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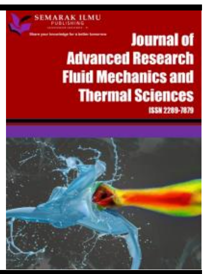

Mixed Convection Flow of a Casson Fluid Past a Thin Needle

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ABSTRACT

This paper deals with a steady mixed convection flow past a horizontal thin needle submerged in Casson fluid. The flow-governing equations are changed into a set of non-linear ordinary differential equations utilizing proper transforms. Employing successive linearization, the resulting equations are linearized, and then the Chebyshev spectral collocation technique is implemented. The effects of needle size and mixed convection parameter on stream on velocity and temperature, together with graphical representations of the coefficient of skin friction and local heat transfer rate, are presented. It is found that temperature decreases as needle size decreases but velocity, the coefficient of skin friction, and the Nusselt number improve for both aiding and opposing flow scenarios.

1. Introduction

The studies on flow and heat transfer across a moving, thin needle have been explored by investigators due to significance of numerous industrial and technical applications. Some of the applications can be found in a wide range of fields, including biomedicine, microstructure electronic tools, hot wire anemometers, lubrication and power generation, aerodynamics, blood flow, microscale cooling devices, cancer therapy, wire coating, and many more. Thin needle geometry refers to the smearing surface produced by rotating a parabola around its axis. Lee [1] conducted the initial research on boundary layer stream across thin moving needle in viscous fluid and described asymptotic behaviour of an approximate solution. Thereafter, Narain and Uberoi [2-4] determined the similarity solutions for convective flow over a needle. Soid *et al.,* [5] analyzed forced laminar convective nanofluid stream past horizontal needle. Qasim *et al.,* [6] examined flow around a small needle with changeable viscosity and thermal conductivity. Prashar *et al.,* [7] considered the consequences of hybrid nanoparticles on the flow across a hot needle. The interactions of viscous dissipation on thermally radiated MHD flow across moving needle in a fluid with hybrid nanoparticles were considered by Nazar *et al.,* [8].

Modern industries and technologies are mostly involved in processes that use mixed convection which is outcome of forced and free convection occurring simultaneously. Examples include heat-

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exchangers maintained in a reduced velocity environment to cool nuclear reactors during emergency shutdowns and fans-driven electrical equipment cooling techniques. Wang [9] calculated the numerical solutions for mixed convection over vertical needle and found that solutions for aiding flows are unique, while result for opposing flows may be single, dual, or nonexistent. In both supporting and opposing flow scenarios, Ahmad *et al.,* [10] examined the problem of constant laminar mixed convection of a viscous incompressible liquid along a moving vertical tiny needle. Ahmad *et al.,* [11] investigated both aiding and opposing stream situations for mixed convection of a viscous incompressible liquid across vertical moving tiny needles with changing heat flux. Trimbitas *et al.,* [12] combined the heat transmission across vertical needle with a changeable wall temperature and mixed convective boundary layer flow by using nanofluids. Salleh *et al.,* [13] discussed both assisting and opposing situations of mixed convection stream of nanofluid caused by motion of tiny vertical needle. Qasim and Afridi [14] looked at how different thermal conductivities and energy dissipation affected the rate at which entropy developed in mixed convection flow. Rehman *et al.,* [15] analyzed the magnetohydrodynamic mixed convection flow of Cross fluid over a movable thin needle with Soret and Dufour effect.

As non-Newtonian fluid flow has applications in a broad range of engineering problems, it has fascinated the curiosity of several investigators. These include tarry fuel the removal of petroleum-based products, the production of plastic substances, material handling, developing crystal, the nuclear reactors cooling, the stream of biological fluids, and the manufacturing of syrup-based pharmaceuticals. Non-Newtonian fluid properties cannot be accurately represented by a single fluid model. As a result, various fluid models, such as power-law fluids, Williamson fluids, Jaffery fluids, couple stress fluids, micro-polar fluids, and so on, were described in the literature over the previous century to explain real fluid dynamics. Casson [16] presented Casson fluid model, which is a shear-thinning fluid. The viscosity of these fluids is infinite at no shear rate and non-existent at infinite shear rates. It is worth noting that if yield stress is less than shear stress, this model reduces to a Newtonian liquid. It offers an easy way to calculate the two parameters, Casson viscosity and apparent yield stress, for use in real-world applications. The Casson fluid model has been used in a huge range of theoretical along with computational studies in recent years due to its extensive applicability in drilling operations, metallurgy, food processing, polymer processing industries, synthetic lubricants, biomedical fields, the preparation of printing ink, etc. Although Casson fluid stream across moving, tiny needle is significant, very little research has been published. Hashim *et al.,* [17] examined the magnetic-Williamson fluid flow in the presence of suspended nanoparticles. over a wedge shape geometry with convective heating mode. Souayeh *et al.,* [18] scrutinized influences of non-linear radiation and cross diffusion on the MHD Casson nanofluid across tiny needle. Ibrar *et al.,* [19] simulated the effects of thermal radiative magneto-nano Casson fluid towards tiny needle under Navier slip condition. Yusof *et al.,* [20] analyzed the steady of stagnation point flow of a Casson fluid over an exponentially permeable slippery Riga plate in presence of thermal radiation, magnetic field, velocity slip, thermal slip, and viscous dissipation effects. Under non-linear thermal radiation and effects of heat source/sink, Hamid [21] examined the physical aspects of a 2D mixed convection stream of magnetized Casson nanofluid above vertical moving tiny needle. In context of simultaneous nonlinear thermal radiation and internally generated heat, Akinshilo *et al.,* [22] scrutinized nanofluid flowing past a narrow needle in a non-Newtonian Casson flow. Bilal and Urva [23] examined the effects of mixed convection and nonlinear radiation on the Casson nanofluid via the tiny needle. Through the use of thermophoretic particles and the Soret/Dufour effect, Kumar *et al.,* [24] examined the two-dimensional stream of an incompressible Casson fluid down horizontal, tiny moving needle. Parvin *et al.,* [25] presented mathematical model and obtained numerical solution for the magnetohydrodynamic Casson fluid flow. Akaje and Olajuwon [26] investigated the impact of the nonlinear radiative heat on the MHD Casson nanofluid flow with stagnation point associated with Thompson and Troian boundary conditions. Rehman *et al.,* [27] investigated the effects of activation energy and thermal radiation on mixed convection striation point flow of Carreau liquid toward the stretchable sheet. Rehman *et al.,* [28] investigated the Soret and Dufour impact on electroosmotic forces in the flow of Casson fluid towards a stretchable sheet with bioconvection, Viscous and Joule dissipation effects. Using a non-Newtonian Casson fluid model, Prashar [29] examined the Blasius and Sakiadis hybrid nanofluid stream all over narrow, heated needle.

