SEMARAK ILMU PUBLISHING

Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage: https://semarakilmu.com.my/journals/index.php/fluid_mechanics_thermal_sciences/index
ISSN: 2289-7879

Using Micropolar Nanofluid under a Magnetic Field to Enhance Natural Convective Heat Transfer around a Spherical Body

Nusayba Yaseen[1], Feras Shatat[2], Firas A. Alwawi[3], Mohammed Z. Swalmeh[1,*], Muhammad Salman Kausar[4], Ibrahim Mohammed Sulaiman[5]

[1] Department of Service Courses, Faculty of Arts and Sciences, Aqaba University of Technology, Aqaba 77110, Jordan
[2] Liwa College of Technology, Abu Dhabi, United Arab Emirates
[3] Department of Mathematics, College of Sciences and Humanities in Al-Kharj, Prince Sattam bin Abdulaziz University, Al-Kharj 11942, Saudi Arabia
[4] Faculty of Informatics and Computing, Universiti Sultan Zainal Abidin (Kampus Gong Badak), 21300 Kuala Terengganu, Terengganu, Malaysia
[5] School of Quantitative Sciences, College of Art and Sciences, Universiti Utara Malaysia, Sintok, 06010, Kedah, Malaysia

ARTICLE INFO

Article history:
Received 14 February 2022
Received in revised form 10 May 2022
Accepted 17 May 2022
Available online 12 June 2022

Keywords:
Magneto-hydrodynamic; natural convection; micropolar nanofluid; Keller box method

ABSTRACT

An analysis is discussed of the heat and mass transfer for micropolar nanofluid in presence of natural convection from a spherical body with magneto-hydrodynamic (MHD) effects. The constant wall temperature boundary condition is also studied. By employing proper similarity transformations, the governing equations are converted into a set of partial differential equations (PDEs) with the used boundary conditions, which can then be solved numerically via the efficient Keller-box implicit numerical finite difference method. The numerical results of impacts of the controlling parameters on heat transfer physical quantities have been presented, tabular and graphically, by MATLAB symbolic software. Comparisons of the current study results to previously published results show good agreement, indicating that our numerical computations are legitimate and accurate. Increasing nanoparticle volume fraction is observed to depress local skin friction, Nusselt number, and angular velocity while the reverse effects are observed for velocity and temperature.

1. Introduction

The analysis of steady laminar boundary-layer flow over a solid sphere surface that is subjected to various boundary conditions in quiescent/moving fluid continues to invigorate the interest of researchers due to applications in industrial and manufacturing environments. Some of these industrial applications of boundary layer flow on a solid sphere include the turbocharged ball bearing in automotive, the spherical storage tanks, and packed beds in a chemical reactor. Abbas *et al.,* [1] discussed the boundary layer flow over a solid sphere considering the impacts of thermal radiation Prandtl number and mixed convection. The mechanism of boundary layer flow with a semipermeable sphere to describe the Happel and Kuwabara cell models through an

[*] *Corresponding author.*
E-mail address: msawalmeh@aut.edu.jo

https://doi.org/10.37934/arfmts.96.1.179193

incompressible fluid has been investigated by Madasu [2]. Mohamed *et al.,* [3] presented a dust nanoparticles model around an isothermal sphere to calculate the heat flux of nanofluids by natural convection through the bvp4c numerical technique. They found an excellent correlation between the model predictions and experimental data. A numerical model, water-based spherical shape nanoparticle with magnetohydrodynamic (MHD), for effective thermal conductivity of nanofluids was developed by Jenifer *et al.,* [4]. They have shown that the vanishing skin friction is delayed by enhancing the mixed convection in both steady and unsteady fluid flow cases. Rodriguez *et al.,* [5] investigated free-stream turbulence natural convective heat transfer flow from the sphere. It was found that the drag coefficient and the Nusselt number are increased with turbulence interruption and momentum transfer is also increased from the surrounding fluid and energizes the separated shear layer. As a result, the recirculation zone shrinks, increasing the heat transfer from the sphere and therefore increasing the Nusselt number in the rear zone of the sphere. As turbulence intensity rises, a greater amount of cooler fluid is entrained from the surrounding environment, which results in a quicker rate of temperature depreciation at higher levels.

Nanofluids are fluids containing nanoparticles, objects with a size of around 100 nanometers. A base fluid contains colloidal suspensions of nanoparticles engineered into these fluids. Metal or metal oxide nanoparticles are typically used in nanofluids. Metal oxides (CuO, TiO_2, Al_2O, SiO_2), nitrides (SiN, AIN), carbides (SiC), or nonmetals (carbon nanotubes, graphite) are also commonly used. There are also lubricants, bio-fluids and polymer solutions that serve as base fluids. In general, nanofluids contain 5% or more nanoparticles to gain an advantage over their base fluid properties. Nanofluids have unique features that could make them useful in several heat transfer applications, such as hybrid engines, fuel cells, pharmaceuticals, and microelectronics. As compared to the base fluid, they are more thermally conductive and have a higher convective heat transfer coefficient. Researchers have shown that average thermal conductivity boosts vary from 15% to 40% above the basic fluid. Other mechanisms include particle agglomeration, nanoparticle size, volume fraction, particle shape/surface area, temperature and liquid layering on the nanoparticle-liquid interface, which attributed to the increase in performance. The development of energy-efficient heat transfer fluids is hindered by a lack of thermal conductivity. The poor heat transfer qualities of common heat transfer fluids including water, ethylene glycol, and motor oil severely limit the heat transfer capabilities of these materials. The thermal conductivities of metals, in contrast to these fluids, can be up to three times higher than those of these fluids. As a result, it is naturally desirable to combine the two substances to produce a heat transfer medium that has the behavior and thermal properties of fluid but the thermal properties of a metal. Choi and Eastman [6] was the pioneer one to use the term nanofluid to describe manufactured colloids made of nanoparticles scattered in a base fluid. In recent years, the improvement of the nanofluid system in the transmission of heat has piqued the interest of researchers and industry representatives from a wide range of disciplines including manufacturing, automotive, and electronics [7-18]. The thermal conductivity behavior of colloidal suspension has been studied by a large number of researchers, and the results have been published several times [19,20]. Zhang *et al.,* [21] evaluated the influence of a three-stage time-varying process on the water-based SiO_2 nanofluid across the boundary layer. The heat and mass transfer investigation under the impact of micropolar nanofluid flow in two parallel plates rotating system by using Adams and explicit Runge–Kutta scheme was considered by Awan *et al.,* [22], and produced the numerical solution of the modelled problem. Recently, Habib *et al.,* [23] worked on the impacts of motile microorganisms and non-linear geometry through thermal radiation micropolar based nanofluids.

The existence of ions and the presence of iron in many physiological fluids cause them to conduct electricity (e.g. hemoglobin in red blood cells). Electrical and magnetic fields have an effect on such fluids. MHD natural convection in the presence of an external magnetic field with water-based nanofluid was studied by Ali *et al.,* [24]. Here, the author develops the governing partial differential equations with a two-component non-homogeneous model and applied Galerkin finite element method to obtain the solution of the models. The simulation outcome shows that the heat transfer rate can be maximized with a suitable combination of governing parameters and minimize the entropy generation as well which are in rational agreement with those of previous literature. Alwawi *et al.,* [25] investigated the free convection flow of Sodium Alginate nanofluid about a solid sphere. The experiment outcome was compared to the other nanoparticles of Sodium Alginate based Casson nanofluid and the result show that the (GO)- Sodium Alginate based Casson nanofluid has the highest velocity profiles, local Nusselt number and local skin friction. Anwar *et al.,* [26] examined the unsteady Casson nanofluid on an infinite vertical plate with ramped wall conditions and MHD flow which incorporates the heat injection/suction and thermal radiation flux. The result shows that the radiative flux leads to an upsurge in the flow while the magnetic field decelerates the flow compared to other existing methods. Under the effect of inclined uniform magnetic field, Sheremet *et al.,* [27] analyze the result of MHD natural convection in a square porous cavity. The study deduced that viscosity parameters increase will likely leads to the convective flow intensification and heat transfer enhancement in addition to intensifying other essential of Rayleigh number with high values. Armaghani *et al.,* [28] studied the entropy generation of Al_2O_3 water alumina nanofluid around of T- shaped baffled cavity and Miroshnichenko *et al.,* [29] investigated the MHD natural convection under the effect of uniform magnetic field of various orientations in a partially open trapezoidal cavity filled with a CuO nanofluid. More so, the heat transfer and MHD natural convection flow was considered in a laterally heated enclosure with an off-centred partition by Kahveci and Öztuna [30]. Later on, Son and Park [31] investigated the two-dimensional laminar natural convection in a uniform magnetic field applied in the horizontal direction in an inversely heated rectangular enclosure with an insulated square block. In the presence of a chemical reaction, Motsa and Shateyi [32] investigated the Soret and Dufour effects on steady MHD natural convection flow over a semi-infinite moving vertical plate. For more details see references [33-43].

