

# Suggestion of Correlations to Obtain Shell and Tube Side Nusselt Numbers in Coiled Heat Exchangers

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ARTICLE INFO	ABSTRACT
Article history: Received 30 October 2023 Received in revised form 20 March 2024 Accepted 29 March 2024 Available online 15 April 2024	In coiled tube heat exchangers increase in curvature ratio and effective contact of shell fluid with coil surface led to better thermal characteristics. This work addresses study of thermal performance, shell and tube heat transfer coefficients ( $h_o$ and $h_i$ ). Also, mathematical correlations are predicted to obtain shell and tube Nusselt numbers (Nu <sub>o</sub> and Nu <sub>i</sub> ). In the designing of compact coiled heat exchangers consisting of straight helical coil (SHC), conical coil (CC) and spiral coil knowledge of Nu <sub>o</sub> and Nu <sub>i</sub> is necessary whereas mathematical correlation to calculate Nu <sub>o</sub> considering variations in shell geometry is not found. Five heat exchangers consisting of three cone coils (angle, $\Theta$ = 30°, 50° and 70°), spiral coil ( $\Theta$ = 0°) and SHC ( $\Theta$ = 90°) are tested. Using Wilson plot method $h_o$ and $h_i$ are obtained for wide range of tube side Reynolds numbers (Re <sub>i</sub> = 3700 - 21000). It is observed that, highest $h_o$ is obtained for $\Theta$ = 30°CC with better thermal performance. Lowest $h_o$ is considered in prediction of single correlation as: Nu <sub>o</sub> = 16.55.Re <sub>o</sub> <sup>0.55</sup> .Pr <sub>o</sub> <sup>0.4</sup> * (H <sub>s</sub> /D <sub>so</sub> ) <sup>0.096</sup> . Comparison with experimental findings shows variation of 0-14%. Similarly two correlations are proposed as: Nu <sub>i</sub> = 0.0512.Re <sub>i</sub> <sup>0.80</sup> .Pr <sub>i</sub> <sup>0.4</sup> .CR <sub>ave</sub> <sup>0.18</sup>
equivalent diameter	researchers.

#### 1. Introduction

Coiled tube heat exchangers are used in the field of heat recovery units, chemical industries, boilers, refrigeration and solar systems. In helical coils curvature ratio causes generation of secondary flow which further leads to increase in tube side heat transfer coefficients. Helical coils have better heat transfer coefficients than straight tubes and are best suitable where enough space is not available. In straight helical coils the curvature ratio remains constant along the vertical axis. In spiral coils the curvature ratio varies along the horizontal axis whereas in cone coils the curvature ratio increases from a base to apex. In the manual design of conical coil heat exchanger (CCHE), for known values of mass flow rates and inlet temperatures of both fluids, to achieve desired outlet

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temperature of one of the fluids the task is to calculate required surface area. This surface area is written as

$$A = \frac{Q}{U.LMTD}$$
(1)

where Q is the heat duty to be achieved and log mean temperature difference, LMTD is obtained from inlet and outlet temperatures. U is overall heat transfer coefficient and given as

$$\frac{1}{U_{i}.A_{i}} = \frac{1}{h_{i}.A_{i}} + R_{t} + \frac{1}{h_{0}.A_{0}}$$
(2)

Without the knowledge of tube side heat transfer coefficient ( $h_i$ ) and shell side heat transfer coefficients ( $h_o$ ), overall heat transfer coefficient U cannot be obtained. Due to which calculation of Area, A is not possible. Rose [1] discussed the traditional Wilson plot method to obtain inner and outer heat transfer coefficients. In this method measurement of wall temperature of coil is not necessary which otherwise may create disturbance to flow patterns. In Eq. (2),  $R_t$  as conductive thermal resistance of the tube can be calculated. As per the Wilson plot method, it is assumed that if tube side mass flow rate is kept constant,  $h_i$  remains constant. Further for this constant tube side mass flow rate, if mass flow rate of shell fluid is varied in the steps,  $h_o$  is directly proportional to shell side fluid velocity,  $v_s^n$ . Now  $h_i$  and  $h_o$  are written in Eq. (3) and Eq. (4).

$$\frac{1}{h_i \cdot A_i} + R_t = C_1 \tag{3}$$

$$h_o = C_2 \cdot v_s^n \tag{4}$$

Eq. (2) can be written as

$$\frac{1}{U_{i}.A_{i}} = C_{1} + \frac{1}{C_{2}.A_{0}.v_{s}^{n}}$$
(5)

