unstable free convection MHD flow by employing a porous material and an exponentially accelerating plate in their experimentation. The current study focuses on the Soret effect as it relates to the MHD convective heat and mass transfer flow of an irregularly shaped viscous incompressible electrically conducting fluid through a vertical plate while the plate is rotating which is explained by Kumar et al., [24]. Vance et al., [25] and Goud et al., [26] included the effects of thermal radiation and chemical reaction. The Soret effect, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight and of medium molecular weight. The Soret and Dufour effects are encountered in many practical applications such as in the areas of geosciences, and chemical engineering. Soret Dufour and radiation effect on MHD flows arise in many areas of engineering and applied physics. The study of such flow has application in MHD
generators, chemical engineering, nuclear reactors, geothermal energy, reservoir, engineering and astrophysical studies.

2. Mathematical Formulation

In this article, flow of an unpredictable viscous incompressible fluid was therefore considered. The taken plate cannot conduct electricity. The $y$-axis is placed perpendicular to the $x$-axis and vertically along the plate as shown in Figure 1. The rotating plate is surrounded by a homogeneous, $B_0$ magnetic field. Initial conditions include the fluid being at the same concentration, $C_\infty$, and the plate being at the same temperature, $T_\infty$. The plate starts to travel vertically in its own plane at time $t > 0$ with velocity $u = u_0 t'$. The fluid is heated to a higher temperature of, $T'_w$ while the fluid concentration is raised to, $C'_\infty$.

![Figure 1. Geometry of the problem](image)

Following are the governing equations using the standard Boussinesq's approximations.

\[ \frac{\partial u'}{\partial t'} - 2\Omega' v' = g\beta (T' - T'_\infty) + g\beta (C' - C'_\infty) + v' \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u' \]  \( \text{(1)} \)

\[ \frac{\partial v'}{\partial t'} + 2\Omega' u' = \frac{\partial^2 v'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} v' \]  \( \text{(2)} \)

\[ \frac{\partial T}{\partial t'} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y'^2} - \frac{\partial^2 \bar{u}'}{\partial y'^2} \]  \( \text{(3)} \)

\[ \rho C_p \frac{\partial C'}{\partial t'} = Dm \frac{\partial^2 C'}{\partial y'^2} + \frac{Dm K_T}{T_m V} \frac{\partial^2 T}{\partial y'^2} \]  \( \text{(4)} \)
When initial and boundary conditions are present

\[ u' = 0, T' = T'_\infty, C' = C'_\infty, \text{ for all } y', t' \leq 0 \]
\[ t' > 0: u' = u_0 t'^2, T' = T'_w, C' = C'_w \text{ at } y' = 0 \]
\[ u' = 0, T' \to T'_\infty, C' \to C'_\infty \text{ as } y' \to \infty \] (5)

On suggesting the subsequent dimensionless quantities

\[ U = \frac{u'}{u_0}, V = \frac{v'}{u_0}, t = \frac{t' u_0^2}{v}, y = y' \frac{u_0}{v} \]
\[ \theta = \frac{T - T'_\infty}{T'_w - T'_\infty}, \text{ Gr} = \frac{g \beta (T'_w - T'_\infty) u_0}{C'_w - C'_\infty}, C = \frac{C'_w - C'_\infty}{C'_w - C'_\infty} \]
\[ Gc = \frac{g \beta (C'_w - C'_\infty) u_0}{C'_w - C'_\infty}, M = \frac{\sigma B_0^2}{\rho u_0^2}, Pr = \frac{\mu c_p}{k} \]
\[ C = \frac{v}{Dm}, Sr = \frac{Dm K_T (T'_w - T'_\infty)}{T_m v (C'_w - C'_\infty)}, R = \frac{16 \alpha (T'_\infty)^3}{3 k^3} \] (6)

Using (6) in the Eq. (1) to Eq. (4), we have derived

\[ \frac{\partial U}{\partial t} - 2\Omega V = \text{Gr} \theta + \text{Gc} C + \frac{\partial^2 U}{\partial y^2} - MU \] (7)

\[ \frac{\partial V}{\partial t} + 2\Omega U = \frac{\partial^2 V}{\partial y^2} - MV \] (8)

\[ \frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - \frac{R}{Pr} \theta \] (9)

\[ \frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + Sr \frac{\partial^2 \theta}{\partial y^2} \] (10)

Now we combine the complex velocity, \( q = U + iV \), from the set of Eq. (7) and Eq. (8) with the boundary condition (11) to form a single equation.