In the current work a mathematical model is developed for natural convection around a solid sphere which would predict the behavior of a micropolar nanofluid flow of magnetic field is imposed through a two-dimensional heat and mass transfer channel under prescribed wall temperature. The spherical rheological model is employed to investigate the micropolar nanofluid. It would be interesting to note that the study explored the action of magnetic parameter; microrotation, and nanoparticle volume fraction parameters on heat transmission-related physical quantities through examining the natural convection flow of a micropolar nanofluid as a host for Aluminum Al and Iron oxide Fe_3O_4 nanoparticles are shown graphically and discussed. This was considered in two different types of base liquids specifically, Sodium Alginate, and kerosene oil on a sphere. This was considered in two different types of base liquids specifically, Sodium Alginate, and kerosene oil on a solid sphere. The Keller box method was employed for the numerical approximation of the governing model through MATLAB symbolic software for local skin friction, Nusselt number, linear velocity, angular velocity and temperature on the boundary layer surface of the solid sphere. The numerical results are investigated and compared with previous published data [36,37]. To understand how MHD biomimetic blood pumps work and how nano-scale robots move water in biomedical devices, the study by Swalmeh *et al.,* [44] could be useful.

2. Problem Description

Suppose we have a steady state laminar 2D incompressible free convection flow of Sodium Alginate and kerosene oil around a sphere of radius a, in the presence of aluminum Al and Iron oxide Fe_3O_4 nanoparticles immersed inside them, and considering the constant surface temperature T_w , in addition to imposing a magnetic force with strength B_0^2 as indicated in Figure 1. refers to the gravity vector, and $(\bar{x}, \bar{y})$ coordinates measured along the circumference of the solid sphere starting from $\bar{x} \approx 0$ and measured normal to the surface of the sphere, respectively. At the start, the temperature of both micropolar nanofluid and spheres are equal. Instantaneously, they are growing to a temperature$T_w > T_\infty$, the surrounding temperature of the liquids which keeps fixed.

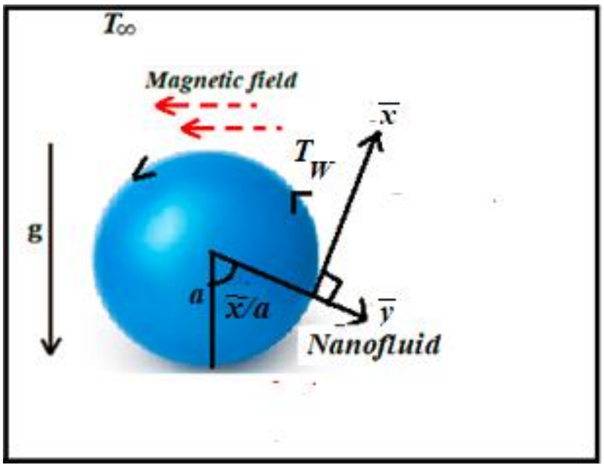

Fig. 1. Configuration model and coordinate system

Depending on the above assumptions, Tiwari and Das [45] model, magnetic effects, and the micropolar equations, we can construct the following dimensional governing equations [45-50]

$$\frac{\partial(\bar{r}\bar{u})}{\partial\bar{x}} + \frac{\partial(\bar{r}\bar{v})}{\partial\bar{y}} = 0, \quad (1)$$

$$\rho_{nf}\left(\bar{u}\,\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right) = \left(\mu_{nf} + \kappa\right)\left(\frac{\partial^2\bar{u}}{\partial\bar{x}^2}\right) + (\beta)_{nf}\,\mathrm{g}\,(T - T_\infty)\sin\frac{\bar{x}}{a} + \kappa\bar{v}\frac{\partial\bar{H}}{\partial\bar{y}} - \sigma_{nf}B_0^2\bar{u} \quad (2)$$

$$\bar{u}\,\frac{\partial T}{\partial\bar{x}} + \bar{v}\,\frac{\partial T}{\partial\bar{y}} = \alpha_{nf}\left(\frac{\partial^2\bar{T}}{\partial\bar{y}^2}\right) \quad (3)$$

$$\rho_{nf}j\left(\bar{u}\,\frac{\partial\bar{H}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{H}}{\partial\bar{y}}\right) = -\kappa\left(2\bar{H} + \frac{\partial\bar{u}}{\partial\bar{y}}\right) + \emptyset_{nf}\frac{\partial^2\bar{H}}{\partial\bar{y}^2} \quad (4)$$

subject to [40]

$$\bar{u} = 0, \bar{v} = 0, T = T_w, \text{as } \bar{y} = 0,$$
$$\bar{u} \to 0, T \to T_\infty,, \text{as } \bar{y} \to \infty, \quad (5)$$

where $(\bar{u}, \bar{v})$ indicate the velocity components along in $\bar{x}$, and $\bar{x}, \bar{y}$ directions, $j = a^2 Gr^{-1/2}, \bar{y}$ is micro-inertia density, $\emptyset_{nf} = \left(\mu_{nf} + \frac{\kappa}{2}\right)j$ is the spin gradient viscosity. $\bar{H}$ indicates the angular velocity, T symbolizes the temperature, and κ refers to vortex viscosity.ρ, μ, α, σ, and β, indicate density, viscosity, thermal diffusivity, electrical conductivity, and thermal expansion coefficient. The subscripts *s, f, nf* symbolize the nanoparticles, base fluid and nanofluid. The properties of nanofluid are (see Alkasasbeh *et al.,* [50])

$$(\beta)_{nf} = (\chi\,(\beta)_s + (1-\chi)(\beta)_f\,), (\mu)_{nf} \;=\; \mu_f\,/(\,1-\,\chi)^{2.5}\,,$$
$$(\rho c_p)_{nf} = (\chi\,(\rho c_p)_s + (1-\chi)(\rho c_p)_f\,), (\alpha)_{nf} = k_{nf}/(\,\rho c_p\,)_{nf}\,,$$
$$(\rho)_{nf} = (\chi(\,\rho)_s + (1-\chi)(\rho)_f\,), \frac{k_{nf}}{k_f} = \frac{(\,k_s+2\,k_f\,)-2\,\chi\,(\,k_f-\,k_s\,)}{(\,k_s+2\,k_f\,)+\chi(\,k_f-\,k_s\,)}, \sigma_{nf} = 1 + \frac{3\left(\frac{\sigma_S}{\sigma_f}-1\right)\chi}{\left(\frac{\sigma_S}{\sigma_f}+2\right)-\chi(\frac{\sigma_S}{\sigma_f}-1\,)} \tag{6}$$

where $\chi, k,$ and $(\,\rho c_p\,)$ refer to the nanoparticle volume fraction, thermal conductivity, and heat capacity. In order to non-dimensionlization process, introducing the following dimensionless variables (see Alwawi *et al.,* [48])

$$r = \left(\frac{\bar{r}}{a}\right), x = \left(\frac{\bar{x}}{a}\right), y = (Gr)^{(1/4)}\,(\frac{\bar{y}}{a}), \theta = \frac{T - T_\infty}{T_w -\, T_\infty}, u = (Gr)^{(-1/2)}\,(\frac{a\bar{u}}{v_f}),$$
$$v = (Gr)^{(-1/4)}\left(\frac{a\bar{v}}{v_f}\right), H = (Gr)^{(-3/4)}\,(\frac{a^2}{v_f})\bar{H} \tag{7}$$

