

Squeezing MHD Flow of Sodium Alginate-Based Casson Hybrid Nanofluid with Soret and Dufour Effects

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ARTICLE INFO	ABSTRACT
Article history: Received 14 October 2023 Received in revised form 11 March 2024 Accepted 23 March 2024 Available online 15 April 2024	Ultrahigh-performance cooling is one of the essential requirements in the industrial technology. Hence, the new heat transfer fluid, hybrid nanofluid is introduced to increase the thermal conductivity of fluid and investigated with various physical parameters. The unsteady magnetohydrodynamics (MHD) flow of Casson hybrid nanofluid through two surfaces in a permeable medium with chemical reaction are explored. The hybrid nanoparticles of Alumina (Al_2O_3) and Copper (Cu) is dispersed in the base fluid of sodium alginate ($C_6H_9NaO_7$). The discretize equations are solved using similarity transformation and Keller-box methods. The comparison of the current results with the published results for validation is conducted and discovered in proper agreement. The impacts of squeeze, magnetic, porous media, chemical reaction, heat sink/source, and Soret and Dufour on behaviour and physical quantities of flow are discussed. The graphical results show the squeeze of two surfaces accelerates the fluid velocity near the upper plate region. Further, the velocity slowing down when β and Ha increases, and it elevates as Da and ϕ_2 rises in the middle of channel. The increment of heat transfer rate and temperature of fluid is shown for increasing Ec , γ and Du , and the opposite behaviour is discovered with raise in ϕ_2 . The fluid concentration decreases and the mass transfer rate enhances for rising Sr .
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1. Introduction

Compression and decompression of fluid within two plates is known as squeezing flow. Many scientists have keen interest on the field of squeeze flow due to the application in hydraulic lifts, lubrication system and injection moulding [1]. The research on the squeeze flow of viscous fluid over horizontal plates is pioneered by Stefan [2]. Then, Cameron [3] continued the study by considering infinite plates. A new similarity variable was introduced by Wang [4] for converting Navier-Stokes equations to ordinary differential equations on the studies of squeeze flow of viscous fluid. Next, the

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researches of Wang [4] was conducted by Bujurke *et al.,* [5], Rashidi *et al.,* [6] and Khan *et al.,* [7] analytically.

The significance of the researches on the non-Newtonian flow has been acknowledged by scientists due to its implementation on engineering industries. Casson fluid is known as shear thinning flow because of high yield stress and shear viscosity [8]. Commonly, the fluid acting as solid, but it starts to flow if the strong force is imposed on the fluid. The motion of pigment oil suspension is explored using Casson fluid model by Casson [9]. Later, Khan *et al.*, [10] introduced the research on time dependent Casson fluid flow by squeezing two surfaces. Ahmed *et al.*, [11] studied the effects of magnetic field on electrically conducted Casson fluid on the squeezing plates. The work from Khan *et al.*, [12] was extended by Ahmed *et al.*, [11] with addition of permeable medium. The joule dissipation impact on squeezing of Casson fluid flow is reviewed by Mohyud-Din *et al.*, [13].

The influences of magnetohydrodynamics (MHD) on the flow is discovered in various geometrical models of fluid. The magnetic field exerted on the electrical conduction fluid produces the Lorentz force. It is often arising in the medical and engineering usages, for instance, it functions as transporter for magnetic drug inside the body, cyclotrons and mass spectrometer [14]. The MHD flow by squeezing of two plates was examined by Siddiqui *et al.*, [15] and resolved via homotopy perturbation method (HPM). The similar problem is solved by Sweet *et al.*, [16] through homotopy analytical method (HAM) with variation of fluid density and magnetic field. The flow of squeeze Casson fluid under influence of MHD was investigated by Ahmed *et al.*, [10]. The new approximate analytical solution according to power series coefficients is used by Al-Saif and Jasim [17] to solve the similar problems. The flow on the porous medium is important in the such as thermal energy storage, nuclear reactors oil extraction, geothermal energy recovery and plasma studies [18]. The impacts of MHD on squeezing Casson flow across permeable medium was explored by Khan *et al.*, [12]. The slip condition on squeezing MHD of Casson flow over permeable medium was done by Qayyum *et al.*, [19]. Later, Sobamowo *et al.*, [20] discovered squeezing Casson nanofluid flow on permeable media under magnetic field impacts via Tiwari and Das model.

The application of heat transfer fluid in various thermal devices is acknowledged by engineers and researchers such as hydraulic analysis of microchannels and pool boiling heat transfer [21]. Conventional fluids have low capability of thermal transfer due to lack of thermal conductivity. Hence, an advanced heat transfer fluid called nanofluid was pioneered by Choi and Eastman [22] by disperse the nanoparticles in the base fluid. Later, the advanced in nanotechnology industry led to the innovation of thermal transfer fluid named hybrid nanofluid. The diffusion of two type of particles in the conventional fluid is discovered by many researchers because of its ability in enhancing the heat properties of nanofluid. The experiment on hybrid nanofluid were done by Turcu *et al.*, [23] and Jana *et al.*, [24] to observe its impact on the thermal conductivity. The production of hybrid nanofluid of Cu-Al₂O₃/H₂O by Suresh *et al.*, [25] is done by thermochemical technique.

