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Presence of Riga Plate on MHD Caputo Casson Fluid: An Analytical Study

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ABSTRACT

Driven by technological advancement, the Riga plate can be seen as a key feature in developing the engineering world. As such, this study aims to investigate the effects of an accelerating semi-infinite Riga plate over a convective flow of MHD Casson fluid incorporated with the Caputo fractional derivative. The obtained governing PDEs are converted in dimensionless form and reduced to systems of ODEs via Laplace transform. Zakian's method of inverse Laplace transform is then utilised to generate graphical results in the time domain. Variations of parameter such as Casson, modified Hartmann number, Grashof number, magnetic parameter and fractional parameters are investigated for velocity profiles. Skin friction coefficient is also calculated and presented numerically. Study shows that Riga plate aids in fluid flow, hence increasing its velocity.

1. Introduction

Riga plates are electromagnetic actuators built on plane surface from electrodes and magnets. From presence of Riga plate a parallel wall of Lorentz force is generated. The phenomenon, also known as electromagnetohydrodynamic (EMHD) force, was first introduced by Gailitis and Lielausis [1]. Research on Riga plate made a significant impact on industrial and technological advancement such as micro-coolers, thermal reactors and submcattemiarines. Specifically, Riga plate is used in submarines to reduce drag force and friction by decreasing turbulence and bypassing boundary layer separation [2]. Research in boundary layer flow involving Riga plate would generate solutions with a modified Hartmann number from the the Grinberg term used in the governing momentum equations [3]. Presence of Riga plate is evaluated through the value of the modified Hartmann number.

Recently, Shah *et al.,* [4] investigated mixed convection stagnation point flow on a Riga plate using the Catteneo-Christov heat flux model. The Cattaneo-Christov heat flux model is used to analyse heat convection within a viscoelastic fluid induced by a stretching sheet [5–8]. The homotopy analysis method is employed in Shah *et al.,* [4] study to generate solutions for the boundary value problem. Authors highlighted that as the modified Hartmann number increases, so does fluid velocity. Rizwana *et al.,* [9] conducted a similar study by considering non-Newtonian nanofluid flow over an oscillating

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plate. Presence of Riga plate was considered. Result of Rizwana *et al.,* [9] were in agreement with Shah *et al.,* [4].

Meanwhile, Loganathan and Deepa [10] conducted a study involving Riga plate over a stratified Casson fluid with heat generation and absorption. Solutions were obtained numerically via implicit finite difference method. Results shows an increase in fluid velocity with an increase in modified Hartmann number. The study was replicated by Nasrin *et al.,* [11] by considering rotational Casson fluid and the results were compatible with that of Loganathan and Deepa [10]. Other honourable mentions on Riga plate studies includes Mallawi *et al.,* [12], Bhatti and Michaelides [13] and Khatun *et al.,* [14].

Up to this point, fractional derivatives have not been addressed for any Riga plate problems. A fractional derivative is a derivative with an order of a non-integer or complex number. Although the geometrical characteristics of fractional derivatives has not been identified, it is highly probable that the solutions obtained by employing fractional derivatives will be useful in future issues. There are many definitions of fractional derivative, Caputo, Caputo-Fabrizio, and Antanga-Baleanu are only a few examples.

Raza [15] studied the effects of fractional derivative on a rotating flow of a second grade fluid inside an infinite cylinder using the Caputo derivative. Solutions were found using the Laplace transform and the Stehfast algorithm method. According to the author, the hybrid method is less time consuming and requires less computations. The findings indicated that increasing the fractional parameter increases fluid velocity. Raza *et al.,* [16] carried on the same research using a new kind of fluid which is Burgers' fluid. The results are in excellent accord with one another. Furthermore, Raza and Ullah [17] conducted a comparison research between Caputo and Caputo-Fabrizio under the same conditions as Raza [15] and Raza *et al.,* [16].

Anwar and Rasheed [18] studied the impact of fractional derivatives on Oldroyd-B nanofluid flow using the Caputo derivative in the same year. Fluid flows through the bottom plate of a restricted non-isothermal plate while thermophoresis and pedesis effects are taken into account. A finite difference-finite element method was used to find numerical solutions. The authors emphasised that increasing the fractional parameters resulted in a greater velocity profile.

Imran *et al.,* [19] went on to investigate fractional models of two kinds of fluid, viscous and second grade. A comparison of the solutions was made, as well as the effect of fractional parameters on the different solutions. The Caputo derivative, as well as the effects of Newtonian heating and chemical reaction, were taken into account. The authors demonstrated that the non-fractional models produced greater velocity profiles for both viscous and second grade fluids than the subsequent fractional models.

Abdullah *et al.,* [20] examined blood flows including nanoparticles inside a circular cylinder in a uniform magnetic field using Caputo derivative the following year. To get the final velocity profile solutions, a similar hybrid approach from Raza [15] was used for the same reason that the hybrid Laplace-Stehfast method is faster and requires less computation. The authors said that the fractional model outperformed the classical model, demonstrating a superior representation of the viscoelastic fluid. It is also said that fractional solutions are considered to be more stable than non-fractional solutions.

In the meanwhile, Aman *et al.,* [21] released a paper on using the Caputo time-fractional derivative model to improve heat transfer of graphene-water nanofluids used in solar panels. The flow of MHD Poiseuille nanofluid across a vertical plate containing graphene nanoparticles was examined. Analyzed fractional PDEs are solved using the Laplace transform and solutions are given as special functions known as Wright functions. The research discovered that increasing the nanoparticle volume fraction and fractional parameters improves heat transmission. Khan *et al.,* [22] investigated the Poiseuille flow of an MHD fluid over a vertical stationary plate with non-uniform wall temperature in a similar manner. That study's results were comparable to those of Aman *et al.,* [21]. Another fractional derivative study that showcases their solutions in special functions is by Tassadiq *et al.,* [23]. However, instead of Wright functions, solution are presented in Mittag-Leffler functions.

