Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage:
https://semarakilmu.com.my/journals/index.php/fluid_mechanics_thermal_sciences/index
ISSN: 2289-7879

Study of Unsteady Two Phase Flow over An Inclined Permeable Stretching Sheet with Effects of Electrification and Radiation

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ARTICLE INFO

Article history:
Received 21 March 2022
Received in revised form 29 May 2022
Accepted 8 June 2022
Available online 4 July 2022

Keywords:
Permeable; volume fraction; radiation; bvp4c method

ABSTRACT

An analysis on flow and heat transfer with in a two-dimensional unsteady radiative boundary layer with fluid-particle interaction has been studied. The flow is occurred due to the suddenly linear movement of an inclined permeable stretching sheet. The flow is considered in a neutral medium where no external electric or magnetic field is supplied. But due to the random motion of particles leads to interaction between fluid-particle, particle-particle and particle-wall, a tribo-electric effect occurs. As a result, both the fluid as well as particles are electrified which creates a major impact on flow field. Hence, a balanced mathematical model has been formulated considering both electrification and radiation parameter on both the phases. Using similarity transformation, the governing equations are transferred to ODEs and solved by built in solver Bvp4c of MATLAB. The impacts of various parameters on the flow field have been discussed and determined the heat transfer characteristics. The stronger electric filed significantly enhances the temperature of both fluid and particle phase, which occurs more heat transfer on the surface.

1.Introduction

The modelling, analysis and applications of two-phase unsteady flow and heat transfer has a large area of applications in diversified industries. Out of the vast areas, the applications of flow phenomena over a stretching sheets narrows down but made it specific in the field industrial applications. Particularly the outcomes of study have abundant applications in manufacturing plants and mechanized units of fibre extraction units, extrusion of polymer sheet in dying industry, formation of plastic thin sheets, sedimentation, and thermal as well as nuclear power plants. The intensity of heat flux over any surface bears an important role in determination of quality of the products. Nevertheless, the rate of stretching, also have great role for getting a desirable qualitative product. In view of such applications, many researchers blended over to study on the topic of flow and heat transfer over a stretching sheet and analysed the solution for different types of flow characteristics.

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https://doi.org/10.37934/arfmts.97.2.2638

The first ever experimental study on boundary layer flow over a stretching surface was made by Sakiadis in 1961 [21]. There after a large number of scholars extended the above experiment with the heat transfer effect over a stretching sheet / surface. Sakiadis's theoretical concepts on Newtonian fluids were later experimented by Tsou *et al.,* and Later Crane [4] extended that work to an extending surface with change of surface velocity. A study on the heat and mass transfer of viscous fluid over an isothermal stretching sheet has been conducted by Gupta and Gupta [12]. Rajgopal *et al.,* [17] have worked on the flow of a viscoelastic fluid over a stretchable sheet. Rashidi *et al.,*[18] have proceeded on the free convective heat and mass transfer for MHD flow over a vertical stretching sheet. Gireesha *et al.,* [8] have modelled and solved numerically the "boundary layer flow of an unsteady dusty fluid" problem. Mohmmod*et et al.,* [15] analyzed the "unsteady boundary layer flow of micro polar fluid over a shrinking surface". Electrification of the solid particles always takes place when the solid particles and wall of similar or different materials are contacted with each other and apart from each other in different surface conditions.

Radiation is a process of transmission of heat energy through an empty space by electromagnetic wave. The heat transfer by radiation plays a vital role in many heating or cooling operations. These knowledges are also used in combustion of fossils fuels, operation of a furnace, thermal cracking etc. Thermal radiation is known as one of the fundamental processes of heat transmission. The features of thermal radiation mainly depend on different properties, like its temperature, its spectacular absorptive and spectral emissive power etc. Also, there is an effect of thermal radiation on heat transfer process of more advanced energy convection systems which are operated in very high temperature. When the variation of surface temperature and the surrounding temperature is very high, the effects of thermal radiation become important. So, the thermal radiation is an important factor for controlling heat and mass transfer. The development thermal diffusivity of the cooling liquid in the stretching sheet problem is made by considering thermal radiation effects of particles. Thus, the facts of radiation of heat transmission in the arrangement can be used in a large number of new engineering work processes like fossil fuel combustion processes, solar power technology etc. First experimental work with consideration of the boundary layer flow in thermal radiation effect was held on 1962 by Viskanta and Grosh [25]. In 1969 England, Emery [5] etc. have conducted a study on the radiation effect on the optically thin gray gas which is bounded by stationary vertical plate. Ansab *et al.,* [14] have studied the effect of heat source on skin friction of vertical stretching sheet. Nadia *et al.,* [19] have studied about the impact of volume fractions of particles on velocity profile of exponentially shrinking Sheet. Chu *et al.,* [1,22] have studied the effects of different parameter on second grade fluid and the impact of various flow parameters on entropy generation.

