

The Evolution of Induced Drag of Multi-Winglets for Aerodynamic Performance of NACA23015

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ARTICLE INFO	ABSTRACT
Article history: Received 28 October 2021 Received in revised form 11 February 2022 Accepted 17 February 2022 Available online 23 March 2022	Eagle is one of the most manoeuvrable and aerodynamically efficient bird capable of soaring for a mile, and it has high gliding ratio that can reach high velocities. Unmanned Aerial Vehicles (<i>UAVs</i>) used in military and civilian applications are required to loiter at significant altitude without being targeted by observers. However, the induced drag is usually held mainly at the wingtip, which affects the performance of the <i>UAVs</i> in steady state condition due to wingtip vortex. Therefore, the objectives of this paper are to study the effect of multi-winglet on different configurations in the performance of lift and drag coefficients and to analyse the flow pattern of multi-winglet with difference configurations. The wing airfoil used was <i>NACA</i> 23015 with chord length (<i>c</i>) of 216.5 <i>mm</i> and wingspan (<i>L</i>) of 866 <i>mm</i> . The multi-winglet device was simulated using <i>ABAQUS/CAE</i> software with three, five and seven multi-winglet configurations at angles of attack between -5° to 20° (with increment of 5°) and at flying speed of 30 <i>m/s</i> (<i>Re</i> = 4 <i>x</i> 10 ⁵). This study found that seven multi-winglets demonstrated better results in lift and drag coefficients can improve the aerodynamic performance of airfoil in reducing the induced drag and increasing the lift coefficient, which is suitable to be implemented at low angles of attack due to the bluffing body of winglet at high angles of attack.

1. Introduction

Man has been engrossed in flight for a long time. The desire to fly has been tested since olden times, and then applied to fundamental science projects [1-3]. The dream of flight never halted there, it proceeded with light flyable machines that were less dense than air and heavier than air, and then used in World War I and II [4,5]. Advancement of unpiloted machines are similar to those made with human command, and military clashes have proven to be progressively productive [6]. Mechanical and structural advancements have impacted the improvement of unpiloted machines, by means of

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achieving unpredictable and complex designs throughout the years [7]. Most of the flying animals flap their wings to generate lift and thrust force. However, their wings actually only produce a lift and not a thrust force when they stop flapping and keep their wings stretched. Thrust can be created by the gravity force on the descending creature. This is known as gliders [6]. Many birds glide like they are hovering in the air with a little jerk of the wing lifting in height. There is winged wildlife such as vultures, albatrosses, pelicans and storks with high ratio of lift to drag.

Winglets is like a small wing at the end of the wing. The justification for winglet development involves an improvement in the aerodynamic efficiency of the wing. Research in this area has shown that winglets have good performance. In the mid-1970s, Richard Whitcomb published research on the invention of winglet in the aeronautics industry [8]. The research focused on high supersonic velocity application for huge aircraft such as Boeing 737. The use of winglets on full-size aircrafts has enhanced the aircraft performance by more than 7% [8]. Without more effective wingspan, the winglets provide more prominent lift force to the flying mechanism [7]. For massive aircraft such as Boeing 747, the lift may be extended while retaining drastic fixed lifting range required at certain global air terminals. Most UAVs operate in much lower Reynolds numbers, and UAVs appear to be limited in wingspan. In both cases, the UAVs will benefit from the winglets. A slight improvement in efficiency would take into consideration more noteworthy in-flight time, higher payload, or an expanded range [1].

The research inspired the development of a productive winglet plan technique and improvement to the UAVs stages. The winglet device can be classified into three categories, namely active, adaptive and passive winglet. Active winglet is the greatest modifications in aerodynamic features, which should enhance maneuverability. The most widely studied technique for active systems is wingtip blowing, in which the jets in the wing plane and the span direction are exhausted [9]. There has been a novel form of blowing device which implemented the concept of *Coanda* effect due to the blowing jets in the unique geometry of the wingtip, whose jets are tangent to the tip tank and the incoming flow deviates against the vortex and requires lower mass flow rates [10].

Moreover, the notion of an adaptive winglet morphing aircraft is one of the potential possibilities to change aircraft design and operation [11]. Unfortunately, the implementation of morphing winglet affects the additional weight of aircraft. Although weight reduction is driven in the aviation industry, the ultimate choice should be made in relation to flying performance, for example, the fuel burned by a passenger aircraft. Eguea *et al.*, [12] revealed that the camber winglet may minimize time to climb so that the airplanes would reach cruise conditions faster. It minimizes fuel usage by decreasing the amount of time available for aircraft operation. The camber wing lowered fuel consumption during cruise condition, leading to a total saving of 300 kg of fuel for maximum range missions. A considerable amount of flight time and fuel was involved in the cruising condition. Consequently, the cruise condition performance was greatly improved in the mission phase of aircraft.