Shah *et al.,* [24] published a research on natural convection flow in a vertical cylinder. In this research, the time-dependent fractional derivative of Caputo was used as well as Laplace transform and the Hankel transform. Stehfest's method is then used to convert solutions in the frequency domain back into the time domain employing the same hybrid methods by Raza [15] and Abdullah *et al.,* [20]. The authors pointed out that the solution for velocity profile with fractional derivatives is quicker than the traditional one. The identical circumstances were then utilised by Shah *et al.,* [25] to investigate the implications of a double convection flow. Imran *et al.,* [26] expanded on Shah *et al.,* [24] research by adding Newtonian heating and arbitrary velocity boundary conditions. Their results are mostly consistent with past literature.

Menwhile, Sarwar *et al.,* [27] investigated the consequences of slip effect with an exponentially moving vertical plate using the Caputo fractional derivative. Answers were found using Laplace transforms, Stehfest's method and Tzu's algorithm. It is noted that when fractional parameters increase, fluid velocity decreases. The hybrid method was also utilized by Saqib *et al.,* [28] in investigating the effects of different fractional models on magnetic bio-nanofluid modelled by the Oldroyd-B model. However, authors replaced the use of Stehfast algorithm with Zakin's method of inverse Laplace transform in the hybrid method. According to Saqib *et al.,* [28] and Hassanzadeh *et al.,* [29], Zakian's method provides a more accurate solution compared to Stehfast's method.

To the best of the authors knowledge, no research has been conducted for a study on effects of Riga plate on MHD Casson fluid flow modelled with fractional derivatives. Although studies on Riga plate and fractional derivatives are abundant, but no research on fluid flow with both effects are available. Thus, the main objective of research will investigate the effects of the Caputo fractional derivative model on the Riga plate with presence of unsteady MHD Casson fluid flow in accelerated semi-infinite by solving it via the Laplace-Zakian hybrid approach. The Laplace-Zakian approach is chosen due to less time consuming, requires less computations and is more accurate. This study will investigate the behavior of Casson fluid, fractional derivative parameter, presence of Riga plate and presence of MHD. Besides, analysis will be done on the behaviour of Casson fluid with and without the presence of Riga plate. A variation of fractional parameters will also be used to investigate the effect of a fractional derivative.

2. Problem Formulation

Heat transfer of a Casson fluid flow with presence of uniform magnetic filed over an accelerated permeable semi-infinite vertical Riga plate is considered. At $t=0$, the plate and fluid are both at rest with constant temperature, T_∞. When $t\geq 0$, the temperature of the plate is increased to T_W and remained constant thereafter. At $t\geq 0$, the plate begins increase in speed uniformly at the rate of, At, where A is the acceleration. A permeated uniform magnetic field, B_0 parallel to the y-axis is applied to the fluid. Due to small Reynold number, effects of induced magnetic field in fluid flow is insignificant enough for it to be ignored. The y coordinate, measured perpendicular to the plate and fluid flow is only considered at $y>0$. Velocity, U and temperature, T are dependent on space variable, y and time, t. Figure 1 shows the geometrical representation of fluid flow as well as Riga plate.

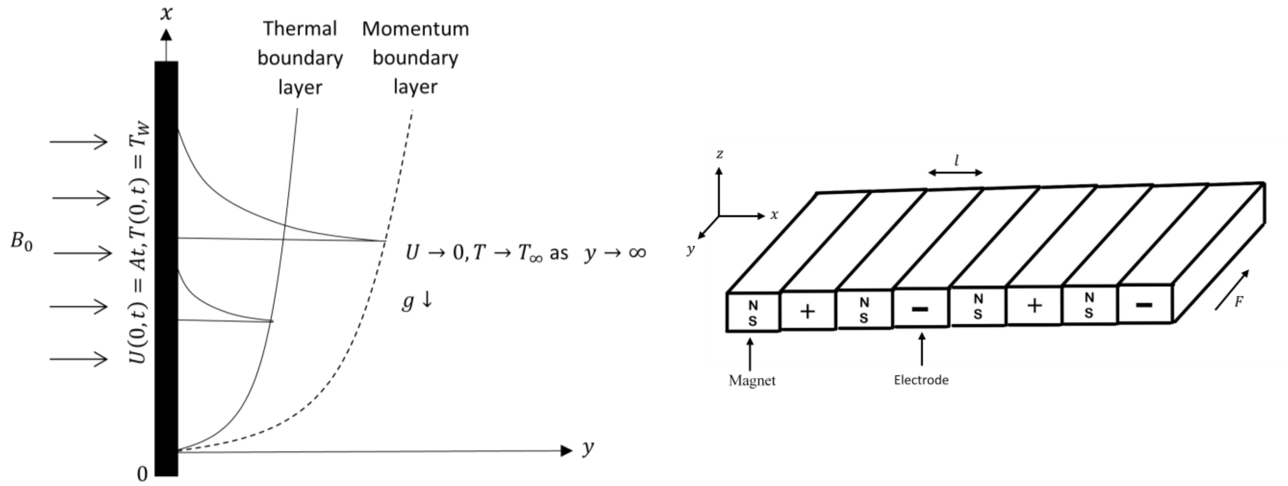

Fig. 1. Physical representation of fluid flow and Riga plate

Using Boussinesq's approximation, the considered environment mentioned produce governing equations for the Casson fluid flow as follows [26,27],

$$\frac{\partial U(y,t)}{\partial t} = \upsilon\left(1+\frac{1}{\beta}\right)\frac{\partial^2 U(y,t)}{\partial y^2} - \frac{\sigma B_0^2}{\rho}U(y,t) + g\beta_T(T-T_\infty) + \frac{\pi J_0 M_0}{8\rho}\exp\left(-\frac{\pi}{l}y\right), \tag{1}$$

$$\frac{\partial T(y,t)}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T(y,t)}{\partial y^2}. \tag{2}$$

where Eq. (1) and (2) are bounded by the following initial and boundary conditions,

$$\begin{aligned} &U(y,0)=0, \quad T(y,0)=T_\infty, \\ &U(0,t)=At, \quad T(0,t)=T_W, \\ &U(\infty,t)\to 0, \quad T(\infty,t)\to T_\infty, \end{aligned} \tag{3}$$

where υ is the kinematic viscosity, β is the Casson parameter, σ is the electrical conductivity, B_0 is the uniform magnetic field, ρ is the density of the fluid, g is the gravitational force, β_T is the volumetric thermal coefficient of expansion, J_0 is density of current, M_0 is magnetisation of magnets, l is the width of electrodes and magnets, k thermal conductivity parameter and C_p is the specific heat at constant pressure.