According to reviews of the literature, no research has yet been published that examine the study mixed convection boundary stream through horizontal tiny needle in Casson fluid in assisting as well as opposite. This study investigates how dimensionless parameters, namely size of needle and mixed convection parameter influence the stream and heat transfer characteristics are significantly explained.

2. Mathematical Formulation

Consider boundary layer stream of mixed convection in Casson fluid with uniform velocity $\bar{U}_{\infty}(\bar{x})$ over a tiny needle moving horizontally with a velocity $\bar{U}_{w}(\bar{x})$. Assume that the stream is steady, laminar, and incompressible. The $\bar{x}$-axis runs horizontally from main edge of needle, and radial axis runs perpendicular to it as shown in Figure 1. The equation for radius of the needle $\bar{r} = \bar{R}(\bar{x})$. It is believed that needle is thin while the needle's thickness is less than that of boundary layer surrounding it. Temperature of the needle is $\bar{T}_{w}(\bar{x})$, whereas temperature of surrounding fluid is T_{∞}, where $\bar{T}_{w} > T_{\infty}$.

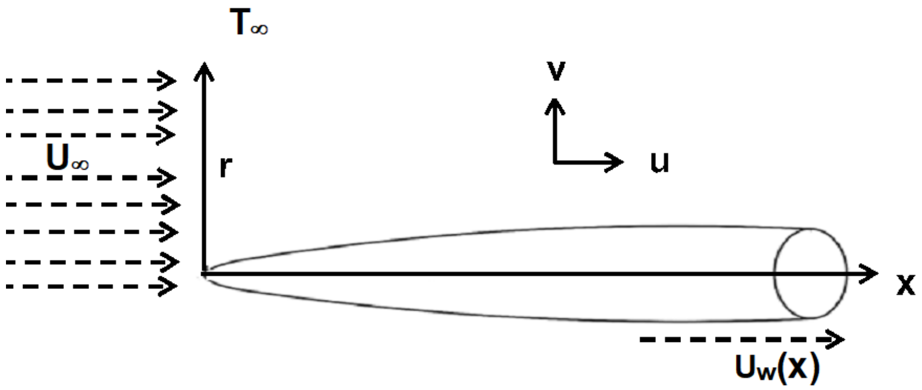


Fig. 1. Schematic representation of the problem

With the above premises, the governing equations becomes

$$\frac{\partial(\bar{r}\bar{u})}{\partial\bar{x}} + \frac{\partial(\bar{r}\bar{v})}{\partial\bar{r}} = 0 \tag{1}$$

$$\bar{v}\frac{\partial\bar{u}}{\partial\bar{r}} + \bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} = \bar{U}\frac{d\bar{U}}{d\bar{x}} + \nu\left(1+\frac{1}{\beta}\right)\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\bar{r}\frac{\partial\bar{u}}{\partial\bar{r}}\right) + g\beta(\bar{T} - T_{\infty}) \tag{2}$$

$$\bar{v}\frac{\partial\bar{T}}{\partial\bar{r}} + \bar{u}\frac{\partial\bar{T}}{\partial\bar{x}} = \alpha\left(\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\bar{r}\frac{\partial\bar{T}}{\partial\bar{r}}\right)\right) \tag{3}$$

where $\bar{u}$ and $\bar{v}$ represents velocity components in the axial and radial directions repectively. T represents temperature of fluid, ν represent kinematic viscosity, β represents Casson parameter,α represents the thermal conductivity, g represents extent of gravity's acceleration.

The conditions on the surface of the needle are

$$\bar{v} = 0, \bar{u} = 0, \ \bar{T} = \bar{T}_w(\bar{x}) \text{ at r} = \bar{R}\ (\bar{x})$$
$$\bar{T} \to T_\infty, \ \bar{u} \to \bar{U}(\bar{x}) \text{ at r} \to \infty \tag{4}$$

The non-dimensional variables listed below

$$r = Re^{1/2}\frac{\bar{r}}{L}, \ x = \frac{\bar{x}}{L}, \ R(x) = Re^{1/2}\frac{\bar{R}(\bar{x})}{L}, \ u = \frac{\bar{u}}{U_\infty},$$
$$U(x) = \frac{\bar{U}(\bar{x})}{U_\infty}, \ v = Re^{1/2}\frac{\bar{v}}{U_\infty}, \ T = \frac{\bar{T}-T_\infty}{\Delta T}, \tag{5}$$

Substituting Eq. (5) into Eq. (1) to Eq. (3), we get

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0 \tag{6}$$

$$v\frac{\partial u}{\partial r} + u\frac{\partial u}{\partial x} = U\frac{dU}{dx} + \left(1+\frac{1}{\beta}\right)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \lambda T \tag{7}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{1}{Pr}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right) \tag{8}$$

where $\lambda = \frac{Gr}{Re^2}$ is mixed convection parameter, Pr is Prandtl number.

Conditions on boundary Eq. (4) become

$$u = 0, v = 0, \ T = T_w(x) \quad \text{at} \quad r = R(x)$$
$$u \to U(x), \ T \to \infty \quad \text{as} \quad r \to \infty \tag{9}$$

Transformations of similarity are described as:

$$\psi = xf(\eta), \ T(x) = x^{2m-1}\theta(\eta), \ \eta = x^{m-1}r^2 \tag{10}$$

where ψ is stream function detailed as $u = \frac{1}{r}\frac{\partial\psi}{\partial r}$ and $v = \frac{-1}{r}\frac{\partial\psi}{\partial x}$.

