

Slippage Flow of Maxwell Fluid Over an Inclined Vertical Plate With Generalized Heat and Mass Transfer

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ARTICLE INFO

Article history:

Received 9 April 2022

Received in revised form 13 August 2022

Accepted 22 August 2022

Available online 18 September 2022

Keywords:

Free convection; Maxwell fluid; slip effect; Sorret effect; CPC fractional derivative

ABSTRACT

In this paper, unsteady flow of fractionalized Maxwell fluid over an inclined vertical plate is considered by using thermo diffusion and slip effects. The flow model is solved using Constant proportional Caputo fractional derivative. Initially, the governing equations are made non-dimensional and then solved by Laplace transform. To see the impact of different flow parameters on the velocity, temperature and concentration, we have drawn some graphs. In addition, it is observed that magnetic field has decreasing effect on fluid motion whereas thermo-diffusion have increasing effect on fluid motion.

1. Introduction

Now a days, magnetohydrodynamic (MHD) has been extended into wide areas of basic and applied research in sciences and engineering. The study of non-Newtonian fluid becomes very interested due to variety of technological applications like making of plastic sheets, lubricant's performance and motion of biological fluid.

Numerous non-Newtonian fluid models have been presented to demonstrate the distinction between Newtonian and non-Newtonian fluids. Kai-Long Hsiao [1] worked on MHD heat transfer thermal extrusion system using non-Newtonian Maxwell fluid with radiative and viscous dissipation effects. Shah *et al.*, [2] discussed the Convective flow of a maxwell hybrid nanofluid due to pressure gradient in a channel. Akage *et al.*, [3] has analyzed the impacts of Nonlinear thermal radiation on stagnation point of an aligned MHD Casson nanofluid flow with Thompson and Troian slip boundary condition. Ali *et al.*, [4] discussed the Convection flow of a Maxwell hybrid nanofluid due to pressure gradient in a channel. Ahmad *et al.*, [5] compared the generalized form of Jeffrey fluid flow acquired by contemplating fractional derivative of singular kernel (Caputo) and non-singular kernel (Caputo-Fabrizio). During the last decade, different generalized fractional derivatives have appeared in the

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literature that are derivatives of Caputo, Caputo-Fabrizio, constant proportional Caputo [6-7]. Some studies of free convection on an inclined plane in various thermal and mechanical situations have recently been presented by mathematicians [8-14]. Some mathematical models of second grade fluids are industrial oils, slurry streams, and dilute polymer solutions with different geometry and boundary conditions. The solution of unsteady second grade fluid at plate with the assistance of the Fourier sine transformation was described by Fetecau *et al.*, [15]. Ahmed *et al.*, [16] has analyzed MHD heat transfer into convective boundary layer with a minimal pressure gradient. Convective mixed MHD flow studied by Narayana [17], while Nadeem *et al.*, [18] worked on MHD stagnation point fluid slanted viscoelastic fluid above the convective field. Because of its rising significance, engineering needs to incorporate non-Newtonian fluid. Khan *et al.*, [19] presented a fractional flow of the second grade fluid on a vertical surface driven by temperature as well as concentration gradients. Khan *et al.*, [20] discussed magnetohydrodynamic flow in the existence of permeable media through plate. Seth *et al.*, [21] discussed the MHD convection flow over a vertical plate with ramped temperature. Tran *et al.*, [22] worked on mandatory stability of fractional derivatives for fractional calculus equations, and the mathematical model used for transference of COVID-19 with Caputo fractional derivatives also discussed by Tuan *et al.*, [23].

Shateyi *et al.*, [24] presented the convection flow of MHD fluid past an infinite vertical plate using a heat generation. Ramzan *et al.*, [25] analyzed the problem of Casson fluid through a channel. Authors in [26] investigated the Brinkman fluid effect between two side walls. Ali *et al.*, [27] worked on the MHD fluid with heat and mass transport immersed in a porous medium. Ramzan *et al.*, [28] discussed the Brinkman fluid over a plate. Authors in [29,31] studied Casson fluid over a vertical plate.

In this problem, an unsteady flow of fractionalized Maxwell fluid over an inclined vertical plate is considered with slip and sorret effects. Initially, the dimensional equations have been made non-dimensional and then solved these equations via Laplace transform. All velocity, temperature, and concentration distribution results have been obtained and evaluated graphically.

2. Mathematical Model

The flow of fractionalized Maxwell fluid over an inclined vertical plate is studied in the presence of thermo-diffusion and slip effect. The fluid is flowing vertically upward along y' -axis and the x' -axis is normal to the plate. The plate is inclined to vertical direction with an angle A . The fluid and plate have concentration C_∞ and temperature T_∞ at time $t' = 0$ with zero velocity. But for $t' > 0$, the plate starts to move in the plane with uniform velocity $U_1 f(t')$. The concentration and temperature of the plate is increased to C_w and $T' = T_w(1 - c_1 e^{-d_1 t'}) + T_\infty$ with time t' as shown in Figure 1.