Afterwards, by taking the dimensionless parameters such as Eq. (4),

$$\begin{aligned} &U^* = \frac{U}{(\upsilon A)^{1/3}}, \quad y^* = \frac{yA^{1/3}}{\upsilon^{2/3}}, \\ &t^* = \frac{tA^{2/3}}{\upsilon^{1/3}}, \quad T^* = \frac{T-T_\infty}{T_W - T_\infty}, \end{aligned} \tag{4}$$

and by abandoning the asterisk notations, the dimensionless form of Eq. (1), (2) and (3) are printed as

$$\frac{\partial U(y,t)}{\partial t}=\frac{1}{\beta_0}\frac{\partial^2 U(y,t)}{\partial y^2}-MU(y,t)+GrT(y,t)+E\exp\left(-Ly\right), \tag{5}$$

$$\frac{\partial T(y,t)}{\partial t}=\frac{1}{\Pr}\frac{\partial^2 T(y,t)}{\partial y^2}, \tag{6}$$

and

$$\begin{aligned} &U(y,0)=0, \quad T(y,0)=0,\\ &U(0,t)=t, \quad T(0,t)=1,\\ &U(\infty,t)\to 0, \quad T(\infty,t)\to 0. \end{aligned} \tag{7}$$

where the parameters of β_0, M, Gr, E, L and $\Pr$ are defined as follows,

$$\beta_0=\frac{\beta}{\beta+1}, \quad M=\frac{\sigma B_0^2\upsilon^{1/3}}{\rho A^{2/3}}, \quad Gr=\frac{g\beta_T\left(T_W-T_\infty\right)}{A}, \quad E=\frac{\pi J_0 M_0}{8A\rho}, \quad L=\frac{\pi\upsilon^{2/3}}{A^{1/3}l}, \quad \Pr=\frac{\upsilon\rho C_P}{k},$$

and β_0 is the dimensionless Casson parameter, M is the magnetic parameter, Gr is the Grashof number, E is the modified Hartmann number, L is a dimensionless constant parameter and $\Pr$ is the Prandtl number.

Then, the time Caputo fractional derivative is employed in Eq. (5) and (6) and the fractional model for Casson fluid flow is generated such as [28,29]

$$D_t^\alpha U(y,t)=\frac{1}{\beta_0}\frac{\partial^2 U(y,t)}{\partial y^2}-MU(y,t)+GrT(y,t)+E\exp\left(-Ly\right), \tag{8}$$

$$D_t^\alpha T(y,t)=\frac{1}{\Pr}\frac{\partial^2 T(y,t)}{\partial y^2}. \tag{9}$$

where $D_t^\alpha f(\cdot)$ is defined as

$$D_t^\alpha f(x,t)=\frac{1}{\Gamma(1-\alpha)}\int_0^t\frac{f'(x,t)}{\left(t-\mu\right)^\alpha}d\mu, \tag{10}$$

and it is known as the Caputo fractional derivative with $\Gamma(\cdot)$ as the gamma function and $0<\alpha<1$ is the fractional parameter. The Laplace transform of Caputo derivative is as follows [30,31],

$$\mathrm{L}\{D_t^\alpha f(x,t)\}=q^\alpha\bar{f}(x,q). \tag{11}$$

Eq. (11) will be employed to generate the final solutions of Eq. (5) and (6) bounded by Eq. (7) in the next section, the Problem Solutions section.

3. Problem Solutions

Final solutions of velocity and temperature profiles are obtained through Laplace transform, and Zakian's inverse Laplace transform. First, the PDEs from Eq. (1) and (2) are reduced to ODEs via Laplace transform and presented in the frequency domain, such as Eq. (12) and (13) below.

$$q^{\alpha}\bar{U}(y,q)=\frac{d^{2}\bar{U}(y,q)}{\partial y^{2}}-M\bar{U}(y,q)+Gr\bar{T}(y,q)+\frac{1}{q}E\exp(-Ly), \tag{12}$$

$$q^{\alpha}\bar{T}(y,q)=\frac{1}{\mathrm{Pr}}\frac{d^{2}\bar{T}(y,q)}{dy^{2}}, \tag{13}$$

and bounded transform initial boundary equations from Eq. (7),

$$\begin{aligned}
&\bar{U}(y,0)=0, \quad \bar{T}(y,0)=0,\\
&\bar{U}(0,q)=\frac{1}{q^{2}}, \quad \bar{T}(0,q)=\frac{1}{q},\\
&\bar{U}(\infty,q)\rightarrow 0, \quad \bar{T}(\infty,q)\rightarrow 0.
\end{aligned} \tag{14}$$

Eq. (15) and (16) below are solutions for the velocity and temperature when solving Eq. (12), (13) and (14).