If  $1/U_iA_i$  is plotted against  $1/v_s^n$  for variation in  $v_s$ , for proper selection of values of n, a straight line fit can be obtained through the data points of the plot. From this straight line fit, y intercept is obtained as C<sub>1</sub> and slope as m. From the values of C<sub>1</sub> and m,  $h_i$  and  $h_o$  are obtained as

$$h_i = \frac{1}{(A_i \cdot (C_1 - R_t))} \tag{6}$$

$$C_2 = \frac{1}{m \cdot A_0};$$
 (7)

$$h_o = C_2 \cdot v_s^n \tag{8}$$

Rennie and Raghavan [2] experimentally studied two double pipe helical heat exchangers in which the size of the inner tube was varied. Heat transfer coefficients were obtained by using Wilson plot in which mass flow rate through tube was kept constant assuming h<sub>i</sub> remained constant. In this way 60 h<sub>i</sub> were obtained from 60 Wilson plots. Fernandez-Seara *et al.,* [3] presented review of application of Wilson plot method to obtain convective heat transfer coefficients in heat exchangers. It was concluded that, if the mass flow rate of cooling liquid was

modified then change in overall heat transfer coefficients would be due to variation in thermal resistance of cooling liquid and other thermal resistances remained constant. Kumar et al., [4] studied double pipe helical coil heat exchanger in which tube side and annulus side heat transfer coefficients were obtained by using Wilson plots. While using Wilson plots, hi were obtained by keeping shell side mass flow rates constant, resulting ho constant. In a similar way for calculation of ho tube side mass flow rate kept constant and shell side mass flow rate was varied. Shokouhmand et al., [5] studied three straight helical coil heat exchangers (SHCHE) and obtained convective heat transfer coefficients using Wilson plot technique. Mass flow rate in tube was kept constant and shell side mass flow rate was varied in five steps. This resulted in 30 Wilson plots, 30 h<sub>i</sub> and 150 h<sub>o</sub>. Hydraulic diameters were considered as a characteristic dimension in the calculation of Re<sub>o</sub>. As Re<sub>o</sub> was increased Nu<sub>o</sub> also increased. It was also predicted that, for higher curvature ratios, higher tube side Nusselt no. were obtained. Salimpour [6] experimentally studied three straight helical coils keeping coil diameter fixed and varying tube diameter, pitch and length. Mass flow rate in the tube was kept constant and shell side mass flow rate was varied to get 15 Wilson plots. From these Wilson plots 15  $h_i$  and 75  $h_o$  were obtained. Also, dimensionless pitch,  $\gamma$  was considered to predict Nu correlations. Jamshidi et al., [7] applied Wilson plot method to determine heat transfer coefficients for SHCHE. Shell side mass flow rate was kept constant and assumed that  $h_0$  remained constant. For three coil diameters, three pitches, three tube side mass flow rates and three shell side mass flow rates, the L<sub>9</sub> orthogonal array was used in Taguchi method. It was predicted as: a) U<sub>0</sub> increased with increase in Nu<sub>o</sub> b) as coil pitch was increased, Nu<sub>i</sub> decreased but Nu<sub>o</sub> increased. Purandare et al., [8] experimentally investigated coiled heat exchangers. On the basis of variation in apex angle five types of coils were studied as: 45°, 90°, 135°, 0° (helical), 180° (spiral). All coils were accommodated one by one in the same shell.  $h_i$  and  $h_o$  were obtained using Wilson plot method. Effectiveness, tube side friction factor and Nui were obtained and correlations were proposed. Sheeba et al., [9] experimentally and numerically investigated conical coil double pipe heat exchangers. Experimentation was carried out for cone apex angle of 72° and heat transfer coefficients were obtained by using Wilson method. In the application of Wilson plot, mass flow rate in inner tube was kept constant and assumed that variation in mass flow rate in annulus caused variation in  $h_0$  keeping  $h_i$  constant. Also, numerical simulation was carried for 72° cone apex angle and following correlation was predicted.

 ${\rm Nu}_i=0.4565$  ,  ${\rm De_i}^{0.7645}.\,{\rm Pr_i}^{-0.4786}$  For  $90\ll {\rm De}_i\ll 1000$  and  $3\ll {\rm Pr}_i\ll 4$ 

Alklaibi *et al.,* [10] experimentally studied horizontal single SHCHE in which hot water was flowing through the shell. Ethylene glycol and water mixture based  $Fe_3O_4$  nanofluid was flowing through the coil as a coolant. Wilson plot method was used to evaluate hi, by keeping shell side mass flow rate constant ( $h_0$  constant). Nu<sub>i</sub> and friction factor correlations were proposed considering Dean number, nanoparticles concentration and Prandtl number. Patil *et al.,* [11] explained the design procedure of SHCHE to obtain required surface area for necessary heat duty. Nu-RePr correlations for calculation of  $h_i$  and  $h_0$  were presented. Following correlation was used to obtain shell side Nusselt No, Nu<sub>0</sub>.