Here, the Grashof number is represented by $Gr = g\,(\beta)_f(T_w - T_\infty)\,a^3/v_f^2$, $\bar{r}(\bar{x})\;a\sin(\bar{x}/a)$ indicates the radial distance from the symmetrical axis to the surface of the sphere. After substituting nanofluid properties (6) and dimensionless variables (7) in Eq. (1) to Eq. (5), and then utilizing the boundary approximations, which are Grashof number Gr → ∞, which can be characterized as the alternative expressions: (1/Gr) →0, we neglected the terms that contain (1/Gr), which goes to 0. We get the following dimensionless model [51]

$$u\,\frac{\partial ru}{\partial x} + v\,\frac{\partial ru}{\partial y} = 0 \tag{8}$$

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \frac{\rho_f}{\rho_{nf}}(D(\chi) + \mathrm{K})\left(\frac{\partial^2 u}{\partial y^2}\right) + \frac{1}{\rho_{nf}}\left((1-\chi)\,\rho_f + \chi\frac{\rho_s\beta_s}{\beta_f}\right)\theta\sin x + \frac{\rho_f}{\rho_{nf}}K\frac{\partial H}{\partial y} - \frac{\rho_f}{\rho_{nf}}\mathrm{M}u \tag{9}$$

$$u\,\frac{\partial\theta}{\partial x} + v\,\frac{\partial\theta}{\partial y} = \frac{1}{\mathrm{Pr}}\left(\frac{k_{nf}/k_f}{(1-\chi) + \chi\frac{(\rho c_p)_S}{(\rho c_p)_f}}\right)\left(\frac{\partial^2\theta}{\partial\bar{y}^2}\right), \tag{10}$$

$$u\,\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y} = -\frac{\rho_f}{\rho_{nf}}\mathrm{K}\left(\,2H + \frac{\partial u}{\partial y}\right) + \frac{\rho_f}{\rho_{nf}}\left(D(\chi) + \frac{\mathrm{K}}{2}\right)\left(\frac{\partial^2 H}{\partial y^2}\right). \tag{11}$$

where $D(\chi) = (1-\chi)^{-2.5}$, $\mathrm{Pr} = \frac{v_f}{\alpha_f}$ is the Prandtl number, $\mathrm{K} = \frac{\kappa}{\mu_f}$ is micro-rotation parameter $\mathrm{M} = \frac{\sigma_f B_0^2 a^2}{\rho_f v_f Gr^{1/2}}$ is magnetic parameter.

The conditions (5) convert to

$$u = 0, v = 0, \theta = 1, H = \frac{1}{2}\frac{\partial u}{\partial y} \text{ as } y = 0,$$
$$u \to 0, \theta \to 0, H \to 0 \text{ as } y \to \infty. \tag{12}$$

In order to transform the above system (8) to (12) to PDEs, introduced the following transformation [52-54]

$$\psi = xr(x)\, f(\,x, y\,), \theta = \theta(\,x, y\,), H = x\, h(\,x, y\,), \tag{13}$$

where ψ is the stream function which given by the following relation

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y}, \text{ and } v = \frac{1}{r}\frac{\partial \psi}{\partial x}, \tag{14}$$

Consequently, Eq. (8) to Eq. (11) turn into

$$\frac{\rho_f}{\rho_{nf}}(D(\chi) + K\,)\frac{\partial^3 f}{\partial x^3} + (1 + xcotx)\, f\frac{\partial^2 f}{\partial x^2} - \left(\frac{\partial f}{\partial y}\right)^2 + \frac{1}{\rho_{nf}}\left(\,\chi\, \rho_s(\beta_s/\beta_f) + (\,1 - \chi)\, \rho_f\right)\theta\frac{\sin x}{x} +$$
$$\frac{\rho_f}{\rho_{nf}}\mathrm{K}\left(\frac{\partial h}{\partial y}\right) - \frac{\rho_f}{\rho_{nf}}\frac{\sigma_f}{\sigma_{nf}}\mathrm{M}f' = x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right) \tag{15}$$

$$\frac{1}{\mathrm{Pr}}\left(\frac{k_{nf}/k_f}{(\,1 - \chi) + \chi\,(\rho c_p)_s\,/\,(\rho c_p)_f}\right)\left(\frac{\partial^2 \theta}{\partial y^2}\right) + (1 + xcotx)f\frac{\partial \theta}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \theta}{\partial y}\right), \tag{16}$$

$$\frac{\rho_f}{\rho_{nf}}\left(D(\chi) + \frac{\mathrm{K}}{2}\right)\left(\frac{\partial^2 h}{\partial y^2}\right) + (1 + xcotx)f\frac{\partial h}{\partial y} - \frac{\partial f}{\partial y}h - \frac{\rho_f}{\rho_{nf}}\mathrm{K}\left(2h + \frac{\partial^2 f}{\partial y^2}\right) = x\left(\frac{\partial f}{\partial y}\frac{\partial h}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial h}{\partial y}\right), \tag{17}$$

subject to

$$f = \frac{\partial f}{\partial y} = 0, \theta = 1, h = -(1/2)\frac{\partial^2 f}{\partial y^2} \text{ as } y = 0,$$
$$\frac{\partial f}{\partial y} \to 0, \theta \to 0, h \to 0 \text{ as } y \to \infty. \tag{18}$$

In this paper, our focus on two physical quantities specifically, the local skin friction coefficien C_f and Nusselt number Nu which can be expressed as

$$C_f = Gr^{-1/4}\left(D(\chi) + \frac{K}{2}\right)x\frac{\partial^2 f}{\partial y^2}\,(x, 0), Nu = -Gr^{1/4}\,\frac{k_f}{k_{nf}}\left(\frac{\partial \theta}{\partial y}\right)(x, 0) \tag{19}$$

3. Results and Discussion

The Discussion section that should describe the relationships and generalizations shown by the results and discuss the significance of the results, making comparisons with previously published work. It may be appropriate to combine the Results and Discussion sections into a single section to improve clarity.

In this work, a used method to approximate the analytical solution to our problem is called the Keller box method. This method has proven its ability and effectiveness for more than three decades in dealing with issues related to boundary layers. Firstly, the implicit finite difference method is used to reduce these equations to the first order; the central differences method is employed to attain the differences equations. Next, the differences equations are linearized by

Newton's method and presented in a matrix-vector form. The resulting linear system is solved by applying the block tri-diagonal elimination technique. Finally, this technique is programmed via MATLAB program codes to get the new numerical results as tables and figures. For more details on this method, see the references [44,55-57].

Graphic outcomes were analyzed and discussed for the effects of magnetic parameter (M > 0) micro rotation parameter (K= 0.1, 0.2, 0.3) and volume fraction (χ = 0.1, 0.15, 0.2) on Nusselt number, local skin friction, temperature, velocity and angular velocity. Table 1 illustrates Thermo-physical properties of employed nanoparticles with based fluid.

Table 1
Thermo-physical properties of used nanoparticles with based fluid [44]

	Based fluids		Nanoparticles	
Physical properties	Kerosene Oil	Sodium Alginate	Al	Fe_3O_4
k (W/mK)	0.145	0.6376	237	80.4
ρ(kg/m$^{3)}$	783	989	2701	5180
C_p (J/kgK)	2090	4175	902	670
β × 10−5/K	99	99	2.31	20.6
σs(Sm^{-1})	5×10^{-11}	2.6 × 10^{-4}	3.5x10^{7}	1.12 x10^{5}

In order to verify and confirm the validity of the results, they were compared with the results published in the previous literature, where they achieved an excellent match, see Table 2 and Table 3.