$$\begin{aligned}
\bar{U}(y,q)=&\left[\frac{1}{q^{2}}+\frac{\beta_0 Gr}{1}\left(\frac{1}{q^{\alpha}d_2-d_1}\right)+\frac{\beta_0 E}{q}\left(\frac{1}{d_3-\beta_0 q^{\alpha}}\right)\right]\exp\left(-y\sqrt{d_1+\beta_0 q^{\alpha}}\right)\\
&-\frac{\beta_0 Gr}{1}\left(\frac{1}{q^{\alpha}d_2-d_1}\right)\exp\left(-y\sqrt{\mathrm{Pr}\,q^{\alpha}}\right)-\frac{\beta_0 E}{q}\left(\frac{1}{d_3-\beta_0 q^{\alpha}}\right)\exp(-Ly),
\end{aligned} \tag{15}$$

$$\bar{T}(y,q)=\frac{1}{q}\exp\left(-y\sqrt{\mathrm{Pr}\,q^{\alpha}}\right), \tag{16}$$

where the constant parameters of d_1, d_2 and d_3 are defined as,

$$d_1=\beta_0 M, \quad d_2=Pr-\beta_0, \quad d_3=L^{2}-d_1.$$

Finally, using Zakian's method of inverse Laplace transform, defined as [25,32,33],

$$f(t)=\frac{2}{t}\sum_{i=1}^{n}\mathrm{Re}\left\{K_i F\left(\frac{\alpha_i}{t}\right)\right\}, \tag{17}$$

the final solutions are generated and presented graphically in the next section, the Results and Discussions section.

3.1 Solution for Skin Friction

Skin friction coefficient for Casson fluid flow is generated via,

$$C_f = -\frac{1}{\beta_0}\frac{\partial U(y,t)}{\partial y}\Big|_{y=0} . \tag{18}$$

Varied values of C_f are then presented numerically in Results and Discussions.

4. Results and Discussions

The presence of an accelerated semi-infinite Riga plate on fluid velocity of an unsteady MHD fractional Caputo Casson fluid is investigated. Solutions are graphically generated by utilising Mathcad-15. Final solutions of velocity profile are obtained using Zakian's algorithm of inverse Laplace transform from Eq. (17) and implementing it to Eq. (15) and (16). Figure 2 displays the validation of current obtained results compared with published results. Meanwhile, Figure 3-8 shows the velocity profiles with various values of α , β, Gr , M, $\Pr$ and t respectively. From each of the figures, the impact on presence of Riga plate for fluid velocity is analysed. Next, skin friction coefficient is generated from Eq. (18) and is presented numerically in Table 1.