Hybrid nanofluid is implemented in many heat transfer usages, for instances improving the coolant performance in vehicle engines, microchips and nuclear power system, and delivery of nanodrug for medical treatment [26,27]. Many scientists have done the researches on hybrid nanofluid flow using various geometric models. The MHD flow of Cu-Al₂O₃/H₂O hybrid nanofluid across a porous stretched plate with suction impact is explored by Devi and Devi [28]. High concentration of nanoparticle volume fraction raises the heat transfer in the flow. Upreti *et al.*, [29] studied the entropy generation of carbon nanotubes (CNTs) on hybrid nanofluid over rotating horizontal squeezed plates. The presence of joule dissipation, heat source/sink and radiative thermal flux were analysed. Waini *et al.*, [30] examined the MHD hybrid nanofluid flow past a porous sensor of squeeze surface. The mixed convective squeeze of hybrid nanofluid flow and non-linear radiative heat transfer past two disks was analysed by Nisar *et al.*, [31]. The magnetized ferroparticles is used in the base fluids of ethylene glycol and water. The MHD squeeze flow of hybrid nanoparticles of Fe_3O_4 and MoS_2 in the influence of heat sink/source is investigated by Salehi *et al.*, [32]. The conventional fluids of ethylene glycol and water are considered. Khashi'ie *et al.*, [33] explored the effects of suction/injection on MHD flow of hybrid Cu-Al₂O₃/H₂O nanofluid over the lower stretching squeezed surface. Wahid *et al.*, [34] investigated the impacts of radiative thermal transfer on MHD hybrid nanofluid flow across a porous vertical surface. The squeezing flow and radiative thermal transfer of copper and alumina hybrid nanoparticles on water-based fluid with joule dissipation and variable viscosity was reviewed by Famakinwa *et al.*, [35]. For Casson hybrid nanofluid, the magnetic flow at a stagnation points and thermal transfer across stretched plate was discovered by Alghamdi *et al.*, [36]. Kamis *et al.*, [37] examined the thermal transfer of the fluid across porous stretched surface with suction impact. Then, Jyothi *et al.*, [38] investigated the squeeze flow of Casson hybrid nanofluid with heat sink/source and thermophoresis. Noor *et al.*, [39] discovered the influences of chemical reaction and heat sink/source on squeeze MHD flow of sodium alginate-based Jeffrey hybrid nanofluid in porous medium with thermal radiation.

The studies of the double diffusion or thermal and mass transfer through fluid flow are important in many practical applications such as in the field of air pollution, disposal of nuclear waste and petroleum reservoirs [40]. It is discovered that the process of double diffusion is more complex caused by the driving potentials of thermal and mass fluxes occurs simultaneously. Soret effect or thermo-diffusion is a heat flux produced due to temperature gradient. Besides, Dufour effect or diffusion-thermo is a mass flux produced due to concentration gradient. Dufour and Soret terms are considered in the energy and concentration equations, respectively. In general, the impacts of Dufour and Soret are not measured because the order of magnitude is not significant based on Fourier and Fick's laws. However, the effects are discovered when the density of particles at fluid surface is lower than the ambient fluid [41]. The researches on the influences of Soret and Dufour on hybrid nanofluid flow are explored in various geometry. For Casson nanofluid, Rafigue et al., [42] explored the numerical solution of MHD flow across a non-linear inclined plate with Soret and Dufour, radiation, Brownian, thermophoresis, and heat sink/source impacts. Bidemi and Ahamed [43] examined the effects of radiation, heat sink/source, and Dufour and Soret on unsteady magnetic flow through an inclined plate through porous medium using Buongiorno model. Sekhar et al., [44] discovered the MHD flow on inclined stretched surface in the presence of Brownian, radiation, heat sink/source, Soret and Dufour, and chemical reaction. Noor et al., [45] discussed the MHD squeeze Casson nanofluid flow for the presence of chemical reaction, porous medium, thermal radiation, and joule dissipation. Later, the prior study was extended by Noor et al., [46] by examining the heat sink and source in the Jeffrey flow. The prior work is continued by Noor et al., [47] for the study of thermal radiation and Dufour and Soret impacts on Jeffrey flow on squeezed plates. For Casson hybrid nanofluid, Hafeez et al., [48] studied the MHD flow through porous medium over a vertical melting surface with Dufour and Soret, Joule heating and viscous dissipation effects. Further, Reddy et al., [49] reported the influences of Soret and Dufour, thermal radiation and magnetic field on hybrid Al₂O₃-Cu based Ethylene glycol nanofluid across a moving thin needle. Deepika et al., [50] analysed the MHD mixed convection flow of Casson hybrid nanofluid on a porous stretching plate with Soret and Dufour, and heat source/sink impacts.

Based on literature review, several studies on the squeeze of nanofluid flow over two surfaces are discovered. The review highlights that limited study are conducted on the squeezing flow for Casson hybrid nanofluid. The thermal and mass transfer of squeezing flow of Casson hybrid nanofluid with the effects of magnetic, porosity, heat sink/source, Soret and Dufour are not discovered yet. Hence, the unsteady magnetohydrodynamics flow of Casson hybrid nanofluid through two squeezed surfaces in permeable medium is investigated in this study. The heat and mass transfer analysis with

the influences of heat sink/source, chemical reaction and Soret and Dufour are observed. Copper and alumina are taken as hybrid nanoparticles with sodium alginate as the conventional fluid.

The potential application of this study is implemented for the modelling of fluid with hybrid nanoparticles as a coolant in the radiator. The large radiator is used to maximize the cooling effects on the engine, which cause high energy is required to maintain the system. Hence, the implementation of hybrid nanofluid is potentially useful in the modelling of small size of radiator and it has presented a good outcome in energy saving and emission reduction. Furthermore, the influence of magnetic field is examined in the flow. It is discovered that the conversion of kinetic energy of particles into a voltage enhances by applying magnetic field and plasma conductivity. The presence of chemical reaction in the mathematical model is to investigate the coolant in a nuclear reactor. It is used to remove heat from the nuclear reactor engine and transfer the heat to electrical generators and the environment.

The present study explores the following research questions

- i. How do the mathematical models for MHD Casson hybrid nanofluid in the problem of unsteady squeezing flow through a porous medium can be formulated?
- ii. How does the combined effects of heat sink/source, chemical reaction, and Soret and Dufour will affect the heat and mass transfer characteristics of the fluid flow?
- iii. How to develop a programming code in MATLAB software to find the numerical solutions of the problems?