 $Nu_o$  = 0.6.  $Re_0^{0.5}.\,Pr_o^{0.31}$  For  $Re_o$  in the range of 50-10000.

Ali [12] experimentally studied natural convection in SHCHE and a constant temperature bath was maintained in the shell. Following Rogers and Mayhew [13] correlation for turbulent flow was used to obtain  $h_i$ .

 $Nu_i = 0.023. Re_i^{0.85}. Pr_i^{0.4}. (d_i/D)^{0.1}$ 

Also, height, H and length of coil,  $L_c$  and  $d_o$  are considered as characteristic dimensions in the calculation of  $Re_o$ . Further  $Nu_o$  correlations were developed. Kakaç *et al.*, [14] discussed  $Nu_i$  correlation given by Manlapaz- Churchill for laminar and turbulent flow in helical coils on the basis of constant temperature and constant heat flux boundary conditions. Also, for turbulent flow, Schmidt correlation was presented as

$$\frac{Nu_{c}}{Nu_{s}} = 1.0 + 3.6. \left[ \left( 1 - \frac{d_{i}}{D} \right) \right] \cdot \left( \frac{d_{i}}{D} \right)^{0.8} \text{ for } 2 \times 10^{4} < Re_{i} < 1.5 \times 10^{5}, 5 < \frac{d_{i}}{D} \right] \le 84$$

In above correlation Nu<sub>c</sub> and Nu<sub>s</sub> stands for Nusselt number for coil and straight tube respectively. Prabhanjan *et al.*, [15] experimentally studied the effect of water bath on straight helical coil (SHC). It was found out that, h<sub>i</sub> of SHC is 1.16 and 1.43 times better than straight tube. Similarly, Naphon and Suwagrai [16] studied the effect of curvature ratio over heat transfer for spiral coils considering 3 curvature ratios. Constant wall temperature was considered on the outer surface. It was predicted that Nu<sub>i</sub> obtained for spiral coils were found to be 1.49 times higher than Nu for a straight tube. Jayakumar *et al.*, [17] numerically and experimentally studied straight helical coils considering various boundary conditions. Schmidt, 1967 formulation was used to determine critical Renault number, Re<sub>crit</sub>. Re<sub>i</sub> above Re<sub>crit</sub> is considered as transition of fluid from laminar to turbulent regime. While estimating following Nu<sub>i</sub> correlation index of Pr<sub>i</sub> was selected as 0.4 and this correlation is proposed as

$$Nu_i = 0.025$$
.  $De_i^{0.9112}$ .  $Pr_i^{0.4}$  For 2000 <  $De_i$  < 12000

Salimpour [18] studied three straight helical coils with constant coil diameter. Hot oil flowing through coils was cooled by cold water flowing in the shells. For the comparison of the results, the following correlation proposed by Dravid *et al.*, [19] was considered.

$$Nu_i = (0.65. De_i^{0.05} + 0.76). Pr_i^{0.175}$$

Also, correlation was predicted for fluids having temperature dependent properties. Ghorbani *et al.*, [20,21] experimentally studied mixed convection in three straight coil heat exchangers considering a combination of two tube diameters, two coil diameters and two pitches. Shell side dimensions were kept constant for all heat exchangers. Nu<sub>i</sub> for laminar and turbulent regimes were calculated by using the following Eq. (10) and Eq. (11) respectively.

$$Nu_{i} = \left[ \left( \frac{48}{11} + \frac{51/11}{1 + \frac{1342}{Pr_{i} \cdot He^{2}}} \right)^{3} + 1.816 \cdot \left( \frac{He}{1 + \frac{1.15}{Pr_{i}}} \right)^{1.5} \right]^{0.333}$$
(10)

$$Nu_{i} = 1 + 3.6. \left(1 - \frac{d_{t}}{D_{c}}\right) \cdot \left(\frac{d_{t}}{D_{c}}\right)^{0.8} \cdot \left[0.0023. \operatorname{Re}_{i}^{0.8} \cdot \operatorname{Pr}_{i}^{0.4}\right]$$
(11)

Two correlations were predicted to estimate Nu<sub>o</sub> on the basis of D<sub>hx</sub> and D<sub>eq</sub>. Moawed [22] experimentally studied forced convection from the outside surface of straight helical coils. Constant heat flux was applied to the coil and air was forced to circulate over the outside surface of coils. Air side heat transfer coefficient was calculated as  $h_o = Q/T_{sm} - T_{\infty}$  where  $T_{sm}$  and  $T_{\infty}$  were

(9)

considered as surface temperature of coil and air stream temperature respectively. Following correlation was predicted.

