Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage:
https://semarakilmu.com.my/journals/index.php/fluid_mechanics_thermal_sciences/index
ISSN: 2289-7879

Bayesian Simultaneous Credible Intervals for the Differences of Coefficients of Variation of Weibull Distributions with Application to Wind Speed Data in Thailand

Manussaya La-ongkaew[1], Sa-Aat Niwitpong[1], Suparat Niwitpong[1,*]

[1] Department of Applied Statistics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Bangkok, Thailand

ARTICLE INFO

Article history:
Received 9 October 2023
Received in revised form 4 January 2024
Accepted 18 January 2024
Available online 15 February 2024

Keywords:
Weibull distribution; coefficient of variation; Bayesian method; gamma prior; uniform prior; OpenBUGS; wind speed

ABSTRACT

The Weibull distribution serves as a versatile tool for modelling various types of data, including reliability analysis, failure rates, survival analysis, and extreme value analysis. Specifically, in weather forecasting, the Weibull distribution is frequently used to represent variables such as wind speed. To examine differences in wind speed dispersion across different areas, the coefficient of variation proves optimal. However, when comparing variations across multiple areas simultaneously, the focus shifts to simultaneous confidence intervals (SCIs). Therefore, this paper focuses on the problem of constructing the SCIs for the differences of coefficients of variation derived from Weibull distributions. The proposed methods in this study are Bayesian approaches that utilize gamma and uniform prior distributions, namely the generalized confidence interval (GCI) and the method of variance estimate recovery (MOVER) based on the Hendricks and Robey interval. To assess the performance of these methods, their coverage probability, expected length, and standard errors were evaluated through a simulation study. The simulation results show that the Bayesian credible interval based on the gamma prior performs satisfactorily in most cases. Finally, to illustrate the SCIs, we present an example concerning wind speed dispersion data in Southern Thailand.

1. Introduction

The coefficient of variation (CV) is a statistical measure that allows the comparison of variability between different datasets, even when they possess varying units of measurement or means. It is computed by dividing the standard deviation by the mean. The standard deviation represents the dispersion of data points around the mean, while the mean represents the average value of the dataset. A higher CV suggests higher relative variability, while a lower CV indicates lower relative variability. The CV finds applications across diverse fields such as finance, economics, engineering, and biology. For instance, it assists investors in assessing investment volatility relative to its average return, and it evaluates the relative variability of economic indicators or variables. Furthermore, it is commonly employed to gauge the dispersion of measurements, including body size, gene expression

* *Corresponding author.*
E-mail address: suparat.n@sci.kmutnb.ac.th

https://doi.org/10.37934/arfmts.114.1.103118

levels, or physiological parameters. Additionally, the CV is often utilized to analyze variability or reliability in various manufacturing or production processes. The CV has garnered attention in both point and interval estimations. Numerous studies have delved into constructing confidence intervals (CIs) for the CV across different models. Koopmans *et al.,* [1] introduced CIs for the CV in normal and log-normal models. Vangel [2] developed CIs for the CV in the normal distribution, while Payton [3] formulated a CI for the CV based on gamma-distributed data in the same year. Mahmoudvand and Hassani [4] presented two new CIs for the CV in a normal distribution. Banik and Kibria [5] examined the CV and its CIs for symmetric and skewed distributions. Liu [6] proposed a generalized CI for CV in the normal distribution, along with three empirical likelihood-based non-parametric intervals for CV in cases of unknown underlying distributions. In 2013, Niwitpong [7] introduced CIs for the CV of the log-normal model within a restricted parameter space. For two-parameter exponential distributions, Sangnawakij and Niwitpong [8] provided CIs for CV, and Yosboonruang *et al.,* [9] established CIs for CV in the delta-lognormal distribution. From 2019 to 2020, CIs for a single CV were presented in several models and applied to real-world data. These encompass the inverse Gaussian distribution, the Weibull distribution, the delta-lognormal distribution, and the inverse gamma distribution [10-13]. Furthermore, numerous researchers have explored different methods to expand the CIs of CVs. In 2012, Buntao and Niwitpong [14] introduced the generalized pivotal approach (GPA) for constructing CIs when dealing with lognormal distributions and the delta-lognormal distribution. They compared their method with the closed form method of variance estimation (CFM). Niwitpong [15] proposed CIs for the difference between CVs of normal distributions with bounded parameters. Yosboonruang *et al.,* [16] presented CIs for the difference of two independent CVs of delta-lognormal distributions using Bayesian methods, the concept of fiducial generalized CI, and the standard bootstrap. In 2021, La-ongkaew *et al.,* [17] employed Bayesian methods based on a gamma prior distribution to estimate CIs for the difference of CVs of Weibull distributions. Finally, for inverse-gamma distributions, CIs for the difference of CVs were proposed using Bayesian methods based on uniform and Jeffreys' priors by Kaewprasert *et al.,* [18]. Additionally, when dealing with multiple independent populations, it becomes possible to calculate the CV by constructing simultaneous confidence intervals (SCIs). In statistical analyses, researchers often compare multiple groups or test multiple hypotheses concurrently. Various techniques have been developed to construct SCIs for comparing CVs of different distributions. Thangjai *et al.,* [19] introduced simultaneous fiducial generalized confidence intervals (SFGCIs) to ascertain differences in CVs for log-normal populations. Their results consistently demonstrated coverage probabilities higher than the nominal confidence level across all sample sizes in simulations. Extending this work, Thangjai and Niwitpong [20] proposed the method of variance estimates recovery (MOVER) approach and a computational method for constructing SCIs for differences in CVs of log-normal distributions. Their findings showcased the satisfactory performance of the MOVER approach across various sample sizes. For two-parameter exponential distributions, multiple approaches were suggested to construct SCIs for differences in CVs: the parametric bootstrap approach, GCI approach, and MOVER [21]. Simulation results indicated that the GCI approach yielded satisfactory performance across all cases, while the MOVER approach was preferable for large sample sizes. Yosboonruang *et al.,* [22] focused on constructing SCIs for pairwise differences of CVs in the delta-lognormal populations. They employed three methods: fiducial generalized CI, Bayesian analysis, and the MOVER. Their findings consistently indicated that the Bayesian credible interval, utilizing Jeffreys' rule prior, outperformed the other methods in almost cases. In 2022, Puggard *et al.,* [23] proposed SCIs for differences of CVs of multiple Birnbaum-Saunders distributions. They employed various methods, including percentile bootstrap, GCI, MOVER using the asymptotic CI (ACI) and GCI, and Bayesian method. Their results showcased that the GCI and MOVER based on the GCI method delivered satisfactory performances.

The construction of CIs for CVs has garnered significant interest in various contexts, including single, difference, and SCIs. Different methods have been employed to estimate these CIs, considering various distributions. However, no previous research has been conducted on constructing SCIs for the differences of CVs of Weibull distributions. Herein, we propose Bayesian methods using gamma and uniform priors and compare them with the GCI and MOVER methods for constructing SCIs for the differences of CVs of Weibull distributions. This paper is structured as follows: In Section 2, we provide an overview of all approaches for constructing SCIs for the differences of CVs of Weibull distributions. Our simulation results are presented in Section 3, followed by the illustration of the proposed approaches using a real wind speed example in Section 4. Finally, we conclude with some remarks in the last section.