2. Mathematical Formulation

Consider the unsteady MHD flow of Casson hybrid nanofluid in permeable medium with chemical reaction, Soret and Dufour, and heat sink/source. The geometrical model presents the squeeze flow of Casson hybrid nanofluid across two surfaces. As illustrated in Figure 1, the length between two surfaces is $y = \pm h(t) = \pm l(1 - \alpha t)^{\frac{1}{2}}$. The external force with velocity, $v_w(t) = \frac{\partial h(t)}{\partial t}$ is exerted on the both surfaces. It moves further when $\alpha < 0$ and moves closer when $\alpha > 0$ until $t = 1/\alpha$. The magnetic field is induced vertically at the lower surface, $B(t) = B_0(1 - \alpha t)^{-1/2}$ [51]. The mixture of two type of hybrid nanoparticles, which are alumina and copper in the conventional fluid of Casson fluid are analysed.



Table 1 [52] and Table 2 [53] demonstrate the correlations on thermophysical properties and values for hybrid nanofluid.

Table 1

 $\frac{\text{Correlations on thermophysical properties of hybrid nanofluid}}{Properties} \qquad Hybrid nanofluid}$ $\frac{Properties}{Properties} \qquad Hybrid nanofluid$ $\frac{Properties}{Properties} \qquad Hybrid nanofluid \qquad Hobrid nanofluid \qquad Hybrid nanofluid \qquad Hobrid nanofluid \qquad Hybrid nanofluid \qquad Hybri$

Table 2 Thermophysical values for the properties of nanoparticles and base fluid

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Properties	Си	Al_2O_3	Sodium alginate $(C_6H_9NaO_7)$
ρ	8933	3970	989
C_p	385	765	4175
σ	5.96×10^{7}	3.69×10^{7}	0.07
k	400	40	0.6367
Pr	_	_	6.5

The continuity, momentum, energy and concentration equations of Casson hybrid nanofluid based on the fluid flow at boundary region are [54].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,\tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{hnf}}{\rho_{hnf}} \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{hnf} B(t)^2}{\rho_{hnf}} u - \frac{\mu_{hnf}}{\rho_{hnf}} \left(1 + \frac{1}{\beta} \right) \frac{\varphi}{k_1(t)} u, \tag{2}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{hnf}}{\left(\rho C_p\right)_{hnf}} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_{hnf}}{\left(\rho C_p\right)_{hnf}} \left(1 + \frac{1}{\beta}\right) \left[4 \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right] + \frac{Q(t)}{\left(\rho C_p\right)_{hnf}} T + \frac{D_m k_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2},$$
(3)

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y^2} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2} - k_c(t)C.$$
(4)

The correlated boundary conditions are [54]

$$u = 0, v = v_w = \frac{\partial h(t)}{\partial t}, T = T_w, C = C_w, \text{ at } y = h(t),$$
(5)

$$\frac{\partial u}{\partial y} = 0, v = 0, \frac{\partial T}{\partial y} = 0, \frac{\partial C}{\partial y} = 0, \text{ at } y = 0.$$
 (6)

The discretization of ordinary differential equations (ODEs) from partial differential equations (PDEs) are done by applying similarity variables as follows [55]

$$u = \frac{\alpha x}{2(1-\alpha t)} f'(\eta), \quad v = -\frac{\alpha l}{2\sqrt{(1-\alpha t)}} f(\eta), \quad \eta = \frac{y}{l\sqrt{(1-\alpha t)}}, \quad \theta = \frac{T}{T_w}, \quad \phi = \frac{C}{C_w}, \tag{7}$$

Substitute the similarity variables Eq. (7) into equations Eq. (2), Eq. (4) and Eq. (9), the nondimensional ODE forms are

$$\frac{\mu_{hnf}}{\mu_{f}} \frac{\rho_{f}}{\rho_{hnf}} \left(1 + \frac{1}{\beta}\right) f^{iv} - S(\eta f^{\prime\prime\prime} + 3f^{\prime\prime} + f^{\prime} f^{\prime\prime} - ff^{\prime\prime\prime}) - \frac{\sigma_{hnf}}{\sigma_{f}} \frac{\rho_{f}}{\rho_{hnf}} Ha^{2} f^{\prime\prime} - \frac{\mu_{hnf}}{\mu_{f}} \frac{\rho_{f}}{\rho_{hnf}} \left(1 + \frac{1}{\beta}\right) \frac{1}{Da} f^{\prime\prime} = 0,$$
(8)

$$\frac{\left(\rho C_{p}\right)_{f}}{\left(\rho C_{p}\right)_{hnf}}\frac{k_{hnf}}{k_{f}}\frac{1}{Pr}\theta^{\prime\prime} + S(f\theta^{\prime} - \eta\theta^{\prime}) + \frac{\left(\rho C_{p}\right)_{f}}{\left(\rho C_{p}\right)_{hnf}}\gamma\theta$$

$$+ \frac{\mu_{hnf}}{\mu_{f}}\frac{\left(\rho C_{p}\right)_{f}}{\left(\rho C_{p}\right)_{hnf}}Ec\left[\left(1 + \frac{1}{\beta}\right)\left[\left(f^{\prime\prime}\right)^{2} + 4\delta^{2}(f^{\prime})^{2}\right]\right] + Du\phi^{\prime\prime} = 0,$$
(9)

$$\frac{1}{sc}\phi^{\prime\prime} + S(f\phi^{\prime} - \eta\phi^{\prime}) + Sr\theta^{\prime\prime} - R\phi = 0,$$
(10)

with the non-dimensional boundary conditions [56]

$$f(\eta) = 1, f'(\eta) = 0, \theta(\eta) = 1, \phi(\eta) = 1,$$
 at $\eta = 1.$ (11)

$$f(\eta) = 0, f''(\eta) = 0, \theta'(\eta) = 0, \phi'(\eta) = 0$$
 at $\eta = 0,$ (12)