Nu<sub>o</sub> = 0.0345. Re<sub>o</sub><sup>0.48</sup>. (
$$^{D}/_{d_{o}}$$
)<sup>0.914</sup>. ( $^{P}/_{d_{o}}$ )<sup>0.281</sup> for 6.6 x 10<sup>2</sup> ≤ Re<sub>o</sub> ≤ 2.3 x10<sup>3</sup>  
7.086 ≤  $^{D}/_{d_{o}}$  ≤ 16.142 and 1.81 ≤  $^{P}/_{d_{o}}$  ≤ 3.205

Sobota [23] discussed various correlations proposed by researchers to calculate Nu<sub>i</sub>. Out of these, Schmidt [24], Mori and Nakayama [25], Rogers and Mayhew [13], and Hewitt *et al.*, [26] are given below. These correlations do not involve terms which depend upon wall temperature. Schmidt [23] correlation for flow in laminar and turbulent regime

$$\begin{split} \text{Nu}_{i} &= 3.65 + 0.08. \left[1 + 0.8. \left(\frac{d_{i}}{D}\right)^{0.9}\right]. \text{Re}_{i}^{\left[0.5 + 0.2903 \left(\frac{d_{i}}{D}\right)^{0.194}\right]}. \text{Pr}_{i}^{-1/3} \\ \text{for } 100 &< \text{Re}_{i} < \text{Re}_{crit} \end{split}$$

$$Nu_{i} = 0.023. \left[ 1 + 14.8. \left( 1 + \frac{d_{i}}{D} \right) . \left( \frac{d_{i}}{D} \right)^{1/3} \right] . Re_{i}^{\left[ 0.8 - 0.22 \left( \frac{d_{i}}{D} \right)^{0.1} \right]} . Pr_{i}^{1/3}$$
for Re<sub>crit</sub> < Re<sub>i</sub> < 22000 (12)

Mori and Nakayama's [24] correlation

$$Nu_{i} = \left[\frac{1}{41.0}\right] \cdot \left[1 + 0.061 / \left(\left(\frac{d_{i}}{D}\right)^{2.5} \cdot Re_{i}\right)^{0.167}\right] \cdot \left(\frac{d_{i}}{D}\right)^{\frac{1}{12}} \cdot Pr_{i}^{0.4} \cdot Re_{i}^{5/6}$$
(13)

Rogers and Mayhew [13], and Hewitt et al., [26] correlation for turbulent flow

$$Nu_{i} = (1 + 3.5. \left(\frac{d_{i}}{D}\right)).0.023 . Re_{i}^{0.80}. Pr_{i}^{0.333}$$
(14)

Also, Schmidt [23] formula of  $Nu_{\text{o}}$  was discussed for tube and tube helical coil heat exchanger. It is given as

$$Nu_{o} = \left(1 + 3.5 \cdot \left(\frac{D_{eq}}{D}\right)\right) \cdot \left[3.66 + 1.2 \cdot \left(\frac{d_{1}}{d_{2in}}\right)^{-0.8} + 1.6 \cdot \left(Re_{o} \cdot Pr_{o} \cdot \left(\frac{D_{eq}}{H_{s}}\right)\right)^{0.33}\right]$$
(15)

where d<sub>1</sub> is the outer diameter of the tube and d<sub>2in</sub> is the inner diameter of annulus. Ke *et al.*, [27] performed numerical simulation of cone coil for variation of cone angle in the range of 55° to 85° keeping base diameter of cone as 250 mm. Three turns were fixed and accordingly the upper radius was varied in the range of 40 to 120 mm. It was predicted that, compared to cone angle helical pitch had a small effect over heat transfer. Elazm *et al.*, [28] investigated that, cone angle and geometry of conical coils had significant effect over coil exit temperature and heat transfer and predicted that, conical coils were found to be better than SHC. Flórez-Orrego *et al.*, [29] performed experimental and CFD study of single horizontal conical coils having zero space between turns. Experimentally h was calculated usingh = Q/A.  $\Delta T_{avg}$ . After calculating Nu<sub>i</sub> correlation for cone coil was proposed as