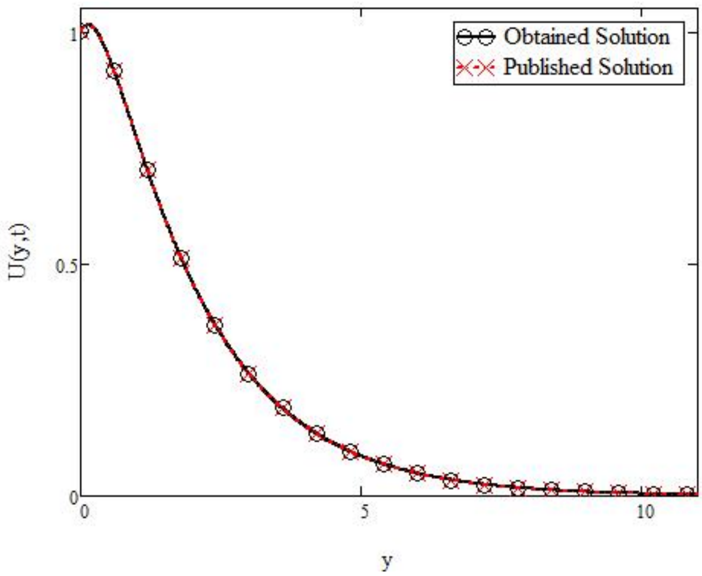

Fig. 2. Validation of obtained results with published results

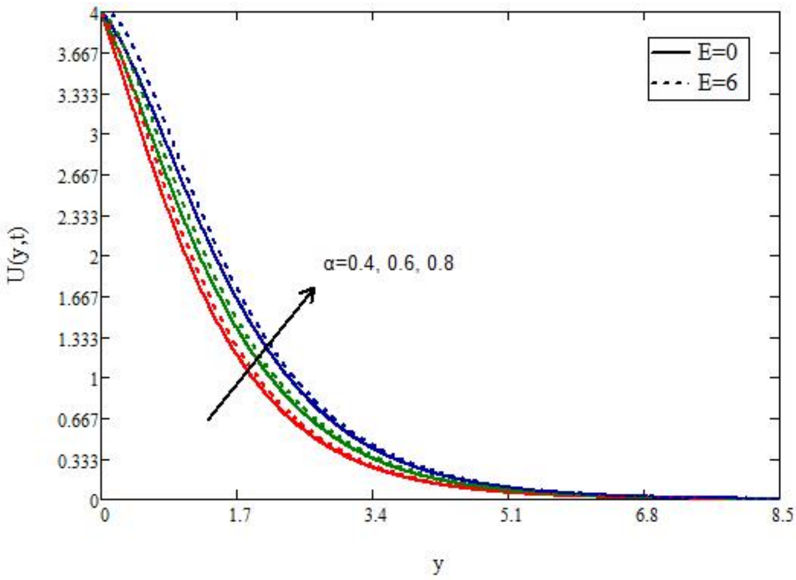

Fig. 3. Impact of α on velocity profile with and without presence of Riga Plate

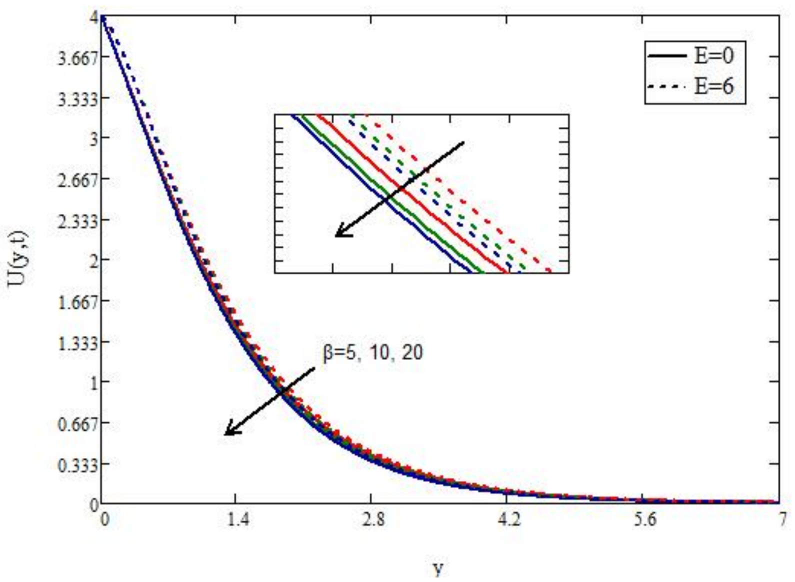

Fig. 4. Impact of β on velocity profile with and without presence of Riga Plate

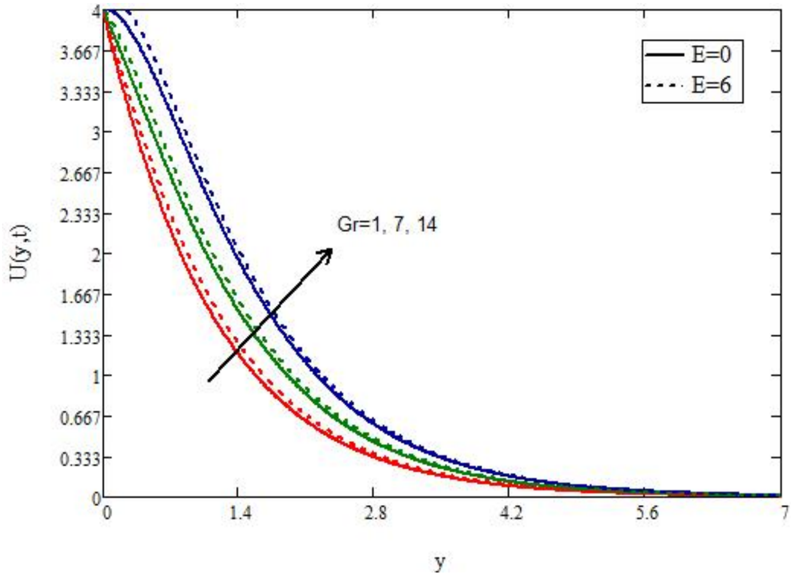

Fig. 5. Impact of Gr on velocity profile with and without presence of Riga Plate

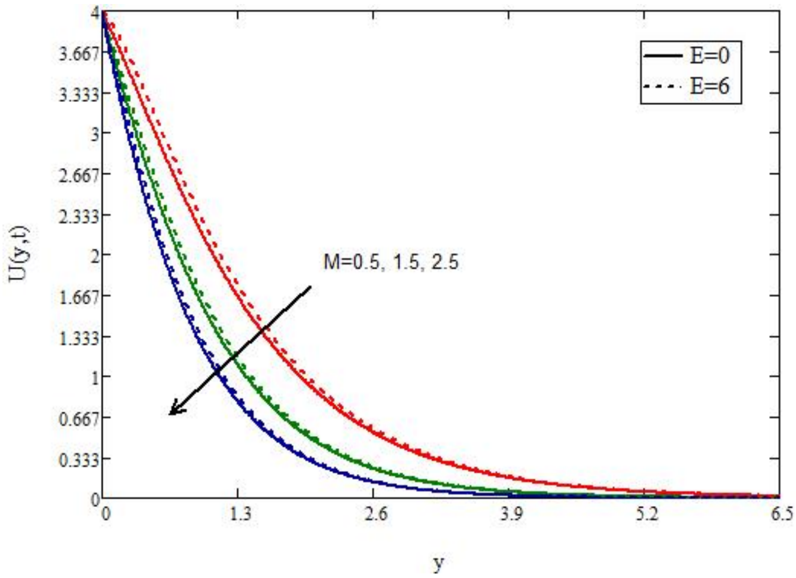

Fig. 6. Impact of M on velocity profile with and without presence of Riga Plate

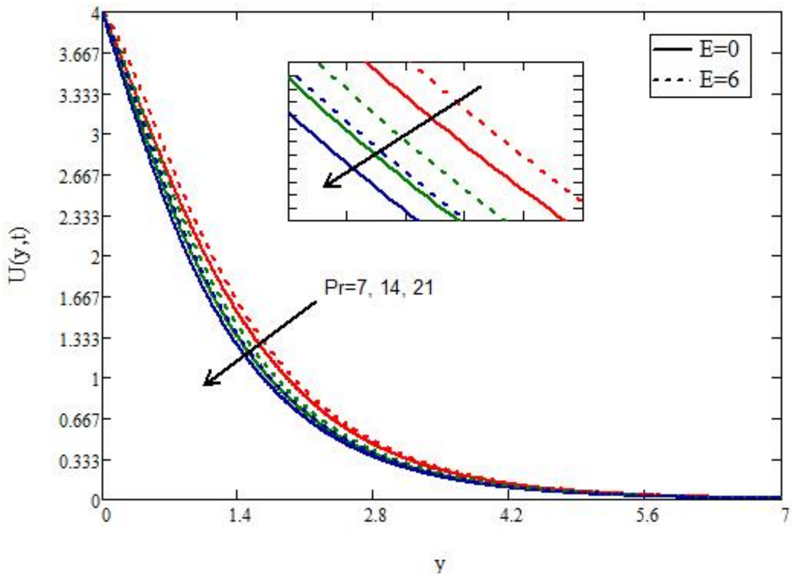

Fig. 7. Impact of Pr on velocity profile with and without presence of Riga Plate

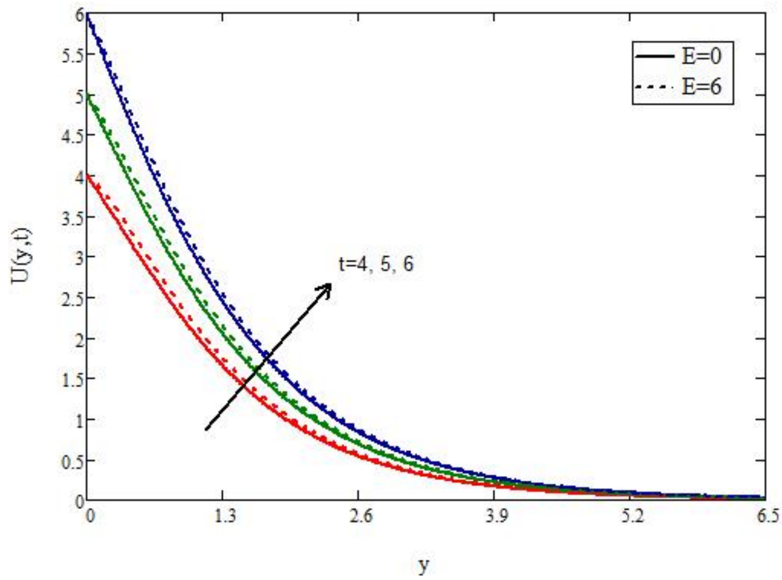

Fig. 8. Impact of t on velocity profile with and without presence of Riga Plate

From Figure 2, current results are compared with published results [34-36]. It is observed that obtained results are in agreement with published results. Thus, current results are valid. Throughout Figure 3-8, it is obvious that fluid velocity is enhanced with the increment of the modified Hartmann number, E. This proves that Riga plates actually assists fluid flow. The reason behind this is that the generated Lorentz force from the Riga plate acts along the Riga wall flowing with the direction of fluid, thus aiding fluid flow. This is further proven with the results from Table 1. Skin friction coefficients decreases within presence of Riga plate, showing that friction between plate and fluid is reduced significantly.

From Figure 3, it is observed that as the value of α increases, fluid velocity also increases. Due to the memory effect of fractional derivative, it can be seen that the fluid is in transitional state, slowly achieving steady state as fractional parameter increases.

Meanwhile, it is shown in Figure 4 that velocity of fluid decreases as Casson parameter, β, increases. β is defined in such a way that it is directly proportional to the fluid's dynamic viscosity. Dynamic viscosity of fluid is enhanced with an increase in Casson value, hence slowing down velocity of fluid.

Figure 5 on the other hand shows a higher fluid velocity due to an increase in Grashof number, Gr. Grashof number is defined as the ratio between buoyancy and viscous force acted on the fluid. Increasing the value of Gr would inadvertently boost buoyancy force on the fluid, thus increasing vertical movement of fluid. Since the fluid is moving along the y-axis, fluid velocity would in turn increase.