2. Methodology

The Weibull distribution is a versatile statistical tool that finds applications in various fields, such as medicine, economics, and social sciences. It effectively models a broad range of phenomena by accommodating various failure patterns and survival characteristics present in real-world data. In particular, it is widely used in reliability engineering and survival analysis to represent the time until system failure or the lifespan of individuals. Introduced by Waloddi Weibull in the 1950s, this distribution is continuous and is defined by Eq. (1), which describes its probability density function for the variable X_i.

$$f\left(x_{ij}, a_i, k_i\right) = \frac{k_i}{a_i}\left(\frac{x_{ij}}{a_i}\right)^{k_i-1} exp\left[-\left(x_{ij}/a_i\right)^{k_i}\right] \tag{1}$$

where $x_{ij} > 0, a_i > 0, k_i > 0$, and $i = 1,2,\ldots p,\ j = 1,2,\ldots,n_i$. According to a Weibull distribution, $X_i = \left(X_{i1}, X_{i2}, \ldots, X_{in_i}\right)$ is the random variable of size n_i, a_i is the scale parameter, and k_i is the shape parameter. The scale parameter determines the location and spread of the distribution, while the shape parameter controls the shape of the distribution curve. Parameter estimation is carried out using maximum likelihood estimation, and the maximum likelihood estimators (MLEs) can be obtained by referencing study of Cohen [24]. The population mean and variance of X_i are

$$E(X_i) = a_i\Gamma(1 + (1/k_i)) \tag{2}$$

and

$$Var(X_i) = {a_i}^2[\Gamma(1 + (2/k_i) - \left(\Gamma(1 + (1/k_i)\right)^2] \tag{3}$$

Consequently, the CV of X_i can be articulated as

$$CV(X_i) = \lambda_i = \sqrt{\Gamma\left(1 + \frac{2}{k_i}\right) / \left(\Gamma\left(1 + \frac{1}{k_i}\right)\right)^2 - 1} \tag{4}$$

Thus, the difference of two independent CVs is as follows:

$$\lambda_1 - \lambda_2 = \sqrt{\Gamma\left(1+\frac{2}{k_1}\right)/\left(\Gamma\left(1+\frac{1}{k_1}\right)\right)^2 - 1} - \sqrt{\Gamma\left(1+\frac{2}{k_2}\right)/\left(\Gamma\left(1+\frac{1}{k_2}\right)\right)^2 - 1} \quad (5)$$

Consider the p parameters of interest, for $i, l = 1,2,\dots,p$ and $i \neq l$, the differences of CVs are

$$\lambda_{il} = \lambda_i - \lambda_l = \sqrt{\Gamma\left(1+\frac{2}{k_i}\right)/\left(\Gamma\left(1+\frac{1}{k_i}\right)\right)^2 - 1} - \sqrt{\Gamma\left(1+\frac{2}{k_l}\right)/\left(\Gamma\left(1+\frac{1}{k_l}\right)\right)^2 - 1} \quad (6)$$

2.1 Bayesian

Prior distributions play a crucial role in Bayesian statistics by allowing the incorporation of prior knowledge or beliefs regarding the parameters of interest before the data is observed. Through the amalgamation of the prior distribution with the likelihood function derived from the observed data, we derive the posterior distribution in Bayesian inference.

$$Posterior\ distribution \propto Prior\ distribution \times Likelihood\ function \quad (7)$$

This updated posterior distribution effectively captures the refined beliefs concerning the parameters subsequent to the data consideration. In Bayesian statistics, two commonly employed methods for constructing CIs are the Bayesian equal-tailed interval and the HPD interval. In this specific case, both methods utilize a gamma prior distribution as well as a uniform prior distribution for establishing the SCIs for the differences of the CVs of Weibull distributions.

2.1.1 Bayesian gamma prior

In Bayesian statistical modelling, a gamma prior denotes the use of a gamma distribution as a prior distribution. The gamma distribution is commonly employed to represent positive-valued random variables. Let a' is a rate parameter which corresponds to $a = (1/a')^{(1/k)}$. The shape and rate parameters of the gamma distribution can be chosen as $\pi(k) \sim gamma(v_1, z_1)$ and $\pi(a') \sim gamma(v_2, z_2)$ with known real numbers as hyperparameters, denoted by (v_1, z_1, v_2, z_2). The joint posterior density function can be determined using the Bayesian approach, as illustrated below.

$$L(a', k|x) = \frac{L(x|a\prime,k)\times\pi(a\prime|k,x)\times\pi(k)}{\int_0^\infty\int_0^\infty L(x|a\prime,k)\times\pi(a\prime|k,x)\times\pi(k)\ dkda\prime} \quad (8)$$

Given that $L(x|a', k)$ is an associated likelihood function. Therefore, Eq. (9) denotes the posterior density function of parameters of a' and k given the data.

$$\pi(a'|k, x) \propto L(a', k|x)\pi(a')\pi(k) \quad (9)$$

Sometimes, the posterior density function cannot be expressed in a closed form. This is the case for parameter estimation in the Weibull distribution, where direct sampling is not possible. Therefore, alternative algorithms are necessary. Gibbs sampling proposed is a well-known Markov chain Monte Carlo (MCMC) algorithm used to approximate the posterior distribution in Bayesian inference problems [25]. It is a powerful technique for conducting Bayesian inference on complex models where direct sampling from the posterior distribution is not feasible. The method enables iterative sampling from simpler conditional distributions. In this research, we applied the Gibbs sampling procedure to generate MCMC samples using an R programming software package. OpenBUGS (Bayesian inference Using Gibbs Sampling) is a software package designed for performing Bayesian analysis with MCMC methods. It offers a flexible and intuitive approach for specifying and fitting Bayesian statistical models. OpenBUGS, an updated version of WinBUGS, is particularly suitable for complex models. The software employs Gibbs sampling and the Metropolis algorithm to generate a Markov chain by sampling from full conditional distributions. To utilize OpenBUGS in R, it is necessary to have the "R2OpenBUGS" package installed, which provides an interface between R and OpenBUGS. The Weibull model in OpenBUGS is given as

```
Weibullmodel <- function(){
 for(i in 1:n){
   x[i] ~ dweib(k,a')
 }
 k ~ dgamma(v1, z1)
 a' ~ dgamma(v2, z2)
}
inits1 <- function(){list( k = initial value of k, a' = initial value of a')}
output <- bugs(data = weibulldata, inits = inits1, parameters.to.save = c("k ","a' "), model.file = Weibullmodel, n.chains = 1, n.iter=20000, debug=FALSE, codaPkg=TRUE, n.burnin = 1000)
```

Considering p parameters of interest, for $i, l = 1,2,\ldots,p$ and $i \neq l$, we estimated the differences of CVs according to Eq. (6). Therefore, the $100(1-\alpha)\%$ equal-tailed simultaneous confidence interval and simultaneous credible interval based on Bayesian using gamma prior distribution for λ_{il} can be expressed as

$$SCI_{il(Baye.Gamma)} = [L_{il(Baye.Gamma)}, U_{il(Baye.Gamma)}] \tag{10}$$

where $L_{il(Baye.Gamma)}$ and $U_{il(Baye.Gamma)}$ represent the lower limit and upper limit of the $100(1-\alpha)\%$ confidence and credible intervals for λ_{il}.

To construct a Bayesian equal-tailed CI, begin by arranging the posterior distribution of the parameter in ascending order. Then, determine the lower and upper boundaries of the interval in a manner that encompasses the desired probability mass. By employing this approach, the specified probability gets equally distributed on both sides of the interval.

The Bayesian credible interval commonly utilized is known as the HPD interval. This interval represents the most densely populated region in the posterior distribution and contains the most probable values of the parameter. An important characteristic of this interval is that the probability density within it is higher compared to values outside of it. As a result, it tends to be the narrowest possible interval.