 $Nu_i = 0.00797. Re_i^{0.82}. Pr_i^{0.4}$  for  $4300 \ll Re_i \ll 18600$  and  $2 \ll Pr_i \ll 6$ 

Also, Nu<sub>i</sub> obtained from experimental and numerical simulation was compared with Nu<sub>i</sub> calculated using Seban and McLaughlin [30] and Xin and Ebadian's [31] correlations. Seban and McLaughlin's [30] correlation is presented below.

$$Nu_{i} = 0.023 . Re_{i}^{0.80} . Pr_{i}^{0.4} \left( Re_{i^{\frac{1}{20}}}^{1} \left( \frac{d_{i}}{D} \right)^{0.01} \right)$$
  
for 5000  $\ll Re_{i} \ll 10^{5}$  and  $Pr_{i} = 5$  (16)

Genić *et al.,* [32] experimentally investigated three coiled heat exchangers.  $Re_0$  was calculated on the basis of  $D_{hx}$  and the following correlation was developed for calculation of  $Nu_0$ .

$$Nu_o = 0.50. Re_o^{0.55}. Re_o^{0.55}. \left(\frac{\mu}{\mu_w}\right)^{0.14}$$

for  $10^3 \le \text{Re}_0 \le 9 \text{ x} 10^3$ , 2.6  $\le \text{Pr} \le 6$  and 9.1  $\le D_{hx} \le 18.3$ 

Pawar et al., [33] experimentally studied two straight coiled heat exchangers with variation in coil diameter and number of turns with a bath of shell side fluid. For calculation of Nui, Rogers and Mayhew [13] correlation (Eq. (9)) was used. It was predicted that Nui, hi and Uo were found to be higher for lower coil diameter. Alimoradi [34] studied SHCHE and correlation for effectiveness was predicted. Ghorbani et al., [20,21] and Purandare et al., [8] developed modified effectiveness correlations based on the ratio of R<sub>m</sub>= m<sub>c</sub>/m<sub>s</sub>. In this type of correlation Alimoradi [34] included group of non-dimensional parameters as: (D/d<sub>i</sub>), (D<sub>s</sub>/d<sub>i</sub>), (H<sub>c</sub>/d<sub>i</sub>), (H<sub>s</sub>/d<sub>i</sub>), (P/d<sub>i</sub>) and (f/d<sub>i</sub>). It was concluded that, for R<sub>m</sub> = 1, effectiveness was found to be minimum. Jamshidi and Mosaffa [35] studied finned cone coil geothermal heat exchanger according to the environmental condition of City Tehran. According to Taguchi method 8 sets of heat exchangers were considered from two variations done in each coil diameter, pitch, angle, number of fins, fin ratio and Reynolds number. Chandrasekar and Kumar [36] experimentally studied heat transfer in single horizontal straight double helical coil using MWCNT/Water nanofluids. A constant heat flux condition was considered. Daghigh and Zandi [37] performed experimentation on CC, SHC and conical- cylindrical- spiral coils in which diameter and height of coils was varied keeping di and P constant. Heat transfer coefficient was calculated from the formulation containing Q, A and LMTD. Results were validated by using correlations suggested by Salimpour [7], Dravid et al., [19], and Kalb and Seader [38]. It was predicted that, cylindrical-conical-spiral coil showed better heat transfer characteristics than other coils. Palanisamy and Kumar [39] experimentally studied single horizontal conical coil heat exchangers having cone angle of 8°. Multiwall carbon nanotube / water nanofluids were used. Q, h<sub>i</sub>, Nu<sub>i</sub> and  $\Delta P_i$  were studied. Ali *et al.*, [40] performed numerical simulation to analyse double pipe conical coils. Heat transfer was studied through the annulus side for laminar and turbulent flow regimes and correlations were suggested for prediction of Nu and friction factor. Also, it was predicted that, as cone angle at apex varied from 0° to 90° friction factor and Nu increased by 15.51% and 37.7% respectively. Al-Salem et al., [41] numerically investigated tube in tube conical coil heat exchanger considering apex of cone coil facing towards ground. It was predicted that, as cone angle at apex varied from 0°to 65° coefficient of exergy performance and exergy efficiency decreased by 11.1% and 32.56% respectively. Heyhat et al., [42] experimentally studied conical tubes under usage of Sio<sub>2</sub>/water nanofluids. Constant heat flux was provided at the outer surface. It was predicted that variation in cone apex angle was more effective than variation in coil pitch. Maghrabie *et al.*, [43] investigated the effect of inclination angle of SHC from horizontal position (0°) to vertical position (90°).  $h_i$  was calculated using formulation between Q, A and  $\Delta T$ .  $U_o$ ,  $Nu_i$  and effectiveness were studied. Considering De<sub>i</sub> and inclination angle,  $\Theta_i$  equations were predicted to estimate  $Nu_i$  and  $\Delta P_i$ . Sharma *et al.*, [44] numerically and experimentally investigated a conical shaped cavity receiver in solar power applications in which a cone coil was provided with coating of nano structured carbon florets (NCF). It was predicted that, for a cone angle of 50° and radial peach diameter ratio equal to 1, maximum efficiency was observed.