\[ \frac{\partial q}{\partial t} = \text{Gr} \theta + \text{Gc} C + \frac{\partial^2 q}{\partial y^2} - mq \] (11)

\[ \frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - \frac{R}{Pr} \theta \] (12)

\[ \frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + Sr \frac{\partial^2 \theta}{\partial y^2} \] (13)

The following are the beginning and limit criteria for utilizing non-dimension quantities.

\[ q = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } y, t \leq 0 \]
\[ t > 0: q = t^2, \quad \theta = 1, \quad C = 1 \quad y = 0 \]
\[ q \to 0, \quad \theta \to 0, \quad C \to 0 \quad y \to 0 \] (14)

Here, \( m = M + 2i\Omega \).
3. Solution of the Problem

Eq. (11), Eq. (12), and Eq. (13) with related starting and limit conditions and a dimensionless administering condition are solved with Laplace transforms. Following a final inverse transform, the solutions are established as follows.

\[
q = \frac{\left(\eta^2 + m^2\right)t}{4m^2} \left[ e^{2\eta \sqrt{mt}} \text{erfc} (\eta + \sqrt{mt}) + e^{-2\eta \sqrt{mt}} \text{erfc} (\eta - \sqrt{mt}) \right] + \frac{\eta \sqrt{t(1 - 4mt)}}{8m^2} \left[ e^{2\eta \sqrt{mt}} \text{erfc} (\eta - \sqrt{mt}) - e^{-2\eta \sqrt{mt}} \text{erfc} (\eta + \sqrt{mt}) - \frac{\eta t}{2m} e^{-(\eta^2 + mt)} \right] 
- \frac{Gr}{b(1 - Pr)} \left[ \frac{1}{2} \left( e^{2\eta \sqrt{mt}} \text{erfc} (\eta + \sqrt{mt}) + e^{-2\eta \sqrt{mt}} \text{erfc} (\eta - \sqrt{mt}) \right) - \frac{e^{bt}}{2} \left( e^{2\eta \sqrt{(m + b)t}} \text{erfc} (\eta + \sqrt{(m + b)t}) + e^{-2\eta \sqrt{(m + b)t}} \text{erfc} (\eta - \sqrt{(m + b)t}) \right) \right] 
+ \frac{Gc}{c(1 - sc)} \left[ \frac{1}{2} \left( e^{2\eta \sqrt{(m + c)t}} \text{erfc} (\eta + \sqrt{(m + c)t}) + e^{-2\eta \sqrt{(m + c)t}} \text{erfc} (\eta - \sqrt{(m + c)t}) \right) + \frac{e^{ct}}{2} \left( e^{2\eta \sqrt{sc} \sqrt{ct}} \text{erfc} (\eta \sqrt{sc} - \sqrt{ct}) + e^{-2\eta \sqrt{sc} \sqrt{ct}} \text{erfc} (\eta \sqrt{sc} + \sqrt{ct}) \right) \right] 
+ \frac{A_1}{c(1 - sc)} \left[ \frac{1}{2} \left( e^{2\eta \sqrt{mt}} \text{erfc} (\eta + \sqrt{mt}) + e^{-2\eta \sqrt{mt}} \text{erfc} (\eta - \sqrt{mt}) \right) - \frac{e^{ct}}{2} \left( e^{2\eta \sqrt{(m + c)t}} \text{erfc} (\eta + \sqrt{(m + c)t}) + e^{-2\eta \sqrt{(m + c)t}} \text{erfc} (\eta - \sqrt{(m + c)t}) \right) \right] 
- \frac{1}{2} \left( e^{2\eta \sqrt{mt}} \text{erfc} (\eta - \sqrt{mt}) + e^{-2\eta \sqrt{mt}} \text{erfc} (\eta + \sqrt{mt}) \right) - \frac{e^{bt}}{2} \left( e^{2\eta \sqrt{Pr(R + b)t}} \text{erfc} (\eta \sqrt{Pr} + \sqrt{(R + b)t}) + e^{-2\eta \sqrt{Pr(R + b)t}} \text{erfc} (\eta \sqrt{Pr} - \sqrt{(R + b)t}) \right) 
+ \frac{A_1}{b(Pr - 1)} \left[ \frac{1}{2} \left( e^{-2\eta \sqrt{PrRt}} \text{erfc} (\eta \sqrt{Pr} - \sqrt{Rt}) + e^{2\eta \sqrt{PrRt}} \text{erfc} (\eta \sqrt{Pr} + \sqrt{Rt}) \right) \right] 
- \frac{1}{2} \left( e^{2\eta \sqrt{mt}} \text{erfc} (\eta - \sqrt{mt}) + e^{-2\eta \sqrt{mt}} \text{erfc} (\eta + \sqrt{mt}) \right) - \frac{e^{bt}}{2} \left( e^{2\eta \sqrt{Pr(R + b)t}} \text{erfc} (\eta \sqrt{Pr} + \sqrt{(R + b)t}) + e^{-2\eta \sqrt{Pr(R + b)t}} \text{erfc} (\eta \sqrt{Pr} - \sqrt{(R + b)t}) \right) 
+ \frac{A_1}{(Sc - 1)(a - c)} \left[ \frac{1}{2} \left( e^{2\eta \sqrt{sc} \sqrt{at}} \text{erfc} (\eta \sqrt{sc} + \sqrt{at}) + e^{-2\eta \sqrt{sc} \sqrt{at}} \text{erfc} (\eta \sqrt{sc} - \sqrt{at}) \right) \right] 
+ \frac{A_1}{(Pr - 1)(a - b)} \left[ \frac{1}{2} \left( e^{2\eta \sqrt{(m + a)t}} \text{erfc} (\eta + \sqrt{(m + a)t}) + e^{-2\eta \sqrt{(m + a)t}} \text{erfc} (\eta - \sqrt{(m + a)t}) \right) \right] 
- \frac{e^{ct}}{2} \left( e^{2\eta \sqrt{(m + c)t}} \text{erfc} (\eta + \sqrt{(m + c)t}) + e^{-2\eta \sqrt{(m + c)t}} \text{erfc} (\eta - \sqrt{(m + c)t}) \right) 
- \frac{e^{ct}}{2} \left( e^{2\eta \sqrt{(m + a)t}} \text{erfc} (\eta + \sqrt{(m + a)t}) + e^{-2\eta \sqrt{(m + a)t}} \text{erfc} (\eta - \sqrt{(m + a)t}) \right) (15)
\[
\begin{align*}
\frac{e^{bt}}{2} \left( e^{2\eta \sqrt{(m+b)t}} \text{erfc} \left( \eta + \sqrt{(m+b)t} \right) + e^{-2\eta \sqrt{(m+b)t}} \text{erfc} \left( \eta - \sqrt{(m+b)t} \right) \right) + \\
\frac{e^{at}}{2} \left( e^{2\eta \sqrt{Pr(R+a)t}} \text{erfc} \left( \eta \sqrt{Pr} + \sqrt{(R+a)t} \right) + e^{-2\eta \sqrt{(R+a)t}} \text{erfc} \left( \eta - \sqrt{(R+a)t} \right) \right)
\end{align*}
\]
\[
\theta = \frac{1}{2} \left[ e^{-2\eta \sqrt{PrRt}} \text{erfc} \left( \eta \sqrt{Pr} - \sqrt{Rt} \right) + e^{2\eta \sqrt{PrRt}} \text{erfc} \left( \eta \sqrt{Pr} + \sqrt{Rt} \right) \right]
\]

