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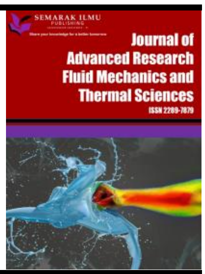

A Numerical Investigation of the MHD Ternary Hybrid Nanofluid (Cu-Al_2O_3-TiO_2/H_2O) Past a Vertically Stretching Cylinder in a Porous Medium with Thermal Stratification

Rupam Shankar Nath[1,*], Rudra Kanta Deka[1]

[1] Department of Mathematics, Gauhati University, Guwahati-781014, Assam, India

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ABSTRACT

This study focuses on examining the impact of thermal stratification on the magnetohydrodynamics (MHD) flow of water-based nano, hybrid, and ternary hybrid nanofluids past a vertically stretching cylinder in a porous medium. The nanoparticles Cu, Al_2O_3, and TiO_2 are suspended in a base fluid H_2O, leading to the formation of a ternary hybrid nanofluid $Cu - Al_2O_3 - TiO_2/H_2O$. The numerical results are calculated with the 3-stage Lobatto IIIa approach, specifically implemented by Bvp4c in MATLAB. The impacts of various parameters are visually depicted through graphs and quantitatively represented in tables. The velocity and temperature of the ternary hybrid nanofluid are lowered by the thermal stratification parameter compared to when there is no stratification. The ternary hybrid nanofluid has a higher heat transfer rate than the hybrid nanofluid, and the hybrid nanofluid has a higher heat transfer rate than ordinary nanofluids.

1. Introduction

The Ternary hybrid nanofluids are a specific kind of fluid that is made up of three distinct kinds of nanoparticles that are scattered throughout a base fluid. This article presents a study on the ternary hybrid nanofluid composed of copper *(Cu)*, aluminum oxide (Al_2O_3), and titanium dioxide (TiO_2) nanoparticles, which are evenly distributed inside a water-based fluid. This ternary hybrid nanofluid has unique qualities that make it suitable for a variety of applications.

The presence of copper *(Cu)* nanoparticles into the nanofluid has been found to improve thermal conductivity, whilst the addition of aluminum oxide (Al_2O_3) and titanium dioxide (TiO_2) nanoparticles has been observed to enhance heat transfer efficiency and stability. The utilization of this nanofluid is applicable in various applications such as heat exchangers, cooling systems, and electronic devices, with the purpose of enhancing heat dissipation and improving thermal management. Copper nanoparticles are known to possess antibacterial capabilities, whereas (TiO_2) nanoparticles have photocatalytic activity against bacteria and other bacteria. The application of the

* *Corresponding author.*
E-mail address: rupamnath23@gmail.com

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ternary hybrid nanofluid, comprising $Cu - Al_2O_3 - TiO_2$, presents a promising avenue for the development of antibacterial coatings on various surfaces, including medical equipment, textiles, and food packaging. These coatings effectively impede bacterial growth and contribute to the preservation of hygiene. Titanium dioxide (TiO_2) nanoparticles have photocatalytic properties, rendering them capable of facilitating the breakdown of organic contaminants and the disinfection of water. The use of the $Cu - Al_2O_3 - TiO_2$ ternary hybrid nanofluid has potential for application in water treatment procedures, facilitating the elimination of pollutants and enhancing the overall quality of water resources.

The term "nanofluid" refers to a type of artificial fluid that is characterized by the presence of very small particles in a base fluid suspended and typically have a size of less than 100 nanometers. The concept of nanofluid, first presented by Choi and Eastman [1] proposes that heat transfer fluids with floating metallic nanoparticles could provide a revolutionary new type of heat transfer fluids. Das and Jana [2] and Das *et al.,* [3] investigated the natural convective flow of nanofluids with radiation for moving vertical plate and vertical channel, respectively. In both of these research articles, the authors explored water-based nanofluids that include titanium dioxide, aluminum oxide, and copper. In the occurrence of heat production or absorption, Rashidi *et al.,* [4] looks for a lie group solution to the problem of how the nanofluid moves past a horizontal plate reacting chemically. Abolbashari *et al.,* [5] uses the homotopy analysis method (HAM) to analyze the entropy of a nanofluid consisting of water as the primary fluid and one of four distinct kinds of nanoparticles: TiO_2, Al_2O_3, *Cu,* and CuO flowing through a stretchable permeable surface. A numerical investigation on the flow of nanofluids in the boundary layer across a moving flat plate was performed by Motsumi and Makinde [6] to study the impacts of thermal radiation, viscous dissipation and thermal diffusion. In the context of velocity slip and temperature leap, Turkyilmazoglu [7] carried out an analytical investigation on the MHD nanofluid flow for a variety of water-based nanoparticles as they passed a continually stretching/shrinking permeable sheet. Sheikholeslami *et al.,* [8] applied the Lattice Boltzmann method for studying the MHD Cu-water nanofluid under the presence of Lorentz forces. Furthermore, Sandeep and Reddy [9] conducted a research on MHD Cu-water nanofluid flow across a cone and a wedge influenced by nonlinear thermal radiation. Reddy *et al.,* [10] and Mahanthesh *et al.,* [11] researched the impacts of heat and mass transfer on nanofluid passing a moving or fixed vertical plate in the presence of a heat source and a chemical reaction, respectively.

Cheng [12,13] examined the impacts of thermal and mass stratification over a vertical wavy truncated cone and a wavy surface, respectively. Furthermore, Paul and Deka [14] have investigated how both stratification effects affect infinite vertical cylinders. Nath and Deka [15] and Kalita *et al.,* [16] conducted a study to investigate the combined impact of thermal stratification and chemical reaction on the flow past an infinite vertical plate and an accelerated vertical plate, correspondingly. Paul *et al.,* [17] investigates the thermal stratification effects of a hybrid nanofluid consisting of $Cu - Al_2O_3 / H_2O$ in a porous medium. Their study focuses on a vertically stretched cylinder and considers the influence of heat sink/source. They found that the thermal conductivity of hybrid nanofluids was significantly higher than that of nanofluids. Hence, the utilization of hybrid nanofluids exhibits a significant influence on enhancing thermal advancements.

Shanmugapriyan and Jakeer [18] aim to investigate the heat transfer characteristics of the magnetohydrodynamic Casson hybrid nanofluid in the presence of a non-Fourier heat flux model and linear thermal radiation along a horizontal porous stretched cylinder. With melting/non-melting heat transfer in mind, Suneetha *et al.,* [19] are interested in the laminar, stable electro magnetohydrodynamic flow and entropy formation of single wall carbon nanotube (SWCNT)-blood nanofluid. The study of magneto *Cu*-Al_2O_3 water hybrid nanofluid flow in a non-Darcy porous square cavity was done by Jakeer *et al.,* [20]. The study conducted by Shaik *et al.,* [21] focuses on investigating the phenomenon of natural convection within a sinusoidal wavy cavity that is filled with a hybrid nanofluid consisting of $TiO_2 - Cu$ particles suspended in water. The analysis takes the influence of internal heat generation into account, an angled magnetic field, and thermal radiation on the convection process. The investigation conducted by Rana and Kumar [22] focused on the nonlinear buoyancy-driven flow of hybrid nano liquid passing a spinning cylinder, taking into account the effects of quadratic thermal radiation.