Amos *et al.,* [2] have analysed the impact of radiation on MHD flow in an inclined plane. Gowdara *et al.,* [10] have concentrated on modelling and computations of an unsteady boundary layer flow and heat transfer of fluid with dust particles and was past an exponentially stretchable sheet withsuction term. Tripathy and Mishra [24] have studied about the distribution of temperature in the bounder layer of heat flow past a horizontal plate. It was noticed that, heat transmits from the surface to fluid and the reason behind this is the positive value of Nusselt number (Nu). The growth of boundary layer has been stabilized by adding buoyancy force in the momentum equation. Gireesh *etal.,* [9] have worked on the effect radiation on boundary layer flow over a stretching sheet in presence of free stream velocity. Akhter *et al.,* [20] have studied the consequence of thermal radiation on the Free Convection boundary layer flow with low Prandtl number.

From the above review of literature, the characteristics of flow field has been studied either the surface is supplied with external electric field or the flow is considered in an electric medium.

Neither the surface nor the flow is considered in an electrified medium but due to tribo-electrification, an electric force is generated which influence flow and temperature field of both the phases, which is hardly studied. Again, in most of the literatures, the radiation parameter is considered in fluid phase only. But due to exchange of energy between fluid and particle phase, both the phases are influenced by radiation, which is also not studied by previous authors. Hence, in the present study, a balanced mathematical model has been formulated for an unsteady boundary layer flow with fluid –particle suspension over a stretching surface with consideration of force due to electrification and radiation in flow and energy field for both the phases and investigated the various boundary layer characteristics and its impacts on the flow field.

2. Model Formulation

Consider an unsteady radiative boundary layer flow over a surface which is permeable and inclined with an angle α. Figure 1 represents the geometry of the flow field, where the abscissa of geometry is assumed along the surface and the ordinate is perpendicular to it. The flow is occurred due the stretching of surface with a velocity of $U_w(x,t) = \frac{cx}{1-at}$. $T_w = T_\infty + T_0 \frac{cx^2}{\nu(1-at)^2}$ is the temperature maintained at the surface and T_∞ is the temperature of the ambient fluid. Neither the surface nor both fluid and particles are supplied by external electric field, but it is generated due to collision particles among themselves as well as with wall within the flow region. Hence, the force due to electrification impacts on both the fluid as well as particle phase. Since the plate is inclined, buoyancy force cannot be neglected in the modulation and simultaneously the force due to electrification plays a major role in temperature field of both the phases.

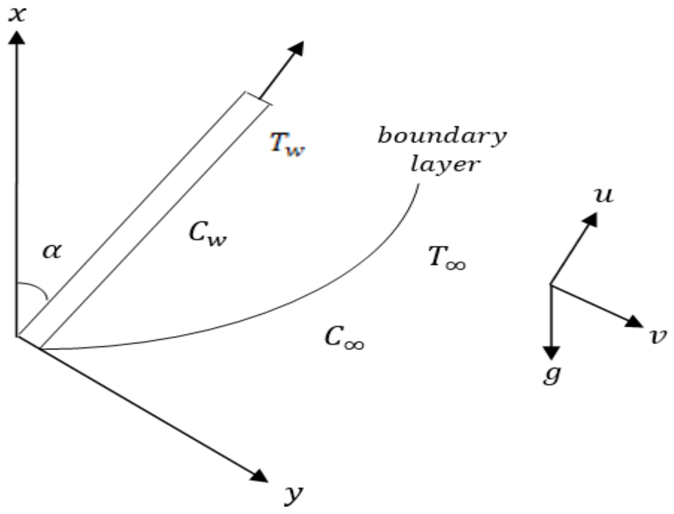

Fig. 1. Geometry of the flow problem

With the above assumption, the Governing Equations within the boundary layer of the flow field is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial \rho_p}{\partial t} + \frac{\partial}{\partial x}\left(\rho_p u_p\right) + \frac{\partial}{\partial y}\left(\rho_p v_p\right) = 0 \tag{2}$$

$$\rho(1-\varphi)\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=(1-\varphi)\mu\frac{\partial^2 u}{\partial y^2}-\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)+\varphi\rho_s\left(\frac{e}{m}\right)E+(1-\varphi)\rho g\beta^*(T-T_\infty)\cos\alpha \tag{3}$$

$$\varphi\rho_s\left(\frac{\partial u_p}{\partial t}+u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial u_p}{\partial y}\right)+\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)+\varphi\rho_s\left(\frac{e}{m}\right)E++\varphi(\rho_s-\rho)g \tag{4}$$

$$\varphi\rho_s\left(\frac{\partial v_p}{\partial t}+u_p\frac{\partial v_p}{\partial x}+v_p\frac{\partial v_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial v_p}{\partial y}\right)+\frac{1}{\tau_p}\varphi\rho_s\left(v-v_p\right) \tag{5}$$

$$(1-\varphi)\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=(1-\varphi)k\frac{\partial^2 T}{\partial y^2}+\frac{1}{\tau_T}\varphi\rho_s c_s\left(T_p-T\right)+\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)^2+(1-\varphi)\mu\left(\frac{\partial u}{\partial y}\right)^2-(1-\varphi)\frac{\partial q_r}{\partial y}+\varphi\rho_s\left(\frac{e}{m}\right)Eu_p \tag{6}$$