Chokphoemphun et al., [45] experimentally studied SHCHE kept in the free board zone of a rice husk fluidized bed combustor. It was proposed that maximum heat transfer was observed for wrapping of steel wire at full length of tube instead of wrapping at two and four positions. Also, Missaoui et al., [46] numerically studied helical coil immersed in heat pump water heater and investigated effect of pitch of coil and storage tank dimensions over heating process. Hasan et al., [47] numerically analyzed SHC using water based nanofluids. Variation in geometrical parameters was studied as multiple head ribbed geometry with coil revolutions. For comparison of Nui, Manlapaz-Churchill's correlations of Nu<sub>i</sub> (Eq. (10) and Eq. (11)) on the basis of constant temperature boundary conditions were used. Missaoui et al., [48] numerically and experimentally compared normal coil with coil having variation in pitch and found out that average heat transfer coefficients of coil having variation in pitch was 16.7 % higher than normal coil. Omri et al., [49] carried out experimentation to study enhancement of thermal performance of single straight helical coil using flow of nanofluids in laminar regime. Hot distilled water based CuO-GP (80 20%) hybrid nanofluid was flowing through the coil and cold water was allowed to flow in the shell. For the calculation of h<sub>i</sub>, Seider-Tate correlation was used. Abdullah and Hussein [50] presented a review of nanofluids and its effect on thermal performance of coiled heat exchangers. Experimental and numerical analysis of straight helical and conical coils using nanofluids was reviewed. Missaoui [51] analysed three types of coils as straight helical coil with constant pitch, conical coil with constant pitch, straight helical coil with variable pitch and predicted that straight helical coil with variable pitch had better thermal performance.

In general, it is observed that approaches used to calculate h<sub>i</sub> and h<sub>o</sub> are: 1) use of Nu-RePr correlations. 2) For known value of experimentation is carried out and values of Q, LMTD and U are obtained. Then  $h_i$  is obtained from Nu-RePr correlations and  $h_o$  is further calculated. 3) Experimentation is carried out to know values of Q, LMTD and U. h<sub>i</sub> and h<sub>o</sub> are obtained using Wilson Plot technique. In comparison with cone coils and spiral coils the majority of work is contributed in the area of deriving Nui correlations for straight helical coils. Also, researchers have undertaken various operating conditions like shell side fluid bath, constant temperature and constant heat flux boundary conditions, fixed shell dimensions, applications of heat exchanger [8,12,14,16,17,20-22,33,35,37,39,42,45]. Very less amount of work is found in the area of application of Wilson plot to analyze forced convection in cone and spiral coils. Also, enough work is not found on calculation of  $h_0$  for spiral and conical coil heat exchangers. In addition to this for forced convection taking place in conical, spiral and straight helical coils derivation of single mathematical correlation to obtain Nu<sub>o</sub> using Wilson plot method is not found. Few researchers developed Nui correlations but separately for SHC, CC and spiral coils. In addition, sufficient work is not found in the area of comparative analysis of straight, conical and spiral coils on the basis of variation in CR and geometry (by varying angle  $\Theta$ ).

Hence it is decided to use Wilson plot method to obtain  $h_i$  and  $h_o$  for straight helical, conical and spiral coil in which forced convection is taking place between coil and shell fluids. Additionally, while using Wilson plot method it is decided to keep mass flow rate of coil fluid constant and mass flow rates of shell fluid is varied in steps to get maximum number of  $h_o$ . To analyze this coil

diameter of straight helical coil ( $\Theta$ =90°) is considered equal to 0.07m. For spiral and cone coils smaller end diameter is considered as 0.07m and for fixed length of coils, variation in bigger end diameters is obtained for variation of angle  $\Theta$  in the steps of 70°, 50°, 30° (CCs) and 0° (spiral coil). Due to this wide range of CR is achieved which is used to propose Nu<sub>i</sub> correlations considering all coils at the same time. To have better comparisons tube side and shell side volumes are kept respectively same in all five heat exchangers. As a result of this variation in shell geometry is done which is considered in the derivation of single Nu<sub>0</sub> correlation for all heat exchangers. These correlations may be used in the designing of coiled heat exchangers. Schematic diagram of conical coil heat exchangers is shown in Figure 1.