Concurrently, a decay in velocity of fluid can be seen with an inflation of M as observed from Figure 6. Induced magnetic force is applied perpendicular to the plate. A bigger magnetic force increases drag of fluid flow due to Lorentz force generated from the magnetic field, resulting in a slower fluid.

Observation from Figure 7 suggests that raising the Prandtl value, Pr, would result in the decline of fluid velocity. Pr is the quotient of momentum diffusivity as well as thermal diffusivity. Hence, the relationship between Pr and thermal diffusivity is described as inversely proportional. An increase in Pr would result in curbing thermal diffusivity, thus reducing kinetic energy stored in fluid. Therefore, a deterioration of fluid velocity is observed.

At the same time, Figure 8 displays behaviour of fluid with increment values of time, t. Changing values of t would result in a change of initial values for this boundary value problem. This is in agreement with the conditions of fluid flow set in Eq. (7). Thus, results obtained in this study are applicable.

Table 1
Skin Friction Analysis with and without Presence of Riga Plate

α	β	Gr	Pr	M	t	C_f, $E=0$	C_f, $E=6$
0.4	2.5	7	7	0.5	4	2.442	1.191
0.6	2.5	7	7	0.5	4	1.705	0.429
0.4	5	7	7	0.5	4	2.128	0.893
0.4	2.5	9	7	0.5	4	1.205	0.311
0.4	2.5	7	14	0.5	4	3.031	1.78
0.4	2.5	7	7	1.5	4	4.591	3.427
0.4	2.5	7	7	0.5	6	4.465	3.206

Skin friction is frictional shear force acting on the plate and is parallel to fluid flow. Since frictional force is considered as a negative direction force, a high value of frictional force would result in the shrinkage of fluid velocity. It is observed from Table 1, skin friction coefficient that is obtained from changes in parametric values are in agreement with behaviour of fluid seen from Figures 3-8 except for the change in β. When Casson parameter is increased, skin friction coefficient decreases. This uncanny behaviour might be the result of the rheological features of a non-Newtonian fluid such as Casson fluid. Conclusions from the aim of this study is presented in the next section, the Conclusions section.

5. Conclusions

Effects for presence of vertical Riga plate under an MHD Casson fluid flow with integrated fractional Caputo derivative is investigated. From the study it can be concluded that

i. Fluid velocity increases with presence of Riga plate.
ii. An increase in α and Gr increases velocity of fluid while increasing β, $\Pr$ and M will decrease it.
iii. Skin friction coefficient tends to decrease with presence of Riga plate.