Table 2
Comparison of the outcomes for *Nu* at Pr = 7, χ = 0, M = 0, and K = 0

x	Huang and Chen [36]	Nazar and Amin [37]	Swalmeh *et al.,* [47]	Present
0	0.9581	0.9595	0.9582	0.9590
$(1/18)\pi$	0.9559	0.9572	0.9561	0.9566
$(1/9)\pi$	0.9496	0.9506	0.9497	0.9498
$(1/6)\pi$	0.9389	0.9397	0.9391	0.9395
$(2/9)\pi$	0.9239	0.9243	0.9241	0.9240
$(5/18)\pi$	0.9045	0.9045	0.9046	0.9044
$(1/3)\pi$	0.8858	0.8801	0.8806	0.8826
$(7/18)\pi$	0.8518	0.8510	0.8519	0.8537
$(4/9)\pi$	0.8182	0.8168	0.8188	0.8173
$(1/2)\pi$	0.7792	0.7792	0.7798	0.7790

Table 3
Comparison of the outcomes for C_f at Pr = 7, χ = 0, M = 0, and K = 0

x	Huang and Chen [36]	Nazar and Amin [37]	Swalmeh *et al.,* [47]	Present
0	0.0000	0.0000	0.0000	0.0000
$(1/18)\pi$	0.0876	0.0875	0.0877	0.0877
$(1/9)\pi$	0.1737	0.1735	0.1739	0.1739
$(1/6)\pi$	0.2566	0.2563	0.2569	0.2567
$(2/9)\pi$	0.3350	0.3345	0.3354	0.3353
$(5/18)\pi$	0.4075	0.4068	0.4079	0.4076
$(1/3)\pi$	0.4727	0.4715	0.4729	0.4731
$(7/18)\pi$	0.5293	0.5380	0.5294	0.5396
$(4/9)\pi$	0.5762	0.5745	0.5773	0.5768
$(1/2)\pi$	0.6123	0.6103	0.6129	0.6131

We start by studying the effect of changing main parameters: magnetic parameter M, micro rotation parameter K, and volume fraction χ on Nusselt number *Nu* and local skin friction C_f, in Figure 2 to Figure 7. Firstly, the increasing the magnetic parameter leads to decreasing both the Nusselt number *Nu* as seen in Figure 2 and the local skin friction C_f as seen in Figure 3. The investigated inverse relation between the magnetic parameter M and Nusselt number may refer to Grashof number Gr, the common factor between them. Gr has a direct relation with Nusselt number *Nu* and reverse relation with M (see Eq. (23)).

Second, the increment of the micro rotation parameter causes a decrement in the Nusselt number values of the micropolar nanofluid as illustrated in Figure 4. In Figure 5 the opposite has happened with local skin friction and micro rotation parameter K, As the values of K increase the skin friction curves are getting raised up to higher values.

Third, growing the volume fraction χ influence is reducing the curves of Nusselt number Nu (Figure 6) and local skin friction C_f (Figure 7). It is notice in previous figures that the micropolar fluid suspended by Al nanoparticles always has the higher values of Nusselt number and local skin friction as compared with the same micropolar fluid suspended by Fe_3O_4 nanoparticle.

The last thing that could be observed in this category of figures (Figure 2 to Figure 7) that if we use a Kerosene oil as a base fluid in the mixture of micropolar nanofluids its Nusselt number quantity results keep superior through all the conditions of changing the parameters M, K, and χ over the results of the micropolar mixture with sodium Alginate base fluid, which means at this case the heat transfers to the solid sphere by convection is more than it by conduction and the skin friction quantity between the Kerosene oil nanofluid mixture and the solid sphere is lower as compared with sodium Alginate nanofluid mixture the possible reason that explains this sole superiority for Kerosene oil is the low value of its thermal conductivity (see Table 1) which effect on Nu (see Eq. (23)).