Alharbi *et al.,* [23] performed an investigation on the computational valuation of Darcy ternary-hybrid nanofluid flow through an extending cylinder with induction effects. In order to improve the heat transfer of a magnetized ternary hybrid nanofluid $Cu - Al_2O_3 - MWCNT$/water, Shanmugapriya *et al.,* [24] investigated the influence of nanoparticle shape. Jakeer *et al.,* [25] conducted a study on the impact of non-linear Darcy-Forchheimer flow in the context of electromagnetic hydrodynamics (EMHD) ternary hybrid nanofluid, namely composed of $Cu - CNT - Ti$ and water. They found that the ternary hybrid nanofluid had a greater impact on the temperature profile than either the nanofluid or the hybrid nanofluid alone. Nasir *et al.,* [26] conducted a study on the heat transport characteristics of ternary hybrid nanofluid flow in the presence of a magnetic dipole with nonlinear thermal radiation. The primary objective of Arif *et al.,* [27] research is to investigate the application of a water-based ternary hybrid nanofluid in the context of advanced cooling for radiators. This nanofluid comprises three distinct types of nanoparticles: spherical aluminum oxide (Al_2O_3) cylindrical carbon nanotubes *(CNT)*, and platelet-shaped particles (Graphene). Patil *et al.,* [28] analyze the behavior of a ternary hybrid nanofluid *(THNF)* with tangent hyperbolic *(T-H)* flow characteristics as it interacts with a rough-yawed cylinder. The motion of the cylinder is induced using impulsive means, and the study focuses on the mixed convection mechanism in conjunction with periodic magnetohydrodynamics. The impact of suction and heat source on MHD stagnation point flow of ternary hybrid nanofluid ($Cu - Fe_3\,O_4 - SiO_2$/polymer) over convectively heated stretching/shrinking cylinder has been researched by Mahmood *et al.,* [29]. Cao *et al.,* [30] conducted research on the similarity solutions of the governing equations that describe the dynamics of a colloidal mixture consisting of water, spherical carbon nanotubes, cylindrical graphene, and platelet alumina nanoparticles. The study considered various levels of partial slip and examined the cases of forced convection, free convection, and mixed convection.

Ayub *et al.,* [31] studied the nanoscale heat transport in Ternary MHD Cross nanofluid across a heated rotating geometry, considering ion-slip and hall current. Furthermore, Riaz *et al.,* [32] discovered new solutions of fractional Maxwell fluid containing ternary-hybrid nanoparticles flowing over an infinitely tall vertical heated wall. Cao *et al.,* [33] and Usman *et al.,* [34] researched the characteristics of a fractional Oldroyd-B fluid between two coaxial cylinders, one containing gold nanoparticles and the other carrying copper nanoparticles, mixed with a blood-based fluid, respectively.

In a doubly stratified fluid non-darcy porous medium, Srinivasacharya and Surender [35-37] investigated mixed convection heat and mass transfer along a vertical plate under various conditions. Additionally, the numerical solution for free convection boundary layer flow past a semi-infinite vertical plate in a doubly stratified fluid-saturated porous medium was derived by Srinivasacharya *et al.,* [38,39]. The thermal and mass stratification effects on mixed and natural convection boundary layer flow were investigated by Srinivasacharya and Surender [40,41], respectively, over a vertical flat plate embedded in a porous medium saturated with a nanofluid. Moreover, Srinivasacharya and Surender [42] obtained the non-similarity solutions for natural convection heat and mass transport down a vertical plate with uniform wall temperature and concentration in a doubly stratified porous media saturated with fluid.

Afzal *et al.,* [43] researched about an engine oil hybrid nanofluid consisting of multi-walled carbon nanotubes, and copper oxide nanoparticles to improve heat transmission, reduce entropy formation, and develop a theoretical model that better matches experimental results. The effects of chemical reactions, thermal density, viscous dissipation, and thermophoresis on the mass and heat transmission of chemically reactive and magnetic nanofluids over the stretched sheet were thoroughly examined by Ullah *et al.,* [44]. Additionally, Bilal *et al.,* [45] studied the physical properties of the Carreau nanofluid over a cylinder that can be extended linearly, as well as the use of the bioconvection phenomenon. Rashid *et al.,* [46] investigated the Al_2O_3 and Cu hybrid nanofluid magnetohydrodynamics in a porous media with the shape effect of nanoparticles. An investigation into the forced convection of nanofluid through a semi-porous channel under the influence of a magnetic field caused by heat radiation, as carried out by Jalili *et al.,* [47]. Wang *et al.,* [48] researched the thermal reaction and behaviour of natural convection in a hexagonal cavity using MWCNT-water nanofluid flow under the influence of a magnetic field.

According to the literature review, as was mentioned in previous research, no one has tried to show the MHD ternary nanofluid past a vertically stretching cylinder in a porous medium. The main objective of the present study is to examine the heat transfer properties of a ternary hybrid nanofluid consisting of $Cu - Al_2O_3 - TiO_2$ particles dispersed in water. This investigation focuses on the heat transfer behavior around a vertically stretching cylinder, taking into account the effects of thermal stratification, and uniform heat sources and sinks. The governing equations of non-linear partial differential equations (PDEs) are transformed into ordinary differential equations (ODEs) by employing suitable self-similarity variables within the bvp4c solver of the MATLAB software. The Bvp4c technique utilized in this study to simulate the problem is widely recognized, as demonstrated by its discussion and application in MATLAB by Hale and Moore [49]. The graphical representation of the results is provided for several parameters such as $\delta, \gamma, \lambda, M, K$ and Pr.

2. Methodology

Consider a two-dimensional steady in-compressible ternary hybrid nanofluid consisting of $Cu - Al_2O_3 - TiO_2/H_2O$, which is immersed in a porous medium over a vertical stretchable cylinder of radius r_0. The system is subjected to the influence of a heat source/sink and thermal stratification. The flow of the ternary hybrid nanofluid is assumed to be in the axial x −direction, with the r coordinate representing the direction normal to the x −axis, as depicted in Figure 1.