$$\varphi\rho_s c_s\left(\frac{\partial T_p}{\partial t}+u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi k_s\frac{\partial T_p}{\partial y}\right)-\frac{1}{\tau_p}\varphi\rho_s c_s\left(T_p-T\right)-\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)^2+\varphi\mu_s\left[u_p\frac{\partial^2 u_p}{\partial y^2}+\left(\frac{\partial u_p}{\partial y}\right)^2\right]-\varphi\frac{\partial q_{r_p}}{\partial y}+\varphi\rho_s\left(\frac{e}{m}\right)Eu_p \tag{7}$$

with boundary conditions

$$u=U_w(x,t)=\frac{cx}{1-at},\ v=V_w(x,t)=-\frac{v_0}{\sqrt{1-at}},\ T=T_w=T_\infty+T_0\frac{cx^2}{\nu(1-at)^2}\ \text{at}\ y=0 \tag{8}$$

$$u=0,\ u_p=0,\ v_p\to v,\ \rho_p=\omega\rho,\ T\to T_\infty,\ T_p\to T_\infty\qquad \text{as}\ y\to\infty \tag{9}$$

The rate of radiative heat flux using the “Rosseland approximation” for fluid and particle phase are given by

$$\frac{\partial q_r}{\partial y}=-\frac{16T_\infty^3\sigma^*}{3K^*}\frac{\partial^2 T}{\partial y^2}\ \text{and}\ \frac{\partial q_{r_p}}{\partial y}=-\frac{16T_\infty^3\sigma^*}{3K^*}\frac{\partial^2 T_p}{\partial y^2} \tag{10}$$

where

σ^*	Stefan Boltzmann constant
K^*	mean absorption co-efficient
ω	density ratio in the main stream

Introducing the following “non dimensional variables”,

$$u=\frac{cx}{1-at}f'(\eta),\quad v=-\sqrt{\frac{c\nu}{1-at}}f(\eta),\ \frac{\varphi\rho_s}{\rho}=\frac{\rho_p}{\rho}=\rho_r=H(\eta)$$

$$u_p=\frac{cx}{1-at}F(\eta),\ v_P=\sqrt{\frac{c\nu}{1-at}}G(\eta),\ \eta=\sqrt{\frac{c}{\nu(1-at)}}y$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \theta_p(\eta)=\frac{T_p-T_\infty}{T_w-T_\infty}, \tag{11}$$

where $T-T_\infty = T_0\frac{cx^2}{\nu(1-at)^2}\theta\,, T_p-T_\infty = T_0\frac{cx^2}{\nu(1-at)^2}\theta_p$

The Eq. (1) to (7) can be transferred to

$$H' = -(HF+HG')/\left(\frac{\eta}{2}A+G\right) \tag{12}$$

$$f''' = A\left(f'+\frac{\eta}{2}f''\right)+f'^2-ff''+\frac{1}{(1-\varphi)}\beta H(f'-F)-\lambda\theta\cos\alpha-\frac{1}{(1-\varphi)}HM \tag{13}$$

$$F'' = \frac{1}{\epsilon}\left[A\left(\frac{\eta}{2}F'+F\right)+F^2+GF'-\beta(f'-F)-\frac{1}{Fr}\left(1-\frac{1}{\gamma}\right)-M\right] \tag{14}$$

$$G'' = \frac{1}{\epsilon}\left[\frac{A}{2}(\eta G'+G)+GG'+\beta(f+G)\right] \tag{15}$$

$$\theta'' = \frac{1}{1+Ra}\begin{bmatrix} Pr(2f'\theta-f\theta')+\frac{2}{3}\frac{1}{1-\varphi}\beta H\left(\theta_p-\theta\right)-\frac{1}{1-\varphi}\beta\,Pr\,Ec\,H(F-f')^2 \\ -PrEcf''^2+\frac{A}{2}Pr(\eta\theta'+4\theta)-\frac{1}{1-\varphi}Pr\,Ec\,HMF \end{bmatrix} \tag{16}$$

$$\theta_p'' = \frac{1}{\frac{\epsilon}{Pr}+\frac{3Ra}{2\,\gamma}}\begin{bmatrix} \frac{A}{2}\left(\eta\theta_p'+4\theta_p\right)+2F\theta_p+G+\beta\left(\theta_p-\theta\right)+\frac{3}{2}\beta\,Pr\,Ec\,(f'-F)^2 \\ -\frac{3}{2}\epsilon\,Pr\,Ec\left(FF''+F'^2\right)-\frac{3}{2}Pr\,Ec\,MF \end{bmatrix} \tag{17}$$

with boundary conditions

$$f=0\,, f'=1\,, F'=0,\ G'=0,\ \theta=1, \theta_p'=0\ at\ \eta=0 \tag{18}$$

$$f'=0, F=0, G=-f_w\,, H=\omega, \theta\to 0, \theta_p\to 0\ as\ \eta\to\infty \tag{19}$$

where

$Pr=\frac{\mu c_p}{k}$, $\beta=\frac{1-at}{c\tau_p}$, $\epsilon=\frac{\nu_s}{\nu}$,$\varphi=\frac{\rho_p}{\rho_s}$,$\mathrm{A}=\frac{a}{c}$,$E_c=\frac{c\nu}{C_pT_0}$,$\nu=\frac{\mu}{\rho}$,