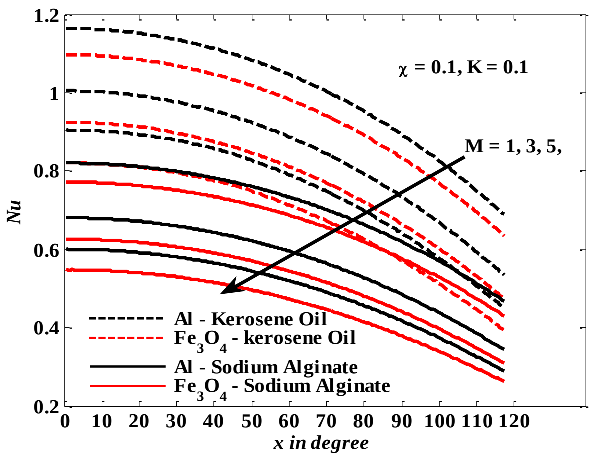

Fig. 2. Variation of Nusselt number for different values of x and M

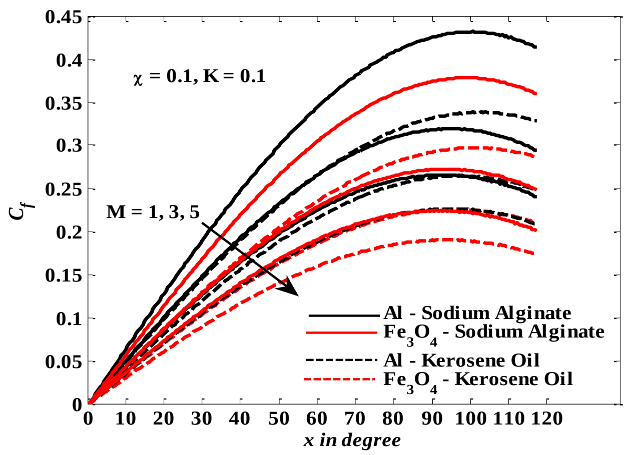

Fig. 3. Variation of local skin friction for different values of x and M

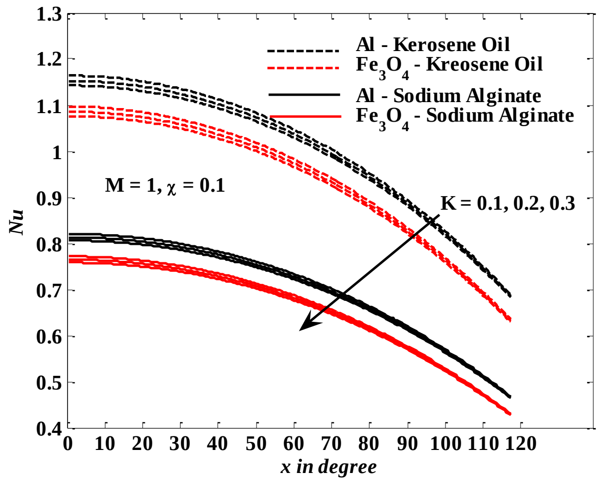

Fig. 4. Variation of Nusselt number for different values of x and K

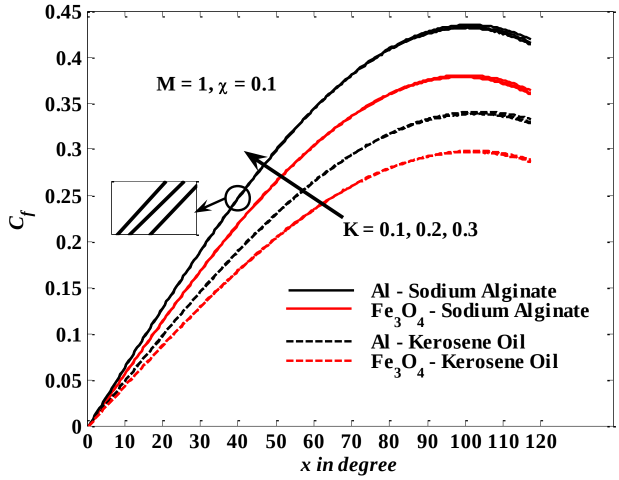

Fig. 5. Variation of local skin friction for different values of x and K

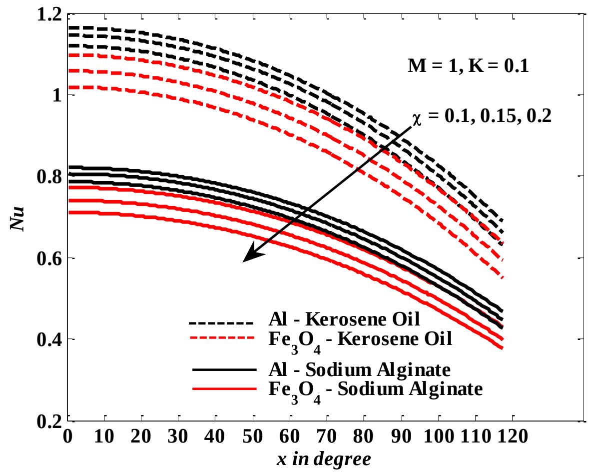

Fig. 6. Variation of Nusselt number for different values of x and χ

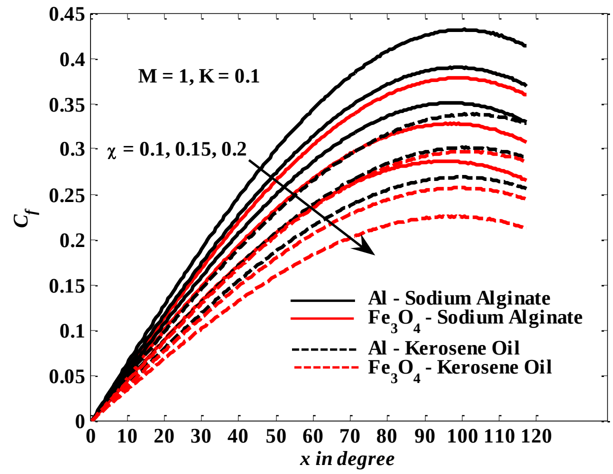

Fig. 7. Variation of local skin friction for different values of x and χ

Figure 8 to Figure 16 exhibit the impact of magnetic parameter M, micro rotation parameter K, and volume fraction χ on temperature, velocity, and angular velocity field. It's found that there is a negative correlation between magnetic parameter M and both velocity, and angular velocity field, while the correlation between magnetic parameter and temperature is positive (see Figure 8 to Figure 10).

Figure 11, Figure 12 and Figure 13 plotted the relation between micro rotation parameter K and the other physical quantities: temperature, velocity, and angular velocity field. The direct relation between them is clear, the increment of K leads to growing up each individual mentioned quantity.

Finally, the variance of temperature, velocity, and angular velocity field due to changing the volume fraction is investigated in Figure 14, Figure 15, and Figure 16. Temperature and velocity react directly to the increment of χ by scaling the curves up. However, the angular velocity behaves inversely proportional to χ as shown in Figure 16. In this category, an interesting note worth mention in magnetite Fe_3O_4 nanoparticles as compared with Aluminum Al nanoparticle, the micropolar fluid containing Fe_3O_4 nanoparticle has the lower values of all the physical quantities that investigated in this study except the temperature profile quantity. Fe_3O_4 Sodium Alginate micropolar nanofluid has the highest values through the changing of all parameters M, K, and χ, and this may attribute to the physical properties of especially Fe_3O_4 the low value of its effective thermal conductivity as seen in Table 1.

