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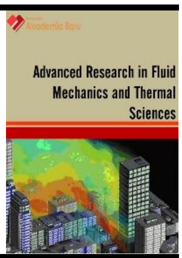

Nanofluid Stagnation-Point Flow Using Tiwari and Das Model Over a Stretching/Shrinking Sheet with Suction and Slip Effects

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ABSTRACT

In this paper, we considered the stagnation point flow and heat transfer of nanofluid over the stretching/shrinking surface by utilizing of Tiwari and Das nanofluid model. Additionally, the impact of suction and the first order slip likewise have been taken into the account. The system of governing partial differential equations (PDEs) is changed into the system of non-linear ordinary differential equations (ODEs) by means of similarity transformation. The resultant ODEs are solved by using BVP solver (bvp4c) in MATLAB software. The impact of some physical parameters, for example the suction parameter and the slip parameter on the skin friction coefficients and the local Nusselt number as well as the temperature and velocity profiles have been investigated, tabulated and graphically presented. These profiles and variations demonstrate that there exist dual solutions for a specific range of the stretching/shrinking parameter. Both suction and slip effects has enhance the local Nusselt number which represent heat transfer rate at the surface. It is also found that inclusion of both suction and slip effects expands the range of the dual solutions exist. The existence of the dual solutions only occurs in in the shrinking region. The flow separation in the boundary layer delay due to suction and slip effects imposed in the boundary condition.

Keywords:
Stagnation point flow; heat transfer; nanofluid; suction; slip; dual solutions

1. Introduction

Nanofluids is a new type of fluids with small particles called nanoparticles dispersed in a liquid with low thermal conductivity, such as water and ethylene glycol in order to increase conductivity of thermal. The example of nanoparticles such as metal or metal oxides to improve conduction and convection coefficient by enabling more heat transfer out from the coolant [1]. It appears that the term "nanofluid" was first coined by Choi [2]. Nanofluid with nanometer sizes of particles have

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unique physical and chemical properties. They can easily flowing passing through microchannels without being clogged due to the fact that they are small to react with liquid molecules [3]. The most common heat transfer fluids such as water and ethylene glycol have limited performance in terms of thermal properties and as consequence can impose restrictions in thermal applications. Most of the solids especially metal, on the contrary, has high thermal conductivity approximately one to three times, by comparison with liquids. Therefore, it is expect that any conventional fluid containing nanoparticles can enhance its conductivity [4]. It was reported by Wong and Leon [5], there are many current and future applications included nanofluids such as in many industrial applications, nuclear reactors, transportation, electronics as well as biomedicine and food. It was reported by several comprehensive review on nanofluids, two nanofluids models have been continuously used by researchers, namely Buongiorno [6] and later proposed by Tiwari and Das [7] with different mechanism, respectively [8-11]. In the pioneering nanofluid model introduced by Buongiorno [6], he considers Brownian motion and the thermophoresis on the heat transfer characteristics to study behaviour of nanofluids. In detail, this model takes into account the Brownian motion and thermophoresis effects in energy equation and found that absolute velocity of the nanoparticles could be estimated as the sum of the base fluid velocity to a relative velocity. On the other hand, the nanofluid model proposed by Tiwari and Das [7] examined nanofluids behaviour by considering the solid volume fraction.

The first paper study on laminar fluid flow caused by a stretching flat surface in a nanofluids was done by Khan and Pop [12]. They used nanofluids model proposed by Buongiorno [6] which combine the effects of Brownian motion and thermophoresis. It is found that the reduced Nusselt number is a decreasing function, while the reduced Sherwood number is an increasing function of each values of the Prandtl number, the Lewis number, the Brownian motion parameter and the thermophoresis parameter considered. They also recommend that their study can be extended to different types of nanofluids as Cu, Al_2O_3 and TiO_2. Buongiorno model later was successfully used in many nanofluids research articles, for example, Nield and Kuznetsov [13,14], Kuznetsov and Neild [15,16], Bachok *et al.,* [17,18], Khan and Aziz [19], Hayat *et al.,* [20], Khan *et al.,* [21] and among others. An extensive investigations on three-dimensional boundary layer flow in nanofluid with different flow and boundary and conditions for example in the presence of a constant applied magnetic field and heat generation/absorption, convective condition and viscoelastic nanofluids was examined by Hayat *et al.,* [22-24] and Muhammad *et al.,* [25,26]. In contrast to the above mentioned model, this present study the problem by means of the nanofluid model proposed by Tiwari and Das [7], which was also used by several researchers [27-31].

Recently, studies on the flow towards a shrinking sheet have received a great attention among researchers. The pioneering study on flow over a shrinking sheet was started by Miklavcic and Wang [32]. They found that the vortex was not confined within a boundary layer and a steady flow could not exist without imposing sufficient suction to the boundary. Since then, numerous research has developed examining different aspects of the problem. In the last several decades, a lot of researchers have explored the boundary layer flow with suction/injection due to its potential applications in the field of aerodynamics and space science [33]. Zhang *et al.,* [34] investigated the effects of wall suction/blowing on two dimensional (2-D) flow past a confined square cylinder. They found that an increase in the Reynolds number destabilizes the flow. For delaying flow separation on a cylindrical surface, Prandtl was the first scientist to employ boundary layer suction. To improve the efficiency and stability of lift systems, suction and blowing approaches have since emerged and been evaluated in a variety of experiments [35]. Sheikholeslami [36] investigated the effect of uniform suction on nanofluid flow and heat transfer through a cylinder. They concluded that the skin friction coefficient has a direct relationship with the Reynolds number and suction parameter. On the other hand, it was reported that slip effect has applications in many industrial developments at boundaries of pipes, walls or curved surfaces [37].