The physical terms in the dimensionless equations are denoted by [57]

$$S = \frac{\alpha l^2}{2\nu_f}, \quad Ha = lB_0 \sqrt{\frac{\sigma}{\rho_f \nu_f}}, \quad Da = \frac{k_0}{\varphi l^2}, \quad \delta = \frac{l}{x} (1 - \alpha t)^{1/2}, \quad Pr = \frac{\nu_f}{\alpha_f}, \quad Ec = \frac{\alpha^2 x^2}{4C_p T_w (1 - \alpha t)^2}, \\ \gamma = \frac{Q_0 l^2}{\nu_f (\rho C_p)_f}, \quad Du = \frac{D_m k_T}{c_s c_p \nu_f} \frac{C_w}{T_w}, \quad Sr = \frac{D_m k_T}{T_m \nu_f} \frac{T_w}{C_w}, \quad Sc = \frac{\nu_f}{D_m}, \quad R = \frac{ak_2 l^2}{\nu_f}.$$

3. Numerical Procedure

The nonlinear ordinary differential Eq. (8) to Eq. (10) with associated boundary conditions Eq. (11) and Eq. (12) are solved numerically using implicit finite difference scheme known as Keller-box method. This method is an unconditionally stable and succeed in obtaining the accurate results. It is also recommended to be used in solving the nonlinear parabolic problems. The four steps involved to obtain the numerical results are as follows:

- i. The ordinary differential equations are reduced to a system of first order equations.
- ii. The first order system is discretized to obtain the equations in the finite difference form by using central difference scheme.
- iii. The nonlinear equations are linearized using Newton's method and then written in matrixvector form.
- iv. Finally, the linear system can be solved via block tri-diagonal elimination technique.

4. Results and Discussion

The main objective of study is to examine the behaviour of velocity, temperature, and concentration of Casson hybrid nanofluid under the effects of *S*, *Da*, β , *Ha*, ϕ_2 , *Ec*, *Sr*, *Du*, γ and *R*. The Keller box techniques is implemented to solve the similarity Eq. (8) to Eq. (10) with boundary conditions Eq. (11) and Eq. (12). The computation and graphical outputs are conducted using MATLAB software. The suitable step size, $\Delta \eta = 0.01$ and width of boundary layer, $\eta_{\infty} = 1$ is applied to attain the accurate and proper results. The computation in MATLAB is ended once the difference of previous and current outputs approaches to 0.00001 [58]. The results obtained using current technique is validated by comparing the values of wall shear stress with Naduvinamani and Shankar [41] and Jyothi *et al.*, [38] as displayed in Table 3.

Table 3 Numerical outputs of $-f''(1)$ for S with $\beta \to \infty$, $Da \to \infty$, $Ha = Ec = \delta = Sr = Du = \gamma = R = \phi_2 = 0$ and $Sc = Pr = 1$						
-f''(1)						
S	Naduvinamani and Shankar [39]	Jyothi <i>et al.,</i> [36]	Present outputs			
-1.0	2.170090	2.170090	2.170255			
-0.5	2.617403	2.617403	2.617512			
0.01	3.007133	3.007133	3.007208			
0.5	3.336449	3.336449	3.336504			
2.0	4.167389	4.167389	4.167412			

The graphical outputs under the effects of S, β , Ha, Da and ϕ_2 on velocity of fluid are presented in Figure 2 to 6. It is noteworthy that squeeze number, S indicates the motion of two plates. For S >0, both plates move nearer one another, while the plates moves away when S < 0. The fluid velocity decelerating nearby lower plates, $\eta < 0.45$, whereas it accelerating in the area at middle plates, $\eta \ge$ 0.45 for S > 0 as displayed in Figure 2. In contrary, the velocity increasing at $\eta < 0.45$, while it declining at $\eta \ge 0.45$ when S < 0. According to the behaviour of fluid velocity, the fluid flow across the medium at the faster rate caused by compressing of both plates. Meanwhile, the velocity declining due to high resistance in the broader channel encountered by the flow.



Figure 3 shows the velocity slowing down when $\eta \leq 0.5$, and it elevating when $\eta > 0.5$ for increment of β . The intermolecular forces within nanoparticles strengthen with the presence of β , which resulting the velocity decelerates nearby the lower plate.



Figure 4 depicts the velocity decreases when $\eta \le 0.5$, whereas it raises when $\eta > 0.5$ with boost in *Ha*. The induction of magnetic field on the lower plate result in the fluid velocity decelerates due to resistance on the flow boost with the impact of Lorentz force.



Figure 5 portrays the velocity rising when $\eta \le 0.5$, while it slowing down at $\eta > 0.5$ for rises *Da*. The enhancement of Darcy number enhances the permeability of medium, which lead to increase the fluid flow through the medium.



The velocity enhances when $\eta \le 0.5$ and it dropping when $\eta > 0.5$ for rising ϕ_2 as shown in Figure 6. The velocity reduces in the middle of channel with addition of nanoparticles. It is discovered that the fluid cannot flow rapidly due to the stronger collision of fluid particles and nanoparticles.



Figure 7 to 11 depicts the impacts of Ec, γ , ϕ_2 , Sr and Du on fluid temperature. The temperature in Figure 7 increases as Ec rises. It implies kinetic energy and heat induced by the motion of nanoparticles in fluid enhances for elevates Ec.



Figure 8 demonstrates the temperature drops with enhance in heat absorption ($\gamma < 0$), while it elevates for rising the heat generation ($\gamma > 0$). The transfer of heat energy from fluid to the plates increasing, which causing the fluid temperature decreases. Meanwhile, the heat sink raises the thermal energy in the fluid and thus, enhancing the temperature of fluid.



Figure 9 presents the deceleration of temperature when ϕ_2 elevates because the volume fraction of nanoparticles promotes the fluid thermal conductivity. Therefore, the rate of thermal transfer from the fluid to surfaces enhances, which result in dropping of fluid temperature.



Figure 10 illustrates the temperature in the flow boosts for increasing Sr. Soret number is inverse proportional to the kinematics viscosity. It indicates that the increment of Sr reduce the viscosity in the flow region, which weaken the resistance encountered in the fluid. Hence, it accelerating the kinetic energy of nanoparticles and increase the fluid temperature.



Figure 11 discovers the temperature boosts when Du rises because the kinematic viscosity in the flow drops, which resulting the resistance decrease and kinetic energy of particles elevates in the fluid.