2.1.2 Bayesian uniform prior

A uniform prior distribution treats every potential value of the parameter with equal probability, indicating no prior information or inclination towards any specific value. Consequently, before any data is observed, all values are considered equally likely. Within the scope of this subsection, we will consider a uniform prior distribution for both the shape and scale parameters, which are $uniform(0,4)$ and $uniform(0,100)$, respectively. For estimating the parameters based on Bayesian with uniform prior, the OpenBUGS software in R programming is discussed. By utilizing Eq. (6) and considering the prior information, an estimation was made to determine the differences in CVs for the p parameters of interest. As a result, the $100(1-\alpha)\%$ equal-tailed simultaneous confidence interval and simultaneous credible interval based on Bayesian using uniform prior distribution for λ_{il} is

$$SCI_{il(Baye.Uniform)} = [L_{il(Baye.Uniform)}, U_{il(Baye.Uniform)}] \tag{11}$$

where $L_{il(Baye.Uniform)}$ and $U_{il(Baye.Uniform)}$ represent the lower limit and upper limit of the $100(1-\alpha)\%$ confidence and credible intervals for λ_{il}.

2.2 GCI

GCI was initially introduced by Weerahandi [26]. Let $X = (X_1, X_2, \ldots, X_n)$ be random variables from a distribution $F_X(x, \varphi, \gamma)$, where φ is a parameter of interest, γ is a nuisance parameter γ_i, and x is the observed value of random variable X. The random quantity $R(X; x, \varphi, \gamma)$ is the generalized pivotal quantity (GPQ) when it fulfils the following two conditions

i. The probability distribution of $R(X; x, \varphi, \gamma)$ is free of all unknown parameters.
ii. The observed value of $R(X; x, \varphi, \gamma)$ at $X = x$, is free of nuisance parameter.

Let $R\left(\frac{\alpha}{2}\right)$ be the $100\left(\frac{\alpha}{2}\right)^{th}$quantiles of $R(X; x, \varphi, \gamma)$, Therefore, $[R\left(\frac{\alpha}{2}\right), R\left(\frac{1-\alpha}{2}\right)]$ becomes the $100(1-\alpha)\%$ GCI for the parameter of interest. For Weibull distribution, the GPQs of k and a are provided by Krishnamoorthy *et al.,* [27]. $R(k)$ and $R(a)$ fulfil the mentioned two conditions. For the shape parameter, the GPQ is given by

$$R_{k_i} = \frac{\hat{k}_{0i}}{\hat{k}_i^*},\ i = 1,2,\ldots,p \tag{14}$$

while the GPQ of scale parameter is given by

$$R_{a_i} = \left(\frac{1}{\hat{a}_i^*}\right)^{\frac{\hat{k}_i^*}{\hat{k}_{0i}}} \hat{a}_{0i},\ i = 1,2,\ldots,p \tag{15}$$

Let $\hat{a}_i$ and $\hat{k}_i$ are the MLEs of the parameters, $\hat{a}_{0i}$ and $\hat{k}_{0i}$ are the observed values of the estimators, and $\hat{a}_i^*$ and $\hat{k}_i^*$ are the MLEs based on a sample of size n_i from $Weibull(1,1)$. Herein, we developed the GCI method to establish SCIs for λ_{il}. Firstly, the GPQ for estimating λ_i is determined as

$$R_{\lambda_i} = \sqrt{\Gamma\left(1+\frac{2}{R_{k_i}}\right) / \left(\Gamma\left(1+\frac{1}{R_{k_i}}\right)\right)^2 - 1} \tag{16}$$

From Eq. (6), we can derive

$$R_{\lambda_{il}} = R_{\lambda_i} - R_{\lambda_l} \tag{17}$$

In consequence, the $100(1-\alpha)\%$ GCI simultaneous confidence interval for λ_{il} can be expressed as

$$SCI_{il(GCI)} = [R_{\lambda_{il}}\left(\frac{\alpha}{2}\right), R_{\lambda_{il}}\left(\frac{1-\alpha}{2}\right)] \tag{18}$$

where $R_{\lambda_{il}}\left(\frac{\alpha}{2}\right)$ be the $100\left(\frac{\alpha}{2}\right)^{th}$ quantiles of the distribution of $R_{\lambda_{il}}$.

2.3 MOVER

The technique introduced by Donner and Zou [28] allows the construction of a CI for a function involving two parameters, $\lambda_1 - \lambda_2$, defined as

$$CI^m = [L^m, U^m] \tag{19}$$

where

$$L^m = (\hat{\lambda}_1 - \hat{\lambda}_2) - \sqrt{\left(\hat{\lambda}_1 - l_1\right)^2 + \left(u_2 - \hat{\lambda}_2\right)^2} \tag{20}$$

and

$$U^m = (\hat{\lambda}_1 - \hat{\lambda}_2) + \sqrt{\left(u_1 - \hat{\lambda}_1\right)^2 + \left(\hat{\lambda}_2 - l_2\right)^2} \tag{21}$$

Hendricks and Robey [29] provided CIs for λ_1 and λ_2 for defined as

$$(l_{1.hr}, u_{1.hr}) = (\hat{\lambda}_1 - t_{(\alpha/2,n_1-1)}\frac{\hat{\lambda}_1}{\sqrt{2n_1}}, \hat{\lambda}_1 + t_{(\alpha/2,n_1-1)}\frac{\hat{\lambda}_1}{\sqrt{2n_1}}) \tag{22}$$

and

$$(l_{2.hr}, u_{2.hr}) = (\hat{\lambda}_2 - t_{(\alpha/2,n_2-1)}\frac{\hat{\lambda}_2}{\sqrt{2n_2}}, \hat{\lambda}_2 + t_{(\alpha/2,n_2-1)}\frac{\hat{\lambda}_2}{\sqrt{2n_2}}) \tag{23}$$

where $t_{(\alpha/2,n_1-1)}$ and $t_{(\alpha/2,n_2-1)}$ represent the $100\left(\frac{\alpha}{2}\right)^{th}$ percentile of t-distribution with $n_1 - 1$ and $n_2 - 1$ degrees of freedom, respectively. To establish a CI for the difference of the CVs of Weibull distributions using MOVER and Hendricks and Robey's CI, we replace l_1, u_1, l_2, and u_2 with the lower and upper limits in Eq. (22) and Eq. (23). When p populations are considered, the $100(1-\alpha)\%$ CI for λ_i of Weibull distribution is provided again.

$$(l_i, u_i) = (\hat{\lambda}_i - t_{(\alpha/2,n_i-1)}\frac{\hat{\lambda}_i}{\sqrt{2n_i}}, \hat{\lambda}_i + t_{(\alpha/2,n_i-1)}\frac{\hat{\lambda}_i}{\sqrt{2n_i}}) \tag{24}$$

where $t_{(\alpha/2, n_i - 1)}$ represent the $100\left(\frac{\alpha}{2}\right)^{th}$ percentile of t-distribution with $n_i - 1$ degrees of freedom. Therefore, the $100(1-\alpha)\%$ MOVER simultaneous confidence interval for λ_{il} can be expressed as

$$SCI_{il(MOVER)} = [L_{il(MOVER)}, U_{il(MOVER)}] \tag{25}$$

The lower limit $L_{il(MOVER)}$ and the upper limit $U_{il(MOVER)}$ are represented by Eq. (26) and Eq. (27), respectively.

$$L_{il(MOVER)} = (\hat{\lambda}_i - \hat{\lambda}_l) - \sqrt{\left(\hat{\lambda}_i - l_i\right)^2 + \left(u_l - \hat{\lambda}_l\right)^2} \tag{26}$$

and

$$U_{il(MOVER)} = (\hat{\lambda}_i - \hat{\lambda}_l) + \sqrt{\left(u_i - \hat{\lambda}_i\right)^2 + \left(\hat{\lambda}_l - l_l\right)^2} \tag{27}$$

3. Parameter Settings and Simulation Results

In this study, we compared the performance of Bayesian methods using gamma and uniform priors with two other methods: the GCI approach and the MOVER approach. The goal was to assess the performance of simultaneous confidence intervals by measuring their coverage probabilities (CPs), expected lengths (ELs), and standard errors (s.e.). We aimed to determine the best method by identifying a CP that was greater than or equal to the nominal confidence level $(1-\alpha)$ of 0.95, while also having the smallest EL. To conduct our simulations, we executed 5,000 iterations. For the GCI approach, we computed the critical value using 2,500 samples. Additionally, we conducted 20,000 replications for each parameter combination in the Bayesian methods. The specific parameter combinations can be found in Table 1.