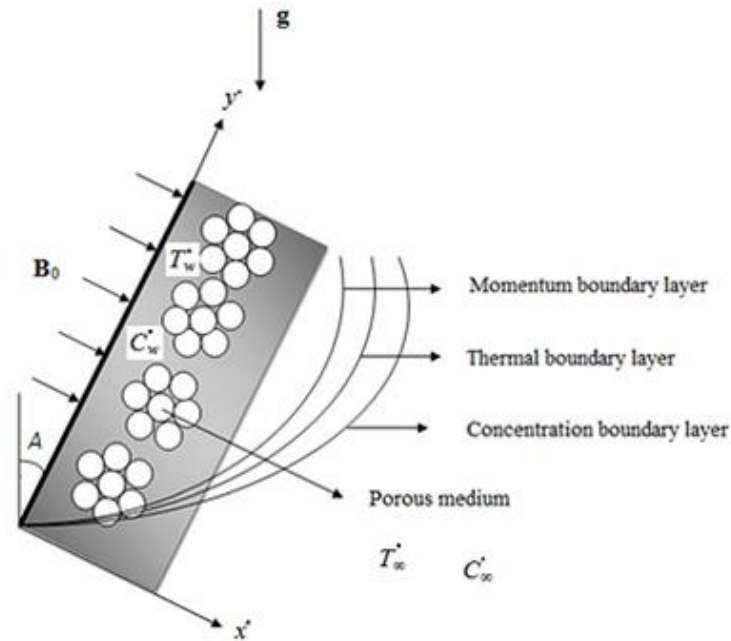


Fig. 1. Flow diagram of physical model

In view of above assumption, the convection flow of viscous fluid with Sorret effect over a plate [28, 31], linear momentum equation is

$$(1 + \lambda_1 \frac{\partial}{\partial t'}) \frac{\partial u_1(x', t')}{\partial t'} = \frac{\partial \tau(x', t')}{\partial x'} + (1 + \lambda_1 \frac{\partial}{\partial t'}) g \beta_T (T' - T_\infty) \cos(A) + (1 + \lambda_1 \frac{\partial}{\partial t'}) g \beta_C (C' - C_\infty) \cos(A) - (1 + \lambda_1 \frac{\partial}{\partial t'}) \frac{\sigma \beta_0^2 u_1(x', t')}{\rho} - (1 + \lambda_1 \frac{\partial}{\partial t'}) \frac{\mu u_1(x', t')}{\rho K_2}, \quad (1)$$

shear stress τ is

$$\tau = \frac{\partial u_1(x', t_1)}{\partial x'} \quad (2)$$

thermal equation is

$$\frac{\partial T'(x', t')}{\partial t'} = -\frac{\partial q(x', t')}{\rho c_p \partial x'} + H_1 (T' - T_\infty) \quad (3)$$

According to Fourier's Law, $q_1(x', t_1)$ is given by

$$q_1(x', t_1) = -\alpha_0 \frac{\partial T'(x', t_1)}{\partial x'} \quad (4)$$

Diffusion equation is

$$\frac{\partial C'(x', t')}{\partial t'} = -\frac{\partial J(x', t')}{\partial x'} - \frac{D_{KT}}{Tm} \frac{\partial q(x', t')}{\partial x'} - S_1 (T' - T_\infty) \quad (5)$$

According to Fick's Law, $J_1(x', t_1)$ is given by

$$J_1(x', t_1) = -D_m \frac{\partial C'(x', t_1)}{\partial x'} \quad (6)$$

The conditions for the model are

$$u_1(x', t') = 0, \quad T'(x', t') = T_\infty, \quad C'(y', t') = C_\infty, \quad y' > 0, \quad t' = 0, \quad (7)$$

$$u_1(0, t') - R_1 \frac{\partial u_1}{\partial x'} = U_1 f(t'), \quad T'(0, t') = T_\infty + T_w'(1 - c_1 e^{-d_1 t'}), \quad C'(0, t') = C_w', \quad t' > 0, \quad (8)$$

$$u_1(x', t') \rightarrow 0, \quad T'(x', t') \rightarrow 0, \quad C'(x', t') \rightarrow 0, \quad x' \rightarrow \infty, \quad t' > 0. \quad (9)$$

3. Generalized Model

Dimensionless form of the variables are

$$x^* = \frac{U^2 x'}{v^2}, \quad t^* = \frac{U t'}{v^2}, \quad T^* = \frac{T' - T_\infty}{T_w' - T_\infty}, \quad u^* = \frac{u_1}{U}, \quad J_1^* = \frac{J_1}{J}, \quad q_1^* = \frac{q_1}{q}, \quad Gr^* = \frac{g \beta_T (T_w' - T_\infty) U^2}{v^2}, \quad C^* = \frac{C' - C_\infty}{C_w' - C_\infty},$$

$$Gm^* = \frac{g \beta_C (C_w' - C_\infty) U^2}{v^2}, \quad M = \frac{\sigma \beta_0^2 U^3}{\rho v^2}, \quad \frac{1}{K} = \frac{\mu U^3}{\rho v^2 K_2}, \quad \lambda = \frac{\lambda_1 U}{v^2}. \quad (10)$$

Eq. (1) is generalized fractionally by [32]

$$\tau = L_\beta D_t^\beta \frac{\partial u(x,t)}{\partial x}, \quad 1 \geq \beta > 0, \quad (11)$$

where $L_\beta = n_1 K_\beta = 1$ when $\beta \rightarrow 1$. Put Eq. (11) into Eq. (1) and using non-dimensional parameters from Eq. (10), we have

$$(1 + \lambda \frac{\partial}{\partial t}) \frac{\partial u(x,t)}{\partial t} = L_\beta D_t^\beta \frac{\partial^2 u(x,t)}{\partial x^2} - (1 + \lambda \frac{\partial}{\partial t})(K^{-1} + M)u(x,t) + (1 + \lambda \frac{\partial}{\partial t})GrT(x,t)\cos(A) + (1 + \lambda \frac{\partial}{\partial t})GmC(x,t)\cos(A), \quad (12)$$

Eq. (2) is generalized fractionally by [33, 34]

$$q = -B_\gamma D_t^\gamma \frac{\partial T(x,t)}{\partial x}, \quad 1 \geq \gamma > 0, \quad (13)$$

where thermal conductivity has generalized coefficient B_γ . Put Eq. (13) into Eq. (2) and making non-dimensional results, we have

$$\frac{\partial T(x,t)}{\partial t} = \frac{1}{Pr} D_t^\gamma \frac{\partial^2 T}{\partial x^2} + HT, \quad (14)$$

where $Pr = \frac{\rho v C_p}{B_\gamma}$.