If $\eta = a$, where a is dimensionless constant, represent needle wall, the surface of the needle, using Eq. (10), can be written as $R = a^{1/2}x^{(1-m)/2}$ which characterizes the shape and size of the needle.

$$8\eta\left[1+\frac{1}{\beta}\right]f''' + 8\left[1+\frac{1}{\beta}\right]f'' + m[1-4(f')^2] + 4ff'' + \lambda\theta = 0 \tag{11}$$

$$\frac{2\eta}{Pr}\theta'' + \frac{2}{Pr}\theta' + f\theta' - (2m-1)f'\theta = 0 \tag{12}$$

The modified conditions on boundary becomes

$$f(\text{a}) = 0, \ f'(a) = 0, \ \theta(a) = 1, f'(\infty) = \frac{1}{2}, \ \theta(\infty) \to 0 \tag{13}$$

The Nusselt number Nu and the coefficient of skin friction C_f are the relevant physical parameters for this model. The non-dimensional form of coefficient of skin friction, and Nusselt number are

$$\sqrt{Re}C_f = 8\sqrt{a}\left(1+\frac{1}{\beta}\right)f''(a) \text{ and } \frac{Nu}{\sqrt{Re}} = -2\sqrt{a}\,\theta'(a) \tag{14}$$

3. Method of Solution

The combined Eq. (11) and Eq. (12) are linearized through the successive linearization method [30]. The solution of resulting linearized equations is attained numerically by Chebyshev collocation method as shown in Figure 2.

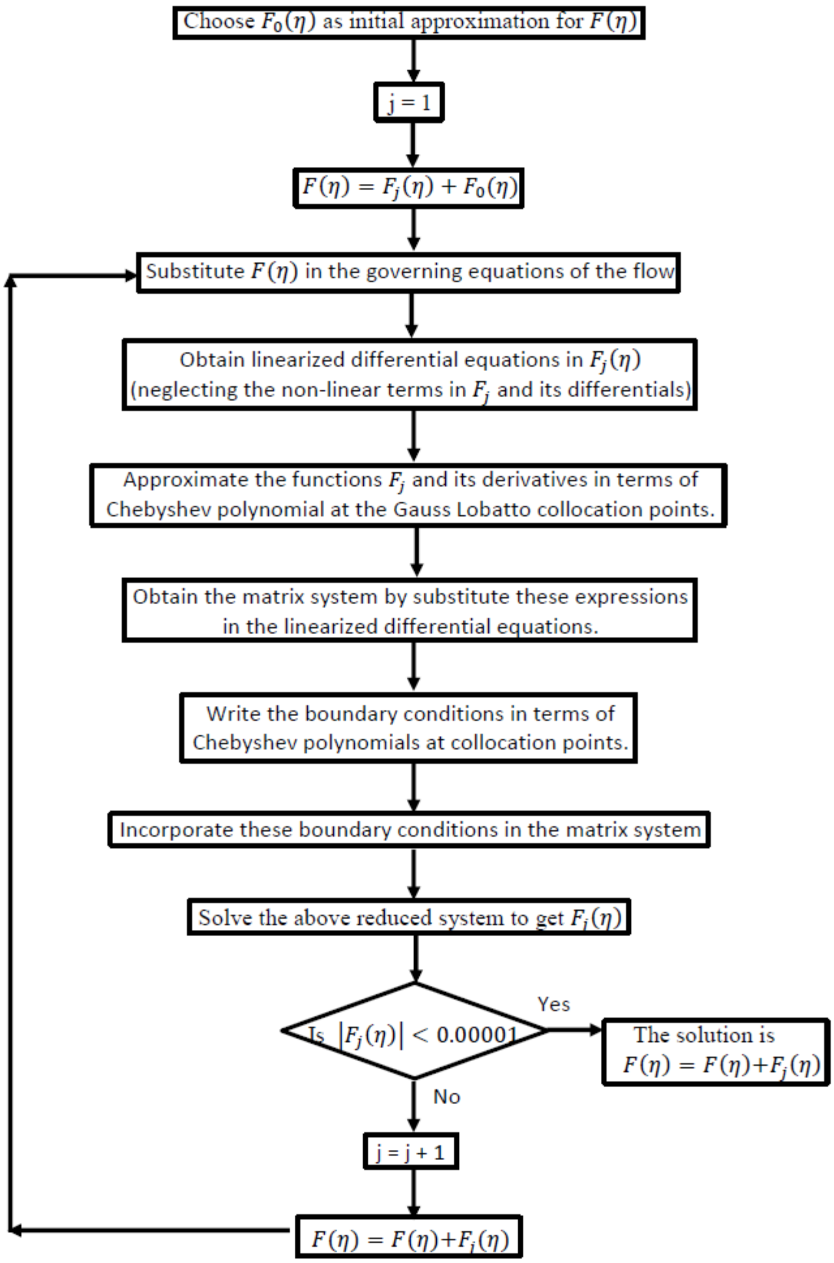


Fig. 2. Flow chart for the solution of the problem

In SLM, it is assumed that the unidentified functions $F(\eta) = [f(\eta), \theta(\eta)]$ can be written as

$$F(\eta) = F_j(\eta) + \sum_{m=0}^{j-1} F_m(\eta) \tag{15}$$

where $F_j(\eta)$ is unknown function and $F_m(\eta)$ is an estimate. This estimate can be calculated by solving the linearized set of equations generated by applying Eq. (15) in the Eq. (11) and Eq. (12). The basic idea is that, even if j becomes large, F_j become very small and hence non-linear terms in F_j and their differentials can be neglected.