Table 1
The parameter settings for the parameter combinations ($p = 5$)

a_i	n_i	λ_i
0.5	(20_5),	(0.5_5), $(0.5_2,1,2_2)$, (1_5), (2_5)
	$(20_3,50_2)$,	(0.5_5), $(0.5_2,1,2_2)$, (1_5), (2_5)
	$(20_2,50,100_2)$,	(0.5_5), $(0.5_2,1,2_2)$, (1_5), (2_5)
	(50_5),	(0.5_5), $(0.5_2,1,2_2)$, (1_5), (2_5)
	$(50_3,100_2)$,	(0.5_5), $(0.5_2,1,2_2)$, (1_5), (2_5)
	(100_5)	(0.5_5), $(0.5_2,1,2_2)$, (1_5), (2_5)
2	(20_5),	(0.5_5), $(0.5_2,1,2_2)$, (1_5), (2_5)
	$(20_3,50_2)$,	(0.5_5), $(0.5_2,1,2_2)$, (1_5), (2_5)
	$(20_2,50,100_2)$,	(0.5_5), $(0.5_2,1,2_2)$, (1_5), (2_5)
	(50_5),	(0.5_5), $(0.5_2,1,2_2)$, (1_5), (2_5)
	$(50_3,100_2)$,	(0.5_5), $(0.5_2,1,2_2)$, (1_5), (2_5)
	(100_5)	(0.5_5), $(0.5_2,1,2_2)$, (1_5), (2_5)

Note: (20_5) stand for (20,20,20,20,20)

Table 2 shows the summarized results for the sample case (p) where the value is equal to 5. For scale parameter (a_i) is equal to 0.5, the CPs of Bayesian credible interval based on gamma prior are higher than the nominal confidence level with the smallest ELs for $\lambda_i = 0.5$, whereas the GCI performed the best in the case of $\lambda_i = 1$ or 2. The equal-tailed simultaneous confidence interval and simultaneous credible interval based on Bayesian using uniform prior distribution produces yields unstable coverage probabilities, meaning that in certain cases, it is significantly lower than the confidence level, while in others, it exceeds it. Considering the larger scale parameter ($a_i = 2$), the Bayesian credible interval based on gamma prior still outperforms other methods when the CVs of each sample are equal. Furthermore, the Bayesian credible interval using uniform prior performs well in many cases especially when $\lambda_i = 1$ for $n_i = (50_3, 100_2), (100_5)$. Finally, the MOVER method yielded the CPs over 0.95 in all scenarios.

Table 2
The coverage probability (CP), expected length (EL) and standard errors (s.e.) of 95% of simultaneous confidence intervals and credible intervals for the differences of CVs Weibull distributions: 5 sample cases

Baye.gamma-E	Baye.gamma-C	Baye.uniform-E	Baye.uniform-C	GCI	MOVER
CP EL (s.e.)	CP EL (s.e.)	CP EL (s.e.)	CP EL (s.e.)	CP EL (s.e.)	CP EL (s.e.)
0.9471 4.1680 (0.3131)	0.9778 4.0684 (0.3001)	0.8797 0.3082 (0.0192)	0.8935 1.0196 (0.0188)	0.9589 8.0773 (1.1421)	0.9987 6.4049 (0.4154)
0.9422 2.3042 (0.4169)	0.9464 2.1427 (0.3843)	0.1911 0.3957 (0.0186)	0.1927 1.3096 (0.0186)	0.7727 3.7837 (0.9682)	0.9989 7.1408 (0.4177)
0.9463 1.1215 (0.0496)	0.9661 1.1118 (0.0488)	0.9975 0.4554 (0.0094)	0.9967 1.5110 (0.0093)	0.9589 0.8382 (0.0502)	0.9998 2.8173 (0.1367)
0.9499 0.4961 (0.0164)	0.9620 0.4933 (0.0164)	0.9999 0.4442 (0.0031)	0.9998 1.4752 (0.0030)	0.9622 0.3128 (0.0104)	0.9999 1.4600 (0.0658)
0.9465 2.9820 (0.2193)	0.9677 2.9295 (0.2109)	0.7204 0.1435 (0.0081)	0.7313 0.4761 (0.0080)	0.9504 4.2960 (0.5554)	0.9994 6.2769 (0.3428)
0.9411 1.9298 (0.3509)	0.9419 1.8152 (0.3298)	0.1659 0.2364 (0.0132)	0.1669 0.7846 (0.0131)	0.7095 2.6806 (0.6465)	0.9997 7.1143 (0.4070)
0.9431 0.8482 (0.0444)	0.9550 0.8402 (0.0437)	0.9748 0.2919 (0.0050)	0.9705 0.9699 (0.0050)	0.9566 0.5964 (0.0384)	1.0000 2.7931 (0.1124)
0.9470 0.3844 (0.0172)	0.9540 0.3819 (0.0171)	0.9965 0.2745 (0.0020)	0.9963 0.9123 (0.0020)	0.9574 0.2412 (0.0107)	1.0000 1.4608 (0.0532)
0.9518 2.4754 (0.2388)	0.9651 2.4028 (0.2257)	0.5943 0.0847 (0.0052)	0.6044 0.2811 (0.0052)	0.9536 3.3637 (0.5196)	0.9999 6.3539 (0.2940)
0.9446 1.8686 (0.3734)	0.9462 1.7491 (0.3498)	0.1504 0.1659 (0.0099)	0.1511 0.5509 (0.0099)	0.6660 2.6574 (0.6655)	0.9998 7.1329 (0.4015)
0.9449 0.7124 (0.0546)	0.9511 0.7026 (0.0533)	0.9301 0.2111 (0.0033)	0.9254 0.7017 (0.0033)	0.9501 0.4945 (0.0441)	1.0000 2.7742 (0.0964)
0.9444 0.3246 (0.0231)	0.9480 0.3214 (0.0227)	0.9727 0.1932 (0.0014)	0.9727 0.6421 (0.0014)	0.9544 0.2045 (0.0144)	1.0000 1.4498 (0.0463)
0.9465 2.1834 (0.0949)	0.9647 2.1669 (0.0937)	0.7983 0.1248 (0.0053)	0.8082 0.4142 (0.0052)	0.9514 2.4839 (0.1813)	1.0000 6.3681 (0.2759)
0.9480 1.2551 (0.2013)	0.9505 1.2182 (0.1951)	0.1807 0.2264 (0.0151)	0.1815 0.7511 (0.0150)	0.5159 1.3035 (0.2621)	1.0000 7.1386 (0.3792)
0.9471 0.6482 (0.0172)	0.9562 0.6457 (0.0171)	0.9958 0.2911 (0.0040)	0.9948 0.9676 (0.0040)	0.9505 0.4303 (0.0144)	1.0000 2.8242 (0.0889)
0.9498 0.2976 (0.0063)	0.9538 0.2965 (0.0062)	0.9998 0.2751 (0.0016)	0.9997 0.9143 (0.0016)	0.9572 0.1867 (0.0037)	1.0000 1.4691 (0.0415)
0.9438 1.8996 (0.1057)	0.9572 1.8802 (0.1039)	0.6638 0.0758 (0.0035)	0.6711 0.2517 (0.0035)	0.9536 2.0733 (0.1739)	1.0000 6.3455 (0.2507)
0.9428 1.2372 (0.2086)	0.9432 1.1988 (0.2023)	0.1630 0.1595 (0.0115)	0.1638 0.5297 (0.0114)	0.5042 1.2839 (0.2641)	1.0000 7.1051 (0.3824)