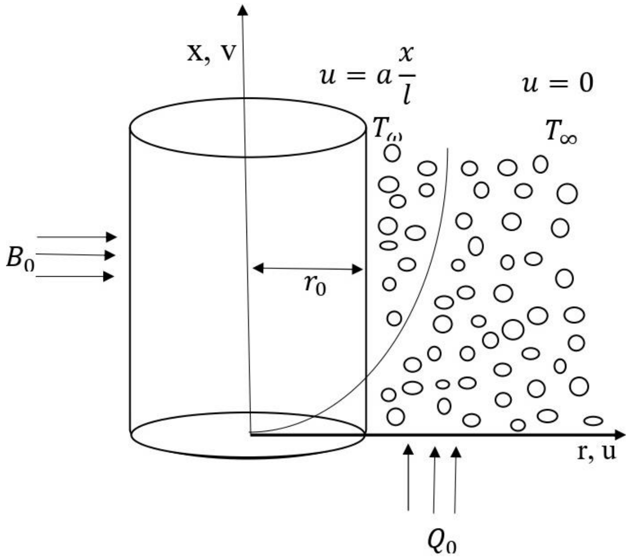

Fig. 1. Physical and coordinate system

In this context, the variables "u" and "v" represent the velocity components of the ternary hybrid nanofluid along the r and $x-$ axes, respectively. In this study, a magnetic field with a magnitude of B_0 is applied in a direction perpendicular to the propagation of the ternary hybrid nanofluid. The flow issue takes into account the thermal buoyancy effect while disregarding the Joule's impact. The velocity that causes linear stretching of the vertical cylinder is denoted as $u = a\frac{x}{l}$ where $'a'$ and $'l'$ represent the velocity and characteristic length of the cylinder, respectively. $T_\omega(x) = T_0 + A\left(\frac{X}{l}\right)$ represents the assumed temperature of the wall, while $T_\infty(x) = T_0 + B\left(\frac{X}{l}\right)$ represents the temperature of the ternary hybrid nanofluid at the ambient condition, where A, B, and T_0 are non-negative constants and the starting temperature, correspondingly. The governing equations for continuity, momentum, and energy in the context of a ternary hybrid nanofluid, as presented by Paul *et al.*, [17], can be expressed as follows.

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{\mu_{thnf}}{\rho_{thnf}}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{(\rho\beta)_{thnf}}{\rho_{thnf}}g(T - T_\infty) - \frac{\sigma_{thnf}}{\rho_{thnf}}B_0^2 u - \frac{\mu_{thnf}}{\rho_{thnf}}\frac{u}{k} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{k_{thnf}}{(\rho c_p)_{thnf}}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{Q_0}{(\rho c_p)_{thnf}}(T - T_\infty) \tag{3}$$

The following are the boundary conditions

$u = a\frac{x}{l}$, $\quad v = 0, \quad T = T_\omega(x)$ when $r = r_0$

$u = 0$, $\quad T \to T_\infty(x)$ when $r \to \infty$

The similarity transformation used in Eq. (1) to Eq. (3) are as follows [17]

$$\eta = \frac{r^2 - r_0^2}{2r_0}\sqrt{\frac{a}{\upsilon_f l}}, \quad \psi = \sqrt{\frac{a\upsilon_f}{l}}\, x r_0 f(\eta), \quad \theta = \frac{T - T_\infty(x)}{T_\omega(x) - T_0}$$

and we provide non-dimensional quantities as the followings

$$M = \frac{B_0^2 l \sigma_f}{a\rho_f},\ K = \frac{l\upsilon_f}{ak},\ \gamma = \sqrt{\frac{l\,\upsilon_f}{a r_0^2}},\ \lambda = \frac{Gr_x}{Re_x^2}$$

$$\delta = \frac{B}{A},\ Q = \frac{Q_0 l}{a\,(\rho c_p)_f},\ Pr = \frac{\nu_f (\rho c_p)_f}{k_f}$$

where, M is the magnetic parameter, K is the porosity parameter, γ is the curvature parameter, λ is the thermal buoyancy parameter, δ is the thermal stratification parameter, Q is the heat source/sink parameter, Pr is the Prandtl number.

The stream function ψ is introduced to satisfy continuity equation Eq. (1) in the manner that $u = \frac{1}{r}\frac{\partial\psi}{\partial r}$ and $v = -\frac{1}{r}\frac{\partial\psi}{\partial x}$. Hence, the non-dimensional forms of the transformed equations are given by

$$f'^2 - ff'' = a_1[2\gamma f'' + (1 + 2\gamma\eta)f'''] + a_2\lambda\theta - (a_3 M + a_1 K)f' \tag{4}$$

$$f'(\theta + \delta) - f\theta' = a_4[2\gamma\theta' + (1 + 2\gamma\eta)\theta''] + a_5\theta \tag{5}$$

where,

$$x_1 = \frac{\mu_{thnf}}{\mu_f},\ x_2 = \frac{\rho_f}{\rho_{thnf}},\ x_3 = \frac{(\rho\beta)_{thnf}}{(\rho\beta)_f},\ x_4 = \frac{(\rho c_p)_f}{(\rho c_p)_{thnf}},\ x_5 = \frac{\sigma_{thnf}}{\sigma_f},\ x_6 = \frac{k_{thnf}}{k_f}$$

$$a_1 = x_1 x_2,\ a_2 = x_2 x_3,\ a_3 = x_2 x_5,\ a_4 = \frac{x_4 x_6}{Pr},\ a_5 = Q x_4$$

Here, the symbols $\mu_{thnf}, \rho_{thnf}, \beta_{thnf}, (\rho c_p)_{thnf}, \sigma_{thnf}, k_{thnf}$ represent the ternary hybrid nanofluid's coefficient of viscosity, electrical conductivity, thermal expansion, heat capacity, density and thermal conductivity, respectively. Also, $\mu_f, \rho_f, \beta_f, (\rho c_p)_f, \sigma_f, k_f$ denote the base fluid's coefficient of viscosity, electrical conductivity, thermal expansion, heat capacity, density and thermal conductivity correspondingly.

The transformed boundary conditions are as follows

$$f(\eta) = 0, \quad f'(\eta) = 1, \quad \theta(\eta) = 1 - \delta \quad \text{at } \eta = 0$$

$$f'(\eta) \to 0, \quad \theta(\eta) \to 0 \quad \text{at } \eta \to \infty$$

The thermo-physical properties of ternary hybrid nanofluid are as follows [25]

$$\mu_{thnf} = \frac{\mu_f}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}(1-\phi_3)^{2.5}}$$

$$\rho_{thnf} = (1-\phi_3)\left[(1-\phi_2)\left\{(1-\phi_1)\rho_f + \phi_1\rho_{s1}\right\} + \phi_2\rho_{s2}\right] + \phi_3\rho_{s3}$$

$$\left(\rho c_p\right)_{thnf} = (1-\phi_3)\left[(1-\phi_2)\left\{(1-\phi_1)\left(\rho c_p\right)_f + \phi_1\left(\rho c_p\right)_{s1}\right\} + \phi_2\left(\rho c_p\right)_{s2}\right] + \phi_3\left(\rho c_p\right)_{s3}$$

