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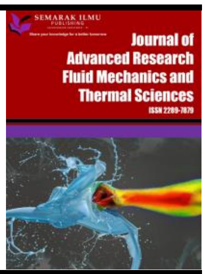

Numerical Analysis of Transient Flow in Polyethylene Pipe

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ABSTRACT

We present a numerical code for calculating transient flow in plastic pipes, especially in the polyethylene pipe, to analysis transient flow in a viscoelastic pipe such as polyethylene. The set partial differential equations to be solved is obtained using conservation laws and behavior for the fluid and the pipe wall, associated with constitutive equations of the two media. A global digital processing is achieved using the method of characteristics. The results obtained are in good agreement with those found in the literature.

1.Introduction

Water hammer, or hydraulic transient, refers to pressure fluctuations caused by a sudden increase or decrease in flow velocity. In this way, many theoretical and experimental studies have, usually, conducted assuming linear elastic behavior of the pipe wall without mass, that is means, time scales of inertia of pipe wall is negligible [1-7].

In view of industry practice, classical water hammer studies are interested in estimating the pressure caused by water hammer. This can, under certain conditions, have disadvantages on pipes conveying fluid, and also, it can be useful for generating energy. For a better approach to analysis physical phenomena of waves propagation involved, several models of fluid structure interactions in transient were developed in the case of conventional materials (steel, copper ...) [1-7].

The interest of this study is the use of increasingly growing plastic tubes (PVC, polyethylene...) in different industries and service life compared to traditional piping materials. This leads us to examine the influence of the viscoelastic behavior of these materials on transient flows. A comparison and validation of our code is done with the measurement results conducted by Güney [8], at INSA Lyon.

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2. Assumptions and Basic Equations

The basic equations are derived from the classical laws of conservation of mass, momentum for the fluid and the pipe wall in the case of isentropic transformations. We assume also that the fluid is barotropic Newtonian and the material of the pipe wall behaves like elastic or viscoelastic, isotropic, Kelvin Voigt type [8-13]. Geometrically, the pipe is assumed cylindrical horizontal and circular. One end is rigidly attached to a reservoir upstream, which imposes a constant pressure and the other is on a fixed support and including an operating valve. The flow is axisymmetric and longitudinal gradients of velocity are assumed to be small compared to transverse gradients.

2.1 Basic Relations

Given these assumptions, the averaged equations of the flow in a cross section of pipe can be written in traditionally a one-dimensional formulation, reflecting the relations of conservation of mass and momentum averaged over a section of a cross section, as hyperbolic system that is suitable for characteristic methods [8-16]

$$\frac{\partial(\rho_f A_f)}{\partial t} + \frac{\partial(\rho_f A_f V)}{\partial x} = 0 \tag{1}$$

$$\rho_f \frac{dV}{dt} + \frac{\partial P}{\partial x} + \rho_f g \frac{\partial z}{\partial x} - \frac{4T_f}{D} = 0 \tag{2}$$

If we introduce $\varepsilon^e = \alpha(p(x,t) - p(x,0))\, D_m J(0)/2e$ the instantaneous elastic deformation of the pipe wall and $\varepsilon^r = \int_0^t \alpha(P(x,t-\tau) - P(x,0))\, \frac{D_m}{2e} \frac{dJ(\tau)}{d\tau}\, d\tau$ its resulting strain retarded creep, the deformation of the wall can be considered as the sum of two terms $\varepsilon^e + \varepsilon^r$, and the Eq. (3) above became

$$\frac{1}{\rho_f} \frac{dP}{dt} + a^2 \frac{\partial V}{\partial x} + 2a^2 \frac{d\varepsilon^r}{dt} = 0 \tag{3}$$

where

$$a = \left(\rho_f \left(1/\kappa + \alpha D_m J(0)/e\right)\right)^{-1/2}$$

α : parameter characterizing the type of anchoring the pipe and which, in the case of a pipe anchored longitudinally, is written [8].

$$\alpha = 1 + e^2/D_m^2 + 2\nu e D_m - \nu^2 \left(1 - e/D_m\right)^2.$$

3. Modelling of the Creep Function

The creep function can be discrezed as [17,18]

$$J(t) = J_0 + \sum_{i=1}^{n} J_i(1 - exp(-t/\tau_i)) \quad (4)$$

It is corresponding to a Kelvin-Voigt model with a generalized representation. in Figure 1. $E_i = 1/J_i$ is the spring module in parallel with the viscous dampers η_i, leading to a relaxation time $\tau_i = \eta_i / E_i$, J_0 representing the instantaneous compliance , the inverse of the instantaneous elastic modulus E_0. If we set, $\varepsilon^r = 0$, that is $J_i = 0$, or, if $\tau_i = \infty$,we find a classical elastic case.