0.9458	0.9525	0.9742	0.9718	0.9520	1.0000
0.5709 (0.0244)	0.5679 (0.0242)	0.2111 (0.0027)	0.7017 (0.0027)	0.3796 (0.0183)	2.8098 (0.0806)
0.9470	0.9517	0.9940	0.9940	0.9523	1.0000
0.2633 (0.0103)	0.2621 (0.0103)	0.1936 (0.0012)	0.6437 (0.0012)	0.1654 (0.0063)	1.4668 (0.0382)
0.9507	0.9605	0.7443	0.7517	0.9506	1.0000
1.4433 (0.0427)	1.4371 (0.0424)	0.0678 (0.0025)	0.2252 (0.0025)	1.4401 (0.0701)	6.3453 (0.1947)
0.9481	0.9473	0.1754	0.1761	0.3090	1.0000
0.8454 (0.1298)	0.8318 (0.1277)	0.1562 (0.0125)	0.5186 (0.0125)	0.7836 (0.1440)	7.1033 (0.3652)
0.9471	0.9520	0.9959	0.9950	0.9554	1.0000
0.4454 (0.0082)	0.4441 (0.0082)	0.2112 (0.0022)	0.7019 (0.0022)	0.2912 (0.0068)	2.8203 (0.0627)
0.9474	0.9488	0.9998	0.9998	0.9541	1.0000
0.2064 (0.0033)	0.2057 (0.0033)	0.1939 (0.0010)	0.6446 (0.0009)	0.1298 (0.0018)	1.4743 (0.0296)
0.9434	0.9769	0.7944	0.8266	0.9579	0.9986
4.1880 (0.3249)	4.0820 (0.3097)	0.6592 (0.0739)	2.1530 (0.0706)	32.4703 (4.7100)	6.4331 (0.4215)
0.9410	0.9422	0.3994	0.3970	0.9339	0.9991
2.2891 (0.4133)	2.1294 (0.3811)	0.6433 (0.0577)	2.1105 (0.0558)	15.1796 (3.8777)	7.1066 (0.4165)
0.9448	0.9641	0.9685	0.9692	0.9600	0.9999
1.1178 (0.0500)	1.1079 (0.0492)	0.8663 (0.0300)	0.8649 (0.0299)	3.3392 (0.2023)	2.7920 (0.1376)
0.9414	0.9542	0.9466	0.9587	0.9567	1.0000
0.4917 (0.0162)	0.4889 (0.0161)	0.4923 (0.0157)	1.6326 (0.0156)	1.2452 (0.0412)	1.4491 (0.0652)
0.9421	0.9669	0.5482	0.5552	0.9514	0.9991
3.0155 (0.2284)	2.9492 (0.2183)	0.2771 (0.0294)	0.9180 (0.0290)	17.6846 (2.3687)	6.3632 (0.3531)
0.9448	0.9468	0.3240	0.3235	0.9286	0.9996
1.9395 (0.3530)	1.8240 (0.3318)	0.3586 (0.0308)	1.1870 (0.0305)	10.8514 (2.6304)	7.1644 (0.4052)
0.9446	0.9568	0.8984	0.8943	0.9555	1.0000
0.8495 (0.0445)	0.8413 (0.0437)	0.5382 (0.0149)	1.7853 (0.0149)	2.3930 (0.1559)	2.7955 (0.1154)
0.9450	0.9514	0.8746	0.8842	0.9558	1.0000
0.3847 (0.0175)	0.3821 (0.0174)	0.2979 (0.0072)	0.9897 (0.0072)	0.9669 (0.0437)	1.4708 (0.0535)
0.9504	0.9630	0.3831	0.3830	0.9503	0.9998
2.4541 (0.2325)	2.3834 (0.2200)	0.1578 (0.0176)	0.5238 (0.0175)	13.2440 (2.0117)	6.2745 (0.2933)
0.9438	0.9440	0.3049	0.3031	0.9154	0.9999
1.8694 (0.3744)	1.7489 (0.3504)	0.2505 (0.0232)	0.8305 (0.0230)	10.6301 (2.6696)	7.0946 (0.4037)
0.9483	0.9535	0.8223	0.8157	0.9549	1.0000
0.7131 (0.0548)	0.7031 (0.0535)	0.3883 (0.0095)	1.2890 (0.0095)	1.9803 (0.1758)	2.7764 (0.0957)
0.9488	0.9520	0.9489	0.9591	0.9501	1.0000
0.3148 (0.0289)	0.3011 (0.0257)	0.2928 (0.0075)	1.0145 (0.0074)	0.4236 (0.1454)	1.4144 (0.0814)
0.9441	0.9633	0.6043	0.6060	0.9532	1.0000
2.1732 (0.0958)	2.1561 (0.0944)	0.2133 (0.0154)	0.7082 (0.0153)	9.8572 (0.7262)	6.3201 (0.2763)
0.9481	0.9483	0.3435	0.3428	0.9078	1.0000
1.2572 (0.2022)	1.2199 (0.1960)	0.3221 (0.0291)	1.0662 (0.0289)	5.2132 (1.0489)	7.1636 (0.3789)
0.9509	0.9592	0.9686	0.9647	0.9552	1.0000
0.6490 (0.0170)	0.6465 (0.0169)	0.5357 (0.0126)	1.7771 (0.0126)	1.7396 (0.0581)	2.8238 (0.0876)
0.9477	0.9528	0.9506	0.9555	0.9506	1.0000
0.2970 (0.0063)	0.2959 (0.0063)	0.2979 (0.0057)	0.9896 (0.0057)	0.7468 (0.0148)	1.4679 (0.0417)
0.9483	0.9576	0.4725	0.4721	0.9538	1.0000
1.4437 (0.0232)	1.4373 (0.0231)	0.1047 (0.0031)	0.1043 (0.0030)	5.7487 (0.1505)	6.3522 (0.0983)
0.9464	0.9457	0.3096	0.3097	0.9131	1.0000
1.2323 (0.2071)	1.1943 (0.2010)	0.2270 (0.0223)	0.7524 (0.0222)	5.1204 (1.0480)	7.0331 (0.3814)
0.9482	0.9528	0.9644	0.9577	0.9535	1.0000
0.4449 (0.0044)	0.4435 (0.0044)	0.3882 (0.0039)	0.3866 (0.0039)	1.1637 (0.0144)	2.8178 (0.0314)
0.9480	0.9508	0.8790	0.8836	0.9515	1.0000
0.2635 (0.0103)	0.2622 (0.0102)	0.2067 (0.0035)	0.6870 (0.0034)	0.6620 (0.0253)	1.4647 (0.0375)
0.9492	0.9580	0.4779	0.4778	0.9500	1.0000
1.4396 (0.0428)	1.4332 (0.0425)	0.1047 (0.0055)	0.3480 (0.0055)	5.7291 (0.2801)	6.3166 (0.1984)
0.9444	0.9464	0.3328	0.3309	0.8766	1.0000
0.8510 (0.1312)	0.8371 (0.1291)	0.2146 (0.0236)	0.7113 (0.0236)	3.1695 (0.5833)	7.1523 (0.3670)
0.9489	0.9526	0.9637	0.9577	0.9513	1.0000