$$(\rho\beta)_{thnf} = (1-\phi_3)\left[(1-\phi_2)\left\{(1-\phi_1)(\rho\beta)_f + \phi_1(\rho\beta)_{s1}\right\} + \phi_2(\rho\beta)_{s2}\right] + \phi_3(\rho\beta)_{s3}$$

$$k_{nf} = \left[\frac{(k_{s1}+2k_f)-2\phi_1(k_f-k_{s1})}{(k_{s1}+2k_f)+\phi_1(k_f-k_{s1})}\right]k_f, \quad k_{hnf} = \left[\frac{(k_{s2}+2k_{nf})-2\phi_2(k_{nf}-k_{s2})}{(k_{s2}+2k_{nf})+\phi_2(k_{nf}-k_{s2})}\right]k_{nf}$$

$$k_{thnf} = \left[\frac{(k_{s3}+2k_{hnf})-2\phi_3(k_{hnf}-k_{s3})}{(k_{s3}+2k_{hnf})+\phi_3(k_{hnf}-k_{s3})}\right]k_{hnf},$$

$$\sigma_{nf} = \left[\frac{(\sigma_{s1}+2\sigma_f)-2\phi_1(\sigma_f-\sigma_{s1})}{(\sigma_{s1}+2\sigma_f)+\phi_1(\sigma_f-\sigma_{s1})}\right]\sigma_f, \quad \sigma_{hnf} = \left[\frac{(\sigma_{s2}+2\sigma_{nf})-2\phi_2(\sigma_{nf}-\sigma_{s2})}{(\sigma_{s1}+2\sigma_{nf})+\phi_2(\sigma_{nf}-\sigma_{s2})}\right]\sigma_{nf}$$

$$\sigma_{thnf} = \left[\frac{(\sigma_{s2}+2\sigma_{hnf})-2\phi_2(\sigma_{hnf}-\sigma_{s2})}{(\sigma_{s1}+2\sigma_{hnf})+\phi_2(\sigma_{hnf}-\sigma_{s2})}\right]\sigma_{hnf}$$

where ϕ_1, ϕ_2, ϕ_3 are volume fraction of Cu (Copper), Al_2O_3(aluminium oxide) and TiO_2 (titanium oxide) nanoparticles respectively. The suffixes *thnf, hnf, nf, f, s1, s2, s3* denote ternary hybrid nanofluid, hybrid nanofluid, nanofluid, base fluid, solid nanoparticles of copper *(Cu),* aluminum oxide (Al_2O_3), and titanium dioxide (TiO_2) correspondingly. Thermo-physical characteristics of Cu, Al_2O_3 and TiO_2 nanoparticles in pure water are presented in Table 1.

The skin friction coefficient and local Nusselt number are defined by

$$C_f Re_x^{1/2} = \frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}(1-\phi_3)^{2.5}} f''(0) \quad \text{and} \quad Nu_x Re_x^{-1/2} = -\frac{k_{thnf}}{k_f}\theta'(0)$$

where, Re_x is the local Reynolds Number.

Table 1
Thermo-physical properties of water and nanoparticles [2]

Physical Properties	H_2O (base fluid)	Cu (s1)	Al_2O_3 (s2)	TiO_2 (s3)
ρ (Kg/m^3)	997.1	8933	3970	4250
c_p (J/KgK)	4179	385	765	686.2
k (W/mK)	0.613	401	40	8.9538
$\beta \times 10^5$ (K^{-1})	21	1.67	0.85	0.9
σ (s/m)	5.5×10^{-6}	59.6×10^6	35×10^6	2.6×10^6

3. Method of Solution

The bvp4c solver, built into the computational platform MATLAB, is used to numerically solve the system of higher-order nonlinear ODEs given by Eq. (4) and Eq. (5) and the boundary conditions. This technique has been extensively utilized by professionals and researchers in order to solve fluid flow problems. The bvp4c solver, created by Jacek Kierzenka and Lawrence F. Shampine of Southern Methodist University in Texas, was introduced by Hale and Moore [49]. The bvp4c solver is a finite modification algorithm that uses the Lobato IIIA implicit Runge-Kutta method to produce numerical solutions with fourth-order accuracy. This method gives the necessary accuracy when a guess is made for the initial mesh points and step-size changes. In the study conducted by Waini *et al.,* [50], it was found that the bvp4c solver yielded satisfactory results in comparison to both the shooting technique and Keller box method. Here, we need to reduce the higher order derivatives with respect to $\boldsymbol{\eta}$. This can be done by introducing the following new variables

$$f = y(1),\ f' = y(2),\ f'' = y(3),\ \theta = y(4),\ \theta' = y(5)$$

$$\frac{d}{d\eta}\begin{bmatrix} y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \end{bmatrix} = \begin{bmatrix} y(2) \\ y(3) \\ \frac{y(2)^2 - y(1)y(3) - a_2\lambda y(4) + (a_3 M + a_1 K)y(2) - 2a_1\gamma y(3)}{(1+2\gamma\eta)a_1} \\ y(5) \\ \frac{y(2)(y(4)+\delta) - y(1)y(5) - 2a_4\gamma y(5) - a_5 y(4)}{(1+2\gamma\eta)a_4} \end{bmatrix}$$

and boundary condition are expressed as

$$ya(1),\ ya(2) - 1,\ ya(4) - 1 + \delta,\ yb(2),\ yb(4)$$

where ya is the condition at $\eta = 0$ and yb is the condition at $\eta \to \infty$.

4. Results and Discussion

The numerical calculations were performed in bvp4c solver of MATLAB, and the outcomes are displayed in Figure 2 to Figure 13, and Table 3 and Table 4. Table 2 presents the skin friction coefficient and local Nusselt number values obtained from the current investigation, which are compared to the findings reported by Paul *et al.,* [17]. This comparison specifically excludes nanoparticle volume fractions of TiO_2. This study reveals that the bvp4c algorithm is capable of generating numerical results that are accurate and in agreement with the results obtained from alternative methods.

Table 2
Comparison of Skin friction Coefficient and local Nusselt Number when $\phi_3 = 0$

δ	Pr	λ	γ	Paul *et al.,* [17] $-C_f Re_x^{1/2}$	Present Study $-C_f Re_x^{1/2}$	Paul *et al.,* [17] $Nu_x Re_x^{-1/2}$	Present Study $Nu_x Re_x^{-1/2}$
0.1	6.2	1.2	0.1	2.9398	2.9795	2.7885	2.7887
0.1	2	1.2	0.1	2.8419	2.8760	1.3264	1.3311
0.1	6.2	2.4	0.1	2.7219	2.7425	2.8528	2.8516
0.1	6.2	1.2	0.2	3.0048	3.0436	2.8283	2.8269

The following values are used in the study: $\delta = 0.2, \gamma = 0.2, M = 1.1, K = 2, Q = 0.1, \lambda = 1.2, Pr = 6.2, \phi_1 = 0.05, \phi_2 = 0.15$ and $\phi_3 = 0.2$. Figure 2 displays the impact of thermal stratification parameter δ on the velocity $f'(\eta)$. The velocity decreases as thermal stratification(δ) increase. If the thermal stratification(δ) parameter rises, the efficient convective potential between the hot wall and the surrounding fluid drops. The buoyancy force is reduced as a result of this, which reduces the flow velocity.