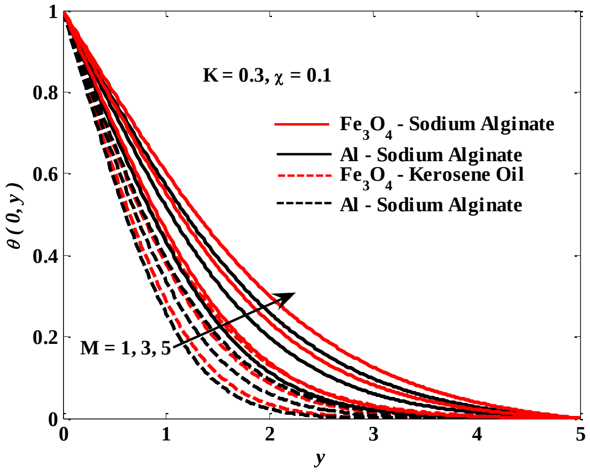

Fig. 8. Variation of temperature for different values of y and M

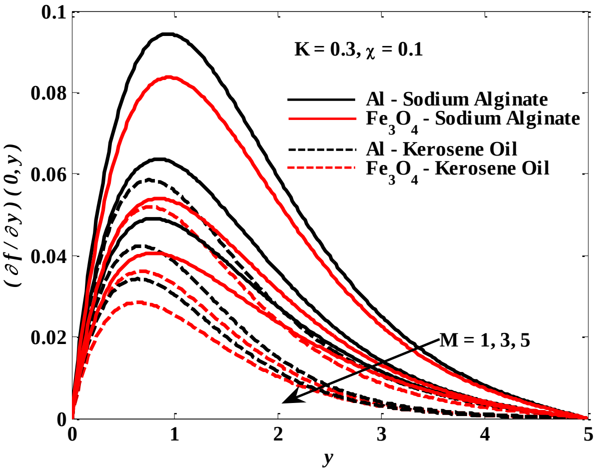

Fig. 9. Variation of velocity for different values of y and M

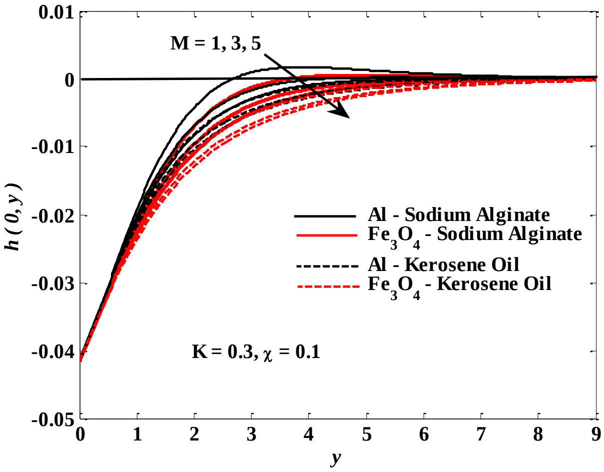

Fig. 10. Variation of angular velocity field for different values of y and M

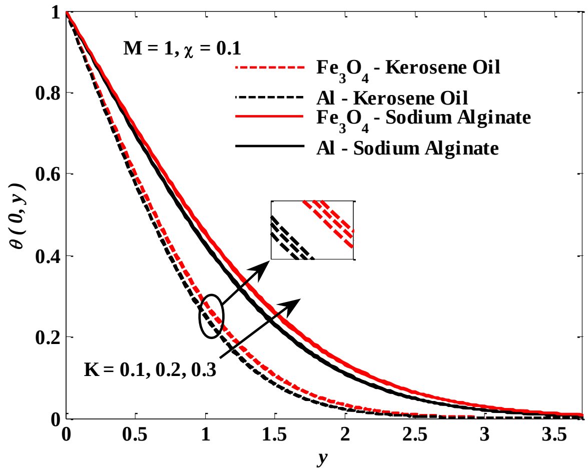

Fig. 11. Variation of Temperature for different values of y and K

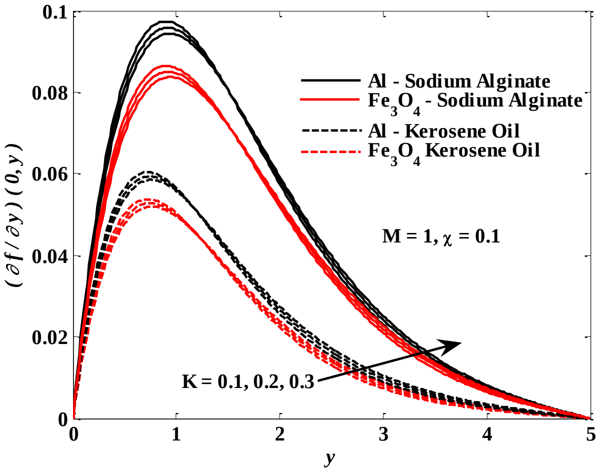

Fig. 12. Variation of velocity for different values of y and K

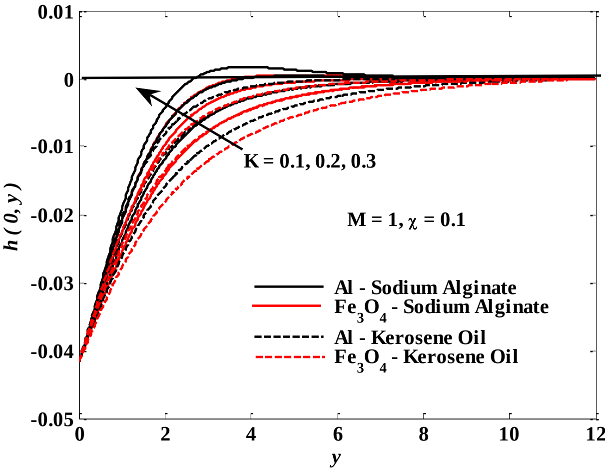

Fig. 13. Variation of angular velocity field for different values of y and K

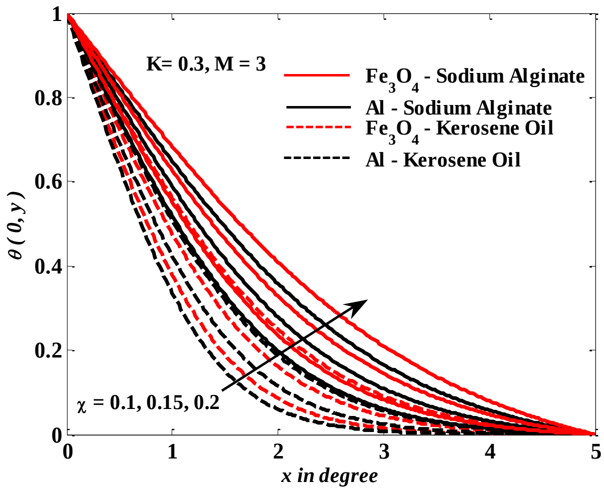

Fig. 14. Variation of temperature for different values of x and χ

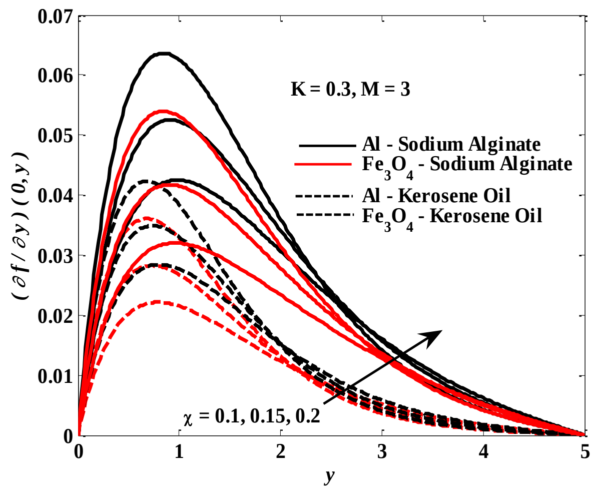

Fig. 15. Variation of velocity for different values of y and χ

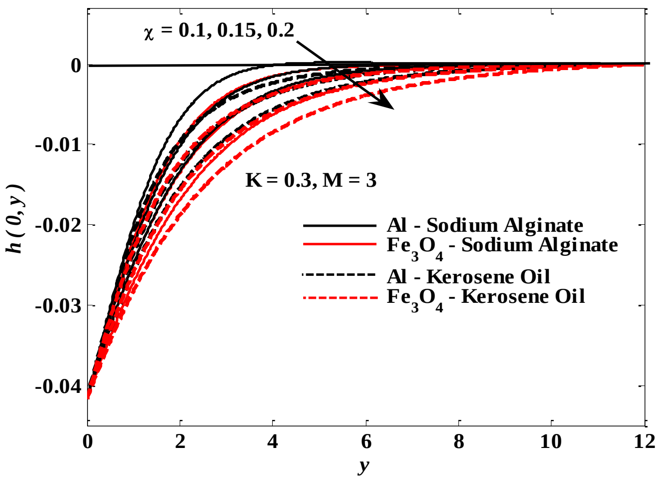

Fig. 16. Variation of angular velocity field for different values of y and χ

4. Conclusions

The current chapter explored the action of the magnetic parameter M, microrotation parameter K, and volume fraction χ on the following physical quantities: Nusselt number, local skin friction, temperature, velocity, and angular velocity field through free convection flow of a micropolar nanofluid of (Al) and (Fe_3O_4) in two different types of base fluids specifically, Sodium Alginate, and kerosene oil on a solid sphere of radius. The points below concluded

i. Kerosene oil as a base micropolar fluid with Al nanoparticle (Al- Kerosene oil) has the upper values of Nusselt number through each parameter variation in this study and that's mean the heat transfer by convection is the maximum at this case. while the Kerosene oil gains the lower values of the rest physical quantities such as local skin friction C_f temperature, velocity, and angular velocity field as compared to sodium alginate.
ii. Adding the magnetite (Fe_3O_4) nanoparticle to the base micropolar fluids leads to getting the highest curves of temperature that varies with y and any other parameters such as M, K, and χ, as a comparison with adding Al nanoparticle to the same fluids.

iii. Increasing the magnetic parameter M causes an increment in the temperature and decrement in Nusselt number Nu, local skin friction, velocity, and angular velocity field.
iv. Raising values of the micro rotation parameter reflects on raising: Nusselt number Nu, local skin friction, velocity, angular velocity field, and decreasing temperature.
v. When the volume fraction of nanoparticles increases, both quantities of temperature and velocity, increase and the quantities of Nusselt number Nu and Angular velocity field decrease.
vi. The influences of studied parameters on the physical quantities can be summarized as shown in Table 4.

Table 4
Summary of the relation between the examined parameters and physical quantities

	Nu	C_f	$\theta(0,y)$	$f'(0,y)$	$h(0,y)$
M↑	↓	↓	↑	↓	↓
K↑	↓	↑	↑	↑	↑
χ↑	↓	↓	↑	↑	↓

Acknowledgement
This research was supported by Aqaba University of Technology.

References

[1] Abbas, Amir, Muhammad Ashraf, and Ali Jawad Chamkha. "Combined effects of thermal radiation and thermophoretic motion on mixed convection boundary layer flow." *Alexandria Engineering Journal* 60, no. 3 (2021): 3243-3252. https://doi.org/10.1016/j.aej.2021.01.038
[2] Madasu, Krisad Prasad. "Boundary effects of a nonconcentric semipermeable sphere using Happel and Kuwabara cell models." *Applied and Computational Mechanics* 15, no. 1 (2021): 19-30. https://doi.org/10.24132/acm.2021.620
[3] Mohamed, R. A., F. M. Hady, A. Mahdy, and Omima A. Abo-zai. "Laminar MHD natural convection flow due to non-Newtonian nanofluid with dust nanoparticles around an isothermal sphere: non-similar solution." *Physica Scripta* 96, no. 3 (2021): 035215. https://doi.org/10.1088/1402-4896/abd795
[4] Jenifer, A. Sahaya, P. Saikrishnan, and Roland W. Lewis. "Unsteady MHD Mixed Convection Flow of Water over a Sphere with Mass Transfer ".*Journal of Applied and Computational Mechanics* 7, no. 2 (2021): 935-943.
[5] Rodriguez, I., O. Lehmkuhl, and M. Soria. "On the effects of the free-stream turbulence on the heat transfer from a sphere." *International Journal of Heat and Mass Transfer* 164 (2021): 120579. https://doi.org/10.1016/j.ijheatmasstransfer.2020.120579
[6] Choi, S. US, and Jeffrey A. Eastman. *Enhancing thermal conductivity of fluids with nanoparticles*. No. ANL/MSD/CP-84938; CONF-951135-29. Argonne National Lab. (ANL), Argonne, IL (United States), 1995.
[7] Sheikholeslami, M., Seyyed Ali Farshad, Z. Ebrahimpour, and Zafar Said. "Recent progress on flat plate solar collectors and photovoltaic systems in the presence of nanofluid: a review." *Journal of Cleaner Production* 293 (2021): 126119. https://doi.org/10.1016/j.jclepro.2021.126119
[8] Jamshed, Wasim. "Numerical investigation of MHD impact on Maxwell nanofluid." *International Communications in Heat and Mass Transfer* 120 (2021): 104973. https://doi.org/10.1016/j.icheatmasstransfer.2020.104973
[9] Qin, Yinghong. "Nanofluid heat transfer within a pipe equipped with external device." *International Communications in Heat and Mass Transfer* 127 (2021): 105487. https://doi.org/10.1016/j.icheatmasstransfer.2021.105487
[10] Muhammad, Khursheed, T. Hayat, A. Alsaedi, and B. Ahmad. "Melting heat transfer in squeezing flow of basefluid (water), nanofluid (CNTs+ water) and hybrid nanofluid (CNTs+ CuO+ water)." *Journal of Thermal Analysis and Calorimetry* 143, no. 2 (2021): 1157-1174. https://doi.org/10.1007/s10973-020-09391-7
[11] Rostami, Sara, Rasool Kalbasi, Nima Sina, and Aysan Shahsavar Goldanlou. "Forecasting the thermal conductivity of a nanofluid using artificial neural networks." *Journal of Thermal Analysis and Calorimetry* 145, no. 4 (2021): 2095-2104. https://doi.org/10.1007/s10973-020-10183-2