Substituting Eq. (15) in the Eq. (11) to Eq. (12) and neglecting nonlinear terms containing f_j and θ_j, we get the following equations

$$a_1 f_j''' + a_2 f_j'' + a_3 f_j' + a_4 f_j + a_5 \theta_j = r_1 \tag{16}$$

$$b_1 f_j' + b_2 f_j + b_3 \theta_j'' + b_4 \theta_j{}' + b_5 \theta_j = r_2 \tag{17}$$

where

$$a_1 = 8\eta\left(1 + \frac{1}{\beta}\right), \quad a_2 = 8\left(1 + \frac{1}{\beta}\right) + 4\sum f_m,$$

$$a_3 = -8m\sum f'_m, \quad a_4 = 4\sum f''_m, \quad a_5 = \lambda,$$

$$r_1 = -8\eta\left(1 + \frac{1}{\beta}\right)(\sum f'''_m) - 8\left(1 + \frac{1}{\beta}\right)(\sum f''_m) + 4m(\sum f'_m)^2 \\ -4(\sum F_m)(\sum f''_m) - \lambda(\sum \theta_m) - m$$

$$b_1 = \sum \theta_m - 2m(\sum \theta_m), \quad b_2 = \sum \theta'_m, \quad b_3 = \frac{2\eta}{Pr},$$

$$b_4 = \frac{2}{Pr} + (\sum f_m), \quad b_5 = \sum f'_m - 2m(\sum f'_m),$$

$$r_2 = -\frac{2\eta}{Pr}(\sum \theta''_m) - \frac{2}{Pr}(\sum \theta'_m) - (\sum f_m)(\sum \theta'_m) + 2m(\sum f'_m)(\sum \theta_m) \\ -(\sum f'_m)(\sum \theta_m)$$

The solution of the set of linearized equations Eq. (16) and Eq. (17) is obtained using the Chebyshev method [31]. Here, the domain of solution $[a, \infty)$, is transformed into the region $[a, B]$, where B is a constant selected to obtain the conditions far away from the body. To implement this approach $[a, B]$ is again shifted to $[-1,1]$ by using the transformation

$$\eta = \frac{(a+B)-(a-B)\xi}{2}, \quad -1 \le \xi \le 1 \tag{18}$$

The functions f_j, θ_j and ϕ_j are predicted at the following Gauss-Lobatto collocation points

$$\xi_i = \cos\frac{\pi i}{N}, \quad i = 0,1,2,3.\ldots.N \tag{19}$$

as

$$f_j(\xi) = \sum_{k=0}^{N} f_j(\xi_k) T_k(\xi_i),\ \theta_j(\xi) = \sum_{k=0}^{N} \theta_j(\xi_k) T_k(\xi_i),\ i = 0,1,2 \dots N \tag{20}$$

where $T_k(\xi) = \cos[k\cos^{-1}(\xi)]$ is the k^{th} degree Chebyshev polynomial.

r^{th} derivative of f_j and θ_j are estimated

$$\frac{d^r f_j}{d\eta^r} = \sum_{k=0}^{N} D_{kj}^r f_j(\xi_k), \quad \frac{d^r \theta_j}{d\eta^r} = \sum_{k=0}^{N} D_{kj}^r \theta_j(\xi_k), \quad i = 0,1,2 \dots N \tag{21}$$

where $D = \frac{2}{L}\mathfrak{D}$ with $\mathfrak{D}$ is the Chebyshev spectral differentiation matrix.

Using Eq. (20) and Eq. (21) into Eq. (16) and Eq. (17), to produce the matrix form equation as

$$A_{j-1} X_j = R_{j-1} \tag{22}$$

where A_{j-1} represents order $2N + 2$ square matrix and X_j and R_{j-1} represents order $(2N + 2)X1$ column matrices given by

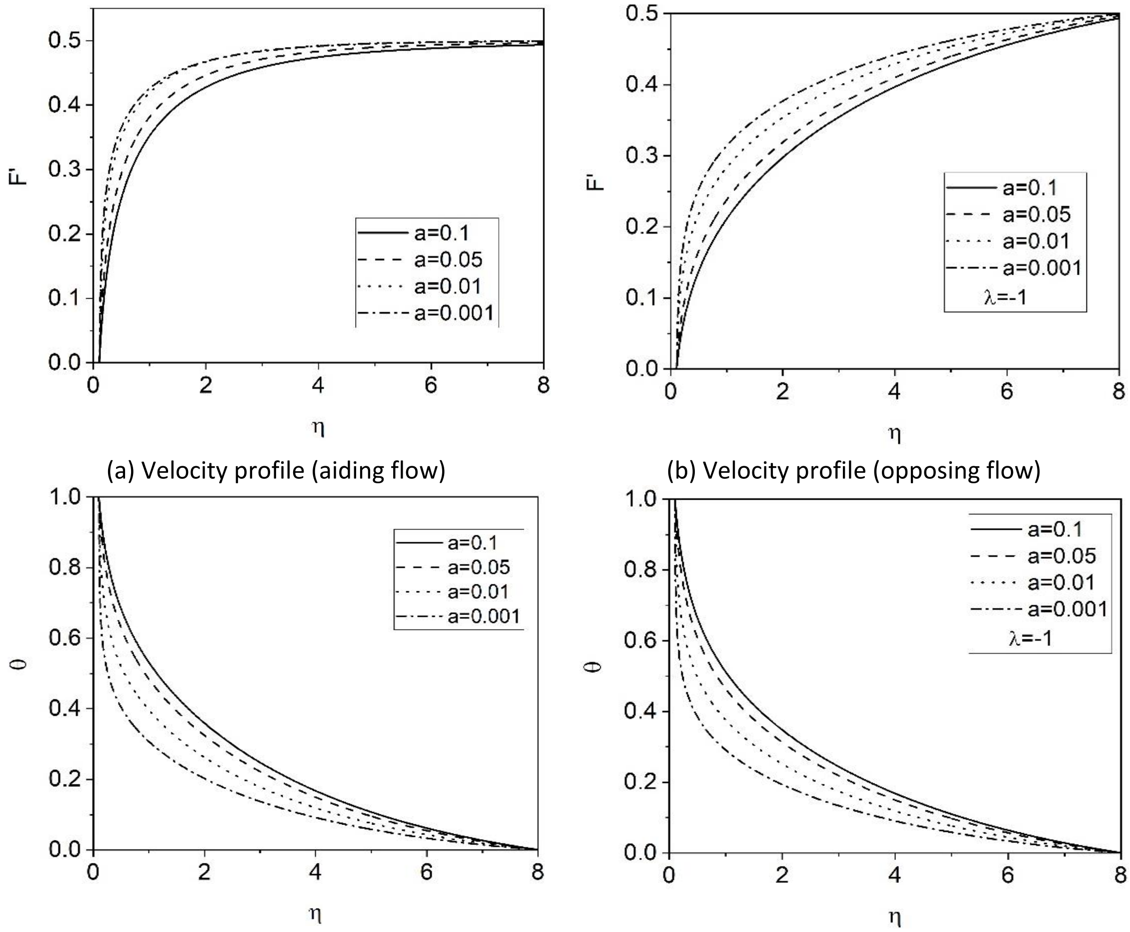
$$A_{j-1} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, X_j = \begin{pmatrix} F_j \\ \Theta_j \end{pmatrix}, R_{j-1} = \begin{pmatrix} r_{1,j-1} \\ r_{2,j-2} \end{pmatrix} \tag{23}$$