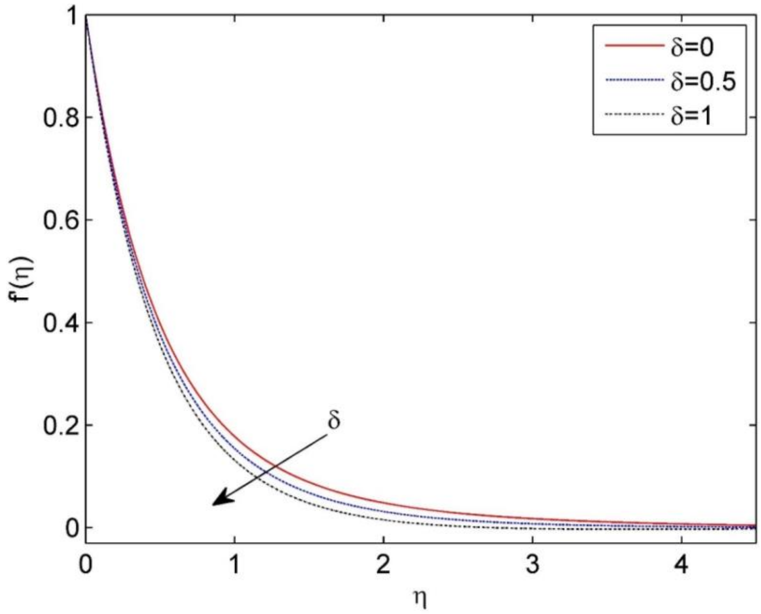

Fig. 2. Effects of δ on velocity profile

As shown in Figure 3, the fluid velocity $f'(\eta)$ decreases as the value of *M* is increased. As magnetic parameter*(M)* grows higher, the thickness of the momentum boundary layer shrinks. This pattern emerges because the Lorentz force produced by the horizontal magnetic field slows down the velocity of the ternary hybrid nanofluid. The impact of the porosity parameter*(K)* on velocity $f'(\eta)$ of the nanofluid is shown in Figure 4. The velocity decreases as the porosity parameter value increases. The relationship between *K* and the diameter of the porous medium is inverse, implying that as *K* grows, the diameter of the porous medium decreases progressively. This reduction in diameter hinders the fluid's ability to flow through the porous medium. As a result of this obstruction, which was induced by porosity parameter*(K)*, the velocity of the fluid dropped.

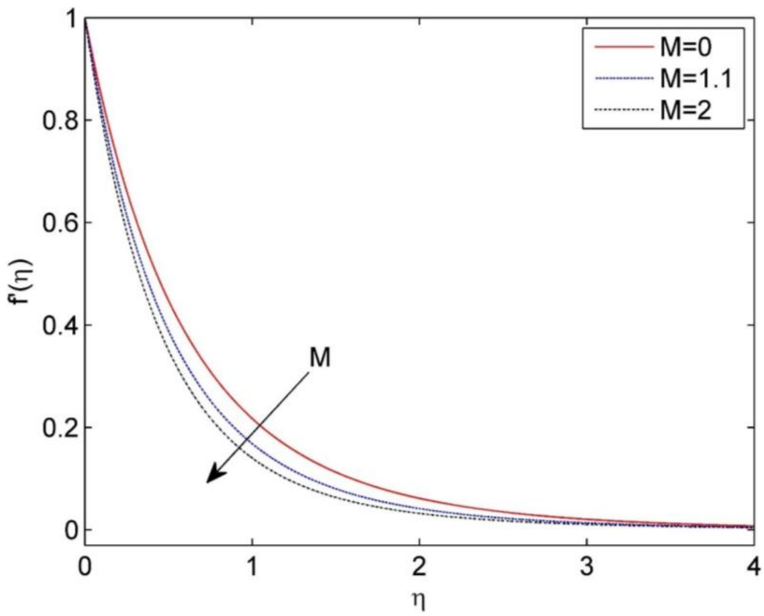

Fig. 3. Effects of M on velocity profile

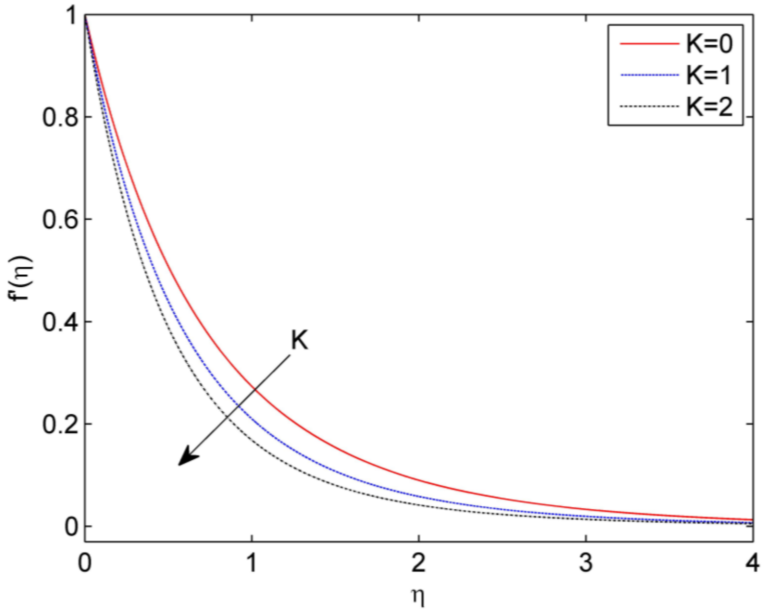

Fig. 4. Effect of K on velocity profile

As depicted in Figure 5, an increase in the curvature parameter(γ) corresponds to an increase in the velocity $f'(\eta)$ of the fluid. The effects of thermal buoyancy parameter on the velocity of the fluid, is represented by Figure 6. It is observed that the velocity will increase as the values of λ is increased. When λ is raised, the thermal buoyancy forces are made stronger. This indicates that the buoyancy forces tend to increase the velocity of the fluid.