[12] Awais, Muhammad, Najeeb Ullah, Javaid Ahmad, Faizan Sikandar, Mohammad Monjurul Ehsan, Sayedus Salehin, and Arafat A. Bhuiyan. "Heat transfer and pressure drop performance of Nanofluid: A state-of-the-art review." *International Journal of Thermofluids* 9 (2021): 100065. https://doi.org/10.1016/j.ijft.2021.100065

[13] Kausar, Muhammad Salman, Abid Hussanan, Muhammad Qasim, and Mustafa Mamat. "Flow of Water Based Nanofluid Containing Different Shapes of Cu Nanoparticles Embedded in a Porous Medium." *International Journal of Applied and Computational Mathematics* 7, no. 3 (2021): 1-16. https://doi.org/10.1007/s40819-021-01042-1

[14] Khoramian, Reza, Riyaz Kharrat, and Saeed Golshokooh. "The development of novel nanofluid for enhanced oil recovery application." *Fuel* 311 (2022): 122558. https://doi.org/10.1016/j.fuel.2021.122558

[15] Kanti, Praveen Kumar, K. V. Sharma, Zafar Said, Mehdi Jamei, and Kyathanahalli Marigowda Yashawantha. "Experimental investigation on thermal conductivity of fly ash nanofluid and fly ash-Cu hybrid nanofluid: prediction and optimization via ANN and MGGP model." *Particulate Science and Technology* 40, no. 2 (2022): 182-195. https://doi.org/10.1080/02726351.2021.1929610

[16] Ahmad, Shafiq, Sohail Nadeem, and Muhammad Naveed Khan. "Enhanced transport properties and its theoretical analysis in two-phase hybrid nanofluid." *Applied Nanoscience* 12, no. 3 (2022): 309-316. https://doi.org/10.1007/s13204-020-01634-1

[17] Wang, Jinyuan, Yi-Peng Xu, Raed Qahiti, M. Jafaryar, Mashhour A. Alazwari, Nidal H. Abu-Hamdeh, Alibek Issakhov, and Mahmoud M. Selim. "Simulation of hybrid nanofluid flow within a microchannel heat sink considering porous media analyzing CPU stability." *Journal of Petroleum Science and Engineering* 208 (2022): 109734. https://doi.org/10.1016/j.petrol.2021.109734

[18] Acharya, N., F. Mabood, S. A. Shahzad, and I. A. Badruddin. "Hydrothermal variations of radiative nanofluid flow by the influence of nanoparticles diameter and nanolayer." *International Communications in Heat and Mass Transfer* 130 (2022): 105781. https://doi.org/10.1016/j.icheatmasstransfer.2021.105781

[19] Karimipour, Arash, Seyed Amin Bagherzadeh, Abdolmajid Taghipour, Ali Abdollahi, and Mohammad Reza Safaei. "A novel nonlinear regression model of SVR as a substitute for ANN to predict conductivity of MWCNT-CuO/water hybrid nanofluid based on empirical data." *Physica A: Statistical Mechanics and its Applications* 521 (2019): 89-97. https://doi.org/10.1016/j.physa.2019.01.055

[20] Sofiah, A. G. N., M. Samykano, S. Shahabuddin, K. Kadirgama, and A. K. Pandey. "A comparative experimental study on the physical behavior of mono and hybrid RBD palm olein based nanofluids using CuO nanoparticles and PANI nanofibers." *International Communications in Heat and Mass Transfer* 120 (2021): 105006. https://doi.org/10.1016/j.icheatmasstransfer.2020.105006

[21] Zhang, Tiancheng, Quanle Zou, Zhiheng Cheng, Zihan Chen, Ying Liu, and Zebiao Jiang. "Effect of particle concentration on the stability of water-based SiO_2 nanofluid." *Powder Technology* 379 (2021): 457-465. https://doi.org/10.1016/j.powtec.2020.10.073

[22] Awan, Saeed Ehsan, Muhammad Asif Zahoor Raja, Faiza Gul, Zuhaib Ashfaq Khan, Ammara Mehmood, and Muhammad Shoaib. "Numerical computing paradigm for investigation of micropolar nanofluid flow between parallel plates system with impact of electrical MHD and Hall current." *Arabian Journal for Science and Engineering* 46, no. 1 (2021): 645-662. https://doi.org/10.1007/s13369-020-04736-8

[23] Habib, Danial, Sohaib Abdal, Rifaqat Ali, Dumitru Baleanu, and Imran Siddique. "On bioconvection and mass transpiration of micropolar nanofluid dynamics due to an extending surface in existence of thermal radiations." *Case Studies in Thermal Engineering* 27 (2021): 101239. https://doi.org/10.1016/j.csite.2021.101239

[24] Ali, Mohammad Mokaddes, Rowsanara Akhter, and Md Alim. "MHD natural convection and entropy generation in a grooved enclosure filled with nanofluid using two-component non-homogeneous model." *SN Applied Sciences* 2, no. 4 (2020): 1-25. https://doi.org/10.1007/s42452-020-2319-x

[25] Alwawi, Firas A., Hamzeh T. Alkasasbeh, A. M. Rashad, and Ruwaidiah Idris. "MHD natural convection of Sodium Alginate Casson nanofluid over a solid sphere." *Results in Physics* 16 (2020): 102818. https://doi.org/10.1016/j.rinp.2019.102818

[26] Anwar, Talha, Poom Kumam, and Wiboonsak Watthayu. "Unsteady MHD natural convection flow of Casson fluid incorporating thermal radiative flux and heat injection/suction mechanism under variable wall conditions." *Scientific Reports* 11, no. 1 (2021): 1-15. https://doi.org/10.1038/s41598-021-83691-2

[27] Sheremet, M. A., M. S. Astanina, and I. Pop. "MHD natural convection in a square porous cavity filled with a water-based magnetic fluid in the presence of geothermal viscosity." *International Journal of Numerical Methods for Heat & Fluid Flow* 28, no. 9 (2018): 2111-2131. https://doi.org/10.1108/HFF-12-2017-0503

[28] Armaghani, Taher, A. Kasaeipoor, Mohsen Izadi, and Ioan Pop. "MHD natural convection and entropy analysis of a nanofluid inside T-shaped baffled enclosure." *International Journal of Numerical Methods for Heat & Fluid Flow* 28, no. 12 (2018): 2916-2941. https://doi.org/10.1108/HFF-02-2018-0041

[29] Miroshnichenko, Igor V., Mikhail A. Sheremet, Hakan F. Oztop, and Khaled Al-Salem. "MHD natural convection in a partially open trapezoidal cavity filled with a nanofluid." *International Journal of Mechanical Sciences* 119 (2016): 294-302. https://doi.org/10.1016/j.ijmecsci.2016.11.001

[30] Kahveci, Kamil, and Semiha Öztuna. "MHD natural convection flow and heat transfer in a laterally heated partitioned enclosure." *European Journal of Mechanics-B/Fluids* 28, no. 6 (2009): 744-752. https://doi.org/10.1016/j.euromechflu.2009.07.001

[31] Son, Jong Hyeon, and Il Seouk Park. "Numerical study of MHD natural convection in a rectangular enclosure with an insulated block." *Numerical Heat Transfer, Part A: Applications* 71, no. 10 (2017): 1004-1022. https://doi.org/10.1080/10407782.2017.1330090

[32] Motsa, Sandile, and Stanford Shateyi. "Soret and Dufour effects on steady MHD natural convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation in the presence of a chemical reaction." In *Evaporation, Condensation and Heat Transfer*. IntechOpen, 2011. https://doi.org/10.5772/19683