Solving the above system of equations, we obtain the values of the unknowns.

4. Results and Discussion

The effect of dimensionless parameters on velocity and temperature together with local heat transfer rate (Nusselt number) $\frac{Nu}{\sqrt{Re}}$ and the coefficient of local skin friction $\sqrt{Re}C_f$ are primarily focus of current model. The dimensionless parameters are: size of the needle (a), mixed convection parameter (λ) and power index (m). A detailed numerical calculation for numerous values of a, λ, and m to assure a greater comprehension of the technical issue, and the findings are presented graphically Figure 3 to Figure 8.

The consequence of size of needle on velocity and temperature for both aiding and opposing cases is represented in Figure 3. It is understood from Figure 3(a) and Figure 3(b) that as a reduces, the velocity improved for both the cases. Since the thickness of the needle is close to the boundary layers of the fluid present inside, the boundary layer separation is delayed and the thickness decreases. Similarity, as a reduces, the temperature decreases in both the cases as presented in Figure 3(c) and Figure 3(d). The increment of the thickness of needle leads to a thicker thermal boundary layer which causes lower needle wall temperature and reduces corresponding temperature profiles.

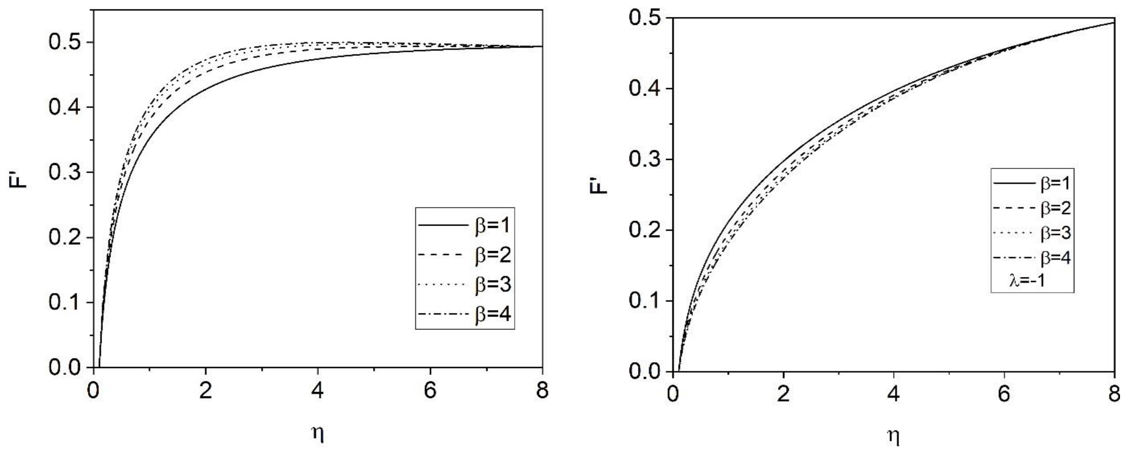


(a) Velocity profile (aiding flow) (b) Velocity profile (opposing flow)

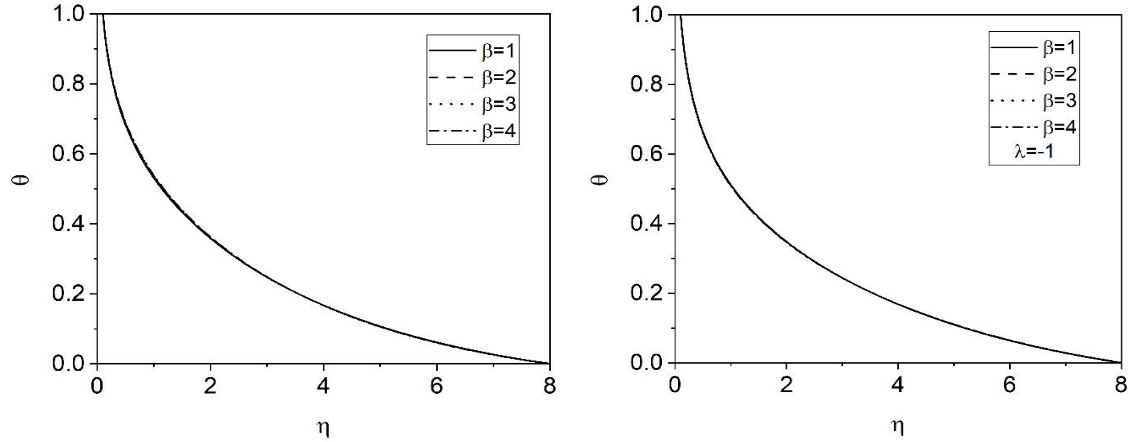
(c) Temperature profile (aiding flow) (d) Temperature profile (opposing flow)

Fig. 3. Effect of size of the needle on the velocity and temperature

The fluctuation of velocity and temperature profiles with Casson fluid parameter β is given in the Figure 4. It is understood from Figure 4(a) that velocity improves as Casson fluid parameter rises for assisting flow. Physical insight escalating β values, gives rise to resistance force which resists the motion of the liquid and this is due to increase in the dynamic viscosity which means reducing the yield stress. This reduces the momentum boundary layer thickness and thereby decreasing the fluid movement and escalating absolute surface velocity gradient. But the velocity reduce as Casson fluid parameter rises in opposing case as depicted in Figure 4(b). Figure 4(c) and Figure 4(d) reveals that the effect of Casson fluid parameter on temperature is almost negligible in both the cases.