$f_0=-\frac{\nu_0}{\sqrt{c\nu}}$, $R_a=\frac{16\sigma^*T_\infty^3}{3K^*k}$, $Re^2=\frac{c^2x^4}{(1-at)^2\nu^2}$,

For most of the gases $\tau_p\approx\tau_T$,$k_s=k\frac{c_s}{c_p}\frac{\mu_s}{\mu}$ if $\frac{c_s}{c_p}=\frac{2}{3P_r}$

where
λ = mixed convection factor
A = unsteady Parameter
C = stretching rate and being a positive constant
c_p = specific heat of fluid phase
K = thermal conductivity
β = fluid particle interaction parameter
a = positive constant

3. Solution of The Problem

The system of nonlinear PDEs (1) to (7) with boundary condition (8) and (9) representing the flow and heat transfer of the flow filed are transferred to a systems of ODEs (12) to (17) with boundary condition (18) and (19) by considering similarity transformation (11). Then the system of ODEs is solved by using bvp4c tool of MATLAB.

The value of rate of heat transfer obtained in this problem have a good agreement with the previous literatures (13), (23), (3), (9), (11), (16) against different values of Pr appended in Table 1, which validate the present programme.

Table 1
Rate of heat transfer

Pr	Ishak *et al.,*	Subhas *et al.,*	Ggiressha	Chen	Gurbka*et al.,*	Mukhpadhya	Present Study
0.72	-	1.0885	1.0885	1.0885	1.0885	1.0885	1.0884
1.0	1.3333	1.3333	1.3333	1.3333	1.3333	1.3333	1.3333
3.0	2.5097	-	2.5097	2.5097		2.5097	2.5097
10.0	4.7969	4.7968	4.7969	4.7968	4.7968	-	-

The flow and heat transfer characteristics are analyzed through the computational outcomes obtained for different values of physical parameter like electrification parameter, radiation parameter, angle of inclination, Prandtl number and Eckret number etc, which are represented through graphs and tables.

4. Result Analysis

4.1 Effect of Prandtl Number (Pr)

The Prandtl number significantly controls the relative thickness of momentum and thermal boundary layer in flow geometry. In present flow analysis, momentum and energy equations are coupled with buoyancy parameter and electrification parameter.

So, both momentum and thermal distributions are changed with variation of Pr, which are shown through the Figure 2 to 5. The values of Pr can be chosen as 0.71, 1.0 and 7.0, which are corresponds to air, electrolyte solution and water respectively. Figure 2 and 3 depict the effect of Prandtl number (Pr) on velocity distribution for fluid and particle phase respectively. The flow of fluid along with particle is very high in case of air rather than electrolyte and water. In air, the thermal diffusion dominant momentum diffusion. For electrolyte solution, the momentum and thermal diffusion are equal, and the momentum diffusion dominant the thermal diffusion in case of water. Hence, the thickness of thermal boundary layer is high for fluids having smaller Pr value, shown in Figure 4 and subsequently the heat transfer is more for higher values of Pr (Table 2). But, the trend is reverse in case of particle thermal distribution, which is clearly indicated in Figure 5.