[33] Hamarsheh, Abdulkareem Saleh, Firas A. Alwawi, Hamzeh T. Alkasasbeh, Ahmed M. Rashad, and Ruwaidiah Idris. "Heat transfer improvement in MHD natural convection flow of graphite oxide/carbon nanotubes-methanol based casson nanofluids past a horizontal circular cylinder." *Processes* 8, no. 11 (2020): 1444. https://doi.org/10.3390/pr8111444

[34] Vijaybabu, T. R., and S. Dhinakaran. "MHD Natural convection around a permeable triangular cylinder inside a square enclosure filled with Al2O3− H2O nanofluid: An LBM study." *International Journal of Mechanical Sciences* 153 (2019): 500-516. https://doi.org/10.1016/j.ijmecsci.2019.02.003

[35] Mebarek-Oudina, Fateh, and Oluwole Daniel Makinde. "Numerical simulation of oscillatory MHD natural convection in cylindrical annulus: Prandtl number effect." In *Defect and Diffusion Forum*, vol. 387, pp. 417-427. Trans Tech Publications Ltd, 2018. https://doi.org/10.4028/www.scientific.net/DDF.387.417

[36] Huang, Mingjer, and Gahokuang Chen. "Laminar free convection from a sphere with blowing and suction." *Journal of Heat Transfer (Transactions of the ASME (American Society of Mechanical Engineers), Series C);(United States)* 109, no. 2 (1987). https://doi.org/10.1115/1.3248117

[37] Nazar, R., and N. Amin. "Free convection boundary layer on an isothermal sphere in a micropolar fluid." *International Communications in Heat and Mass Transfer* 29, no. 3 (2002): 377-386. https://doi.org/10.1016/S0735-1933(02)00327-5

[38] Swalmeh, M. Z., Alkasasbeh, H. T., Hussanan, A., and Mamat, M. "Numerical Study of Mixed Convection Heat Transfer in Methanol based Micropolar Nanofluid about a Horizontal Circular Cylinder." In *Journal of Physics: Conference Series* 1366, No. 1, (2019), p. 012003. IOP Publishing. https://doi.org/10.1088/1742-6596/1366/1/012003

[39] Alwawi, Firas A., Mohammed Z. Swalmeh, Amjad S. Qazaq, and Ruwaidiah Idris. "Heat Transmission Reinforcers Induced by MHD Hybrid Nanoparticles for Water/Water-EG Flowing over a Cylinder." *Coatings* 11, no. 6 (2021): 623. https://doi.org/10.3390/coatings11060623

[40] Gul, Taza, Saleem Nasir, Saeed Islam, Zahir Shah, and M. Altaf Khan. "Effective Prandtl Number Model Influences on the γAl2O3-H2O and γAl2O3-C2H6O2 Nanofluids Spray Along a Stretching Cylinder." *Arabian Journal for Science & Engineering (Springer Science & Business Media BV)* 44, no. 2 (2019).

[41] Li, Yi-Xia, Taseer Muhammad, Muhammad Bilal, Muhammad Altaf Khan, Ali Ahmadian, and Bruno A. Pansera. "Fractional simulation for Darcy-Forchheimer hybrid nanoliquid flow with partial slip over a spinning disk." *Alexandria Engineering Journal* 60, no. 5 (2021): 4787-4796. https://doi.org/10.1016/j.aej.2021.03.062

[42] Ahmadian, Ali, Muhammad Bilal, Muhammad Altaf Khan, and Muhammad Imran Asjad. "The non-Newtonian maxwell nanofluid flow between two parallel rotating disks under the effects of magnetic field." *Scientific Reports* 10, no. 1 (2020): 1-14. https://doi.org/10.1038/s41598-020-74096-8

[43] Zuhra, Samina, Noor Saeed Khan, Muhammad Altaf Khan, Saeed Islam, Waris Khan, and Ebenezer Bonyah. "Flow and heat transfer in water based liquid film fluids dispensed with graphene nanoparticles." *Results in Physics* 8 (2018): 1143-1157. https://doi.org/10.1016/j.rinp.2018.01.032

[44] Swalmeh, Mohammed Z., Hamzeh T. Alkasasbeh, Abid Hussanan, T. Nguyen Thoi, and Mustafa Mamat. "Microstructure and inertial effects on natural convection micropolar nanofluid flow about a solid sphere." *International Journal of Ambient Energy* 43, no. 1 (2022): 666-677. https://doi.org/10.1080/01430750.2019.1665582

[45] Tiwari, Raj Kamal, and Manab Kumar Das. "Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids." *International Journal of Heat and Mass Transfer* 50, no. 9-10 (2007): 2002-2018. https://doi.org/10.1016/j.ijheatmasstransfer.2006.09.034

[46] Swalmeh, Mohammed Zaki. "Numerical solutions of hybrid nanofluids flow via free convection over a solid sphere." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 83, no. 1 (2021): 34-45. https://doi.org/10.37934/arfmts.83.1.3445

[47] Swalmeh, Mohammed Z., Hamzeh T. Alkasasbeh, Abid Hussanan, and Mustafa Mamat. "Heat transfer flow of Cu-water and Al_2O_3-water micropolar nanofluids about a solid sphere in the presence of natural convection using Keller-box method." *Results in Physics* 9 (2018): 717-724. https://doi.org/10.1016/j.rinp.2018.03.033

[48] Alwawi, Firas A., Hamzeh T. Alkasasbeh, A. M. Rashad, and Ruwaidiah Idris. "Heat transfer analysis of ethylene glycol-based Casson nanofluid around a horizontal circular cylinder with MHD effect." *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* 234, no. 13 (2020): 2569-2580. https://doi.org/10.1177/0954406220908624

[49] Bhat, Ashwini, and Nagaraj N. Katagi. "Magnetohydrodynamic flow of micropolar fluid and heat transfer between a porous and a non-porous disk." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 75, no. 2 (2020): 59-78. https://doi.org/10.37934/arfmts.75.2.5978

[50] Alkasasbeh, Hamzeh T., Mohammed Z. Swalmeh, Abid Hussanan, and Mustafa Mamat. "Effects of mixed convection on methanol and kerosene oil based micropolar nanofluid containing oxide nanoparticles." *CFD Letters* 11, no. 1 (2019): 55-68.

[51] Nazar, Roslinda Mohd. "Mathematical models for free and mixed convection boundary layer flows of micropolar fluids." *PhD diss., Universiti Teknologi Malaysia*, 2004.

[52] Alzgool, Husein A., Hamzeh T. Alkasasbeh, Sana Abu-ghurra, Zain Al-houri, and Mohammed Z. Swalmeh. "Numerical solution of heat transfer in MHD mixed convection flow micropolar Casson fluid about solid sphere with radiation effect." *International Journal of Engineering Research and Technology* 12, no. 4 (2019): 519-529.

[53] Swalmeh, Mohammed Z., Hamzeh T. Alkasasbeh, Abid Hussanan, and Mustafa Mamat. "Numerical investigation of heat transfer enhancement with Ag-GO water and kerosene oil based micropolar nanofluid over a solid sphere." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 59, no. 2 (2019): 269-282.

[54] Qadan, Hani, Hamzeh Alkasasbeh, Nusayba Yaseen, Mohammed Z. Sawalmeh, and Shaima ALKhalafat. "A Theoretical study of steady MHD mixed convection heat transfer flow for a horizontal circular cylinder embedded in a micropolar casson fluid with thermal radiation." *Journal of Computational Applied Mechanics* 50, no. 1 (2019): 165-173.

[55] Cebeci, Tuncer, and Peter Bradshaw. *Physical and computational aspects of convective heat transfer*. Springer Science & Business Media, 2012.

[56] Malik, M. Y., Mair Khan, T. Salahuddin, and Imad Khan. "Variable viscosity and MHD flow in Casson fluid with Cattaneo-Christov heat flux model: Using Keller box method." *Engineering Science and Technology, an International Journal* 19, no. 4 (2016): 1985-1992. https://doi.org/10.1016/j.jestch.2016.06.008

[57] Prasad, Kerehalli Vinayaka, Hanumesh Vaidya, Gudekote Manjunatha, Kuppalapalle Vajravelu, Choudhari Rajashekhar, and V. Ramanjini. "Influence of Variable Transport Properties on Casson Nanofluid Flow over a Slender Riga Plate: Keller Box Scheme." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 64, no. 1 (2019): 19-42.