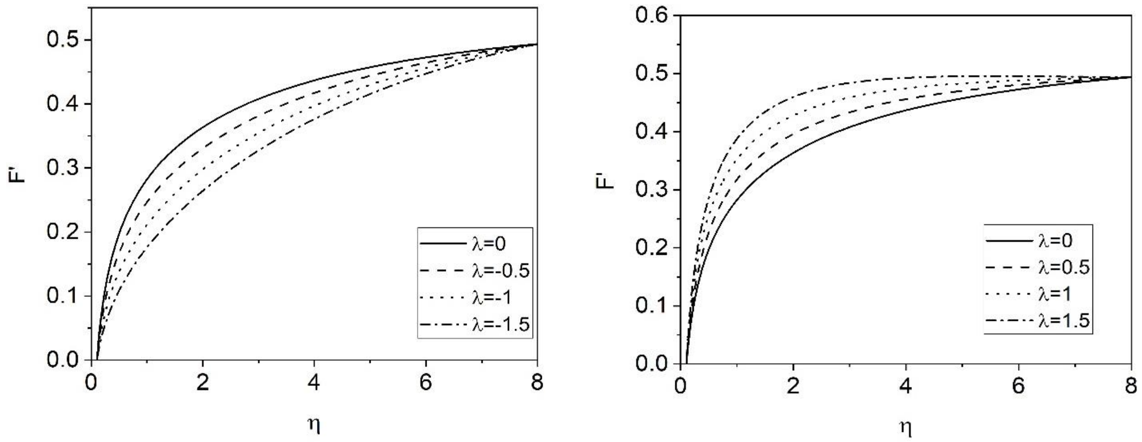
(a) Velocity profile (aiding flow) (b) Velocity profile (opposing flow)



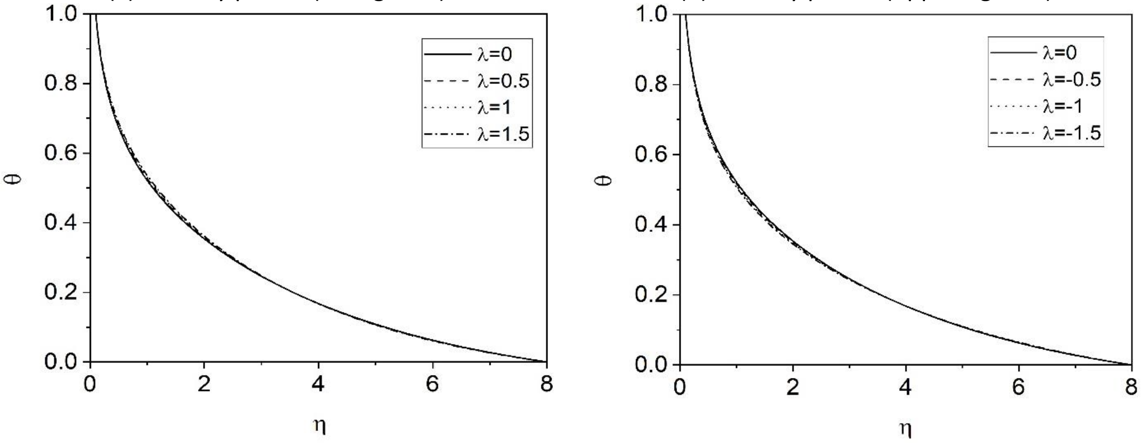
(c) Temperature profile (aiding flow) (d) Temperature profile (opposing flow)

Fig. 4. Effect of β on the velocity and temperature

The variation of velocity and temperature with mixed convection parameter λ is given in the Figure 5. As presented in Figure 5(a), intensifying λ increase the velocity in assisting case. Mixed convection parameter represents the buoyancy force which has correlation with momentum equation. The increase of mixed convection parameter causes the decrease of fluid density. Hence, the velocity of the fluid decreases. The velocity is decreasing in opposing case as given in Figure 5(b). The effect of mixed convection parameter on temperature is negligible in both cases as presented in Figure 5(c) and Figure 5(d).