0.4448 (0.0082)	0.4434 (0.0082)	0.3883 (0.0071)	1.2892 (0.0072)	1.1641 (0.0275)	2.8258 (0.0625)
0.9443	0.9562	0.9465	0.9485	0.9539	1.0000
0.2065 (0.0033)	0.2058 (0.0033)	0.2069 (0.0028)	0.6877 (0.0028)	0.5196 (0.0075)	1.4718 (0.0298)

Note: Bold means the CP greater than or equal to the nominal confidence level of 0.95 together with the smallest EL

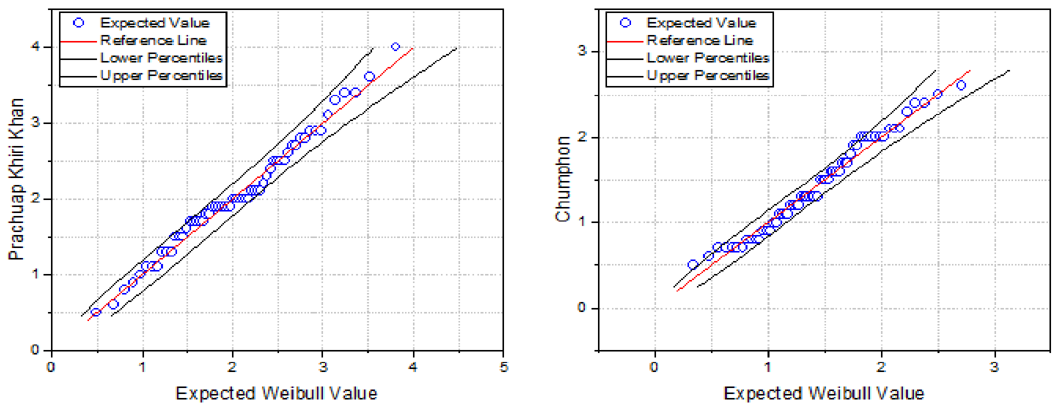
4. An Empirical Verification of the Methods

Wind speeds are often modelled using the Weibull distribution because it provides a suitable fit for wind speed data and incorporates the varying probabilities of different wind speeds. It is important to recognize that wind speeds can exhibit different Weibull distributions in different locations due to variations in local climate conditions. Numerous statistical methods can be used to fit the Weibull distribution to wind speed data, and several studies have been conducted to explore its use for estimating wind speed parameters, which can be seen from the previous studies [30-32].

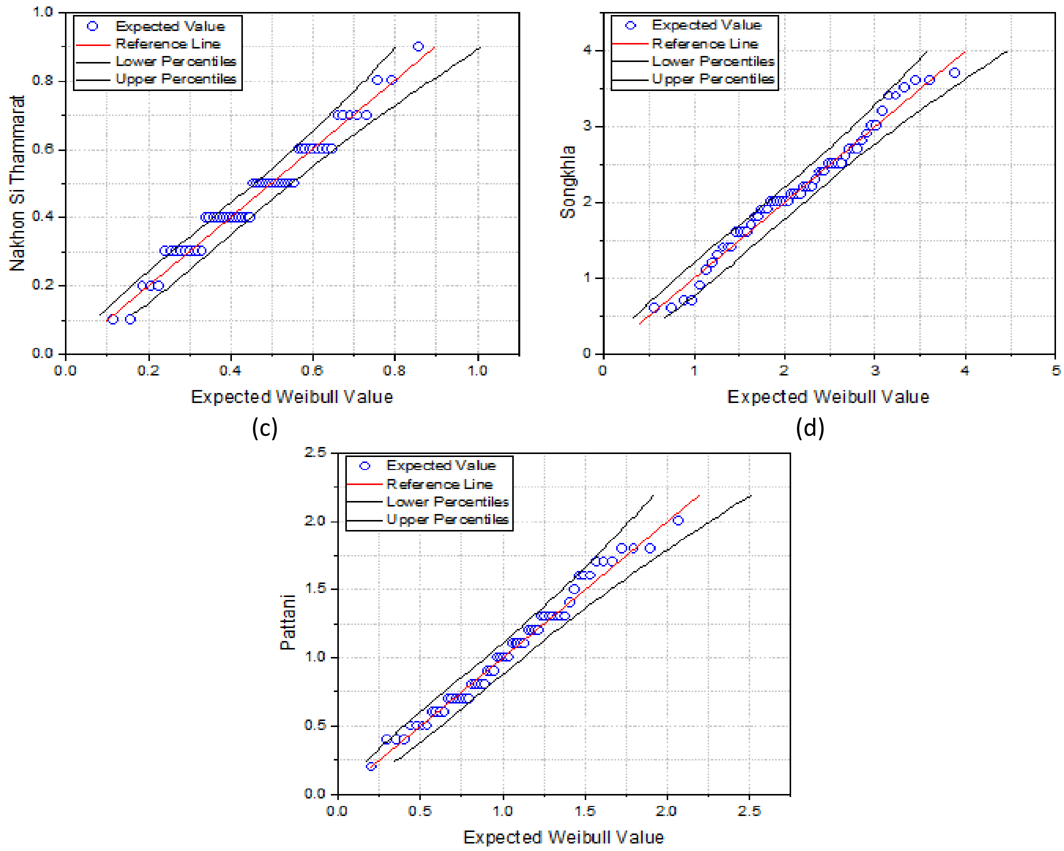
To conduct this research, we utilized monthly wind speed data from five provinces in southern Thailand during the period of 2008-2012. This data was sourced from the Meteorological Department of Thailand and served as an illustrative dataset. Initially, we examined the data distribution using the minimum Akaike information criterion (AIC), and Table 3 displays the AIC values for different distributions. Notably, the Weibull distribution exhibited the smallest AIC values, indicating its compatibility with the dataset. Furthermore, we confirmed this compatibility by conducting a Weibull qq-plot (Figure 1), which yielded p-values of 0.8157, 0.1107, 0.1025, 0.2761, and 0.5379. Summary statistics for the wind speed datasets of the five provinces in southern Thailand can be found in Table 4. These statistics provide the number of samples, estimates of scale parameters, shape parameters, and the CVs for each province. Additionally, Table 5 summarizes the 95% two-sided confidence intervals and credible intervals for the pairwise differences of CVs. This table includes the lower (L) and upper (U) bounds, as well as the interval length for each method.

Table 3
The AIC measurements of the wind speed datasets collected from five provinces in southern Thailand

	Distributions				
	Weibull	Gamma	Lognormal	Normal	Exponential
Prachuap Khiri Khan	137.2638	139.5352	145.2464	138.2018	208.0432
Chumphon	97.6350	98.1604	100.3659	100.2891	165.7571
Nakhon Si Thammarat	-41.3078	36.6078	-28.3861	-41.1056	30.1138
Songkhla	139.7312	145.5709	153.1028	140.2439	213.5767
Pattani	68.7226	70.1450	74.6099	71.7907	125.9347



(a) (b)



(e)

Fig. 1. Graphs of Weibull Q-Q plot of wind speed datasets from five provinces in southern Thailand: (a) Prachuap Khiri Khan: (b) Chumphon: (c) Nakhon Si Thammarat: (d) Songkhla: (e) Pattani

Table 4
The summary statistics outline the key characteristics of the wind speed datasets collected from five provinces in southern Thailand

	Parameters			
	n_i	a_i	k_i	λ_i
Prachuap Khiri Khan	60	2.2936	2.9923	0.3642
Chumphon	60	1.6187	2.9376	0.3703
Nakhon Si Thammarat	60	0.5198	3.0339	0.3597
Songkhla	60	2.3964	3.1376	0.3491
Pattani	60	1.1653	2.6355	0.4081

According to Table 5, the Bayesian credible intervals using gamma and uniform priors had the smallest length. However, the Bayesian credible interval with a gamma prior outperformed in terms of CP values. Therefore, when considering both CP and EL, it is recommended to use the Bayesian credible interval using the gamma prior to estimate the SCIs for the differences of CVs of wind speed data from the five provinces in southern Thailand.