Inspired by above mentioned studies and applications, we extend the previous study by Bachok *et al.,* [4] by considering both suction and slip effects in nanofluid. In the present study, the influence of the suction and slip effects on the coefficient of skin friction and the local Nusselt number as well as the related profiles are will be examined. For both stretching/shrinking case with inclusion of suction and slip effects which was not considered by Bachok *et al.,* [4]. To our best information, the present investigation has not been reported before.

2. Methodology

Let us consider a steady incompressible nanofluid in the region $y > 0$ driven by a permeable stretching/shrinking surface located at $y = 0$ near the stagnation point at $x = 0$ with slip effect as shown in Figure 1, where x and y are the Cartesian coordinates measured along the surface and normal to it, respectively. It is assumed that the velocity of stretching/shrinking sheet is $U_w(x) = ax$ and the ambient fluid velocity is $U_\infty(x) = bx$ are vary linearly from the stagnation point, where a and b are positive constants. The corresponding stretching and shrinking sheet depend upon the conditions of $a > 0$and $a < 0$, respectively.

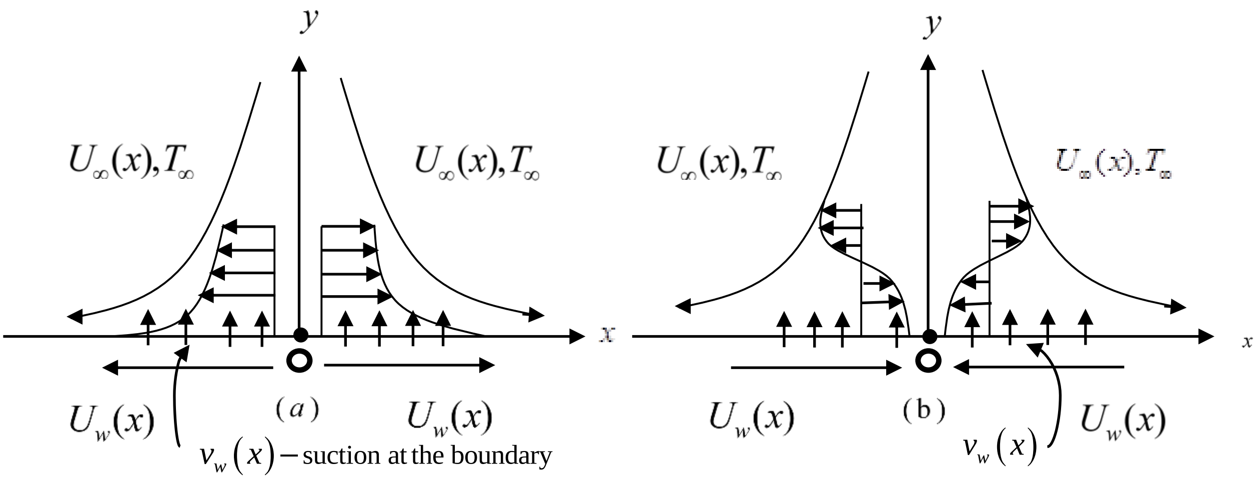

Fig. 1. Physical model and coordinate system: (a) Stretching sheet; (b) Shrinking sheet

With all above mentioned conditions the boundary layer governing equations of mass, momentum and energy can be written as follows [4].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_\infty \frac{dU_\infty}{dx} + \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

subject to the initial and boundary conditions

$$u = U_w(x) + L\left(\frac{\partial u}{\partial y}\right),\ v = v_w(x),\ T = T_w \text{ at } y = 0$$
$$u \to U_\infty(x),\ T \to T_\infty \text{ as } y \to \infty. \tag{4}$$

Here, $u$and v are velocity components corresponding to the along $x$and y axes, respectively, L is the velocity slip factor, T is the temperature of the nanofluid, μ_{nf} is the viscosity of the nanofluid, α_{nf} denotes the thermal diffusivity of the nanofluid, ρ_{nf} denotes the density of a nanofluid which are given by Oztop and Abu-Nada [38].

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},$$

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s, \frac{k_{nf}}{k_f} = \frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)} \tag{5}$$

Here, k_{nf} is the thermal conductivity of the nanofluid, $(\rho C_p)_{nf}$ is the heat capacity of the nanofluid, $\phi$is the nanoparticle volume fraction, ρ_f and ρ_s are the densities of the fluid and of the solid fractions, respectively, k_f and k_s are the thermal conductivities of the fluid and of the solid fractions, respectively. It should be stated that the use of the above expression for k_{nf} is restricted to spherical nanoparticles where it does not account for other shapes of nanoparticles [4,27]. The viscosity of the nanofluid μ_{nf} has been approximated by Brinkman [39] as viscosity of a base fluid μ_f containing dilute suspension of fine spherical particles.