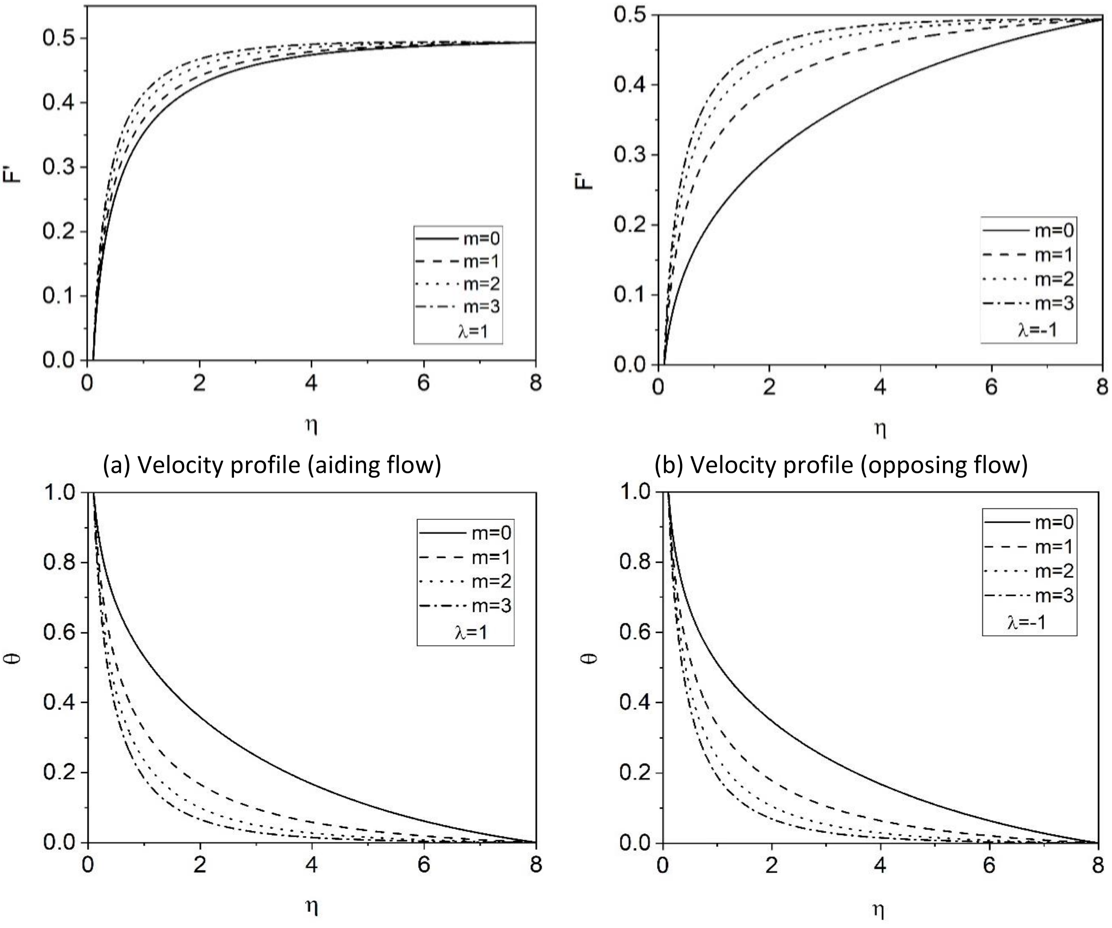
(a) Velocity profile (aiding flow) (b) Velocity profile (opposing flow)



(c) Temperature profile (aiding flow) (d) Temperature profile (opposing flow)

Fig. 5. Effect of λ on the velocity and temperature

Figure 6 presents, effect of power index on velocity and temperature. Figure 6(a) and Figure 6(b) exhibits that the velocity rises as power index rises for both aiding and opposing cases. The temperature decreases as m rises for both aiding and opposing cases as depicted in Figure 6(c) and Figure 6(d).

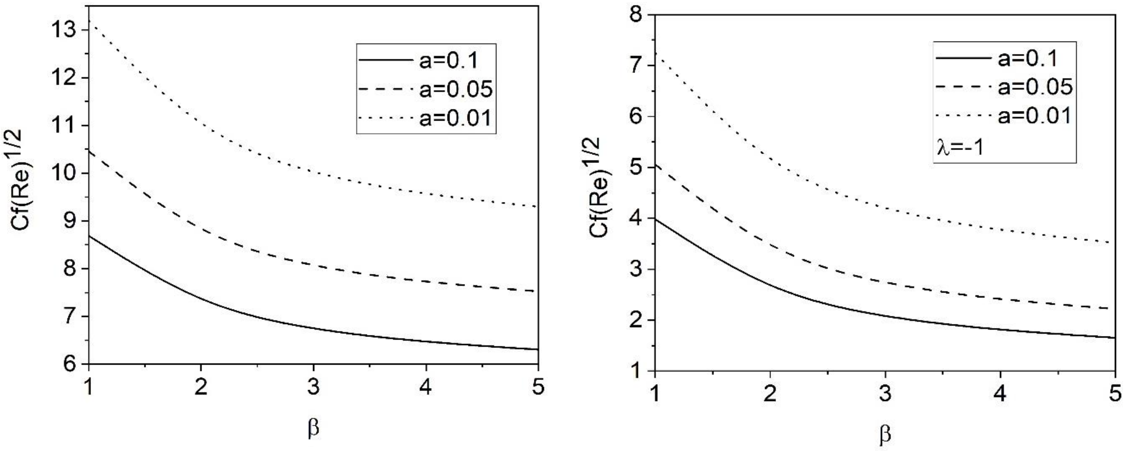


(a) Velocity profile (aiding flow) (b) Velocity profile (opposing flow)

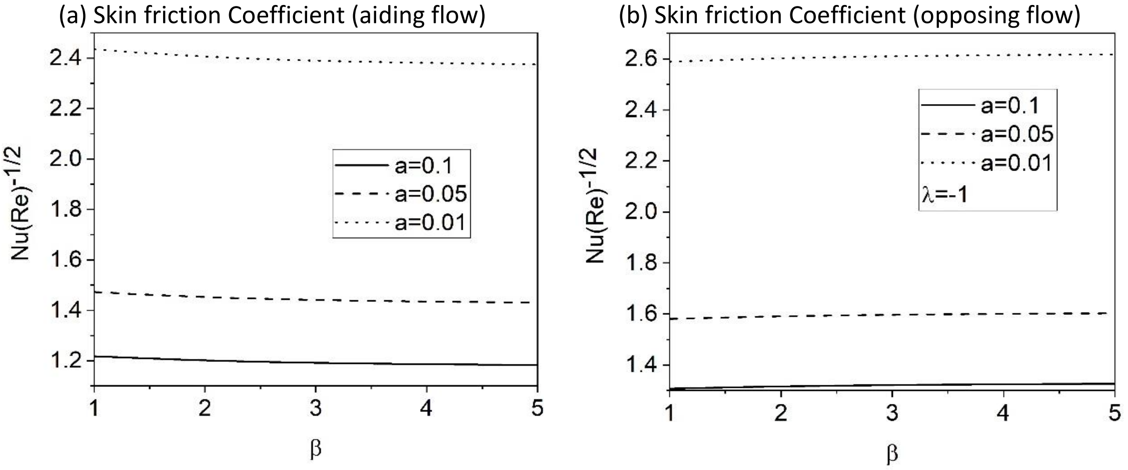
(c) Temperature profile (aiding flow) (d) Temperature profile (opposing flow)

Fig. 6. Effect of m on the velocity and temperature

The impact of size of needle a on coefficient of skin friction $\sqrt{Re}C_f$ and Nusselt Number $\frac{Nu}{\sqrt{Re}}$ is depicted in Figure 7. As depicted in Figure 7(a) and Figure 7(b), the skin friction coefficient is improved by decreasing a for both aiding and opposing cases. As a decreasing, local Nusselt number is also enhancing in both cases, as presented in Figure 7(c) and Figure 7(d).