Fig. 1. Schematic diagram of conical coil heat exchanger

# 2. Wilson Plot Method and Heat Transfer Coefficients

In this study, hot water and cold water are forced through coil and shell respectively. It is decided to vary the mass flow rates of both the fluids into four steps. Mass flow rate of coil fluid is fixed at some specific value and mass flow rate of shell fluid is varied in four steps. After plotting Wilson plot for this case single  $h_i$  and four  $h_o$  are obtained. Thus, for the entire range of coil hot water and shell cold water mass flow rates 8  $h_i$  and 32  $h_o$  are obtained for parallel and counter flow arrangements. Here 05 heat exchangers are studied on the basis of variation CR and geometry. Thus, for all heat exchangers 40 Wilson plots are obtained. Also, most of the researchers have obtained Wilson plots in the range of 15-60. Thus 40  $h_i$  and 160  $h_o$  are obtained. During experimentation inlet-exit temperatures, mass flow rates of both fluids are recorded and energy balance equation is written as:

$$Q_{ch} = (mCp)_{ch} (T_{hi} - T_{ho})$$
,  $Q_{ch} = Q_{sc} = (mCp)_{sc} (T_{co} - T_{ci})$ 

 $Q_{ch}$  and  $Q_{sc}$  are heat rejected by hot water and heat absorbed by cold water respectively. Also,  $T_{hi}$ ,  $T_{ci}$ , and  $T_{ho}$ ,  $T_{co}$  are temperatures of hot and cold water recorded at entrance and exit of coil and shell respectively. Readings were recorded when steady state is achieved and energy balance is observed for  $Q_{ch}/Q_{sc}$ =1 +0.5.  $Q_{ave}$  is calculated as [32]

$$Q_{ave} = \frac{Q_{ch} + Q_{sc}}{2}$$

Overall heat transfer coefficient U is calculated on the basis of LMTD.

 $Q_{ave} = U_{i/o} . A_{i/o} . LMTD$ 

LMTD is given as: LMTD =  $\frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}}$ 

For parallel flow,  $\Delta T_1 = T_{hi} - T_{ci}$  and  $\Delta T_2 = T_{ho} - T_{co}$ 

For counter flow,  $\Delta T_1 = T_{hi} - T_{co}$  and  $\Delta T_2 = T_{ho} - T_{ci}$ 

U is the overall heat transfer coefficient and is written as per Eq. (2). Wilson plot is obtained by plotting 1/UA vs.  $1/v_s^n$ . Where  $v_s$  is shell water velocity. Proper values of exponent, n are selected to get linear fit between data points in the plot. From this linear fit slope and y axis intercept are noted and  $h_i$ ,  $h_o$  are obtained as per the Eq. (3) to Eq. (7). Curvature ratio is obtained considering  $D_{ave}$ . For SHC i.e.  $\Theta = 90^\circ$ ,  $D_{ave} = D_{cb} = D_{ct}$ . For conical coils;  $D_{ave} = D_{cb} + D_{ct}/2$ . And for  $\Theta = 0^\circ$  HE,  $D_{ave}$  is obtained as average of inner and outer coil diameters of horizontal spiral coil. Formulations to calculate non-dimensional parameters are given below.

$$CR_{ave} = \frac{d_i}{D_{ave}}$$

$$Re_i = \frac{\rho_i \cdot v_i \cdot d_i}{\mu_i}; De_i = Re_i \cdot CR_{ave}^{0.5}; Re_o = \frac{\rho_o \cdot v_s \cdot D_{eq}}{\mu_o}$$
(17)

For the calculation of shell side fluid velocity and  $Re_s$ , required  $D_{eq}$  is obtained using following formulation [11,32]

$$D_{eq} = ((D_{so}^2 - D_{si}^2).H_s - d_o^2.L_c)/(d_o.L_c)$$

Tube side and shell side Nusselt numbers are given as

$$Nu_i = \frac{h_i \,.\, d_i}{k_i} \, \text{and} \, Nu_o = \frac{h_o . \, D_{eq}}{k_o}$$

Schimidt's formula for Recrit is preferred [52].

$$Re_{crit} = 2300. (1 + 8.6. (CR^{0.45}))$$

Tube side flow having  $Re_i$  less than  $R_{crit}$  is considered as laminar and  $Re_i$  greater than  $R_{crit}$  is considered as flow in turbulent regime.