In order to reduce the Eq. (1)-(3) into ODEs, the following similarity transformation variables are used [4].

$$\eta = \left(\frac{b}{v_f}\right)^{\frac{1}{2}} y, \psi = (v_f b)^{\frac{1}{2}} x f(\eta), \theta(\eta) = \left(\frac{T-T_\infty}{T_w-T_\infty}\right) \tag{6}$$

where η is the similarity variable, v_f denotes the kinematic viscosity of the fluid and ψ is the stream function defined as $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$ which identically satisfies the continuity Eq. (1). Substituting Eq. (6) into Eq. (2)-(3), may be written as [4]

$$\frac{1}{(1-\phi)^{2.5}(1-\phi+\phi\rho_s/\rho_f)} f''' + ff'' - f'^2 + 1 = 0 \tag{7}$$

$$\frac{1}{\Pr} \frac{\left(\frac{k_{nf}}{k_f}\right)}{[(1-\phi)+\phi(\rho C_p)_s/(\rho C_p)_f]} \theta'' + f\theta' = 0. \tag{8}$$

The boundary conditions (4) are then becomes

$$f'(0) = \lambda + \delta f''(0), f(0) = \gamma, \theta(0) = 1$$

$$f'(0) \to 1, \theta(\eta) \to 0 \text{as} \eta \to \infty \tag{9}$$

where $\lambda = b/a$ is the stretching/shrinking parameter or velocity ratio parameter with $\lambda > 0$ for a stretching sheet and $\lambda < 0$ for a stretching sheet, respectively, $\gamma = \left(\nu_w/\left(\nu_f b\right)^{1/2}\right) > 0$ is the suction parameter and $\delta = L\left(b/\nu_f\right)^{1/2}$ is the slip parameter and prime denotes differentiation with respect to η.

The physical quantities of interest are the skin friction coefficient, C_f and the local Nusselt number, Nu_x which can be defined as [4]

$$C_f = \frac{\tau_w}{\rho_f U_\infty^2},\ Nu_x = \frac{x q_w}{k_f (T_w - T_\infty)}, \tag{10}$$

where τ_w is the surface shear stress along the plate and q_w is the heat flux from the plate, as in Bachok *et al.,* [4]

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0},\ q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{11}$$

Substituting (6) into (11) and using (10), the following expression can be obtained

$$C_f\, {Re_x}^{1/2} = \frac{1}{(1-\phi)^{2.5}} f''(0)\ (12) \qquad Nu_x/\, {Re_x}^{1/2} = -\left(\frac{k_{nf}}{k_f}\right)\theta'(0) \tag{13}$$

where $Re_x = \frac{U_\infty x}{\nu_f}$ is the local Reynolds number.

3. Results and Discussion

In this section, we discussed the numerical solutions of the transformed ODEs (7-8) along boundary condition (8). In order to solve these highly non-linear ODEs, bvp4c solver built in MATLAB software has been used. The results are revealed that there is range of non-uniqueness solutions which depends upon the stretching and shrinking parameters. In Figure 2 to 9, the solid lines denote the first solution, while the dash lines denote the second solution. It was initiated by Merkin [40] followed by several researchers for examples, Weidman *et al.,* [41], Harris *et al.,* [42], Rosca and Pop [43,44] and Awaludin *et al.,* [45] that the common similarity equations for different problems generally accept the existence of multiple solutions, where the first solutions is stable, whereas the second solutions is unstable. Therefore, the procedure for proving solution stability analysis is not repeated here. Dual solutions exist for shrinking case is because of backward flow caused by shrinking surface. By using error and trial base technique in order to find the initial guesses of $f''(0)$ and $-\theta'(0)$. The thickness of boundary layer is improved until unless profiles of velocity and temperature satisfy the far filed boundary conditions asymptotically. The effect of suction parameter γ, slip parameter δ and solid volume fraction ϕ on the coefficient of skin friction and Nusselt number have been examined and analyzed. Three different nanoparticles have been considered to examine the water based nanofluid specifically Cu −water, Al_2O_3 −water and TiO_2 −water. As working base fluid temperature is considered as $25°C$, therefore, $Pr = 6.2$ is kept fix as it is mentioned in the study of Oztop and Abu-Nada [38]. The thermophysical properties of nanomaterials and water are given in the Table 1.

Table 1
Thermophysical properties of fluid and nanoparticles [38]

Physical properties	Fluid phase (water)	TiO_2	Al_2O_3	Cu
$\rho\left(\frac{kg}{m^3}\right)$	997.1	4250	3970	8933
$C_p\left(\frac{J}{kgK}\right)$	4179	686.2	765	385
$k\left(\frac{W}{mK}\right)$	0.613	8.9538	40	400

In order to validate our results, the results of present study in term of $C_f\,{Re_x}^{1/2}$ and $Nu_x/\,{Re_x}^{1/2}$ have been compared with the results from previous study by Bachok *et al.,* [4] for specific case as tabulated in Table 2 and 3. Table 2 and 3 display the values of $C_f\,{Re_x}^{1/2}$ and $Nu_x/\,{Re_x}^{1/2}$for different values of λ and ϕ when the suction and slip effect are neglected by setting $\gamma = \delta = 0$ in Eq. (9). It is found that the present numerical results are in an excellent agreement with the solutions obtained by Bachok *et al.,* [4]. It is analyzed from Table 3 that heat transfer rate of Cu is higher than Al_2O_3andTiO_2 basically because of the thermal conductivity of Cu is greater than Al_2O_3 and TiO_2.