Eq. (3) is generalized by using Fick's Law defined by

$$J = -C_\alpha D_t^\alpha \frac{\partial C(x,t)}{\partial x}, \quad 1 \geq \alpha > 0. \quad (15)$$

where molecular diffusion has generalized coefficient C_α . Put Eq. (15) into Eq. (3) and making non-dimensional results, we have

$$\frac{\partial C(x,t)}{\partial t} = \frac{1}{Sc} D_t^\alpha \frac{\partial^2 C(x,t)}{\partial x^2} + SC(x,t) + Sr D_t^\gamma \frac{\partial^2 T(x,t)}{\partial x^2}, \quad (16)$$

where $Sc = \frac{\nu}{C_\alpha}$ is the generalized Schmidt number.

Initial and boundary conditions are

$$u(x,t) = T(x,t) = C(x,t) = 0, \quad t = 0, \quad (17)$$

$$u(0,t) - R \frac{\partial u}{\partial x} = f(t), \quad T(0,t) = 1 - ce^{-dt}, \quad C(0,t) = t, \quad t > 0, \quad (18)$$

$$u(x,t) \rightarrow 0, \quad T(x,t) \rightarrow 0, \quad C(x,t) \rightarrow 0, \quad x \rightarrow \infty, \quad t > 0, \quad (19)$$

where Gm, R, M, H, Gr, λ , and u represents the mass Grashof number, slip parameter, magnetic field, non-dimensional heat generation parameter, mass Grashof number, Maxwell parameter, and motion of fluid respectively and $D_t^\beta u(x,t)$ is the CPC derivative of $u(x,t)$ given by

$$D_t^\beta u(x,t) = \frac{1}{\Gamma(1-\beta)} \int_0^t [K_1(\beta)u(x,\tau) + K_0(\beta)u'(x,\tau)](t-\tau)^{-\beta} d\tau \quad (20)$$

4. Solution of Problem

Eqs. (12, 14, 16) with conditions have been solved semi-analytically.

4.1 Temperature Profile

From Eq. (14), we have

$$s\underline{T}(x,s) = \frac{1}{Pr} \left[\frac{K_1(\gamma)}{s} + K_0(\gamma) \right] s^\gamma \frac{\partial^2 \underline{T}(x,s)}{\partial x^2} + H\underline{T}(x,s) \quad (21)$$

Eq. (21) is satisfied by

$$\underline{T}(0,s) = \frac{1}{s} - \frac{c}{s+d}, \quad \underline{T}(x,s) \rightarrow 0, \quad x \rightarrow \infty \quad (22)$$

Put Eq. (22) in Eq. (21)

$$\underline{T}(x,s) = \left(\frac{1}{s} - \frac{a}{s+b} \right) e^{-x \sqrt{\frac{Pr(s-H)}{\left(\frac{K_1(\gamma)}{s} + K_0(\gamma) \right) s^\gamma}}} \quad (23)$$

4.2 Calculation of Concentration

Solution of Eq. (16) with conditions

$$s\underline{C}(x,s) = \frac{1}{Sc} \left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha \frac{\partial^2 \underline{C}(x,s)}{\partial x^2} + S\underline{C}(x,s) + Sr \left[\frac{K_1(\gamma)}{s} + K_0(\gamma) \right] s^\gamma \frac{\partial^2 \underline{T}(x,s)}{\partial x^2} \quad (24)$$

$$\underline{C}(0, s) = s^{-2}, \quad \underline{C}(x, s) \rightarrow 0, \quad x \rightarrow \infty \tag{25}$$

Using Eq. (25) in Eq. (24) for $\alpha = \gamma$,

$$\underline{C}(x, s) = \left[s^{-2} + \frac{SrScPr(s-H)\left(\frac{1}{s} - \frac{c}{s+d}\right)}{(s-H)Pr-(s+S)Sc} \right] e^{-x \sqrt{\frac{Sc(s+S)}{\left[\frac{K_1(\alpha)}{s} + K_0(\alpha)\right]s^\alpha}} - \frac{SrScPr(s-H)\left(\frac{1}{s} - \frac{c}{s+d}\right)}{(s-H)Pr-(s+S)Sc}} e^{-x \sqrt{\frac{Pr(s-H)}{\left[\frac{K_1(\alpha)}{s} + K_0(\alpha)\right]s^\alpha}} \tag{26}$$

4.3 Calculation of Velocity

Solution of Eq. (12) with conditions

$$s(1 + \lambda s)\underline{u}(x, s) = L_\beta \left[\frac{K_1(\beta)}{s} + K_0(\beta) \right] s^\beta \frac{\partial^2 \underline{u}(x, s)}{\partial x^2} - (1 + \lambda s)(K^{-1} + M)\underline{u}(x, s) + (1 + \lambda s)Gr\underline{T}(x, s)\cos(A) + (1 + \lambda s)Gm\underline{C}(x, s)\cos(A), \tag{27}$$

$$\underline{u}(0, s) - R \frac{\partial \underline{u}(0, s)}{\partial x} = f(s), \quad \underline{u}(x, s) \rightarrow 0, \quad x \rightarrow \infty \tag{28}$$