The influences of Sr, Du, ϕ_2 and R on concentration are shown in Figures 12 to 15. The concentration reduces as observed in Figure 12 for increasing Sr. It is noticed that the transfer of mass in the fluid to the upper surface rises due to the presence of thermal diffusion or Soret effect.



Figure 13 portrays the concentration declines with elevate in Du. The generation of energy flux caused by concentration differences enhances result from the effect of Dufour parameter. It has caused the fluid concentration drops due to the increment of temperature in the flow region.



Figure 14 shows the concentration increasing when ϕ_2 elevates due to the fluid viscosity escalates with high volume fraction of nanoparticles.

Figure 15 displays the decrease in concentration for destructive chemical reaction, R > 0, while it enhancing for constructive chemical reaction, R < 0. The increment of constructive reaction promotes the chemical reaction rate, whereas adverse effect is examined in destructive reaction.



The effects of Ha and Da on wall shear stress are illustrated in Figure 16. The skin friction increasing when Ha boosts, while it declining as Da rises. It is caused by the Lorentz force in flow escalates with raise in Ha and therefore, it enhances the friction force close to the plates. In contrast, the flow moving rapidly close to the plate for increasing the permeability of medium. Hence, the skin friction at the boundary area decelerates due to the friction force in the flow decreases.

Figure 17 illustrates the increment of Nusselt number when Du and γ elevates. The ratio of convective and conductive thermal transfer is denoted by Nusselt number. The fluid temperature escalates because the generation of heat flux boost with higher Du and γ values. It indicates that the thermal convection is greater than conduction, which lead to the convective thermal transfer and Nusselt number increases.



Fig. 16. Effect of *Ha* and *Da* on $(Re_x)^{1/2}Cf_x$



Figure 18 shows the Sherwood number increases for R > 0 and it decreasing for R < 0. Sherwood number is the ratio of convection and diffusion mass transfer. The constructive reaction (R < 0) promotes the fluid concentration, which result in the diffusive mass transfer increasing. Hence, it indicates that the convective mass transfer and Sherwood number drops in the flow. The contrary behaviour is noticed on destructive chemical reaction (R > 0) caused by low fluid concentration. Consequently, the Sherwood number increases due to the convective mass transfer in fluid more dominant. Furthermore, the Sherwood number elevates with raise in Sr. The reduction of fluid concentration in Figure 12 highlights that the diffusive mass transfer drops when Sr increases. Sherwood number is inversely proportional to mass transfer by diffusion. Thus, it concludes that the increase in Sr promotes the convective mass transfer and boosts the Sherwood number in the fluid flow.



4. Conclusions

In this study, the impacts of Soret and Dufour on squeezing MHD flow Casson hybrid nanofluid through permeable medium with heat sink/source and chemical reaction was analysed. The resulting equations are discretized using Keller-box technique by developing the algorithm in MATLAB. The present outputs are compared with prior outputs from selected papers and shown in good agreement. The physical behaviours and quantities of the flow were affected by different values of S, Da, β , Ha, ϕ_2 , Ec, Sr, Du, γ and R. The main results of Casson hybrid nanofluid according to the discussions are concluded as

- i. The acceleration of axial velocity occurs when the surfaces is squeezing (S > 0) and it decelerating when the surfaces is separating (S < 0) near the upper plate.
- ii. The velocity of fluid decreasing in the middle of channel as β and Ha elevates, and it increasing when Da and ϕ_2 rises.
- iii. The wall shear stress escalates for rises of *Ha*, while it reduces for enhancing *Da*.
- iv. The heat transfer rate and temperature of fluid boosts as Ec, γ and Du increases, and the contrary behaviour is shown for enhancing ϕ_2 .
- v. The fluid concentration declines and the mass transfer rate enhances with elevate in *Sr*.
- vi. The fluid concentration elevates, while the convective mass transfer declining when R < 0. The adverse behaviour of fluid is observed for R > 0.

For future works, it is recommended to further this study by investigating the ternary hybrid nanofluid in the flow. The importance of ternary nanoparticles is acknowledged as it improves the thermal performance of nanofluid at low cost. The implementation of ternary hybrid nanofluid as a coolant in thermal based devices is important because it has result in energy saving and emission reduction.

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References

[1] Das, Subhendu, and Sanatan Das. "EDL aspects in swirling ionic tribological fluid flow in a squeezed/split channel underlie a high-power magnetic field." *Forces in Mechanics* 11 (2023): 100196. <u>https://doi.org/10.1016/j.finmec.2023.100196</u>