Table 2
Values of $\boldsymbol{C_f\,{Re_x}^{1/2}}$ for some values of $\boldsymbol{\lambda}$and $\boldsymbol{\phi}$

λ	ϕ	Bachok *et al.,* [4]			Present results		
		$Cu-$ water	Al_2O_3 −water	TiO_2 −water	$Cu-$ water	Al_2O_3 −water	TiO_2 −water
-0.5	0.1	2.2865	1.9440	1.9649	2.286512	1.943998	1.964912
	0.2	3.1826	2.4976	2.5413	3.182538	2.497651	2.541209
-0.3	0.1				2.182412	1.855492	1.875454
	0.2				3.037645	2.383939	2.425514
0	0.1	1.8843	1.6019	1.6192	1.884324	1.602057	1.619292
	0.2	2.6226	2.0584	2.0942	2.622743	2.058324	2.094220
0.3	0.1				1.447449	1.230625	1.243864
	0.2				2.014668	1.581109	1.608682
0.5	0.1	1.0904	0.9271	0.9371	1.090453	0.927106	0.937079
	0.2	1.5177	1.1912	1.2118	1.517774	1.191147	1.211919

Table 3
Values of $\boldsymbol{Nu_x/\,{Re_x}^{1/2}}$ for some values of $\boldsymbol{\lambda}$and $\boldsymbol{\phi}$

λ	ϕ	Bachok *et al.,* [4]			Present results		
		$Cu-water$	Al_2O_3 −water	TiO_2 −water	$Cu-water$	Al_2O_3 −water	TiO_2 −water
-0.5	0.1	0.8385	0.7272	0.7082	0.838510	0.727149	0.708157
	0.2	1.0802	0.8878	0.8423	1.080308	0.887849	0.842242
-0.3	0.1				1.078584	0.982354	0.958912
	0.2				1.330904	1.162202	1.107817
0	0.1	1.4043	1.3305	1.3010	1.404327	1.330508	1.301085
	0.2	1.6692	1.5352	1.4691	1.669338	1.535160	1.469033
0.3	0.1				1.694789	1.639754	1.604959
	0.2				1.971910	1.867288	1.790581
0.5	0.1	1.8724	1.8278	1.7898	1.872386	1.827847	1.789738
	0.2	2.1577	2.0700	1.9867	2.157690	2.069987	1.986723

The values of $f''(0)$ and $-\theta'(0)$ for some values of γ with $\lambda = -2$, $\phi = 0.1$ and $\delta = 1$ are given in Table 4. Meanwhile, Tables 5 and 6 are tabulated for some values of δ when $\lambda = -2$, $\gamma = 1$, $\phi = 0.1$ and $\phi = 0.2$ in order to see the influence of the solid volume fraction $f''(0)$ and $-\theta'(0)$.

Table 4
Values of $f''(0)$ and $-\theta'(0)$ for some values of γ with $\lambda = -2$, $\phi = 0.1$ and $\delta = 1$

	$f''(0)$			$-\theta'(0)$		
γ	$Cu - water$	Al_2O_3 −water	TiO_2 −water	$Cu - water$	Al_2O_3 −water	TiO_2 −water
0	1.685164	1.494822	1.508228	0.797086	0.546989	0.566981
	(0.755326)	(0.767193)	(0.765917)	(0.000595)	(0.000273)	(0.000217)
0.5	1.949752	1.774183	1.786290	2.737066	2.544629	2.634112
	(0.506110)	(0.524721)	(0.523560)	(0.002988)	(0.000517)	(0.000458)
1	2.122986	1.954583	1.966227	4.913762	4.789090	4.957726
	(0.272737)	(0.330425)	(0.327212)	(0.072111)	(0.010419)	(0.010571)

( ) dual solution

Table 5
Values of $f''(0)$ and $-\theta'(0)$ for some values of δ with $\lambda = -2$, $\phi = 0.1$ and $\gamma = 1$

	$f''(0)$			$-\theta'(0)$		
δ	$Cu - water$	Al_2O_3 −water	TiO_2 −water	$Cu - water$	Al_2O_3 −water	TiO_2 −water
0.05	3.539007	1.790801	2.038780	3.158007	1.876948	2.330443
	(1.071590)	(1.844711)	(1.850102)	(0.699207)	(1.967930)	(2.008518)
0.1	3.837117	2.487392	2.580516	3.524661	2.839105	3.038627
	(0.892603)	(1.238326)	(1.205657)	(0.507445)	(0.818328)	(0.786365)
0.5	3.131171	2.690649	2.720173	4.588271	4.369896	4.538909
	(0.430420)	(0.523101)	(0.517713)	(0.141028)	(0.046977)	(0.046924)
1	2.122985	1.954582	1.966226	4.913762	4.789089	4.957725
	(0.272736)	(0.330425)	(0.327212)	(0.072111)	(0.010419)	(0.010571)