Meanwhile, passive winglet or more commonly known as fixed winglet is a conventional winglet attached at the end of the wing tip to enhance aerodynamic performance and reduce the influence of wingtip vortex resulting in induced drag reduction. Whitcomb [13] seriously considered a winglet for large and heavy aircraft. Since Whitcomb's breakthrough on wings, several variants were created, and all of them were passive or fixed to the wingspan. The indicator to passive winglet is its cant angle; the inclination or declination angle from the wing axis, which is fixed to one position only.

Investigations on the winglet device have been intensively conducted to reduce induced drag in aircraft application. Cosin *et al.*, [7] attempted to reduce the effect of induced drag on low-speed aircraft by applying a multi-winglet. The experimental study was conducted at a wind tunnel facility with six tip-sail device configurations for half wing-body at $Re = 4 \times 10^5$. The result indicated that the device led to a 32% improvement in the Oswald efficiency factor, which represented a maximum

aerodynamic performance increase of 7%. Moreover, Céron-Muñoz *et al.*, [14-16] conducted three studied related to tip-sail device aimed to study the potential of multi-winglet use to reduce induced drag through variation of winglet cant angles. Subsequently, study on tip-sail device was also conducted by Catalano and Céron-Muñoz [17] to investigate the potential of wing tip-sail device in reducing induced drag with various cant angles and winglet incidence angles. The model tested was a low-aspect ratio rectangular wing of NACA 653-018 airfoil at Re = 650,000 the wind tunnel facility. The parameters of interest were lift, drag and mapping of wake through hot-wire anemometry. The results showed that the adaptive multi-winglet system for maneuver operation may result in an improvement of the overall flight envelope from the climb to the maximum range.

In addition, wingtip-sail device is commonly implemented on agricultural aircraft to reduce offtarget potential in pesticide activity due to the influence of wingtip vortex. In a study, Santos *et al.*, [18] implemented tip-sails device to study aerodynamic performance in producing a new concept of agricultural aircraft capable of reducing the effect of induced drag. A numerical study was also conducted using ANSYS CFX 14.5 software with an experimental approach at the wind tunnel. A test flight was conducted to verify the potential of the new concept of agricultural aircraft with wingtipsail application. A dramatic increase in the lift-to-drag ratio in the study suggested a decrease in fuel and time consumption, retention of a nearly identical concept of aspect ratio, and minimization of induced drag. This indicated an improved agricultural aircraft to retain the basic design of wing load and provide excellent maneuverability without any major structural problems for high aerodynamic efficiency.

The limitation of previous studies related to the winglet device especially on the tip-sail device is that there were no studies revealed the characteristics of lift and drag, and flow pattern of the wingtip-sail device with more than three tip-sail configurations. Many researchers have conducted studies to reduce the effect of induced drag on aircraft and UAV applications. However, most previous studies have focused more on wingtip-sail device with three configurations to examine the winglet characteristics. This reveals the limitation to understand the actual characteristics of the tipsails device inspired by the avian creature with high maneuverability. Therefore, this paper aims to study the effect of multi-winglet on difference configurations in the performance of lift and drag coefficients, and to analyse the flow patterns of multi-winglet with different configurations. Therefore, the contribution of this study is apparent as the results can be used to conduct an experimental study or to manufacture a multi-winglet for UAVs application as a reference to other researchers.

2. Methodology

2.1 Overview of Multi-Winglet

The diversity of the multi-winglet was inspired by the wing structure of the Eagle which was selected based on its long-term flight characteristics to glide without compromising flight efficiency. The selection of wing type also plays an important role in ensuring that the simulation results could as closely as possibly represent the actual scenario. The airfoil concept was derived based on the NACA 23015 as its design best suited UAVs or MAVs. The drawing reference considered all relevant details along with the NACA proportions.

Appropriate form for the winglet design can be determined from the corresponding journals and previous studies. Due to its efficiency in lift-to-drag coefficient and aerodynamic performance, Bald Eagle was selected as a reference for this stimulation study. Researchers have divided the Eagle wing feathers into three classes as shown in Figure 1. To enable an Eagle to fly, each part has different position to play. "Green Feathers" coverts make the wing thicker at the front, so that air will flow

faster at the top of the wing. "Grey Feathers" or primary feathers can be spread out like fingers on the hand to reduce drag. "Blue Feathers" or secondary feathers can be moved down to increase drag, or up to reduce it. The multi-winglet design in this study was conducted based on the "Grey Feathers".