- [2] Stefan, M. "Experiments on apparent adhesion." *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 47, no. 314 (1874): 465-466. <u>https://doi.org/10.1080/14786447408641062</u>
- [3] Cameron, Alastair. "Basic lubrication theory." (No Title) (1976).
- [4] Wang, C-Y. "The squeezing of a fluid between two plates." (1976): 579-583. https://doi.org/10.1115/1.3423935
- [5] Bujurke, N. M., P. K. Achar, and N. P. Pai. "Computer extended series for squeezing flow between plates." *Fluid Dynamics Research* 16, no. 2-3 (1995): 173-187. <u>https://doi.org/10.1016/0169-5983(94)00058-8</u>
- [6] Rashidi, Mohammad Mehdi, Hamed Shahmohamadi, and Saeed Dinarvand. "Analytic approximate solutions for unsteady two-dimensional and axisymmetric squeezing flows between parallel plates." *Mathematical problems in* engineering 2008 (2008). <u>https://doi.org/10.1155/2008/935095</u>
- [7] Khan, Umar, Naveed Ahmed, Sheikh Irfanullah Khan, Zulfiqar Ali Zaidi, Yang Xiao-Jun, and Syed Tauseef Mohyud-Din. "On unsteady two-dimensional and axisymmetric squeezing flow between parallel plates." *Alexandria Engineering Journal* 53, no. 2 (2014): 463-468. <u>https://doi.org/10.1016/j.aej.2014.02.002</u>
- [8] Noor, Nur Azlina Mat, Sharidan Shafie, and Mohd Ariff Admon. "Unsteady MHD Flow of Cassonnano Fluid with Chemical Reaction, Thermal Radiation and Heat Generation/Absorption." *Matematika* 35 (2019).
- [9] Casson, N. "Flow equation for pigment-oil suspensions of the printing ink-type." *Rheology of disperse systems* (1959): 84-104.
- [10] Khan, U., Ahmed, N., Khan, S. I., Bano, S., and Mohyud-Din, S. T. "Unsteady squeezing flow of a Casson fluid between parallel plates." *World Journal of Modelling and Simulation 10*, no. 4 (2014): 308-319.
- [11] Ahmed, Naveed, Umar Khan, Sheikh Irfanullah Khan, Saima Bano, and Syed Tauseef Mohyud-Din. "Effects on magnetic field in squeezing flow of a Casson fluid between parallel plates." *Journal of King Saud University-Science* 29, no. 1 (2017): 119-125. <u>https://doi.org/10.1016/j.jksus.2015.03.006</u>
- [12] Khan, Hamid, Mubashir Qayyum, Omar Khan, and Murtaza Ali. "Unsteady squeezing flow of Casson fluid with magnetohydrodynamic effect and passing through porous medium." *Mathematical Problems in Engineering* 2016 (2016). <u>https://doi.org/10.1155/2016/4293721</u>
- [13] Mohyud-Din, Syed Tauseef, Muhammad Usman, Wei Wang, and Muhammad Hamid. "A study of heat transfer analysis for squeezing flow of a Casson fluid via differential transform method." *Neural Computing and Applications* 30 (2018): 3253-3264. <u>https://doi.org/10.1007/s00521-017-2915-x</u>
- [14] D'Emilia, E., L. Giuliani, A. Lisi, M. Ledda, S. Grimaldi, L. Montagnier, and A. R. Liboff. "Lorentz force in water: Evidence that hydronium cyclotron resonance enhances polymorphism." *Electromagnetic Biology and Medicine* 34, no. 4 (2015): 370-375. <u>https://doi.org/10.3109/15368378.2014.937873</u>
- [15] Siddiqui, Abdul M., Sania Irum, and Ali R. Ansari. "Unsteady squeezing flow of a viscous MHD fluid between parallel plates, a solution using the homotopy perturbation method." *Mathematical Modelling and Analysis* 13, no. 4 (2008): 565-576. <u>https://doi.org/10.3846/1392-6292.2008.13.565-576</u>
- [16] Sweet, Erik, K. Vajravelu, Robert A. Van Gorder, and I. Pop. "Analytical solution for the unsteady MHD flow of a viscous fluid between moving parallel plates." *Communications in Nonlinear Science and Numerical Simulation* 16, no. 1 (2011): 266-273. <u>https://doi.org/10.1016/j.cnsns.2010.03.019</u>
- [17] Al-Saif, Abdul-Sattar JA, and Abeer Majeed Jasim. "A novel algorithm for studying the effects of squeezing flow of a Casson Fluid between parallel plates on magnetic field." *Journal of Applied Mathematics* 2019 (2019). <u>https://doi.org/10.1155/2019/3679373</u>
- [18] Ahmad, Shakeel, Muhammad Farooq, Aisha Anjum, and Nazir Ahmad Mir. "Squeezing flow of convectively heated fluid in porous medium with binary chemical reaction and activation energy." *Advances in Mechanical Engineering* 11, no. 10 (2019): 1687814019883774. <u>https://doi.org/10.1177/1687814019883774</u>
- [19] Qayyum, Mubashir, Hamid Khan, and Omar Khan. "Slip analysis at fluid-solid interface in MHD squeezing flow of Casson fluid through porous medium." *Results in physics* 7 (2017): 732-750. <u>https://doi.org/10.1016/j.rinp.2017.01.033</u>
- [20] Sobamowo, Gbeminiyi, Lawrence Jayesimi, David Oke, Ahmed Yinusa, and Oluwatoyin Adedibu. "Unsteady Casson nanofluid squeezing flow between two parallel plates embedded in a porous medium under the influence of magnetic field." *Open Journal of Mathematical Sciences* 3, no. 1 (2019): 59-73. <u>https://doi.org/10.30538/oms2019.0049</u>
- [21] Mat Noor, Nur Azlina, Sharidan Shafie, and Mohd Ariff Admon. "Slip effects on MHD squeezing flow of Jeffrey nanofluid in horizontal channel with chemical reaction." *Mathematics* 9, no. 11 (2021): 1215. <u>https://doi.org/10.3390/math9111215</u>
- [22] Choi, S. US, and Jeffrey A. Eastman. Enhancing thermal conductivity of fluids with nanoparticles. No. ANL/MSD/CP-84938; CONF-951135-29. Argonne National Lab.(ANL), Argonne, IL (United States), 1995.
- [23] Turcu, R., A. L. Darabont, A. Nan, N. Aldea, D. Macovei, D. Bica, L. Vekas *et al.,* "New polypyrrole-multiwall carbon nanotubes hybrid materials." *Journal of optoelectronics and advanced materials* 8, no. 2 (2006): 643-647.