( ) dual solution

Table 6
Values of $f''(0)$ and $-\theta'(0)$ for some values of δ with $\lambda = -2$, $\phi = 0.2$ and $\gamma = 1$

	$f''(0)$			$-\theta'(0)$		
δ	$Cu - water$	Al_2O_3 −water	TiO_2 −water	$Cu - water$	Al_2O_3 −water	TiO_2 −water
0	3.311506	2.267005	1.469663	2.004908	2.440068	1.487628
	(1.258851)	(1.350643)	(1.929177)	(0.734018)	(1.095494)	(2.038339)
0.5	3.226885	2.569098	2.617122	3.547750	3.247783	3.489868
	(0.405568)	(0.544453)	(0.536163)	(0.208344)	(0.063147)	(0.061493)
1	2.158114	1.906088	1.925355	3.809717	3.638109	3.882787
	(0.256777)	(0.342877)	(0.338100)	(0.140305)	(0.016545)	(0.016158)

( ) dual solution

Figure 2 and 3 illustrate the velocity and temperature profiles for different values of γ by keeping $Pr = 6.2$, $\phi = 0.1$ and $\delta = 1$. From these Figures 2 and 3, it is noticed that the existence of the dual solutions in the shrinking case. For the first solution, it is clear that the velocity is increased and temperature is decreased with the increasing of γ. Imposing suction parameter has cause to reduction in momentum boundary layer thickness and thus increases the flow velocity near the surface as depicted in Figure 2.

It is also observed from Figure 3 that thickness of thermal boundary layer boundary layer are reduced for higher values of the suction and consequently decreases the temperature near the surface in the first solution. On the other hand, opposite trend are shown for the second solution when the values of $\gamma$is increases as presented in Figures 2 and 3.

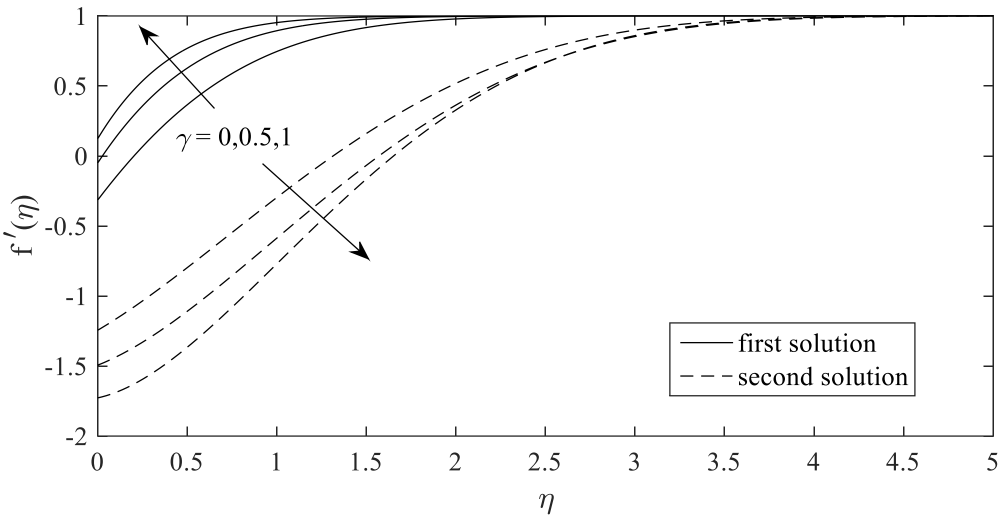

Fig. 2. The velocity profiles $f'(\eta)$for different values of γ when $Pr = 6.2$, $\phi = 0.1$, $\delta = 1$and $\lambda = -2$ (shrinking case) for Cu —water base fluid