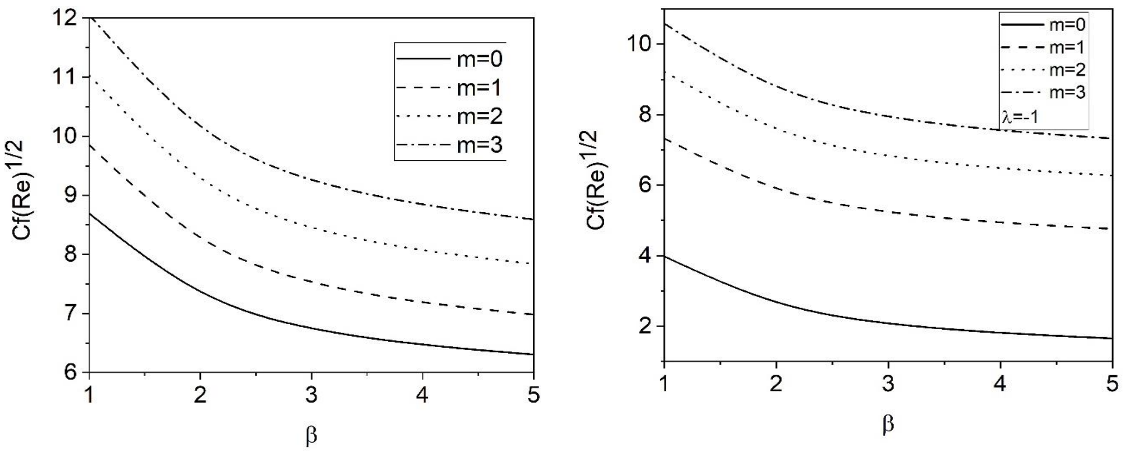
(a) Skin friction Coefficient (aiding flow) (b) Skin friction Coefficient (opposing flow)



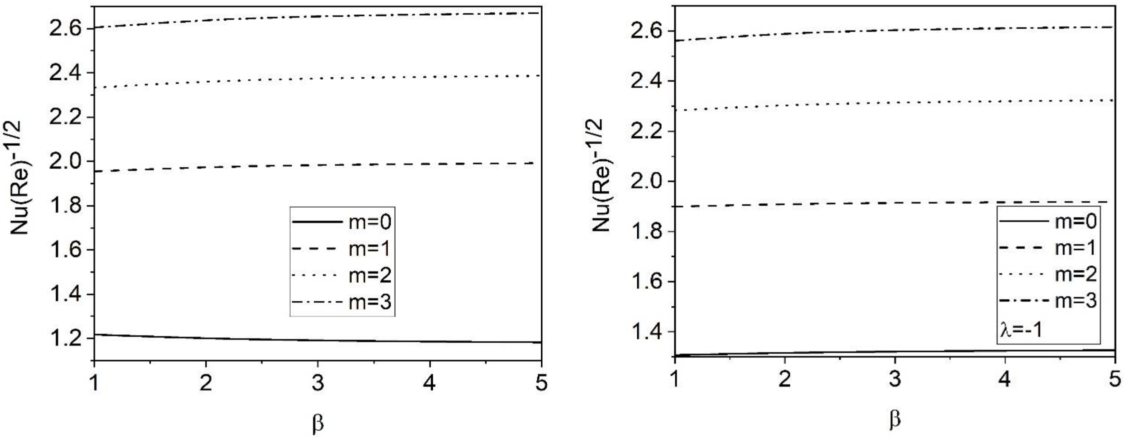
(c) Nusselt Number (aiding flow) (d) Nusselt Number (opposing flow)

Fig. 7. Effect of a on $\sqrt{Re}C_f$ and $\frac{Nu}{\sqrt{Re}}$

The effect of power index on coefficient of skin friction $\sqrt{Re}C_f$ and Nusselt Number $\frac{Nu}{\sqrt{Re}}$ is given in the Figure 8. It is understood from Figure 8(a) and Figure 8(b) that the $\sqrt{Re}C_f$ enhances as power index rises in both assisting and opposing case of the flow. As exhibited in Figure 8(c) and Figure 8(d), $\frac{Nu}{\sqrt{Re}}$ is also enhancing with an enhancement in m in both aiding and opposing cases.



(a) Skin friction Coefficient (aiding flow) (b) Skin friction Coefficient (opposing flow)



(c) Nusselt Number (aiding flow) (d) Nusselt Number (opposing flow)

Fig. 8. Effect of m on $\sqrt{Re}C_f$ and $\frac{Nu}{\sqrt{Re}}$

5. Conclusions

A mixed convection stream past a horizontal needle in Casson fluid is analyzed. The flow-equations are changed into a set of non-linear ordinary differential equations utilizing appropriate transforms and then linearized utilising successive linearization. Chebyshev spectral collocation technique is implemented to find the resulting equations.

i. As size of needle reduces, the velocity, the coefficient of skin frictin and Nusselt number are improved for both the cases, whereas temperature decreases in both the cases.
ii. Intensifying λ increase the velocity in assisting case and decrease in opposing case whereas effect of mixed convection parameter on temperature is negligible.
iii. The velocity rises and temperature decreases as the power index rises for both aiding and opposing cases.

The findings presented in this study open up intriguing possibilities for future research in the domain of fluid flow in porous media. This research explores multi-physics and multi-scale modeling techniques to account for a broader range of phenomena, including cross diffusion effects, chemical reactions or the heterogeneity of the porous medium.

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