Putting Eq. (28) in Eq. (27) for $\alpha = \beta = \gamma$

$$\underline{u}(x, s) = \frac{\frac{1}{s^{e+1}}}{1+R \sqrt{\frac{(s+K^{-1}+M)(1+\lambda s)}{L_\beta \left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha}}} e^{-x \sqrt{\frac{(s+K^{-1}+M)(1+\lambda s)}{L_\beta \left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha}} + (1 + \lambda s)\cos(A) \left[\frac{1}{s} - \frac{c}{s+d} \right] \left[\frac{Gr - \frac{PrScGmSr(s-H)}{Pr(s-H)-(s+S)Sc}}{L_\beta Pr(s-H)-(s+k^{-1}+M)(1+\lambda s)} \right] + \left[\frac{1+R \sqrt{\frac{Pr(s-H)}{\left[\frac{K_1(\alpha)}{s} + K_0(\alpha)\right]s^\alpha}}}{1+R \sqrt{\frac{(s+K^{-1}+M)(1+\lambda s)}{L_\beta \left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha}} e^{-x \sqrt{\frac{(s+K^{-1}+M)(1+\lambda s)}{L_\beta \left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha}} - e^{-x \sqrt{\frac{Pr(s-H)}{\left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha}} \right] + \left[\frac{(1+\lambda s)\cos(A) \left[\frac{Gm}{s^2} + \left[\frac{1}{s} - \frac{c}{s+d} \right] \right] \frac{PrScGmSr(s-H)}{Pr(s-H)-(s+S)Sc}}{L_\beta (s+S)Sc - (s+k^{-1}+M)(1+\lambda s)} \right] \times \left[\frac{1+R \sqrt{\frac{(s+S)Sc}{\left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha}}}{1+R \sqrt{\frac{(s+K^{-1}+M)(1+\lambda s)}{L_\beta \left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha}} e^{-x \sqrt{\frac{(s+K^{-1}+M)(1+\lambda s)}{L_\beta \left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha}} - e^{-x \sqrt{\frac{(s+S)Sc}{\left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha}} \right] \tag{29}$$

5. Results and Discussion

The solution for the impact of thermo-diffusion, magnetic field, and heat generation on flow of Maxwell fluid past over a vertical plate are developed by using Laplace transform technique. The effect of numerous parameters used in the governing equations of velocity fields have been analyzed in the figures.

Figure 1(a) represent the result of Gr on $u(x, t)$. The fluid motion rises up with maximizing the values of Gr , and it represents the impact of thermal buoyancy force to viscous force. Therefore maximizing the values of Gr exceed the temperature gradient due to which velocity field rises. Figure. 1(b) represent the result of Gr on $u(x, t)$ with slippage. The impact of Gm on fluid velocity $u(x, t)$ without slippage is illustrate in Figure 2(a). It is highlighted that fluid motion raises as values of Gm increasing. Physically higher the values of Gm increase the concentration gradients which make the buoyancy force significant and hence it is examined that velocity field is raising. The impact of Gm on $u(x, t)$ with slip effect is reported in Figure 2(b).

The behavior of λ on $u(x, t)$ with non slippage is reported in Figure 3(a). It is highlighted that fluid motion raises as values of λ decreases. Figure 3(b) display the effect of λ on $u(x, t)$ with slippage. The behavior of K on $u(x, t)$ with non slippage is reported in Figure 4(a). It is highlighted that fluid motion decays down as values of K decreases. Figure 4(b) display the effect of K on $u(x, t)$ with slippage. The impact of M on $u(x, t)$ without slip effect is reported in Figure 5(a). Graph shows that fluid speed $u(x, t)$ is reduced with accelerating values of parameter M . Resistivity becomes dominant with raising M which reduced the speed of fluid. The impact of M on $u(x, t)$ with slip effect is reported in Figure 5(b). The effect of Sr on $u(x, t)$ without slippage is depicted in Figure 6(a). The $u(x, t)$ increases with increasing the values of Sr . Physically, mass buoyancy force is significant with raising effect of Sr which raises the fluid motion. Figure 6(b) represents the effect of Sr on $u(x, t)$ with slippage. The impact of inclination angle A on $u(x, t)$ with non-slip effect is shown in Figure 7(a). Fluid motion is speed up with decreasing values of inclination angles. The impact of inclination angle A on $u(x, t)$ with slip effect is shown in Figure 7(b). Figure 8(a) indicates the impact of Pr on $T(x, t)$. Figure 9(b) indicates the effect of H on $T(x, t)$. The $T(x, t)$ increases with increasing values of H as reported in the figure. The behavior of Sc on $C(x, t)$ are shown in Figure 9(a). The behavior of S on $C(x, t)$ are shown in Figure 10(b).

Figure 10(a) shows the comparison of present work with Siyal *et al.*, [30]. If we put $\beta = \gamma = \alpha \rightarrow 1, A = Sr = R = S\lambda = 0$, and $B = \omega = 0$ of Siyal *et al.*, [30] work, the both fluid are identical. Figures 10(b) to 11(b) are drawn for for authenticity of inverse algorithms.