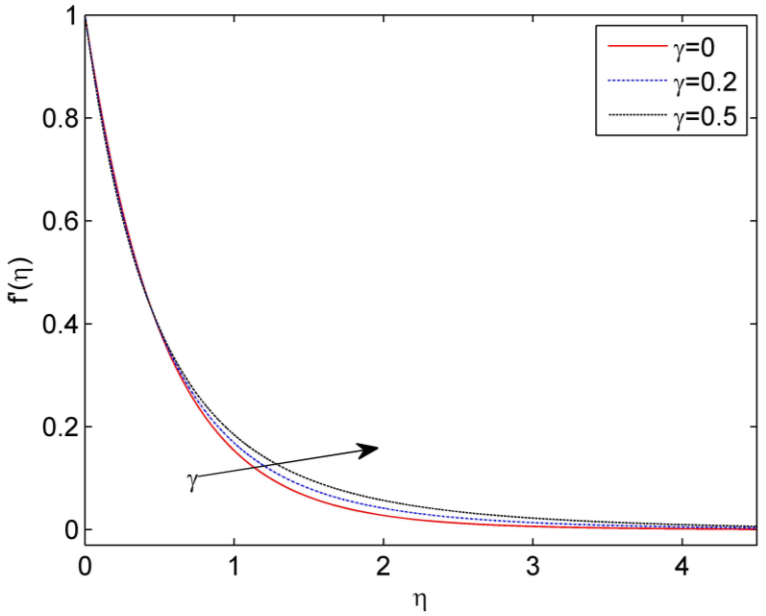

Fig. 5. Effects of γ on velocity profile

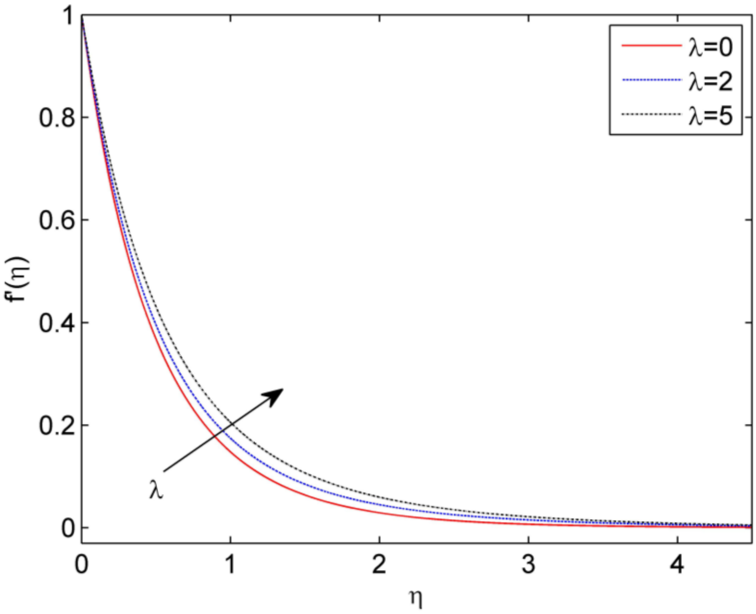

Fig. 6. Effects of λ on velocity profile

The Figure 7 displays that the temperature of the fluid goes down as the thermal stratification (δ) goes up. The temperature gradient between the heated wall and the surrounding fluid will decrease when the thermal stratification (δ) is present. Hence, the thermal boundary layer thickens and the temperature falls. The temperature of the fluid exhibits a positive correlation with the increasing values of the curvature parameter(γ) as depicted in Figure 8.

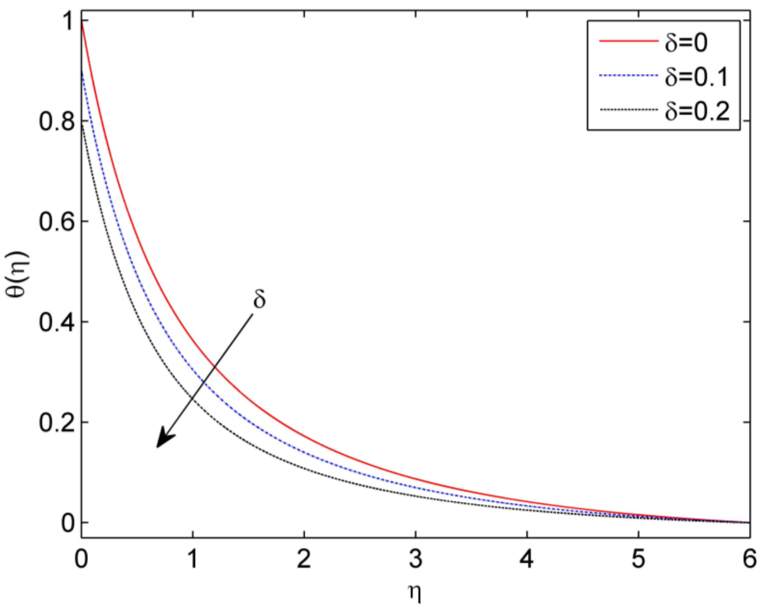

Fig. 7. Effects of δ on temperature profile

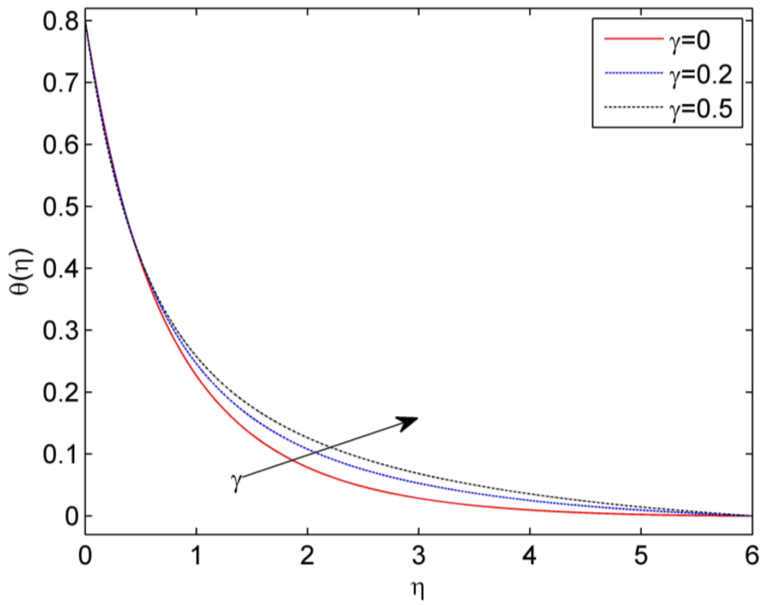

Fig. 8. Effects of γ on temperature profile

The impact of a heat source parameter*(Q)* on temperature profile is seen in Figure 9. As the values of heat source parameter *(Q)* raises, the fluid temperature also increase. This characteristic matches how the fluid behaves physically in general. Figure 10 illustrates the influence of *Pr* on the temperature profile of the fluid. As the *Pr* goes up, the temperature of the fluid goes down. It's clear that a fluid with a high Prandtl number has a low thermal conductivity. This means that heat doesn't move as easily through the fluid, so the transfer rate goes down and the thermal boundary layer gets thinner. Hence, the temperature of the fluid drops. Engine oil has a Prandtl number of 21, while water has a Prandtl value of 6.2. As a result, engine oil typically transfers heat more slowly than water due to its lower thermal diffusivity.