Table 5
The 95% confidence and credible intervals for the pairwise differences of CVs of wind speed datasets from five provinces in southern region

	Baye.gamma-E		Baye.gamma-C		Baye.uniform-E		Baye.uniform-C		GCI		MOVER	
	L	U	L	U	L	U	L	U	L	U	L	U
	(Length)		(Length)		(Length)		(Length)		(Length)		(Length)	
$\lambda_2 - \lambda_1$	-0.0918 (0.1853)	0.0934	-0.0919 (0.1852)	0.0932	-0.0869 (0.1868)	0.0998	-0.0873 (0.1867)	0.0994	-0.0888 (0.1951)	0.1063	-0.5110 (1.0369)	0.5259
$\lambda_3 - \lambda_1$	-0.0884 (0.1794)	0.0910	-0.1013 (0.1790)	-0.5202	-0.5202 (0.2052)	0.3744	0.1737 (0.2048)	0.3785	-0.1013 (0.1902)	0.0888	-0.5202 (1.0371)	0.5168
$\lambda_4 - \lambda_1$	-0.0971 (0.1732)	0.0760	-0.1104 (0.1725)	-0.4901	-0.4901 (0.1815)	0.0767	-0.1063 (0.1807)	0.0744	-0.1104 (0.1886)	0.0781	-0.4901 (0.9413)	0.4511
$\lambda_5 - \lambda_1$	-0.0605 (0.1965)	0.1359	-0.0542 (0.1952)	-0.5049	-0.5049 (0.1928)	0.1476	-0.0490 (0.1925)	0.1435	-0.0542 (0.2020)	0.1477	-0.5049 (1.1020)	0.5970
$\lambda_3 - \lambda_2$	-0.1044 (0.1921)	0.0876	-0.1061 (0.1900)	-0.5393	-0.5393 (0.2069)	0.3690	0.1617 (0.2064)	0.3681	-0.1061 (0.1903)	0.0842	-0.5393 (1.0603)	0.5210
$\lambda_4 - \lambda_2$	-0.1219 (0.1979)	0.0759	-0.1225 (0.1967)	-0.5103	-0.5103 (0.1858)	0.0715	-0.1144 (0.1857)	0.0713	-0.1225 (0.1935)	0.0709	-0.5103 (0.9668)	0.4564
$\lambda_5 - \lambda_2$	-0.0629 (0.1973)	0.1344	-0.0619 (0.1969)	-0.5232	-0.5232 (0.1948)	0.1431	-0.0450 (0.1937)	0.1486	-0.0619 (0.2025)	0.1406	-0.5232 (1.1239)	0.6006
$\lambda_4 - \lambda_3$	-0.1078 (0.1919)	0.0841	-0.1038 (0.1912)	-0.5014	-0.5014 (0.2016)	-0.1849	-0.3872 (0.2012)	-0.1859	-0.1038 (0.1861)	0.0823	-0.5014 (0.9670)	0.4656
$\lambda_5 - \lambda_3$	-0.0639 (0.2057)	0.1417	-0.0485 (0.2048)	-0.5141	-0.5141 (0.2120)	-0.1163	-0.3299 (0.2109)	-0.1189	-0.0485 (0.1995)	0.1509	-0.5141 (1.1241)	0.6100
$\lambda_5 - \lambda_4$	-0.0470 (0.1981)	0.1511	-0.0367 (0.1972)	-0.4528	-0.4528 (0.1881)	0.1601	-0.0280 (0.1880)	0.1599	-0.0367 (0.1955)	0.1588	-0.4528 (1.0363)	0.5835

5. Discussions and Conclusion

La-ongkaew *et al.,* [17] presented Bayesian approaches that relied on a gamma prior distribution to estimate CIs for the difference of the CVs of Weibull populations. They compared these methods with the GCI method, MOVER method, and bootstrap method. The results of their investigation demonstrated that both the Bayesian HPD-interval and the GCI method outperformed the others in different scenarios. Based on these methods, we extended our work to construct CIs for the differences of CVs of various populations simultaneously. Consequently, we employed Bayesian approaches constructed under the equal-tailed CIs and credible intervals using gamma and uniform priors to estimate SCIs for the differences of CVs of Weibull distributions. To evaluate their effectiveness, we compared these SCIs with the GCI method and the MOVER approach in our research, using criteria based on CPs, ELS, and s.e. under various scenarios. Our findings indicate that the Bayesian credible interval, utilizing a gamma prior distribution, is recommended for establishing SCIs for the differences of CVs of Weibull distributions.

For this particular study, we chose scale parameters of 0.5 and 2. Notably, the Bayesian credible interval with a gamma prior approach exhibited a higher CP at 0.95 in more cases. On the other hand, for the GCI method, the EL and s.e. appeared to increase as the scale parameter grew larger. As we considered all approaches using different sample sizes, the EL and s.e. decreased as the sample size increased. Additionally, we conducted a more in-depth analysis of the results for different sample cases ($p = 3$, 5, or 10). The simulation outcomes for these scenarios exhibited similarities, suggesting that the number of sample cases does not significantly affect the construction of the CI. As a result, we have chosen not to include these specific results in this study. Finally, we employed monthly wind speed records from five provinces in southern Thailand to gauge the efficacy of the suggested approaches. In this instance, the Bayesian credible interval derived from the gamma prior demonstrated superior performance, serving as a reliable technique for estimating the SCIs related to differences in CVs of Weibull distributions. With knowledge of the difference in the dispersion of wind speed in two areas, related agencies can better understand, utilize, prepare for, and even predict appropriate wind speed levels.

Acknowledgement
The authors are grateful to Faculty of Applied Science, King Mongkut's University of Technology North Bangkok for financial support. Moreover, the first author is supported by King Mongkut's University of Technology North Bangkok (Grant No. KMUTNB-PHD-62-04).

References

[1] Koopmans, Lambert H., Donald B. Owen, and Judah I. Rosenblatt. "Confidence intervals for the coefficient of variation for the normal and log normal distributions." *Biometrika* 51, no. 1/2 (1964): 25-32. https://doi.org/10.1093/biomet/51.1-2.25
[2] Vangel, Mark G. "Confidence intervals for a normal coefficient of variation." *The American Statistician* 50, no. 1 (1996): 21-26. https://doi.org/10.1080/00031305.1996.10473537
[3] Payton, Mark E. "Confidence intervals for the coefficient of variation." *Conference on Applied Statistics in Agriculture* (1996). https://doi.org/10.4148/2475-7772.1320
[4] Mahmoudvand, Rahim, and Hossein Hassani. "Two new confidence intervals for the coefficient of variation in a normal distribution." *Journal of Applied Statistics* 36, no. 4 (2009): 429-442. https://doi.org/10.1080/02664760802474249
[5] Banik, Shipra, and B. M. Golam Kibria. "Estimating the population coefficient of variation by confidence intervals." *Communications in Statistics-Simulation and Computation* 40, no. 8 (2011): 1236-1261. https://doi.org/10.1080/03610918.2011.568151
[6] Liu, Shuang. "Confidence interval estimation for coefficient of variation." *Master's thesis, Georgia State University* (2012).