- [24] Jana, Soumen, Amin Salehi-Khojin, and Wei-Hong Zhong. "Enhancement of fluid thermal conductivity by the addition of single and hybrid nano-additives." *Thermochimica acta* 462, no. 1-2 (2007): 45-55. https://doi.org/10.1016/j.tca.2007.06.009
- [25] Suresh, S., K. P. Venkitaraj, P. Selvakumar, and M. Chandrasekar. "Synthesis of Al2O3–Cu/water hybrid nanofluids using two step method and its thermo physical properties." *Colloids and Surfaces A: Physicochemical and Engineering Aspects* 388, no. 1-3 (2011): 41-48. <u>https://doi.org/10.1016/j.colsurfa.2011.08.005</u>
- [26] Ahmed, Naveed, Fitnat Saba, Umar Khan, Ilyas Khan, Tawfeeq Abdullah Alkanhal, Imran Faisal, and Syed Tauseef Mohyud-Din. "Spherical shaped (A g– F e 3 O 4/H 2 O) hybrid nanofluid flow squeezed between two Riga plates with nonlinear thermal radiation and chemical reaction effects." *Energies* 12, no. 1 (2018): 76. <u>https://doi.org/10.3390/en12010076</u>
- [27] Madhukesh, J. K., G. K. Ramesh, RS Varun Kumar, B. C. Prasannakumara, and M. Kbiri Alaoui. "Computational study of chemical reaction and activation energy on the flow of Fe3O4-Go/water over a moving thin needle: Theoretical aspects." *Computational and Theoretical Chemistry* 1202 (2021): 113306. https://doi.org/10.1016/j.comptc.2021.113306
- [28] Devi, SP Anjali, and S. Suriya Uma Devi. "Numerical investigation of hydromagnetic hybrid Cu–Al2O3/water nanofluid flow over a permeable stretching sheet with suction." *International Journal of Nonlinear Sciences and Numerical Simulation* 17, no. 5 (2016): 249-257. <u>https://doi.org/10.1515/ijnsns-2016-0037</u>
- [29] Upreti, Himanshu, Alok Kumar Pandey, and Manoj Kumar. "Unsteady squeezing flow of magnetic hybrid nanofluids within parallel plates and entropy generation." *Heat Transfer* 50, no. 1 (2021): 105-125. <u>https://doi.org/10.1002/htj.21994</u>
- [30] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Squeezed hybrid nanofluid flow over a permeable sensor surface." *Mathematics* 8, no. 6 (2020): 898. <u>https://doi.org/10.3390/math8060898</u>
- [31] Nisar, Kottakkaran Sooppy, Umair Khan, A. Zaib, Ilyas Khan, and Dumitru Baleanu. "Numerical simulation of mixed convection squeezing flow of a hybrid nanofluid containing magnetized ferroparticles in 50%: 50% of ethylene glycol–water mixture base fluids between two disks with the presence of a non-linear thermal radiation heat flux." *Frontiers in Chemistry* 8 (2020): 792. https://doi.org/10.3389/fchem.2020.00792
- [32] Salehi, Sajad, Amin Nori, Kh Hosseinzadeh, and D. D. Ganji. "Hydrothermal analysis of MHD squeezing mixture fluid suspended by hybrid nanoparticles between two parallel plates." *Case Studies in Thermal Engineering* 21 (2020): 100650. <u>https://doi.org/10.1016/j.csite.2020.100650</u>
- [33] Khashi'ie, Najiyah Safwa, Iskandar Waini, Norihan Md Arifin, and Ioan Pop. "Unsteady squeezing flow of Cu-Al2O3/water hybrid nanofluid in a horizontal channel with magnetic field." *Scientific reports* 11, no. 1 (2021): 14128. <u>https://doi.org/10.1038/s41598-021-93644-4</u>
- [34] Wahid, Nur Syahirah, Norihan Md Arifin, Najiyah Safwa Khashi'ie, Ioan Pop, Norfifah Bachok, and Mohd Ezad Hafidz Hafidzuddin. "MHD mixed convection flow of a hybrid nanofluid past a permeable vertical flat plate with thermal radiation effect." *Alexandria Engineering Journal* 61, no. 4 (2022): 3323-3333. <u>https://doi.org/10.1016/j.aej.2021.08.059</u>
- [35] Famakinwa, O. A., O. K. Koriko, and K. S. Adegbie. "Effects of viscous dissipation and thermal radiation on time dependent incompressible squeezing flow of CuO- Al2O3/water hybrid nanofluid between two parallel plates with variable viscosity." *Journal of Computational Mathematics and Data Science* 5 (2022): 100062. https://doi.org/10.1016/j.jcmds.2022.100062
- [36] Alghamdi, Wajdi, Taza Gul, Mehranullah Nullah, Ali Rehman, S. Nasir, A. Saeed, and E. Bonyah. "Boundary layer stagnation point flow of the Casson hybrid nanofluid over an unsteady stretching surface." *AIP Advances* 11, no. 1 (2021). <u>https://doi.org/10.1063/5.0036232</u>
- [37] Kamis, Nur Ilyana, Md Faisal Md Basir, Sharidan Shafie, Taufiq Khairi Ahmad Khairuddin, and Lim Yeou Jiann.
 "Suction effect on an unsteady Casson hybrid nanofluid film past a stretching sheet with heat transfer analysis." In *IOP Conference Series: Materials Science and Engineering*, vol. 1078, no. 1, p. 012019. IOP Publishing, 2021. https://doi.org/10.1088/1757-899X/1078/1/012019
- [38] Jyothi, A. M., R. S. Varun Kumar, J. K. Madhukesh, B. C. Prasannakumara, and G. K. Ramesh. "Squeezing flow of Casson hybrid nanofluid between parallel plates with a heat source or sink and thermophoretic particle deposition." *Heat Transfer* 50, no. 7 (2021): 7139-7156. <u>https://doi.org/10.1002/htj.22221</u>
- [39] Noor, Nur Azlina Mat, and Sharidan Shafie. "Magnetohydrodynamics squeeze flow of sodium alginate-based Jeffrey hybrid nanofluid with heat sink or source." *Case Studies in Thermal Engineering* 49 (2023): 103303. https://doi.org/10.1016/j.csite.2023.103303
- [40] Ullah, Imran, Ilyas Khan, and Sharidan Shafie. "Soret and Dufour effects on unsteady mixed convection slip flow of Casson fluid over a nonlinearly stretching sheet with convective boundary condition." *Scientific Reports* 7, no. 1 (2017): 1113. <u>https://doi.org/10.1038/s41598-017-01205-5</u>