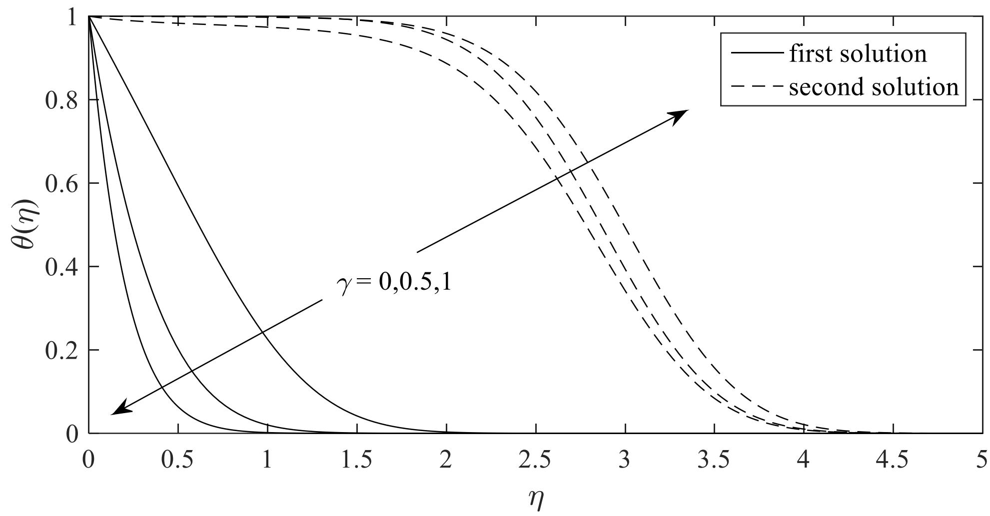

Fig. 3. The temperature profiles $\theta(\eta)$for different values of γ when $Pr = 6.2$, $\phi = 0.1$, $\delta = 1$and $\lambda = -2$ (shrinking case) for Cu —water base fluid

Effect of the slip parameter δon the velocity and temperature distributions are shown in the Figure 4 and 5. It is clearly noticed from Figure 4 and 5 that velocity and temperature of fluid are increase and decrease in the first solutions, respectively with the increase of δ. For the first solution, the velocity is increased with an increase in the values of δ as illustrated in Figure 4. An increasing in slip parameter reflects to reduction in momentum boundary layer thickness and in turn increases the flow near the surface.

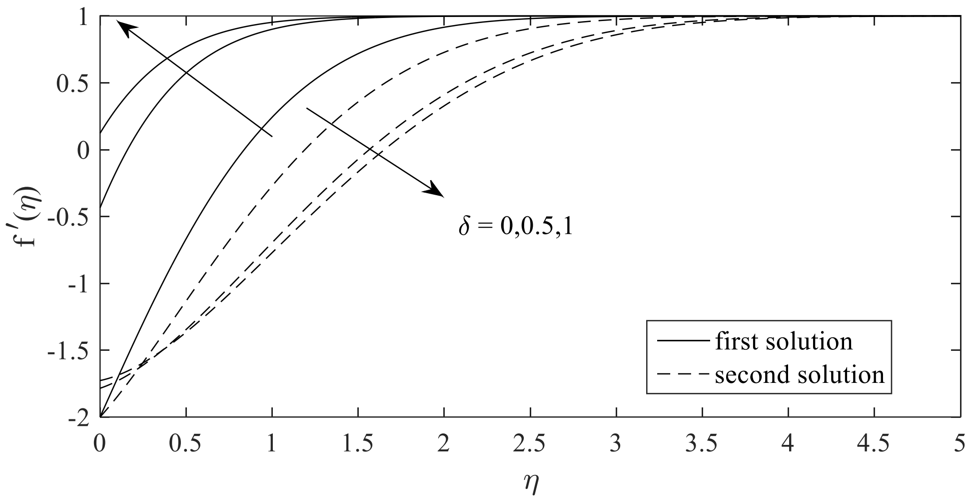

Fig. 4. The velocity profiles $f'(\eta)$for different values of δ when $Pr = 6.2$, $\phi = 0.1$, $\gamma = 1$and $\lambda = -2$ (shrinking case) for Cu −water base fluid

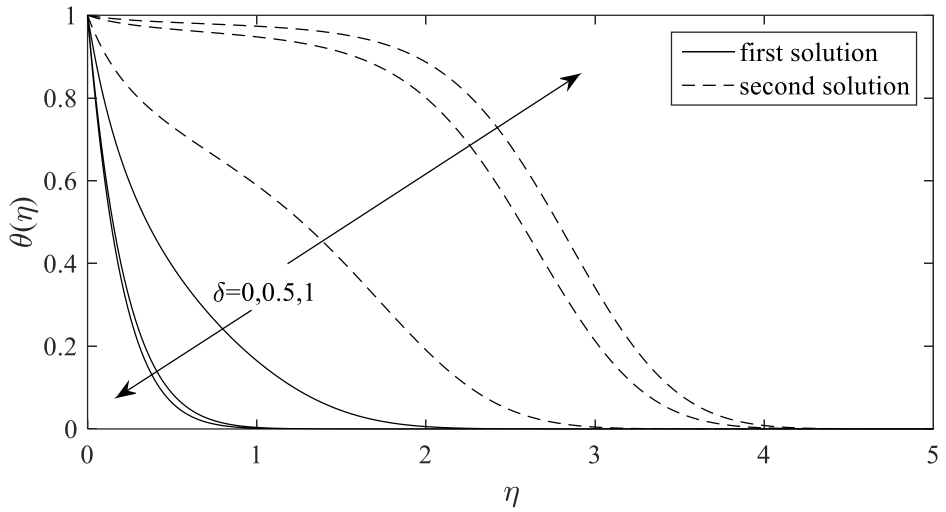

Fig. 5. The temperature profiles $\theta(\eta)$for different values of δ when $Pr = 6.2$, $\phi = 0.1$, $\gamma = 1$and $\lambda = -2$ (shrinking case) for Cu −water base fluid

In Figure 5, it is seen that the temperature drop as the slip effect is imposed. In physical, slip effect has enhanced the competency of the diffusion process. As more heat is removed, the temperature is decreasing and the rate of heat transfer is getting higher as illustrated in Figure 9.