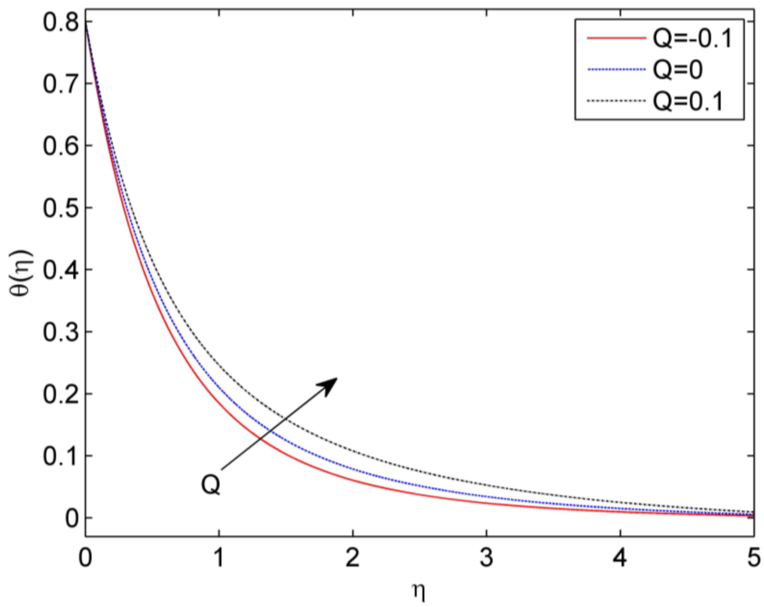

Fig. 9. Effects of Q on temperature profile

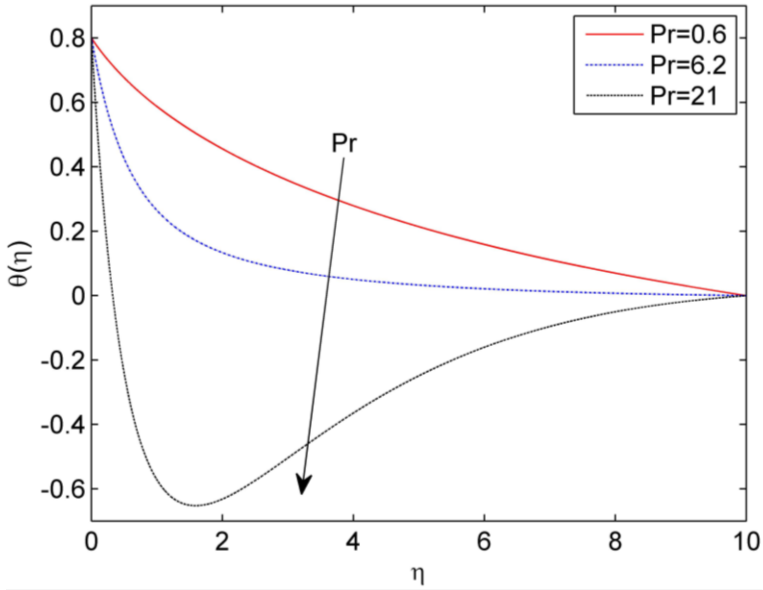

Fig. 10. Effects of Pr on temperature profile

As the solid volume fraction of Cu nanoparticles increases, while the volume fractions of Al_2O_3 and TiO_2 remain constant, the temperature profile increases, as seen in Figure 11. Furthermore, it can be observed from Figure 12 that the temperature is increased with the increasing volume fraction of Al_2O_3 while the volume fraction of Cu and TiO_2 remains constant. Similarly, ϕ_3 also increases the temperature profile as seen in Figure 13.