- [41] Naduvinamani, N. B., and Usha Shankar. "Thermal-diffusion and diffusion-thermo effects on squeezing flow of unsteady magneto-hydrodynamic Casson fluid between two parallel plates with thermal radiation." Sādhanā 44, no. 8 (2019): 175. <u>https://doi.org/10.1007/s12046-019-1154-5</u>
- [42] Rafique, Khuram, Muhammad Imran Anwar, Masnita Misiran, Ilyas Khan, S. O. Alharbi, Phatiphat Thounthong, and K. S. Nisar. "Numerical solution of casson nanofluid flow over a non-linear inclined surface with soret and dufour effects by keller-box method." *Frontiers in Physics* 7 (2019): 139. <u>https://doi.org/10.3389/fphy.2019.00139</u>
- [43] Bidemi, Olumide Falodun, and MS Sami Ahamed. "Soret and Dufour effects on unsteady Casson magneto-nanofluid flow over an inclined plate embedded in a porous medium." *World Journal of Engineering* 16, no. 2 (2019): 260-274. <u>https://doi.org/10.1108/WJE-04-2018-0144</u>
- [44] Sekhar, P. Raja, S. Sreedhar, S. Mohammed Ibrahim, and P. Vijaya Kumar. "Radiative heat source fluid flow of MHD Casson nanofluid over a non-linear inclined surface with Soret and Dufour effects." *CFD Letters* 15, no. 7 (2023): 42-60. <u>https://doi.org/10.37934/cfdl.15.7.4260</u>
- [45] Noor, Nur Azlina Mat, Sharidan Shafie, and Mohd Ariff Admon. "Effects of viscous dissipation and chemical reaction on MHD squeezing flow of Casson nanofluid between parallel plates in a porous medium with slip boundary condition." *The European Physical Journal Plus* 135, no. 10 (2020): 855. <u>https://doi.org/10.1140/epjp/s13360-020-00868-w</u>
- [46] Noor, Nur Azlina Mat, Sharidan Shafie, and Mohd Ariff Admon. "Unsteady MHD squeezing flow of Jeffrey fluid in a porous medium with thermal radiation, heat generation/absorption and chemical reaction." *Physica Scripta* 95, no. 10 (2020): 105213. <u>https://doi.org/10.1088/1402-4896/abb695</u>
- [47] Mat Noor, Nur Azlina, Sharidan Shafie, Y. S. Hamed, and Mohd Ariff Admon. "Soret and Dufour effects on MHD squeezing flow of Jeffrey fluid in horizontal channel with thermal radiation." *Plos one* 17, no. 5 (2022): e0266494. <u>https://doi.org/10.1371/journal.pone.0266494</u>
- [48] Hafeez, Muhammad Bilal, Wojciech Sumelka, Umar Nazir, Hijaz Ahmad, and Sameh Askar. "Mechanism of solute and thermal characteristics in a Casson hybrid nanofluid based with ethylene glycol influenced by Soret and Dufour effects." *Energies* 14, no. 20 (2021): 6818. <u>https://doi.org/10.3390/en14206818</u>
- [49] Reddy, Vinodh Srinivasa, Jagan Kandasamy, and Sivasankaran Sivanandam. "Impacts of casson model on hybrid nanofluid flow over a moving thin needle with dufour and soret and thermal radiation effects." *Mathematical and Computational Applications* 28, no. 1 (2022): 2. <u>https://doi.org/10.3390/mca28010002</u>
- [50] Deepika, A. R., Kamatam Govardhan, Amalendu Rana, and Motahar Reza. "Soret and Dufour effects on MHD mixed convection flow of Casson hybrid nanofluid over a permeable stretching sheet." *International Journal of Ambient Energy* 44, no. 1 (2023): 2115-2127. <u>https://doi.org/10.1080/01430750.2023.2224337</u>
- [51] Noor, Nur Azlina Mat, Sharidan Shafie, and Mohd Ariff Admon. "Impacts of chemical reaction on squeeze flow of MHD Jeffrey fluid in horizontal porous channel with slip condition." *Physica Scripta* 96, no. 3 (2021): 035216. <u>https://doi.org/10.1088/1402-4896/abd821</u>
- [52] Takabi, Behrouz, and Saeed Salehi. "Augmentation of the heat transfer performance of a sinusoidal corrugated enclosure by employing hybrid nanofluid." *Advances in Mechanical Engineering* 6 (2014): 147059. <u>https://doi.org/10.1155/2014/147059</u>
- [53] Oztop, Hakan F., and Eiyad Abu-Nada. "Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids." *International journal of heat and fluid flow* 29, no. 5 (2008): 1326-1336. <u>https://doi.org/10.1016/j.ijheatfluidflow.2008.04.009</u>
- [54] Khashi'ie, Najiyah Safwa, Iskandar Waini, Norihan Md Arifin, and Ioan Pop. "Unsteady squeezing flow of Cu-Al2O3/water hybrid nanofluid in a horizontal channel with magnetic field." *Scientific reports* 11, no. 1 (2021): 14128. <u>https://doi.org/10.1038/s41598-021-93644-4</u>
- [55] Mat Noor, Nur Azlina, Sharidan Shafie, and Mohd Ariff Admon. "Heat and mass transfer on MHD squeezing flow of Jeffrey nanofluid in horizontal channel through permeable medium." *Plos one* 16, no. 5 (2021): e0250402. <u>https://doi.org/10.1371/journal.pone.0250402</u>
- [56] Noor, Nur Azlina Mat, Mohd Ariff Admon, and Sharidan Shafie. "Unsteady MHD squeezing flow of Casson fluid over horizontal channel in presence of chemical reaction." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 92, no. 2 (2022): 49-60. <u>https://doi.org/10.37934/arfmts.92.2.4960</u>
- [57] Mat Noor, Nur Azlina, Sharidan Shafie, and Mohd Ariff Admon. "Heat Transfer on Magnetohydrodynamics Squeezing Flow of Jeffrey Fluid Through Permeable Medium with Slip Boundary." *Journal of Nanofluids* 11, no. 1 (2022): 31-38. <u>https://doi.org/10.1166/jon.2022.1814</u>
- [58] Noor, Nur Azlina Mat, Sharidan Shafie, and Mohd Ariff Admon. "MHD squeezing flow of Casson nanofluid with chemical reaction, thermal radiation and heat generation/absorption." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 68, no. 2 (2020): 94-111. <u>https://doi.org/10.37934/arfmts.68.2.94111</u>