The physical quantities of interest of the present study are the skin friction coefficient and the local Nusselt number. Thus, it is important to sketch Figures 6- 9 by plotting the variations of $f''(0)$ and $-\theta'(0)$ withλ for some values of suction parameter γ and slip parameter $\delta$to see the influence of suction and slip parameter on the heat transfer characteristics. The variations of $f''(0)$ and $-\theta'(0)$ corresponding to stretching/shrinking parameter λ for some values of suction parameter γ as shown in the Figures 6 and 7, respectively in Cu-water nanofluid. It can be observed from the Figures 6 and 7 that there exist three ranges of the solutions namely no similarity solution, unique solution and dual solution ranges. These Figure 6 and 7 indicate that there are dual solutions for $\lambda_c < \lambda < \lambda_l$, unique solutions for $\lambda > \lambda_l$ and no solution found for $\lambda < \lambda_c$, where λ_c and λ_l are the critical value and lower critical value of λ, respectively for which Eq. (7) and (8) have no solution and the full Navier-Stokes and energy equations should be considered. Based on analysis, the critical values λ_c

for $\gamma = 0,0.5$ and 1 are -2.5696, -3.7420 and -5.4552 while the lower critical values are -1.1, -1 and -0.9.

Figure 6 demonstrates the values of $f''(0)$ increases as γ increases. Physically, this is caused by the suction effect increasing the surface shear stress, delay the fluid flow and therefore, increase the velocity gradient at the surface which is consistence with the graph in Figure 2. From Figure 6 also, it can be noted that the critical values stretching/shrinking parameter λ for which the solution exist increase as increases, proposes that suction expands the range of the dual solutions of the similarity Eq. (7) and (8).

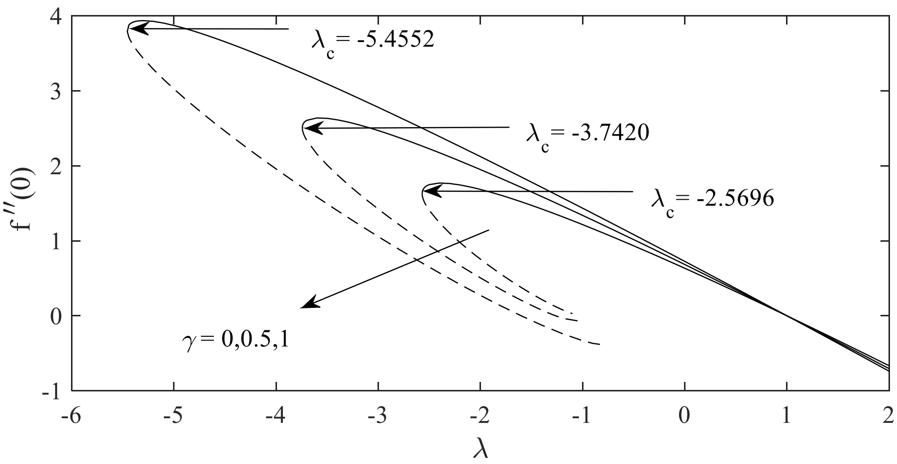

Fig. 6. Variation of $f''(0)$ with λ for different values of γ in Cu-water nanofluid with $Pr = 6.2$, $\delta = 1$and $\phi = 0.1$

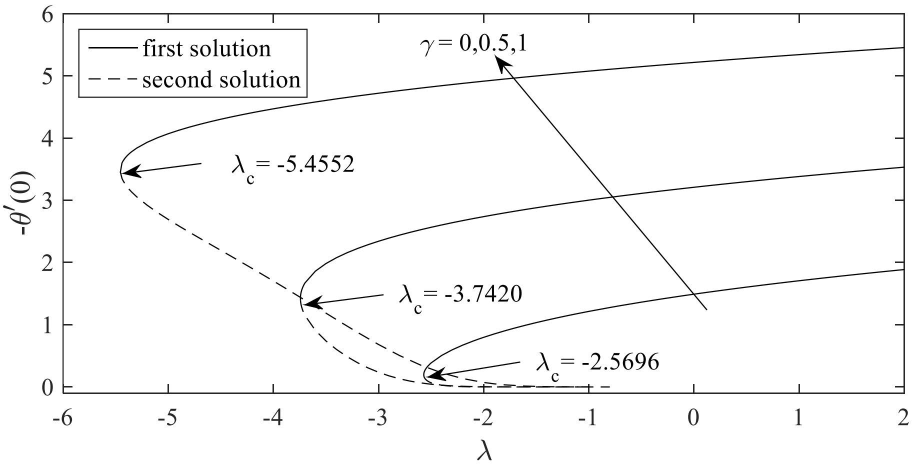

Fig. 7. Variation of $-\theta'(0)$ with λ for different values of γ in Cu-water nanofluid with $Pr = 6.2$, $\delta = 1$and $\phi = 0.1$

Figure 7 exhibits the variation of $-\theta'(0)$ as a function of the stretching/shrinking parameter λ for certain value of γ. The local Nusselt number which represents the heat transfer rate of the surface tends to increase as γ increases. The suction effect has cause the reduction in the thermal boundary layer thickness, increasing the temperature gradient on the surface and in consequence, enhance the heat transfer rate at the surface which consistent with the temperature profile $\theta(\eta)$ presented in Figure 3.