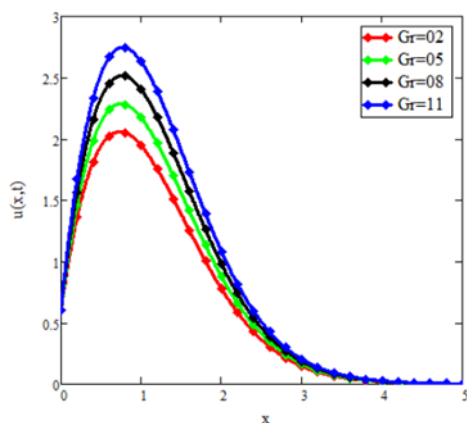


Fig. 1(a). Velocity profile $u(x,t)$ for various values of Gr at $\lambda=0.6, Sr=0.5, M=0.3, R=0.0, K=3.5, Sc=6.2, Gm=10$

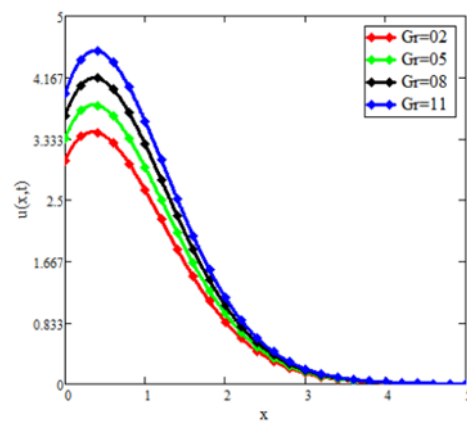


Fig. 1(b). Velocity profile $u(x,t)$ for various values of Gr at $\lambda=0.6, Sr=0.5, M=0.3, R=0.6, K=3.5, Sc=6.2, Gm=10$

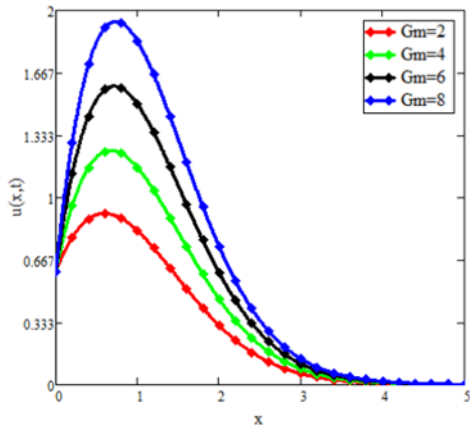


Fig. 2(a). Velocity profile $u(x,t)$ for various values of Gm at $\lambda=0.6$, $Sr=0.5$, $M=0.3$, $R=0.0$, $K=3.5$, $Sc=6.2$, $Gr=10$

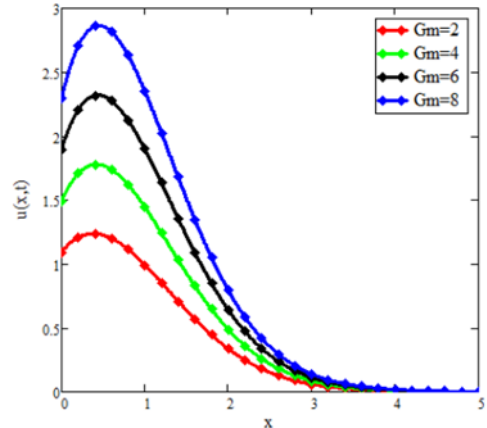


Fig. 2(b). Velocity profile $u(x,t)$ for various values of Gm at $\lambda=0.6$, $Sr=0.5$, $M=0.3$, $R=0.6$, $K=3.5$, $Sc=6.2$, $Gr=10$

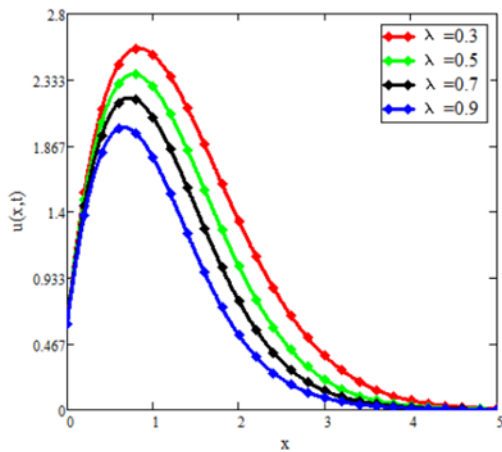


Fig. 3(a). Velocity profile $u(x,t)$ for various values of λ at $Gr=5$, $Sr=0.5$, $M=0.3$, $R=0.0$, $K=3.5$, $Sc=6.2$, $Gm=10$

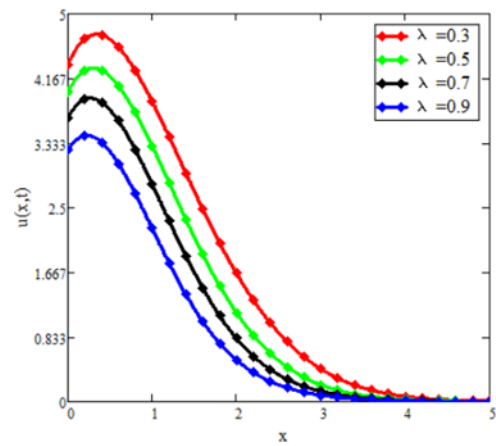


Fig. 3(b). Velocity profile $u(x,t)$ for various values of λ at $Gr=5$, $Sr=0.5$, $M=0.3$, $R=0.6$, $K=3.5$, $Sc=6.2$, $Gm=10$

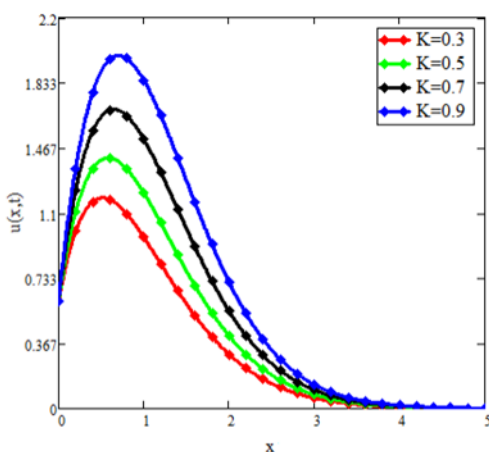


Fig. 4(a). Velocity profile $u(x,t)$ for various value of K at $Gr=5$, $Sr=0.5$, $M=0.3$, $R=0.0$, $\lambda=0.6$, $Sc=6.2$, $Gm=10$