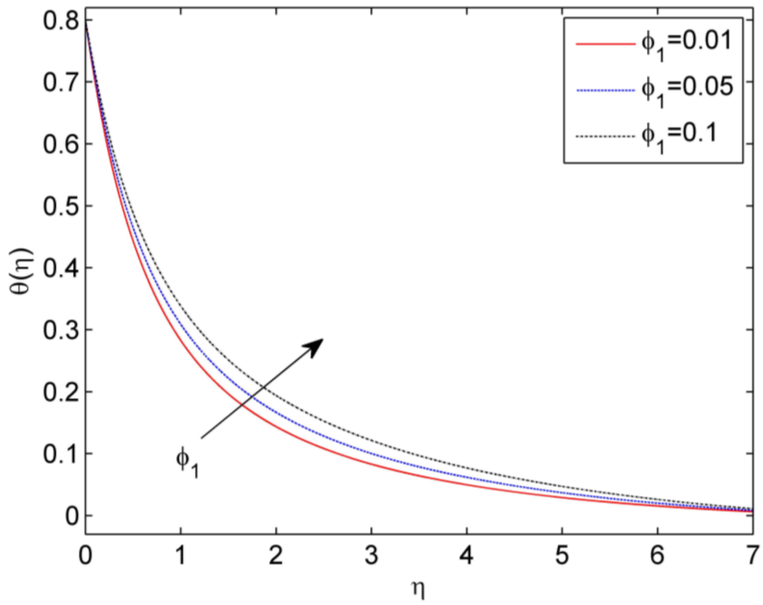

Fig. 11. Effects of ϕ_1 on temperature profile

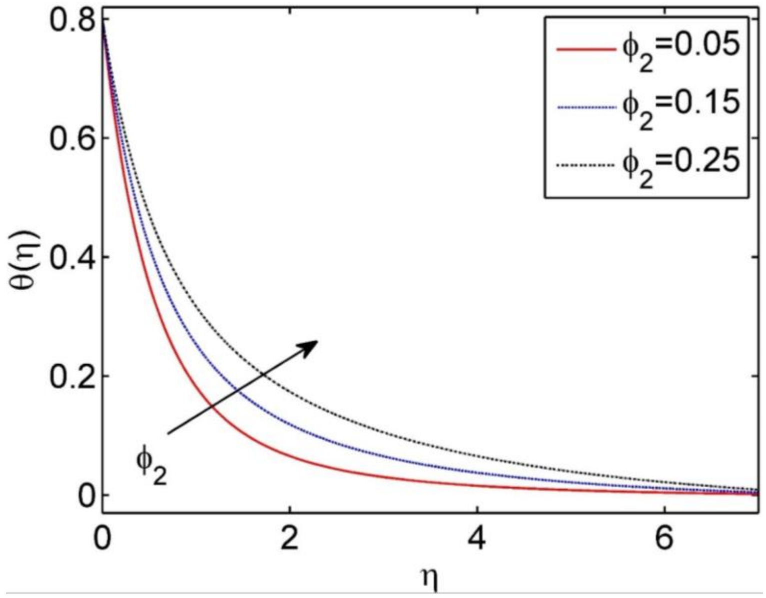

Fig. 12. Effects of ϕ_2 on temperature profile

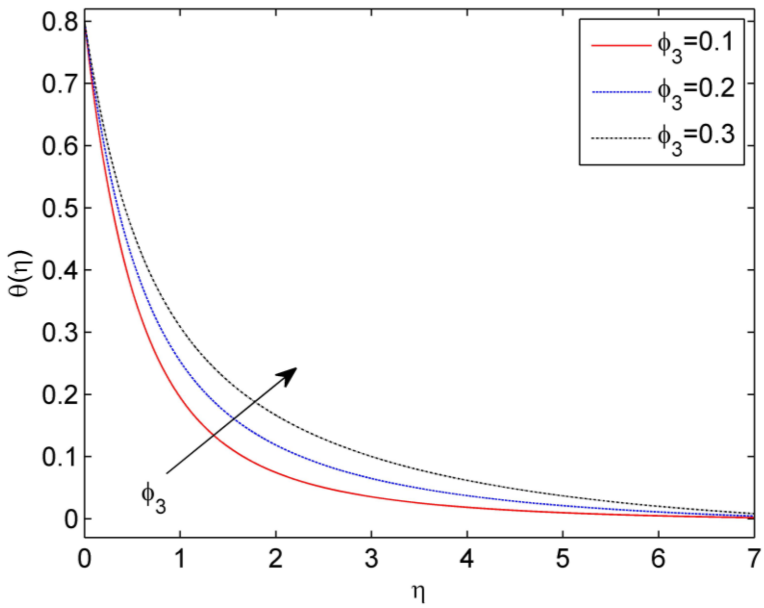

Fig. 13. Effects of ϕ_3 on temperature profile

Table 3 presents the effects on absolute skin friction and local Nusselt number for different flow parameters. As the thermal stratification value (δ) grows, there is a decrease in the local Nusselt number, but the absolute skin friction experiences an increase. Table 3 reveals that an increase in the magnitude of absolute skin friction and a decrease in the local Nusselt number are observed when the values of the magnetic parameter *(M)* and porosity parameter *(K)* are increased. As the value of the thermal bouyanacy parameter(λ) grows, there is a drop in the absolute skin friction and an increase in the Nusselt number. An increase in the Prandtl number *(Pr)* leads to a rise in both absolute skin friction and local Nusselt number. It is evident that an increase in the heat source*(Q)* results in a reduction in the rate of heat transfer and the absolute skin friction. The magnitude of skin friction and the value of the local Nusselt number both exhibit an upward trend when the parameter γ increases.

Table 3
Skin friction Coefficient and Local Nusselt number for different values of $\delta, M, K, \lambda, Pr, Q, \gamma$

δ	M	K	λ	Pr	Q	γ	$-C_f Re_x^{1/2}$	$Nu_x Re_x^{-1/2}$
0	1.1	2	1.5	6.2	0.1	0.2	5.8976	3.3791
0.2							5.9937	3.1437
0.4							6.0895	2.9029
0.2	0	2	1.5	6.2	0.1	0.2	5.0625	3.3897
	0.5						5.5047	3.2716
	1.5						6.2992	3.0654
0.2	1.1	0	1.5	6.2	0.1	0.2	4.2386	3.6142
		1					5.1882	3.3559
0.2	1.1	2	0	6.2	0.1	0.2	6.2814	3.0258
			3				5.7207	3.2339
0.2	1.1	2	1.5	0.6	0.1	0.2	5.8739	0.9577
				4			5.9541	2.3516
0.2	1.1	2	1.5	6.2	-0,1	0.2	6.0190	3.6293
					0		6.0083	3.4083
0.2	1.1	2	1.5	6.2	0.1	0	5.7202	3.0029
						0.1	5.8582	3.0723

The experimental results shown in Table 4 indicate that the ternary hybrid nanofluid exhibits a higher absolute skin friction compared to the hybrid nanofluid. In a similar manner, it can be noted that the skin friction of hybrid nanofluids demonstrates an increase in comparison to that of standard nanofluids. Moreover, it has been observed that the ternary hybrid nanofluid exhibits a greater heat transfer rate in comparison to the hybrid nanofluid, while the hybrid nanofluid, in turn, demonstrates a higher heat transfer rate when compared to conventional nanofluids.

Table 4
Comparison of Skin friction Coefficient and Local Nusselt number

δ	Cu Nanofluid $-C_f Re_x^{1/2}$	$Cu-Al_2O_3$ Hybrid Nanofluid $-C_f Re_x^{1/2}$	$Cu-Al_2O_3-TiO_2$ Ternary Hybrid Nanofluid $-C_f Re_x^{1/2}$	Cu Nanofluid $Nu_x Re_x^{-1/2}$	$Cu-Al_2O_3$ Hybrid Nanofluid $Nu_x Re_x^{-1/2}$	$Cu-Al_2O_3-TiO_2$ Ternary Hybrid Nanofluid $Nu_x Re_x^{-1/2}$
0	1.9645	3.3981	5.8976	2.3250	3.0662	3.3791
0.2	2.1869	3.5020	5.9937	2.5539	2.8668	3.1437
0.5	2.3329	3.6531	6.1372	2.2778	2.5361	2.7805

4. Conclusions

The comprehensive investigation of the effects of thermal stratification on the flow of a ternary hybrid nanofluid with magnetohydrodynamics (MHD) properties along a vertical stretchable cylinder, considering the presence of a thermal bouncy effect and heat source/sink inside a porous medium, has been studied. The analysis additionally considers the flow characteristics and their effects on the velocity and temperature profiles, skin friction, and local Nusselt number. The key outcomes of the current investigation are outlined below

i. The velocity profile exhibits a decreasing trend as the parameters δ, M and K are incresed, whereas it demonstrates an increasing trend with increasing values of γ and λ.
ii. The temperature shows a reduction as the values of δ and Pr grow, whereas it demonstrates an increase with the increase of γ and Q.

iii. The absolute skin friction is increased for parameters δ, M, K, Pr and γ while it decreases for parameters λ and Q.
iv. An increase in the local Nusselt number is observed with increasing values of λ, γ and Pr, while it decreases with increasing δ, M, K and Q.
v. The ternary hybrid nanofluid shows a greater absolute skin friction when compared to the hybrid nanofluid. Likewise, the skin friction of hybrid nanofluids shows an increase in comparison to that of regular nanofluids.
vi. The ternary hybrid nanofluid shows a higher heat transfer rate in comparison to the hybrid nanofluid, and the hybrid nanofluid exhibits a higher heat transfer rate when compared to traditional nanofluids.

The future scope of ternary hybrid nanofluids—comprising copper (Cu), aluminum oxide (Al_2O_3), and titanium dioxide (TiO_2)—holds immense potential across various scientific and engineering domains. Ternary hybrid nanofluids have the potential to significantly improve heat transfer efficiency in applications including heat exchangers, radiators, and cooling devices. Enhanced heat transfer qualities could be advantageous for applications in solar energy systems, geothermal power extraction, and high-temperature processes.

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