Figure 8 and 9 illustrate the variation of $f''(0)$ and $-\theta'(0)$ with λ, for some values of slip parameter δ, respectively. From the Figures 6 and 8, the values of $f''(0) = 0$ at $\lambda = 1$, therefore there is no friction at the fluid-solid interface when the boundaries of the fluid and solid move with same velocity.

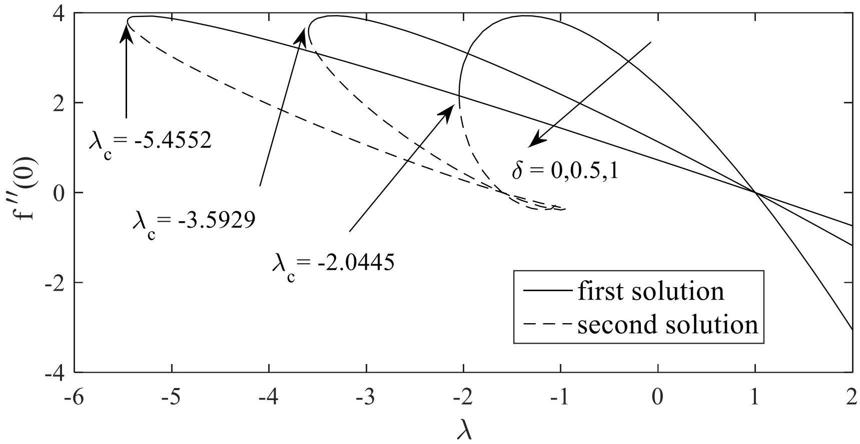

Fig. 8. Variation of $f''(0)$ with λ for different values of δ in Cu-water nanofluid with $Pr = 6.2$, $\gamma = 1$ and $\phi = 0.1$

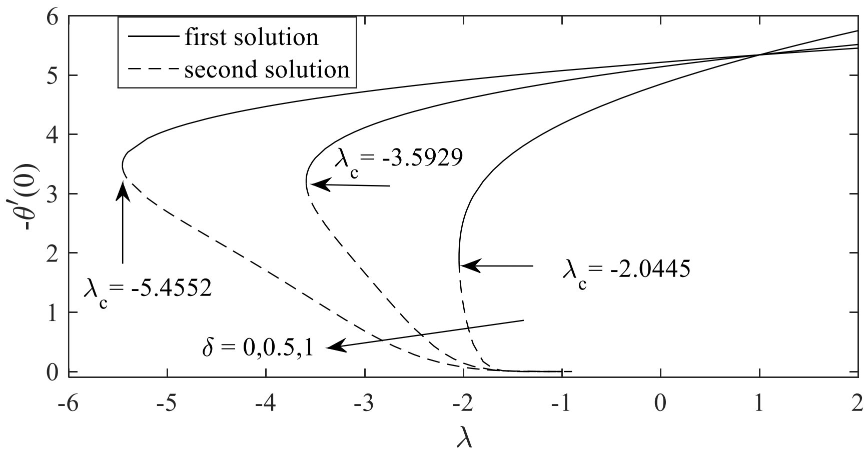

Fig. 9. Variation of $-\theta'(0)$ with λ for different values of δ in Cu-water nanofluid with $Pr = 6.2$, $\gamma = 1$ and $\phi = 0.1$

It is observed that the increases in slip parameter δ has increase the local Nusselt number, as shown in Figure 9. This phenomenon occurs due to fact that slip effect tends to reduce in the thermal boundary layer thickness, increasing the temperature gradient on the surface and as a result, enhance the heat transfer rate at the surface which consistent with the temperature profile $\theta(\eta)$ presented in Figure 5. The velocity and temperature profiles which have been shown in Figure 2-5 satisfy the far field boundary conditions (9) asymptotically, which leads to the confidence to the present numerical results and the existence of the dual solutions obtained.

4. Conclusions

In this study, fluid flow and heat transfer of a nanofluid on a stretching/shrinking surface is investigated numerically. The main contribution of this study was considering both suction and slip effects in the original work done by Bachok *et al.,* [4] which was not considered before. This present has been motivated by the fact there are numerous applications included nanofluids such as in many industrial applications, nuclear reactors, transportation, electronics as well as biomedicine and food. In solving this problem, the partial differential equations are reduced to ordinary differential equations by using similarity transformation, before being solve using the bvp4c solver in Matlab software. Dual solutions were found to exist for the certain range of shrinking case and the unique solution was exist for the stretching case. The impact of the parameters namely suction parameter and slip parameter on fluid flow and heat transfer characteristics are graphically presented and discussed. Both suction and slip effects has enhance the local Nusselt number which represent heat transfer rate at the surface. It is also found that inclusion of both suction and slip effects expands the range of the dual solutions exist. The existence of the dual solutions only occurs in in the shrinking region. The flow separation in the boundary layer delay due to suction and slip effects imposed in the boundary condition.

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