Fig. 1. Different Parts of a Bald Eagle Wings [19]

2.2 Multi-Winglet Considerations

The most convincing concept for applying the multi-winglets lies at the wingtips of UAVs or MAV, as referred to the literature analysis performed. However, the simulation did not imitate the Eagle wing configuration but attempted to use the advantage of cant angle exhibited by this bird species. To achieve a clear contrast between performance, the multi-winglet configuration including the number of multi-wings (n) and the cant angle (θ) between each winglet should be varied. Figure 2 shows the use of cant angle in this study.



2.3 Multi-Winglet Designing

The multi-winglet modelling process was performed using the ABAQUS/CAE program. The concept of multi-winglet design referred to previous findings on simulation analysis of aerodynamic performance on multi-winglet. This helped in comparing the data obtained from the preceding and recent simulation studies.

Table 1 illustrates different multi-winglet configurations conducted in this study. Table 2 shows all the details of the simulation model, while Table 3 shows the differences of each configuration in terms of cant angles.

Multi-winglet configuration			
Configuration	Model	Abaqus Model	
Baseline			
$n = 3$ $a = 15^{\circ}$	15°		
n = 5 a = 7.5°	7.5°		
n = 7 $a = 5^{\circ}$	5°		

Table 1 Multi-winglet configuration

Table 2

Wing shape parameter and variable of wingtip	
Part of model design	Dimension
Chord Length of the Wingtip, (mm)	216.5
Wingspan of the Wingtip, (mm)	866
Sweep Angle, °	0
Angle of attack of the Wingtip, °	0
Chord Length of the Winglet Root, (mm)	30
Chord Length of the Winglet Tip, (mm)	13.5
Wingspan of the Winglet, (mm)	81

Table 3 Multiple winglet model and cant angle configuration			
Configuration	Label	Winglet model	
Baseline	Model Zero (M0)	Airfoil wingtip without multi-winglets	
n = 3 α = 15°	Model One (M1)	Airfoil wingtip with multi-winglets	
<i>n</i> = 5 α = 7.5°	Model Two (M2)		
n = 7 α = 5°	Model Three (M3)		

2.3 Multi-Winglet Testing

Experimental works were conducted to evaluate aerodynamic performance of multi-winglet wing models. The aerodynamic elements of interest in this field of work were the lift coefficient (C_L) and drag coefficient (C_D), which are dimensionless parameters in measuring aerodynamic performances. To measure C_L and C_D , other parameter needs to be considered in this simulation study. The C_L and

 C_D can be expressed, particularly when considering wings in different geometries, in Eq. (1) and Eq. (2).

$$C_L = \frac{2F_L}{\rho V^2 A} \tag{1}$$

$$C_D = \frac{2F_D}{\rho V^2 A} \tag{2}$$

The parameters influencing the simulation result were velocity of air flow (V), density of air (ρ), area of wingspan (A) used in the simulation study. Meanwhile, lift force (F_L) and drag force (F_D) were obtained while conducting the simulation using ABAQUS/CAE software integrated with Computational Fluid Dynamic (CFD). Table 4 shows the boundary condition of the air flow used in the simulation.

Table 4

Boundary Condition	s of air flow			
Simulation Velocity,	Surrounding	Dynamic Viscosity, μ	Kinetic Viscosity, v	Density of air, $ ho$
V (m/s)	Temperature, T (K)	(kg/ms)	(m²/s)	(kg/m³)
30	293.2	1.825 x 10 ⁻⁵	1.562 x 10 ⁻⁵	1.225

Using the K-epsilon turbulence model, the dissipation rate and turbulence kinetic energy were determined using a turbulence calculator. The turbulence kinetic energy and dissipation rate generated by the ABAQUS/CAE software were 337.5 J/Kg and $22863.6 m^2/s^3$, respectively. Figure 3 shows the schematic diagram of the boundary condition for this simulation study using ABAQUS/CAE software.



Fig. 3. Schematic diagram of boundary condition in ABAQUS/CAE

2.4 Simulation Run

The use of ABAQUS/CAE only allowed the simulation to focus on Fluid family analysis. As a result of grid-independent testing, the optimal grid for each winglet configuration reached about 300,000 tetrahedrons mesh elements. The symmetrical wall and side wall were designated as the symmetric boundary condition and the slip surface boundary condition, respectively. The symmetric boundary condition refers to the symmetric condition as reported in previous research conducted to study the flow pattern visualization on the research model [21].