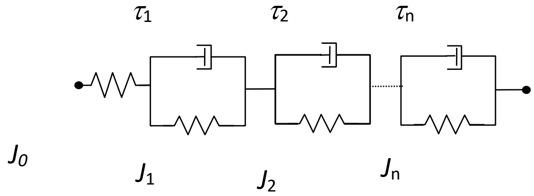

Fig. 1. Generalized Kelvin-tVoigt model

4. Initial and Boundary Conditions

The initial conditions are those for steady f low and the balance for the pipe wall. The boundary conditions are in addition to the pressure imposed by the tank on the upstream and the instantaneous closing of a free valve on the downstream, the conditions of fluid-conduit interfaces requiring, in viscous flow, equal velocities and stresses as well as: $P(0,t) = \rho\, g\, H_0$, as shown in Figure 2.

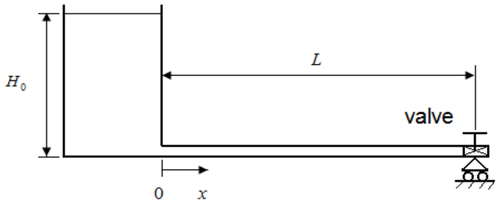

Fig. 2. Diagram of the system studied

5. Numerical Solution

The numerical solution of hyperbolic systems of equations (2), (3) associated to initial and boundary conditions can easily be obtained by the usual method of characteristics [19-23]. In this case, we have three characteristic directions and the relationships along these characteristic curves are , respectively.

For characteristics of slopes: $\frac{dx}{dt} = a$ and $\frac{dx}{dt} = a$

$$\frac{\delta P}{\delta t} + \rho a \frac{\delta V}{\delta t} + 2\frac{\rho a^2}{V+a}\left(V \frac{\delta \varepsilon^r}{\delta t} + a\frac{\partial \varepsilon^r}{\partial t}\right) + \rho g a \frac{\partial z}{\partial x} - a\frac{4T_f}{D} = 0 \quad (5)$$

$$\frac{\delta P}{\delta t} - \rho a \frac{\delta V}{\delta t} + 2\frac{\rho a^2}{V-a}\left(V\frac{\delta \varepsilon^r}{\delta t} - a\frac{\partial \varepsilon^r}{\partial t}\right) - \rho g a \frac{\partial z}{\partial x} + a\frac{4T_f}{D} = 0 \quad (6)$$

and, $$\frac{\partial \varepsilon^r}{\partial t} = \int_0^t \frac{\alpha D_m}{2e}\frac{\partial P(x,t-\tau)}{\partial t}\frac{dJ(\tau)}{d\tau}d\tau \quad (7)$$

along the characteristic curve of slope: $\frac{\mathrm{dx}}{\mathrm{dt}} = 0$

The numerical solution of the Eq. (5) and (6) can be made along the characteristic curves and the unknown parameters problem; P ,V can be calculated, numerically, at each point of the pipe, and over time.

6. Application and Results

In this application, we consider steady-state flow value V_0 =0.5m/s. The flow is in the horizontal pipe anchored to the upstream to a tank filled with water and of height H_0, ending at the downstream to a valve that closes abruptly. The parameters of the fluid and the pipe are summarized in Table 1.