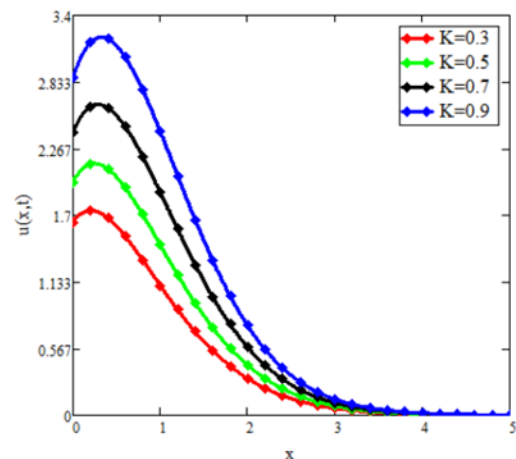


Fig. 4(b). Velocity profile $u(x,t)$ for various value of K at $Gr=5$, $Sr=0.5$, $M=0.3$, $R=0.8$, $\lambda=0.6$, $Sc=6.2$, $Gm=10$

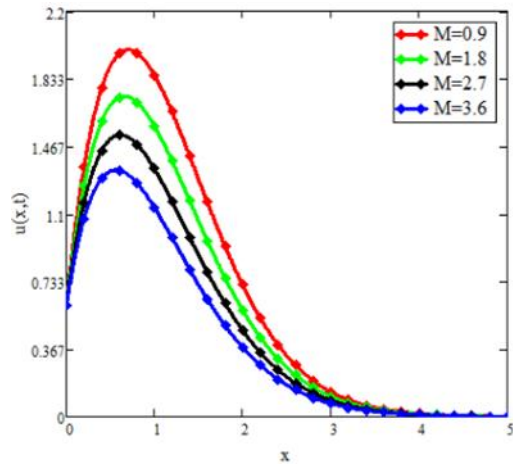


Fig. 5(a). Velocity profile $u(x,t)$ for various value of M at $Gr=5$, $Sr=0.5$, $K=3.5$, $R=0.0$, $\lambda=0.6$, $Sc=6.2$, $Gm=10$

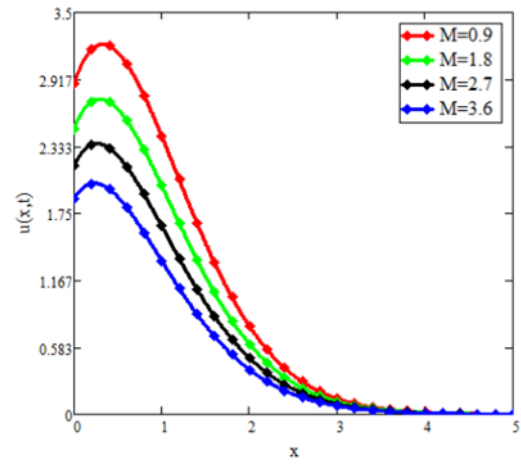


Fig. 5(b). Velocity profile $u(x,t)$ for various value of M at $Gr=5$, $Sr=0.5$, $K=3.5$, $R=0.8$, $\lambda=0.6$, $Sc=6.2$, $Gm=10$

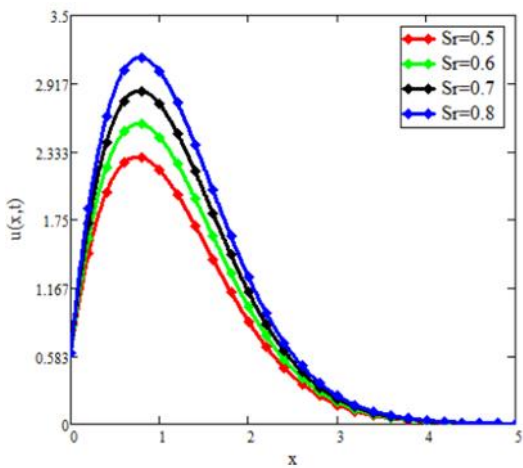


Fig. 6(a). Velocity profile $u(x,t)$ for various value of Sr at $Gr=5$, $M=0.3$, $K=3.5$, $R=0.0$, $\lambda=0.6$, $Sc=6.2$, $Gm=10$

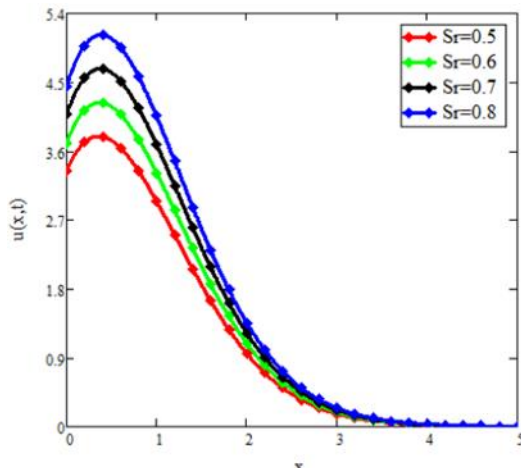


Fig. 6(b). Velocity profile $u(x,t)$ for various value of Sr at $Gr=5$, $M=0.3$, $K=3.5$, $R=0.8$, $\lambda=0.6$, $Sc=6.2$, $Gm=10$