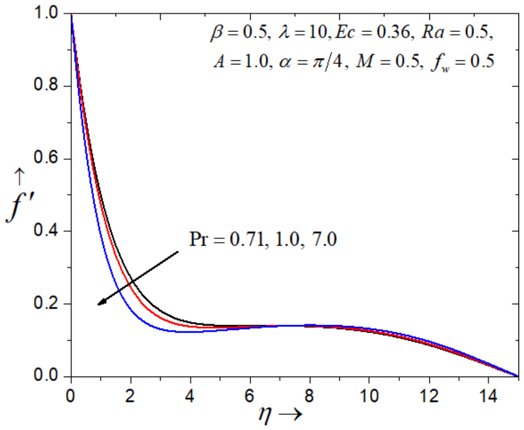

Fig. 2. Effect of Pr on fluid velocity

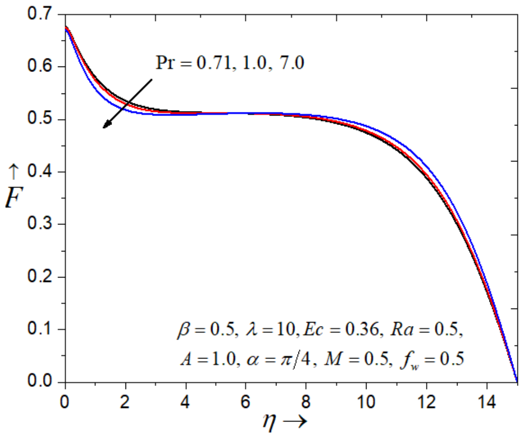

Fig. 3. Effect of Pr on particle velocity

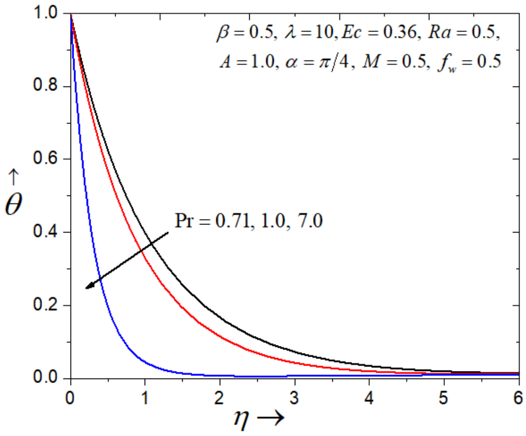

Fig. 4. Effect of Pr on fluid temperature

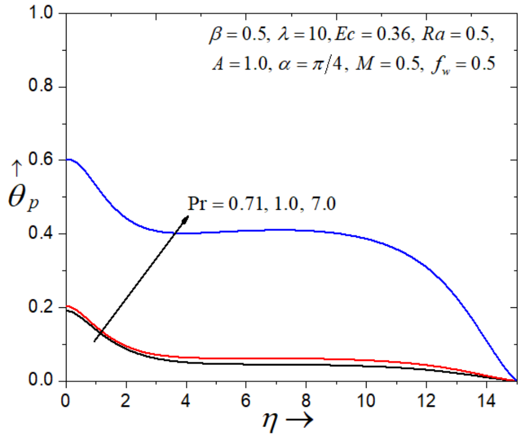

Fig. 5. Effect of Pr on particle temperature

4.2 Effect of Eckret number (Ec)

Figure 6 to 9 depicts the impact of Eckret number (Ec) on flow and temperature distribution for fluid and particles. The computational outcomes for flow and heat transfer characteristics can be obtained for $0.36 \le Ec \le 1.0$. A negligible impact has been seen in flow properties.

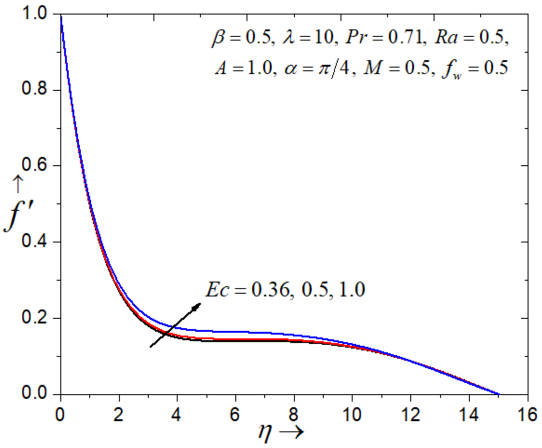

Fig. 6. Effect of Ec on fluid velocity

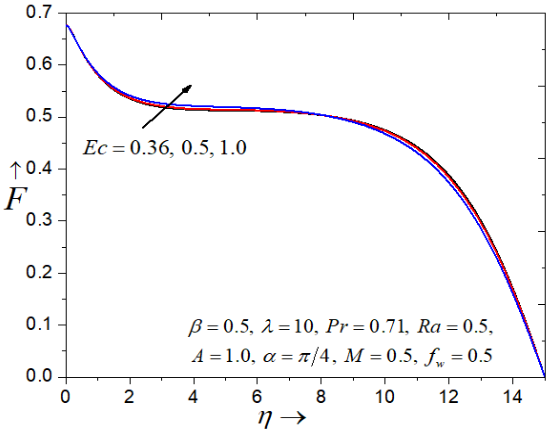

Fig. 7. Effect of Ec on particle velocity

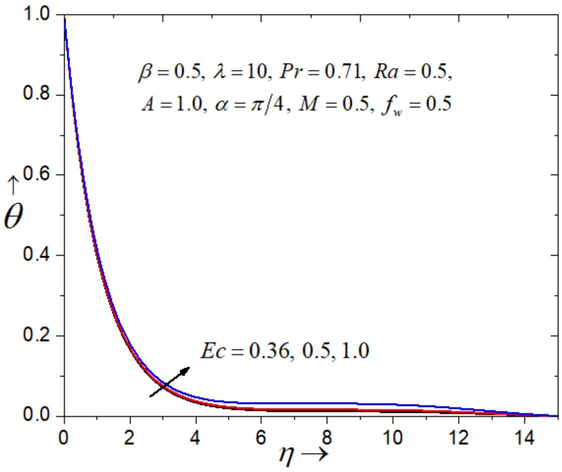

Fig. 8. Effect of Ec on fluid temperature

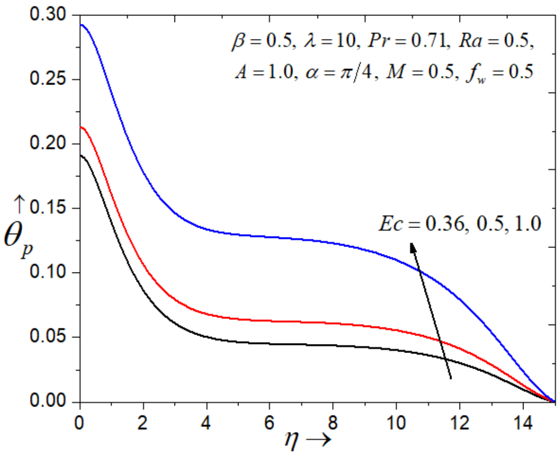

Fig. 9. Effect of Ec on particle temperature

As expected, the temperature of fluid as well as particle phase being enhanced with increasing values of Ec. It resembles to the fact that, due to frictional heating, more heat energy is stored for higher values of Ec, depicted through the Figure 8 and 9 and hence rate of heat transfer decays for higher Ec, shown in Table 2. Again, the flow velocity is high for Ec~o(1). So, the magnitude of fluid and particle velocity is maximum for Ec=1, observed from Figure 6 and 7.