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References

[1] Khashi'ie, Najiyah Safwa, Norihan Md Arifin, and Ioan Pop. "Mixed convective stagnation point flow towards a vertical Riga plate in hybrid Cu-Al2O3/water nanofluid." *Mathematics* 8, no. 6 (2020): 912. https://doi.org/10.3390/math8060912
[2] Ganesh, N. Vishnu, Qasem M. Al-Mdallal, Sara Al Fahel, and Shymaa Dadoa. "Riga–Plate flow of γ Al2O3-water/ethylene glycol with effective Prandtl number impacts." *Heliyon* 5, no. 5 (2019): e01651. https://doi.org/10.1016/j.heliyon.2019.e01651
[3] Bilal, S., Kanayo K. Asogwa, Hammad Alotaibi, M. Y. Malik, and Ilyas Khan. "Analytical treatment of radiative Casson fluid over an isothermal inclined Riga surface with aspects of chemically reactive species." *Alexandria Engineering Journal* 60, no. 5 (2021): 4243-4253. https://doi.org/10.1016/j.aej.2021.03.015
[4] Shah, Faisal, M. Ijaz Khan, Tasawar Hayat, Shaher Momani, and M. Imran Khan. "Cattaneo-Christov heat flux (CC model) in mixed convective stagnation point flow towards a Riga plate." *Computer Methods and Programs in Biomedicine* 196 (2020): 105564. https://doi.org/10.1016/j.cmpb.2020.105564
[5] Tassaddiq, Asifa. "Impact of Cattaneo-Christov heat flux model on MHD hybrid nano-micropolar fluid flow and heat transfer with viscous and joule dissipation effects." *Scientific Reports* 11, no. 1 (2021): 1-14. https://doi.org/10.1038/s41598-020-77419-x
[6] Abbasi, F. M., M. Mustafa, S. A. Shehzad, M. S. Alhuthali, and T. Hayat. "Analytical study of Cattaneo–Christov heat flux model for a boundary layer flow of Oldroyd-B fluid." *Chinese physics B* 25, no. 1 (2015): 014701. https://doi.org/10.1088/1674-1056/25/1/014701
[7] Gireesha, B. J., B. M. Shankaralingappa, B. C. Prasannakumar, and B. Nagaraja. "MHD flow and melting heat transfer of dusty Casson fluid over a stretching sheet with Cattaneo–Christov heat flux model." *International Journal of Ambient Energy* (2020): 1-9. https://doi.org/10.1080/01430750.2020.1785938
[8] Hayat, Tasawar, Faisal Shah, and Ahmed Alseadi. "Cattaneo-Christov double diffusions and entropy generation in MHD second grade nanofluid flow by a Riga wall." *International Communications in Heat and Mass Transfer* 119 (2020): 104824. https://doi.org/10.1016/j.icheatmasstransfer.2020.104824
[9] Rizwana, Rizwana, and S. Nadeem. "Series solution of unsteady MHD oblique stagnation point flow of copper-water nanofluid flow towards Riga plate." *Heliyon* 6, no. 10 (2020): e04689. https://doi.org/10.1016/j.heliyon.2020.e04689
[10] Loganathan, P., and K. Deepa. "Stratified Casson Fluid Flow Past a Riga-plate with Generative/Destructive Heat Energy." *International Journal of Applied and Computational Mathematics* 6, no. 4 (2020): 1-20. https://doi.org/10.1007/s40819-020-00863-w
[11] Nasrin, Sonia, Rabindra Nath Mondal, and Md Mahmud Alam. "Impulsively Started Horizontal Riga Plate Embedded in Unsteady Casson Fluid Flow with Rotation." *Journal of Applied Mathematics and Physics* 8, no. 9 (2020): 1861-1876. https://doi.org/10.4236/jamp.2020.89140
[12] Mallawi, F. O. M., M. Bhuvaneswari, S. Sivasankaran, and S. Eswaramoorthi. "Impact of double-stratification on convective flow of a non-Newtonian liquid in a Riga plate with Cattaneo-Christov double-flux and thermal radiation." *Ain Shams Engineering Journal* 12, no. 1 (2021): 969-981. https://doi.org/10.1016/j.asej.2020.04.010
[13] Bhatti, M. M., and Efstathios E. Michaelides. "Study of Arrhenius activation energy on the thermo-bioconvection nanofluid flow over a Riga plate." *Journal of Thermal Analysis and Calorimetry* 143, no. 3 (2021): 2029-2038. https://doi.org/10.1007/s10973-020-09492-3