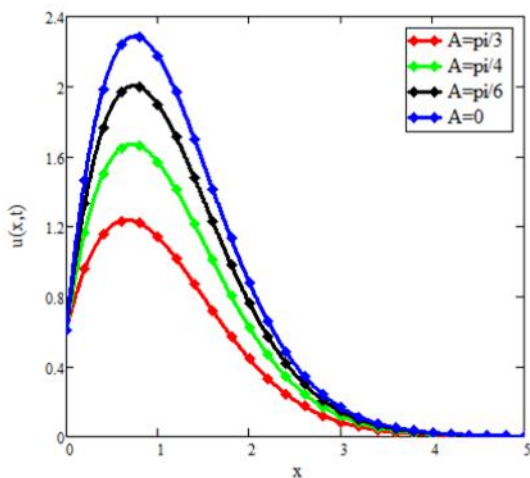


Fig. 7(a). Velocity profile $u(x,t)$ for various value of angle (A) at $Gr=5$, $M=0.3$, $K=3.5$, $R=0.0$, $\lambda=0.6$, $Sc=6.2$, $Gm=10$

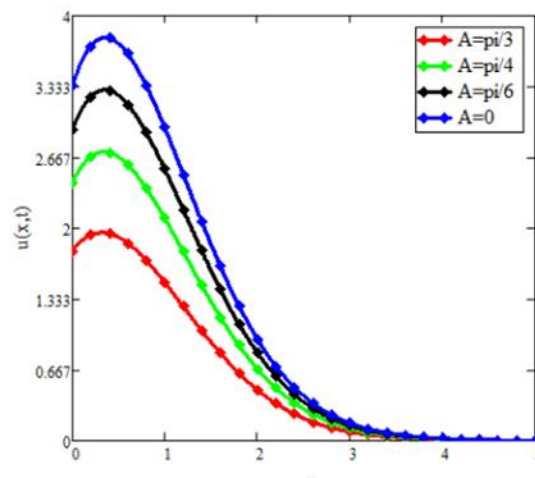


Fig. 7(b). Velocity profile $u(x,t)$ for various value of angle (A) at $Gr=5$, $M=0.3$, $K=3.5$, $R=0.8$, $\lambda=0.6$, $Sc=6.2$, $Gm=10$