4.3 Effect of Radiation Parameter (Ra)

The impact of radiation parameter (*Ra*) on velocity and temperature profiles for fluid and particle phases are depicted through Figure 10 to 13. The thermal boundary layer thickness for both fluid and particle phases are thicker, when radiation is more, shown in Figure 12 and Figure 13.

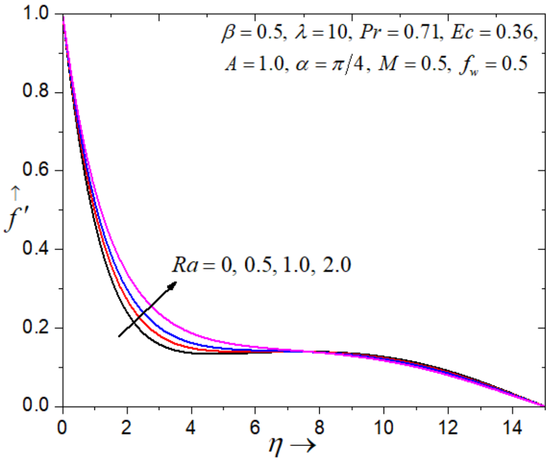

Fig. 10. Effect of Ra on fluid velocity

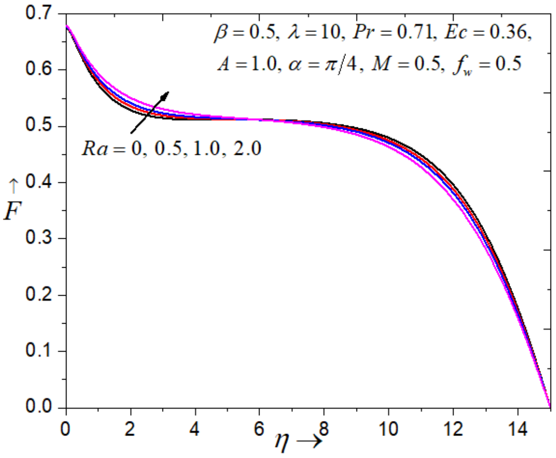

Fig. 11. Effect of Ra on particle velocity

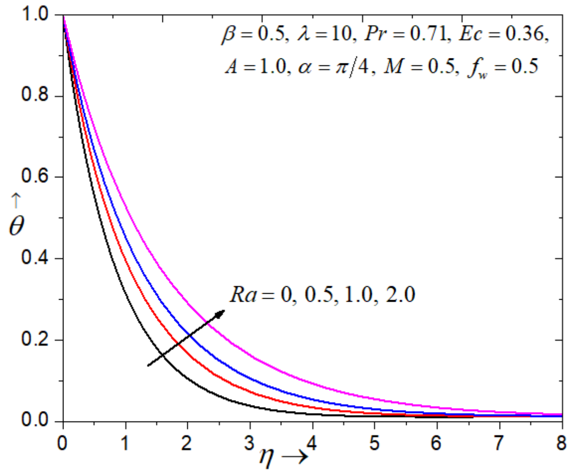

Fig. 12. Effect of Ra on fluid temperature

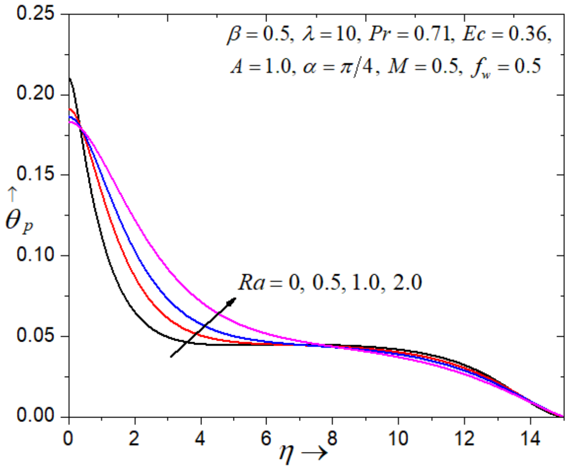

Fig. 13. Effect of Ra on particle temperature

Since, the temperature impacted on velocity boundary layer through buoyancy force. So, the velocity of both the phases increases with increase in Ra value near the wall, shown in Figure. 10 and 11. But, for high radiation, the magnitude of velocity for both the phases decreases. It is due to the decay in particle temperature and rate of heat transfer, shown in Table 2. Hence, Radiation parameter has greater impact on boundary layer characteristics.