[7] Niwitpong, Sa-aat. "Confidence intervals for coefficient of variation of lognormal distribution with restricted parameter space." *Applied Mathematical Sciences* 7, no. 77 (2013): 3805-3810. https://doi.org/10.12988/ams.2013.35251

[8] Sangnawakij, Patarawan, and Sa-aat Niwitpong. "Confidence intervals for coefficients of variation in two-parameter exponential distributions." *Communications in Statistics-Simulation and Computation* 46, no. 8 (2017): 6618-6630. https://doi.org/10.1080/03610918.2016.1208236

[9] Yosboonruang, Noppadon, Sa-Aat Niwitpong, and Suparat Niwitpong. "Confidence intervals for the coefficient of variation of the delta-lognormal distribution." In *Econometrics for Financial Applications*, pp. 327-337. Springer International Publishing, 2018. https://doi.org/10.1007/978-3-319-73150-6_26

[10] Chankham, Wasana, Sa-Aat Niwitpong, and Suparat Niwitpong. "Confidence intervals for coefficient of variation of inverse Gaussian distribution." In *Proceedings of the 3rd International Conference on Vision, Image and Signal Processing*, pp. 1-6. 2019. https://doi.org/10.1145/3387168.3387254

[11] La-ongkaew, Manussaya, Sa-Aat Niwitpong, and Suparat Niwitpong. "Confidence intervals for single coefficient of variation of Weibull distribution." In *Proceedings of the 3rd International Conference on Vision, Image and Signal Processing*, pp. 1-6. 2019. https://doi.org/10.1145/3387168.3387253

[12] Yosboonruang, Noppadon, Sa-aat Niwitpong, and Suparat Niwitpong. "Measuring the dispersion of rainfall using Bayesian confidence intervals for coefficient of variation of delta-lognormal distribution: a study from Thailand." *PeerJ* 7 (2019): e7344. https://doi.org/10.7717/peerj.7344

[13] Kaewprasert, Theerapong, Sa-Aat Niwitpong, and Suparat Niwitpong. "Confidence interval for coefficient of variation of inverse gamma distributions." In *Integrated Uncertainty in Knowledge Modelling and Decision Making: 8th International Symposium, IUKM 2020*, Phuket, Thailand, November 11-13, 2020, Proceedings 8, pp. 407-418. Springer International Publishing, 2020. https://doi.org/10.1007/978-3-030-62509-2_34

[14] Buntao, Nitaya, and Sa-aat Niwitpong. "Confidence intervals for the difference of coefficients of variation for lognormal distributions and delta-lognormal distributions." *Applied Mathematical Sciences* 6, no. 134 (2012): 6691-6704.

[15] Niwitpong, Sa-aat. "Confidence intervals for the difference between coefficients of variation of normal distribution with bounded parameters." *Far East Journal of Mathematical Sciences* 98, no. 5 (2015): 649. https://doi.org/10.17654/FJMSNov2015_649_663

[16] Yosboonruang, Noppadon, Sa-Aat Niwitpong, and Suparat Niwitpong. "The Bayesian confidence intervals for measuring the difference between dispersions of rainfall in Thailand." *PeerJ* 8 (2020): e9662. https://doi.org/10.7717/peerj.9662

[17] La-ongkaew, Manussaya, Sa-Aat Niwitpong, and Suparat Niwitpong. "Confidence intervals for the difference between the coefficients of variation of Weibull distributions for analyzing wind speed dispersion." *PeerJ* 9 (2021): e11676. https://doi.org/10.7717/peerj.11676

[18] Kaewprasert, Theerapong, Sa-Aat Niwitpong, and Suparat Niwitpong. "Confidence intervals for the coefficient of variation and the difference between coefficients of variation of inverse-gamma distributions." *Songklanakarin Journal of Science & Technology* 44, no. 2 (2022). https://doi.org/10.14416/j.asep.2021.12.002

[19] Thangjai, Warisa, Sa-Aat Niwitpong, and Suparat Niwitpong. "Simultaneous fiducial generalized confidence intervals for all differences of coefficients of variation of log-normal distributions." In *Integrated Uncertainty in Knowledge Modelling and Decision Making: 5th International Symposium, IUKM 2016*, Da Nang, Vietnam, November 30-December 2, 2016, Proceedings 5, pp. 552-561. Springer International Publishing, 2016. https://doi.org/10.1007/978-3-319-49046-5_47

[20] Thangjai, W., and Suparat Niwitpong. "Simultaneous confidence intervals for all differences of coefficients of variation of log-normal distributions." *Hacettepe Journal of Mathematics and Statistics* 48, no. 5 (2019): 1505-1521.

[21] Thangjai, Warisa, and Suparat Niwitpong. "Simultaneous confidence intervals for all differences of coefficients of variation of two-parameter exponential distributions." *Thailand Statistician* 18, no. 2 (2020): 135-149.

[22] Yosboonruang, Noppadon, Sa-Aat Niwitpong, and Suparat Niwitpong. "Simultaneous confidence intervals for all pairwise differences between the coefficients of variation of rainfall series in Thailand." *PeerJ* 9 (2021): e11651. https://doi.org/10.7717/peerj.11651

[23] Puggard, Wisunee, Sa-Aat Niwitpong, and Suparat Niwitpong. "Simultaneous Confidence Intervals for All Pairwise Differences between the Coefficients of Variation of Multiple Birnbaum-Saunders Distributions." *Symmetry* 14, no. 12 (2022): 2666. https://doi.org/10.3390/sym14122666

[24] Cohen, A. Clifford. "Maximum likelihood estimation in the Weibull distribution based on complete and on censored samples." *Technometrics* 7, no. 4 (1965): 579-588. https://doi.org/10.1080/00401706.1965.10490300

[25] Geman, Stuart, and Donald Geman. "Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images." *IEEE Transactions on Pattern Analysis and Machine Intelligence* 6 (1984): 721-741. https://doi.org/10.1109/TPAMI.1984.4767596

[26] Weerahandi, Samaradasa. "Generalized confidence intervals." *Exact Statistical Methods for Data Analysis* (1995): 143-168. https://doi.org/10.1007/978-1-4612-0825-9_6
[27] Krishnamoorthy, K., Shubhabrata Mukherjee, and Huizhen Guo. "Inference on reliability in two-parameter exponential stress-strength model." *Metrika* 65 (2007): 261-273. https://doi.org/10.1007/s00184-006-0074-7
[28] Donner, Allan, and G. Y. Zou. "Closed-form confidence intervals for functions of the normal mean and standard deviation." *Statistical Methods in Medical Research* 21, no. 4 (2012): 347-359. https://doi.org/10.1177/0962280210383082
[29] Hendricks, Walter A., and Kate W. Robey. "The sampling distribution of the coefficient of variation." *The Annals of Mathematical Statistics* 7, no. 3 (1936): 129-132. https://doi.org/10.1214/aoms/1177732503
[30] Al-Ghriybah, Mohanad, Mohd Fadhli Zulkafli, Djamal Hissein Didane, and Sofian Mohd. "Wind energy assessment for the capital city of Jordan, Amman." *Journal of Applied Engineering Science* 17, no. 3 (2019): 311-320. https://doi.org/10.5937/jaes17-20241
[31] Shu, Z. R., and Mike Jesson. "Estimation of Weibull parameters for wind energy analysis across the UK." *Journal of Renewable and Sustainable Energy* 13, no. 2 (2021). https://doi.org/10.1063/5.0038001
[32] Mondal, Mithun, Djamal Hissein Didane, Alhadj Hisseine Issaka Ali, and Bukhari Manshoor. "Wind Energy Assessment as a Source of Power Generation in Bangladesh." *Journal of Advanced Research in Applied Sciences and Engineering Technology* 26, no. 3 (2022): 16-22. https://doi.org/10.37934/araset.26.3.1622