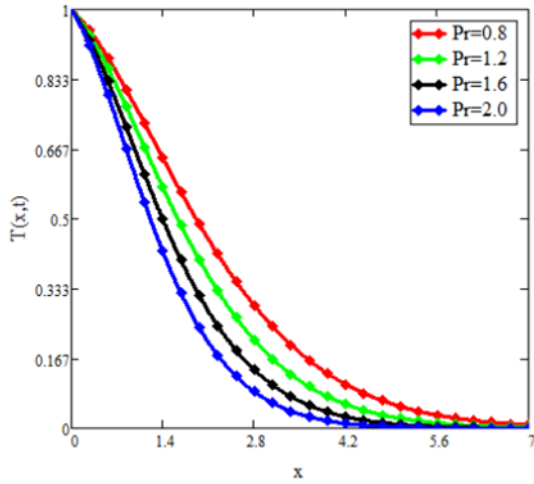


Fig. 8(a). Temperature profile $T(x,t)$ for various values of Pr

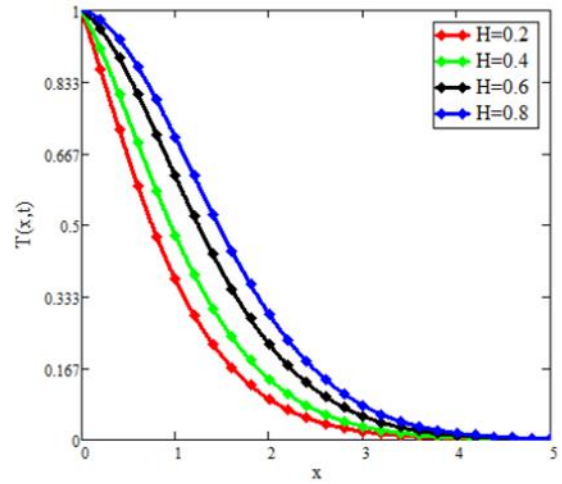


Fig. 8(b). Temperature profile $T(x,t)$ for various values of H

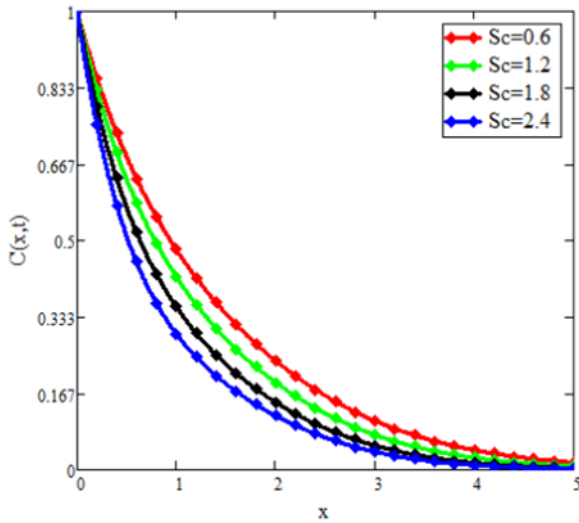


Fig. 9(a). Concentration profile $C(x,t)$ for various values of Sc

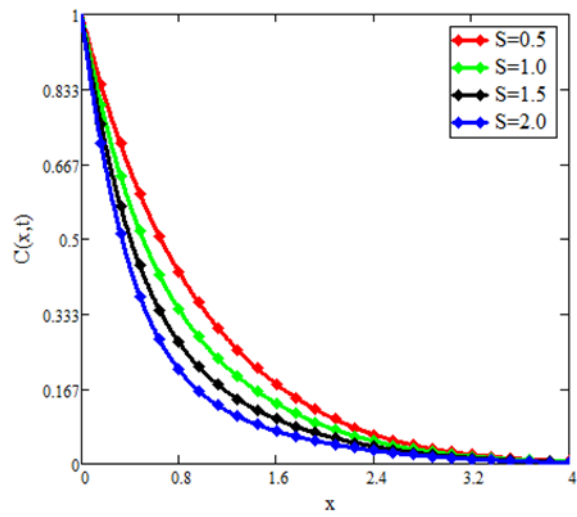


Fig. 9(b). Concentration profile $C(x,t)$ for various values of S

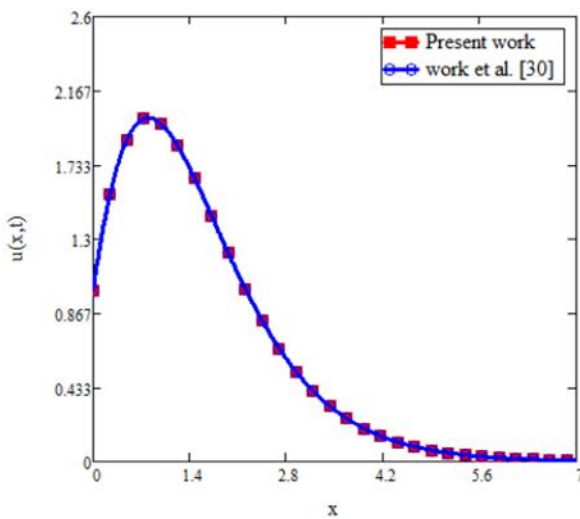


Fig. 10(a). Comparison of present work with [29]

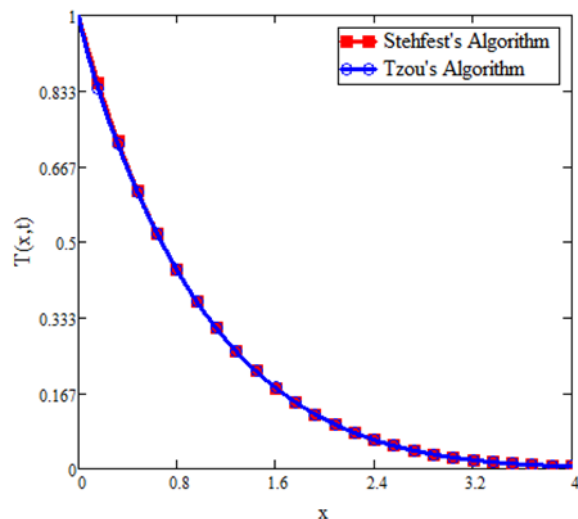


Fig. 10(b). Temperature obtained by Stehfest's and Tzou's algorithms [35,36]

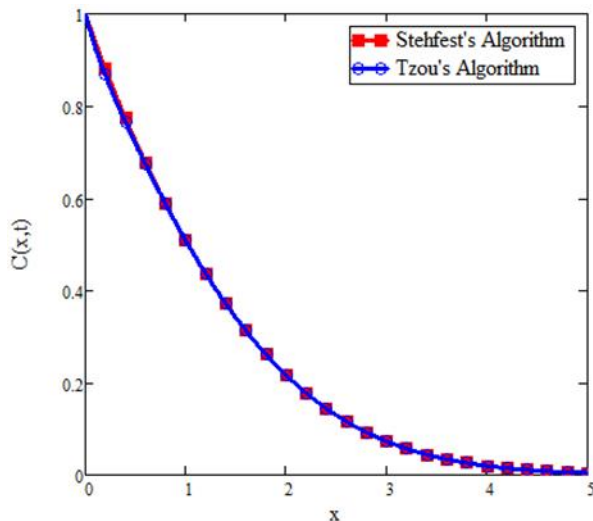


Fig. 11(a). Comparison obtained by Stehfest's and Tzou's algorithms [35,36]

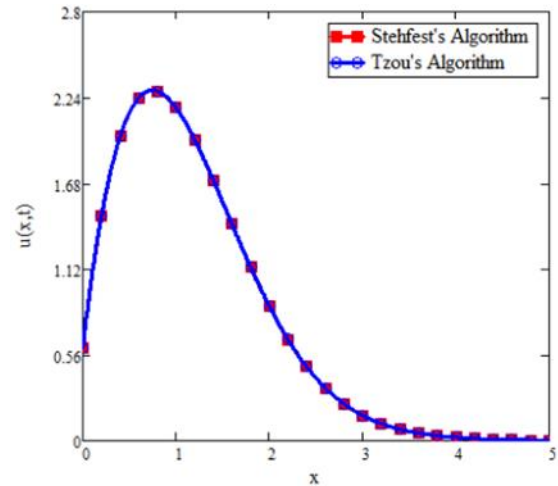


Fig. 11(b). Velocity obtained by Stehfest's and Tzou's algorithms [35,36]

6. Findings

The flow of fractional Maxwell fluid model has been taken and solved using Laplace transform with solution. The conditions of flow problem are satisfied by the results. Different graphs have been plotted for flow parameters and then discussed.

The key points of this flow model are

- i. With higher Magnetic values, the velocity distribution slows down.
- ii. Thermal buoyancy forces accelerate the fluid velocity.
- iii. The fluid velocity increased for higher values of sorret effect.
- iv. The fluid velocity increased for decreasing inclination angle.
- v. The Temperature of fluid decays down for larger values of Pr .
- vi. The Temperature of fluid rises up for larger values of H .
- vii. The concentration of fluid is a decreasing function of S .
- viii. The concentration level is an decreasing function of smidth number.

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