4.4 Effect of Angle of Inclination (α)

The influences of surface inclination on flow and heat transfer of both the phases are presented through graphs in Figure 14 to 17. From the figures it is concluded that the rise of value of angular inclination will decrease the velocity profile of both the phases, which agrees with the term $\rho g\beta^*(T - T_\infty)\cos\alpha$present in momentum equation as $\cos\alpha$ value decreases with increase of the value of angle α

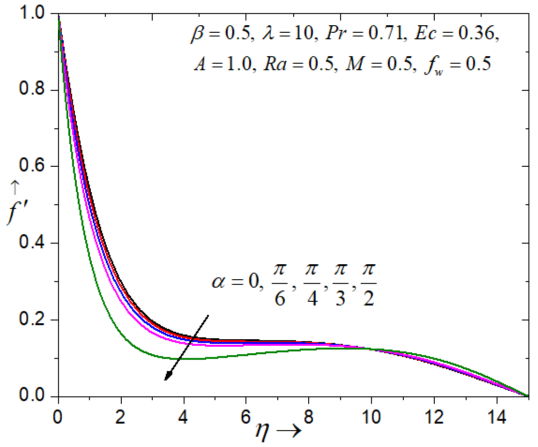

Fig. 14. Effect of α on fluid velocity

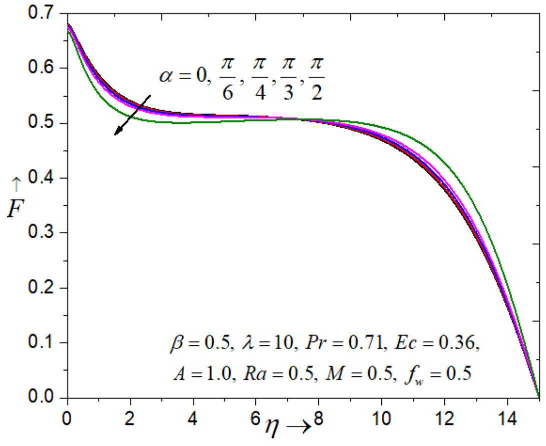

Fig. 15. Effect of α on particle velocity

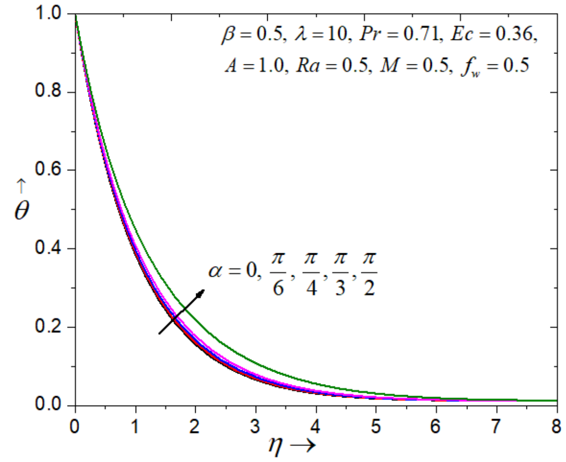

Fig. 16. Effect of α on fluid temperature

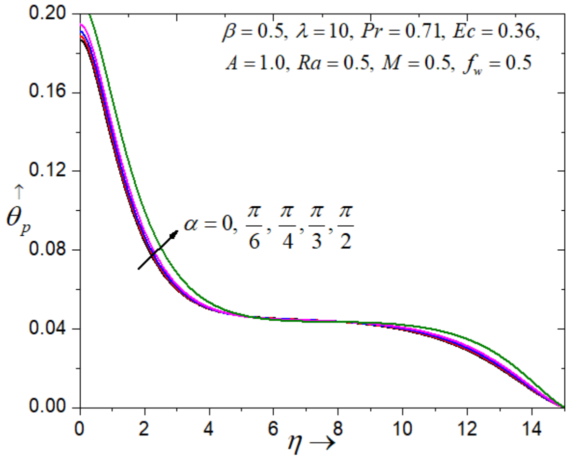

Fig. 17. Effect of α on particle temperature

But in case of temperature profile, the effect is reverse of velocity profile. From the figure it is inferred that as the angle increases the temperature of both the phases also increases. By increasing value of angle, the sheet becomes obstacle to flow and generate heat due to friction of fluid with the sheet. The result quite agrees with the concept of generation of heat due to friction.

4.5 Effect of Electrification parameter (M)

Figure 18 to 21 indicate the result of electrification parameter on momentum and temperature distribution for both the phases. It is noticed that, the flows of both the phases are accelerates for higher electrification parameter. It is due to introduce of electric force, known as Lorentz force helps in accelerating body force which acts in the direction of electric field. So it results in reducing the skin friction on the surface of stretching sheet.