Table 1
Parameter values of flow

Components	Values
Tank height H_0 (m)	0.55
Internal pipe diameter (mm)	50
Length of pipe (m)	43.1
Thickness of the pipe (mm)	4.2
Dynamic viscosity of water (Pa.s)	1.11×10^{-3}
Poisson’s ratio	0.43
Bulk modulus of water (GPa)	2.2
Density of water (Kg/m^3)	1000

We conduct the calculation in the case of a polyethylene pipe tested at the Laboratory of Fluid Mechanics at INSA Lyon and with experimental records of compliance values shown in Table 2 below [8] as function of temperature. This study shows that the viscoelastic behaviour of the pipe wall is even more pronounced as the temperature is high. At low temperatures, little change explains that one can admit behaviour substantially elastic. But when the temperature rises, the effect of viscoelasticity is dominant and must be taken into account in the calculations.

Table 2
Experimental values of compliances and relaxation times for different temperatures [8]

(°C)	$J_0 \times 10^{-9}$ $(Pa)^{-1}$	$J_1 \times 10^{-9}$ $(Pa)^{-1}$	$J_2 \times 10^{-9}$ $(Pa)^{-1}$	$J_3 \times 10^{-9}$ $(Pa)^{-1}$	$\tau_1(s)$	$\tau_2(s)$	τ_3(s)
13.8	1.144	0.516	0.637	0.871	0.56×10^{-4}	0.0166	1.747
25	1.542	0.754	1.046	1.237	0.89×10^{-4}	0.0222	1.864
31	1.791	1.009	1.397	1.628	1.15×10^{-4}	0.0221	1.822
35	1.995	1.235	1.797	2.349	1.38×10^{-4}	0.0265	2,34
38.5	2.239	1.479	2.097	3.570	1.24×10^{-4}	0.0347	3,07

following, we consider an application for both temperatures 25°C and 38.5°C and correspondent values in the table above, for example, we compared, for Figure 3 and Figure 4 below, the result of calculations assuming an elastic material and viscoelastic with those experimentally obtained by Güney [8], in a registration water-hammer at the valve for the instantaneous closing (10 ms) on the downstream of a polyethylene pipe.

The results presented in Figure 3 and Figure 4 display the change in pressure at the valve versus time. They correspond to

Curve (a): experimental results,

Curve (b): theoretical calculation conducted, under the conditions described above, by the losses being those corresponding to a flow steady state and elastic pipe,

Curve(c): theoretical calculation conducted in linear elastic pipe with neglecting the viscoelasticity of material, but with taking into account a transient shear stress of the flow,

Curve (d): theoretical calculation with taking into account in the latest case, always, under the same conditions, the transient nature of the flow, the viscoelasticity of pipe wall. The latest correction improves strongly the agreement between numerical and experiment results. The viscoelasticity of pipe wall has a significant damping effect, the conventional calculation assumed elastic behaviour, do not realize, in plastic material.

Figure 5 displays always in the same condition, the evolution of the average pressure in pipe versus time for different values of the polyethylene temperature. It shows that the effect of the viscoelasticity of the material becomes greater with increasing its temperature.

This can be an effective tool for controlling the pressure in a hydraulic system by adding a section of pipe in plastic material. This study can be, also, under assumptions, adapted to arterial conducts.