\[
C = \text{erfc} \left( \eta \sqrt{Sc} \right) + \frac{Sc \cdot Pr}{Sc-Pr} \left[ - \left( \frac{R}{2a} \right) \text{erfc} \left( \eta \sqrt{Sc} + \sqrt{at} \right) + \frac{e^{2\eta \sqrt{Sc}at}}{2} \text{erfc} \left( \eta \sqrt{Sc} + \sqrt{at} \right) + e^{-2\eta \sqrt{Sc}bt} \text{erfc} \left( \eta \sqrt{Sc} - \sqrt{at} \right) - \left( \frac{R}{2a} \right) \left( e^{2\eta \sqrt{PrRt}} \text{erfc} \left( \eta \sqrt{Pr} + \sqrt{Rt} \right) + e^{-2\eta \sqrt{PrRt}} \text{erfc} \left( \eta \sqrt{Pr} - \sqrt{Rt} \right) \right) - \left( \frac{a+R}{a} \right) e^{at} \left( e^{2\eta \sqrt{Pr(R+a)t}} \text{erfc} \left( \eta \sqrt{Pr} + \sqrt{(R+a)t} \right) + e^{-2\eta \sqrt{Pr(R+a)t}} \text{erfc} \left( \eta \sqrt{Pr} - \sqrt{(R+a)t} \right) \right) \right]
\]

\[
erfc(a + ib) = \text{erf}(a) + \frac{\exp (-a^2)}{2\pi} \sum_{n=1}^{\infty} \frac{\exp (-\eta^2/4)}{\eta^2 + 4a^2} \left[ f_n(a, b) + ig_n(a, b) \right] + \epsilon(a, b)
\]

where

\[
a = \frac{Pr(R+S)}{Sc}, \quad b = \frac{m-Pr}{Pr-1}, \quad c = \frac{m}{Sc-1}, \quad A_1 = S_r S_c G_c \quad \text{and} \quad \eta = \frac{\gamma}{2\sqrt{t}}
\]

\[
f_n = 2a - 2a \cosh(nb) \cos(2ab) + n \sinh(nb) \sin(2ab)
\]

and

\[
g_n = 2a \cosh(nb) \sin(2ab) + n \sinh(nb) \cos(2ab)
\]

\[|\epsilon(a, b)| \approx 10^{-16} |\text{erf}(a + ib)|\]