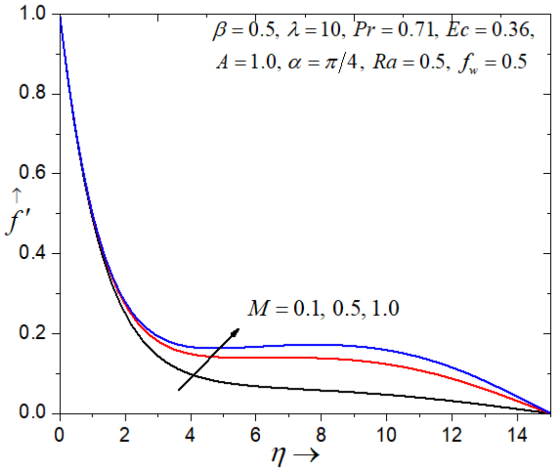

Fig. 18. Effect of M on fluid velocity

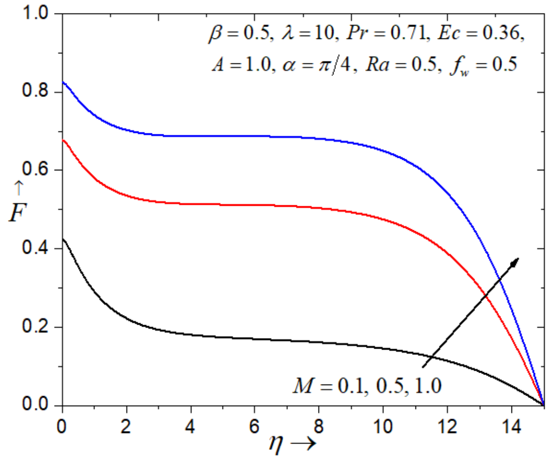

Fig. 19. Effect of M on particle velocity

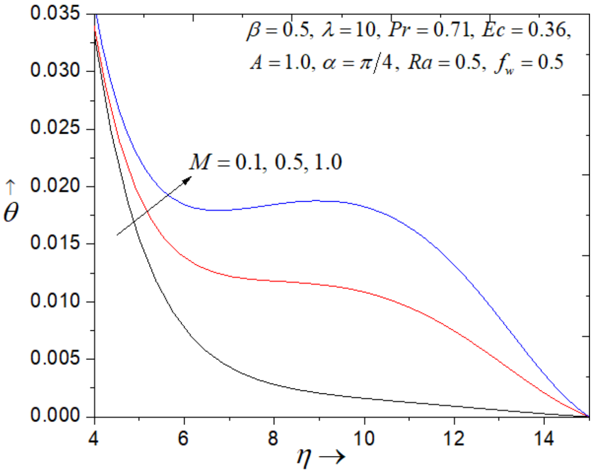

Fig. 20. Effect of M on fluid temperature

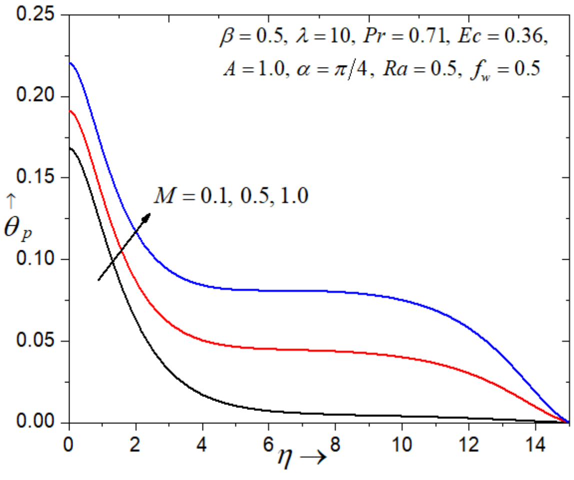

Fig. 21. Effect of M on particle temperature

Due to accelerating flow field, the temperature of both the phases rapidly increases along the surface. i.e. the fluid-particle interaction is more for accelerated fluid and causes the stronger electric field, which significantly enhances the temperature and it occurs more heat transfer on the surface of the stretching surface, shown in Table 2.

Table 2
Effect of Prandtl number, Eckret number, Radiation parameter, angle of inclination of the sheet and Electrification parameter on skin friction and Nusselt number

Pr	Ec	Ra	α	M	$f''(0)$	$-\theta'(0)$
0.71	0.36	0.5	$\pi/4$	0.5	-0.70052	0.95802
1.0					-0.73621	1.15040
7.0					-0.90060	3.26596
0.71	0.36	0.5	$\pi/4$	0.5	-0.70052	0.95802
	0.5				-0.69711	0.94442
	1.0				-0.68557	0.89775
0.71	0.36	0.5	$\pi/4$	0.5	-0.70052	0.95802
		1.0			-0.67037	0.82171
		2.0			-0.62867	0.66245
0.71	0.36	0.5	0	0.5	-0.56578	0.98516
			$\pi/6$		-0.62664	0.97332
			$\pi/4$		-0.70052	0.95802
			$\pi/3$		-0.80015	0.93548
			$\pi/2$		-1.06634	0.85917
0.71	0.36	0.5	$\pi/4$	0.1	-0.71345	0.95527
				0.5	-0.70052	0.95802
				1.0	-0.69862	0.95891

5. Conclusion

From above graphs and result discussions, it concludes that presence of particles in fluid has greater impacts on flow and heat transfer profiles as follows.

i. The presence of particles in fluid collides with each other and with the wall, thus generates electric force that helps in accelerating body force which acts in the direction of electric field.It results in reducing the skin friction on the surface of stretching sheet.
ii. It significantly enhances the temperature and it occurs more heat transfer on the surface of the stretching surface.
iii. The rate of heat transfer became less in case of maximum inclination of the surface.

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