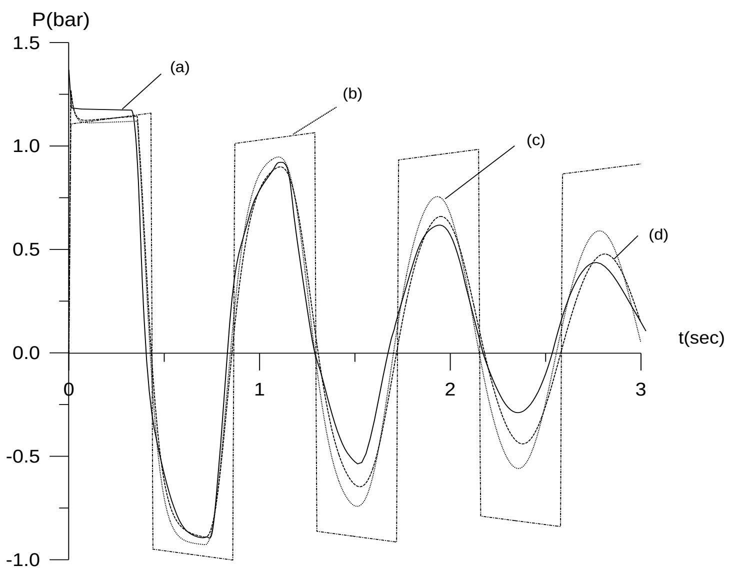

Fig. 3. Comparison of numerical and experiment -results at temperature 25°C

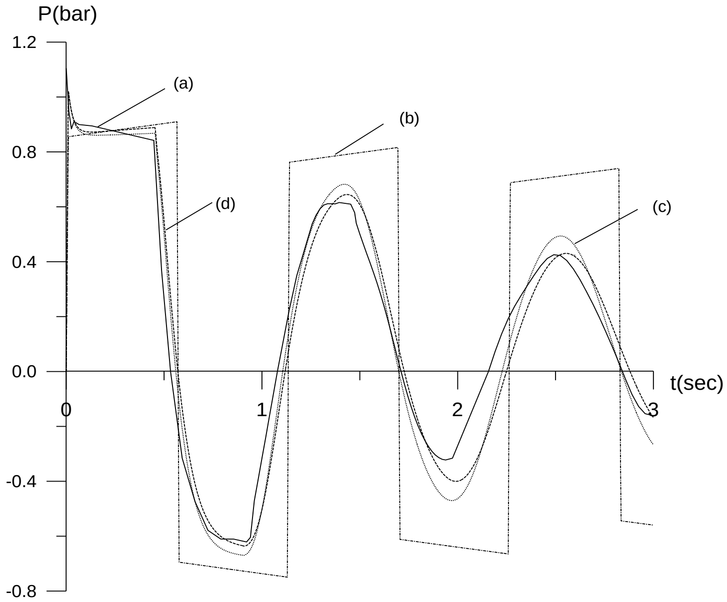

Fig. 4. Comparison of numerical and experiment –results at temperature 38.5°C

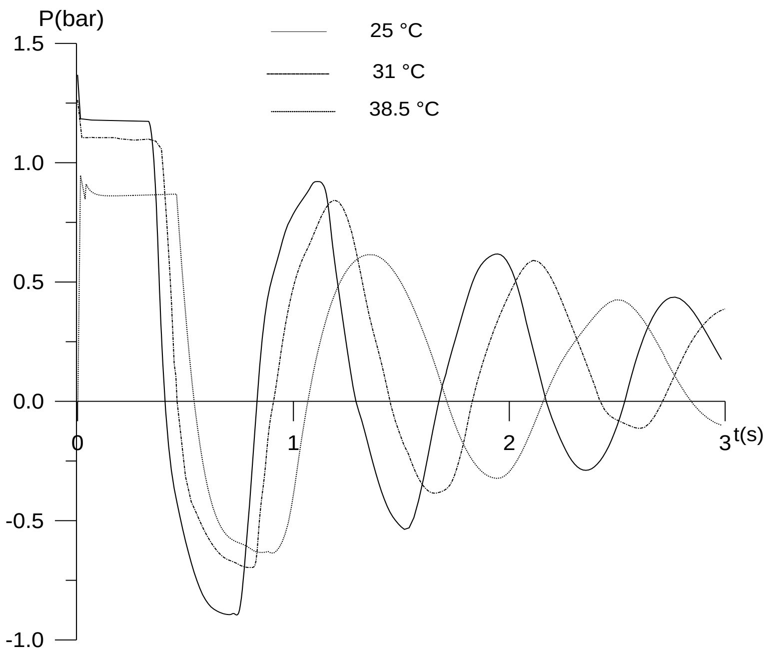

Fig. 5. Effect of the viscoelasticity with temperature

7. Conclusion

We have attempted in this study, to analysis, in addition to the viscosity of the fluid, the rheology of the pipe wall on transient flow in plastic pipe. This approach allows predicting liquid pressure response based on the method of characteristics.

This study highlights that the viscosity of the fluid and the viscoelastic nature of the wall pipe material have the effect of dissipation and damping of pressure waves. This code can be generalized to predict the acoustic vibrations and dynamic stability of pipes conveying fluid in transient flow and in addition, be adapted to simulate, in hemodynamic, some arterial disease. The extension of this work to a flexible pipe will be the subject to a future paper.

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