4. Results and Discussions

To analyse the results and better understand the problem, calculations for numerical are performed for a number of physical factors, depending on the kind of flow and transport, Sr, M, t, Sc, Gr, and Gc. The Pr Prandtl number, which represents air, is chosen with 0.71 value. The important parameters' numerical values for temperature, concentration, and velocity are calculated.

Figure 2 states how the Pr Prandtl number affects the temperature profiles. It is evident that rising the Prandtl number causes the temperature profiles to fall.
Figure 3 illustrates how the radiation parameter $R$ affects profiles of temperature. In profiles of temperature, the impact of the parameter of thermal radiation is crucial. It displayed that when the radiation parameter dropped, temperature rises. This demonstrates that a decline in temperature results from increased thermal radiation.

The impact of temperature profiles for various time values (0.2, 0.3, 0.4, and 0.5) is shown in Figure 4. It has been shown that the wall temperature rises when it is improved.
Figure 5 exhibits that the fluid velocity rises as the parameter of radiation $R$ decreases drops. This is because the greater dominance of conduction-less radiation caused by modest values of $R$ escalates the buoyancy force and the thickness of the momentum boundary layer.

Figure 6 shows how the Schmidt number $Sc$ affects the concentration profiles. It is evident that concentration drops as the Schmidt number rises. According to physical laws, a rise in the Schmidt number causes the molecular diffusivity to fall, which lowers the concentration boundary layer. From the findings, the species concentrations are greater for small values of $Sc$ and lower for large $Sc$ values.
Figure 7 shows how the concentration patterns vary with Soret number Sr. It has been noted that concentration distribution in the boundary layer is significantly affected by rising Soret number Sr values. It is observed for concentration profiles rise as the levels of Sr rise.

Figure 8 depicts how the time parameter t affects the concentration C. It could be seen that the concentration profiles become bigger as the time parameter t gets bigger.
Figure 9 exhibits how the magnetic parameter $M$ affects the velocity profiles. The velocity curves demonstrate that the rate of transport significantly reduces as the magnetic parameter's value rises, indicating that the magnetic field slows the fluid's mobility. The flow properties may be controlled by a magnetic field.

Figure 10 depicts how the Soret number $Sr$ affects the profiles of velocity. As the Soret number rises, the fluid's velocity is also seen to inflate.
Figure 11 depicts how the Sc is the Schmidt number affects the profiles of velocity. It is evident that when Sc values increase, the fluid’s velocity drops. This is because increasing Sc causes molecular diffusivity to decrease, it ultimately results in the concentration and velocity boundary layer thickness to drop.

Figure 12 depicts the fluctuation of the velocity profiles with the Grashof number Gr. It can be seen that when the Grashof number rises, the fluid velocity does as well. This is because when the value of the Grashof number rises, the buoyancy force rises fluid velocity and thickens the boundary layer.
Figure 13 displays the fluctuation of the velocity profiles with mass Grashof number $Gc$. It could be shown that the velocity of fluid rises as the mass Grashof number rises. This is due to the buoyancy force’s ability to improve fluid velocity and thicken the boundary layer as the mass Grashof number rises.

5. Conclusions

This study examines how rotation affects radiation-affected unstable MHD free convection mass and heat transfer flow across infinite vertical plate. By utilizing the Laplace transform method, the dimensionless leading equations of the flow have been numerically to be solved. It is shown that the problem’s material properties have an impact on the flow characteristics. The lists of discoveries of the analysis are as follows:
i. The rise in the Prandtl number reasons for the drop in fluid temperature & velocity, but elevation in the parameter radiation causes a reduction in fluid velocity & temperature in the boundary layer.

ii. As the number of Soret rises, the fluid velocity rises as well. Both the fluid concentration and velocity drop as the Schmidt number rises.

iii. When the time parameter is extended, so do the temperature, concentration, and velocity.

The current investigation into the mechanics of fluid flow has potential implications in both science and engineering. Fluid flow is well documented to strongly influence microstructural evolution and its dominant effect is to speed up the transport of heat (or mass) on the large scale. Different means of incorporation of this effect into the phase field model have been developed.

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References