3. Results and Discussion

3.1 Lift and Drag Coefficients Influence

From the graph, the relationship between lift coefficient and angle of attack for each configuration can be determined as shown in Figure 4. Based on the graph obtained, it can be seen that the number of winglets affected the lift coefficient on the simulation model. At angles of attack starting from -5° to 10°, the lift coefficients improved as the number of winglets increased. Specifically, at angle of attack of 10°, the lift coefficient increased from the baseline value of 1.0968 to the highest lift coefficient of 1.2 on 7 winglet configurations, indicating an increase in lift performance by 9.4%.



Lift Coefficient at Re = 40 000 (30 m/s)

3.2 Induced Drag and Flow Visualization

The results obtained from the simulation demonstrated that there was a direct correlation between the induced drag and the number of winglets as shown in Table 5.

Table 5				
Induced drag of different models at V = 30 m/s				
Angle of	Induced Dr	ag		
Attack (°)	M0	M1	M2	M3
-5	-0.0828	-0.0799	-0.0764	-0.0732
0	0.5502	0.5456	0.5809	0.6253
5	1.0640	1.0391	1.0782	1.0603
10	1.3966	1.4602	1.4447	1.4133
15	1.2278	1.2038	1.2050	1.1370
20	1.1851	1.1210	1.0910	1.0591

In addition, the relationship on the drag coefficient and square of lift coefficient was also analyzed in the simulation study. Figure 5 shows the relationship between the induced drag and square of lift coefficient for all multi-winglet configurations.



Fig. 5. Induced drags of different models at V = 30m/s

From the graph above, it indicated that the induced drag decreased significantly as the number of winglets increased. The slope of the multi-winglet models became less steep as the angle of attack increased. The lowest induced drag at 7-winglet models with 20° angle of attack was 1.0591, and the baseline had a maximum value of 1.1851, which represented an increase by 10.63%. Despite the reduction in induced drag, the zero-lift drag increased due to the bluffing body configuration. This restricted the angle of attacks which can be used by the models for multi-winglet application. This phenomenon has also been discussed by Al-Atabi [22] in conducting an experimental study on different tip-sail device configurations. Based on the findings, when the lift coefficient was less than 0.4, all wing configurations with sails exhibited greater drag coefficients than wing without sails. On the other hand, the tip sails have lower drag values when the lift coefficient was greater than 0.4. These findings were anticipated given the plan area of the wing with sails was greater than the wing without sails. Greater plan area increased the skin friction drag, which was more pronounced at low lift coefficients, and thus explained the drag reduction at higher lift coefficients. In addition, the longitudinal static stability of the wing was also reported to have improved significantly after the addition of the tip-sail device.

Another previous study related to the differences in the configuration of the tip sail device has been reported by Narayan and John [23], who conducted a simulation study on two, three and four tip sail configurations of. The findings revealed that the addition of multi-tipped winglet improved the aerodynamic performance with an increase the number of tips. Although it can only be used for lower angles of attack, the models with multi-winglets were observed to increase significantly from the baseline. The improvement could be due to the size of the vortex formation. The 7-winglet model has smaller induced vortex formation compared to the baseline that has larger vortex formation at the angle of attack at 20° as shown in Table 6.



4. Conclusions

To learn about the influence of multi-winglets on UAVs flight performance, an analysis of various winglet configurations was conducted. Simulation was conducted using ABAQUS/CAE program with various angle of attacks from -5° to 20° with a constant speed of 30 m/s. A few conclusions can be drawn at the end of this study. The multi-winglet concept was inspired by the slotted wingtips of the Bald Eagle as they could hover at the same altitude without moving or flapping its wings. The numbers of winglet of 3, 5 and 7 were derived from multiple observations of the Bald Eagle. It was evidenced that the lift coefficient improved with increasing number of winglets but the airfoil stalled at a lower angle compared to the baseline model. This enhanced the drag coefficient due to the increase in the zero lift drag coefficient but reduced the induced drag caused by the vortex produced by multi-winglets. The flow pattern was also analyzed and identified. The size of the vortex decreased with the increase in winglets. However, the zero lift drag coefficient became worse at higher angle of attack as the bluff body of the multi-winglet models made it easily to stall at high angle of attack. Thus, multi-winglets were determined to reduce the induced drag and to increase the lift coefficient but can only be used at low angles of attack when the body geometry of the model is bluffed at high angle of attack.

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