[14] Khatun, Sheela, Muhammad Minarul Islam, Md Mollah, Saykat Poddar, and Md Alam. "EMHD radiating fluid flow along a vertical Riga plate with suction in a rotating system." *SN Applied Sciences* 3, no. 4 (2021): 1-14. https://doi.org/10.1007/s42452-021-04444-4
[15] Raza, Nauman. "Unsteady rotational flow of a second grade fluid with non-integer Caputo time fractional derivative." *Punjab University Journal of Mathematics* 49, no. 3 (2020): 15-25.
[16] Raza, Nauman, Aziz Ullah Awan, Ehsan Ul Haque, Muhammad Abdullah, and Muhammad Mehdi Rashidi. "Unsteady flow of a Burgers' fluid with Caputo fractional derivatives: A hybrid technique." *Ain Shams Engineering Journal* 10, no. 2 (2019): 319-325. https://doi.org/10.1016/j.asej.2018.01.006
[17] Raza, Nauman, and Muhammad Asad Ullah. "A comparative study of heat transfer analysis of fractional Maxwell fluid by using Caputo and Caputo–Fabrizio derivatives." *Canadian Journal of Physics* 98, no. 1 (2020): 89-101. https://doi.org/10.1139/cjp-2018-0602
[18] Anwar, Muhammad Shoaib, and Amer Rasheed. "Simulations of a fractional rate type nanofluid flow with non-integer Caputo time derivatives." *Computers & Mathematics with Applications* 74, no. 10 (2017): 2485-2502. https://doi.org/10.1016/j.camwa.2017.07.041
[19] Imran, M. A., I. Khan, M. Ahmad, N. A. Shah, and M. Nazar. "Heat and mass transport of differential type fluid with non-integer order time-fractional Caputo derivatives." *Journal of Molecular Liquids* 229 (2017): 67-75. https://doi.org/10.1016/j.molliq.2016.11.095
[20] Abdullah, M., Asma Rashid Butt, Nauman Raza, Ali Saleh Alshomrani, and A. K. Alzahrani. "Analysis of blood flow with nanoparticles induced by uniform magnetic field through a circular cylinder with fractional Caputo derivatives." *Journal of Magnetism and Magnetic Materials* 446 (2018): 28-36. https://doi.org/10.1016/j.jmmm.2017.08.074
[21] Aman, Sidra, Ilyas Khan, Zulkhibri Ismail, Mohd Zuki Salleh, and I. Tlili. "A new Caputo time fractional model for heat transfer enhancement of water based graphene nanofluid: An application to solar energy." *Results in physics* 9 (2018): 1352-1362. https://doi.org/10.1016/j.rinp.2018.04.007
[22] Khan, Ilyas, Nehad Ali Shah, Niat Nigar, and Yasir Mahsud. "MHD mixed convection Poiseuille flow in a porous medium: New trends of Caputo time fractional derivatives in heat transfer problems⋆." *The European Physical Journal Plus* 133, no. 8 (2018): 1-14. https://doi.org/10.1140/epjp/i2018-12105-0
[23] Tassaddiq, Asifa, Ilyas Khan, Kottakkaran Sooppy Nisar, and Jagdev Singh. "MHD flow of a generalized Casson fluid with Newtonian heating: A fractional model with Mittag–Leffler memory." *Alexandria Engineering Journal* 59, no. 5 (2020): 3049-3059. https://doi.org/10.1016/j.aej.2020.05.033
[24] Shah, Nehad Ali, Thanaa Elnaqeeb, I. L. Animasaun, and Yasir Mahsud. "Insight into the natural convection flow through a vertical cylinder using caputo time-fractional derivatives." *International Journal of Applied and Computational Mathematics* 4, no. 3 (2018): 1-18. https://doi.org/10.1007/s40819-018-0512-z
[25] Shah, Nehad Ali, Ilyas Khan, Maryam Aleem, and M. A. Imran. "Influence of magnetic field on double convection problem of fractional viscous fluid over an exponentially moving vertical plate: New trends of Caputo time-fractional derivative model." *Advances in Mechanical Engineering* 11, no. 7 (2019): 1-11. https://doi.org/10.1177/1687814019860384
[26] Imran, M. A., Nehad Ali Shah, Ilyas Khan, and Maryam Aleem. "Applications of non-integer Caputo time fractional derivatives to natural convection flow subject to arbitrary velocity and Newtonian heating." *Neural Computing and Applications* 30, no. 5 (2018): 1589-1599. https://doi.org/10.1007/s00521-016-2741-6
[27] Sarwar, Shakila, Mudassar Nazar, and M. A. Imran. "Influence of slip over an exponentially moving vertical plate with Caputo-time fractional derivative." *Journal of Thermal Analysis and Calorimetry* 145, no. 5 (2021): 2707-2717. https://doi.org/10.1007/s10973-020-09700-0
[28] Saqib, Muhammad, Ilyas Khan, Yu-Ming Chu, Ahmad Qushairi, Sharidan Shafie, and Kottakkaran Sooppy Nisar. "Multiple fractional solutions for magnetic bio-nanofluid using Oldroyd-B model in a porous medium with ramped wall heating and variable velocity." *Applied Sciences* 10, no. 11 (2020): 3886. https://doi.org/10.3390/app10113886
[29] Hassanzadeh, Hassan, and Mehran Pooladi-Darvish. "Comparison of different numerical Laplace inversion methods for engineering applications." *Applied mathematics and computation* 189, no. 2 (2007): 1966-1981. https://doi.org/10.1016/j.amc.2006.12.072
[30] Loganathan, Parasuraman, and Krishnamurthy Deepa. "Electromagnetic and radiative Casson fluid flow over a permeable vertical Riga-plate." *Journal of Theoretical and Applied Mechanics* 57, no. 4 (2019): 987–998. https://doi.org/10.15632/jtam-pl/112421
[31] Yusof, Nur Syamila, Siti Khuzaimah Soid, Mohd Rijal Illias, Ahmad Sukri Abd Aziz, and Nor Ain Azeany Mohd Nasir. "Radiative Boundary Layer Flow of Casson Fluid Over an Exponentially Permeable Slippery Riga Plate with Viscous Dissipation." *Journal of Advanced Research in Applied Sciences and Engineering Technology* 21, no. 1 (2020): 41-51. https://doi.org/10.37934/araset.21.1.4151

[32] Shakeel, Abdul, Sohail Ahmad, Hamid Khan, and Dumitru Vieru. "Solutions with Wright functions for time fractional convection flow near a heated vertical plate." *Advances in Difference Equations* 2016, no. 1 (2016): 1-11. https://doi.org/10.1186/s13662-016-0775-9

[33] Vieru, Dumitru, Constantin Fetecau, and Corina Fetecau. "Time-fractional free convection flow near a vertical plate with Newtonian heating and mass diffusion." *Thermal science* 19, no. suppl. 1 (2015): 85-98. https://doi.org/10.2298/TSCI15S1S85V

[34] Ali, Farhad, Nadeem Ahmad Sheikh, Ilyas Khan, and Muhammad Saqib. "Solutions with Wright function for time fractional free convection flow of Casson fluid." *Arabian Journal for Science and Engineering* 42, no. 6 (2017): 2565-2572. https://doi.org/10.1007/s13369-017-2521-3

[35] Khan, Ilyas, Nehad Ali Shah, and Dumitru Vieru. "Unsteady flow of generalized Casson fluid with fractional derivative due to an infinite plate." *The European physical journal plus* 131, no. 6 (2016): 1-12. https://doi.org/10.1140/epjp/i2016-16181-8

[36] Saqib, Muhammad, Ilyas Khan, and Sharidan Shafie. "Application of fractional differential equations to heat transfer in hybrid nanofluid: modeling and solution via integral transforms." *Advances in Difference Equations* 2019, no. 1 (2019): 1-18. https://doi.org/10.1186/s13662-019-1988-5