#### 3. Experimentation

Test rig mainly consists of a hot and cold water tank, heat exchanger unit, temperature measurement systems, data collection and storage unit. Hot and cold water tanks are made up of steel and are provided with solenoid, float valves to maintain constant head of water during test run. Hot water is heated by the heater and temperature of hot water is maintained at set value by and automatic on-off of heater is done by thermostat. This temperature is displayed at section 11,

shown in schematic diagram of the test rig, Figure 2. Insulations are provided to the hot water tank and heat exchanger units. Heat exchanger unit consists of a copper coil fitted in a steel shell. Piping arrangements are made on each heat exchanger unit to measure inlet-exit temperatures and pressure drops. RTD thermocouples are used for temperature measurement and are displayed at section I2. Cold water is forced into the shell from the top side only. Also, hot water is forced into the coil and its entry is done from top side or bottom side to have parallel flow and counter flow configurations. Steady state heat transfer between coil hot water and shell cold water is observed. After achieving steady state readings are recorded. Heat exchanger unit is replaced and again readings are taken for variation in mass flow rates of both fluids.



Fig. 2. Schematic diagram of Heat exchanger experimental set up

Copper coils of different shapes are made up by winding the copper tube on wooden blocks. Wooden blocks are converted into Cylindrical and conical shape as per the dimensions to produce straight helical, conical and spiral coils. Sand is filled into the copper tube to ensure a circular cross section. Other heat exchangers are manufactured by varying angle  $\Theta = 70^{\circ}$ ,  $50^{\circ}$ ,  $30^{\circ}$ , and  $0^{\circ}$  keeping smaller diameter  $D_{ct} = 0.07$  m and length of coil fixed. As a result of this, the diameter of another end ( $D_{cb}$ ) is increased. Additionally, the volume of the shell side is kept fixed and accordingly shell dimensions are obtained. For  $\Theta = 90^{\circ}$  SHCHE,  $D_{ave}$  becomes equal to  $D_{ct} = 0.07$  m and it is close to  $D_{si} = 0.02$  m. For  $\Theta = 70^{\circ}$ ,  $50^{\circ}$ ,  $30^{\circ}$ , and  $0^{\circ}$ ,  $D_{cb}$  is increasing causing  $D_{ave}$  remains at the centre of shell. As a result of this, variation of coil curvature ratio takes place in the range of 0.14 to 0.048. Geometrical dimensions of the heat exchangers are shown in Table 1.

Parameter	Dimension	Parameter	Dimension			
Cone angle, O	90°, 70°, 50°, 0°, 30°	Average diameter, Dave	0.07, 0.12, 0.16, 0.17, 0.21			
Tube diameter, d <sub>i</sub> , m	0.01	Curvature ratio, CRave	0.14, 0.083, 0.063, 0.059, 0.048			
Top diameter, D <sub>ct</sub> , m	0.07	Tube length, L <sub>c</sub> , m	3.3			
Pitch, P, m	0.018	Shell inner diameter, D <sub>si</sub> , m	0.02			
Base diameter, D <sub>cb</sub>	0.07, 0.17, 0.25,	Shell outer diameter, D <sub>so</sub> , m	0.24, 0.3, 0.33, 0.42, 0.38			
	0.27,0.35					

# Table 1 Dimension of Heat Exchangers

Hot water flow into coils is considered in laminar and turbulent regimes. Reynolds numbers for coil fluid is in the range of 3700 to 21000. Accordingly fluid flow parameters are obtained and mentioned in Table 2. Method proposed by Kline and McClintock [53] is used to calculate uncertainties. Uncertainties involved in measurements and calculations of parameters are reported in Table 3. Experimental uncertainty is less than 4.5 % for all runs [8,10,20,21].

Table 2				
Flow parameters used in heat exchangers				
Parameter	Range			
Mass flow rates (Coil- Hot water), mch, kg/s	0.02 - 0.1			
Mass flow rates (Shell- Cold water), m <sub>sc</sub> , kg/s	0.02-0.1			
Inlet temperature (Coil- Hot water), Tch, °C	42 ±0.5			
Inlet temperature (Shell- Cold water), Tsc, °C	27.5 ±0.5			
Tube side Reynolds number, Rei	3700-21000			
Tube side Dean number, Dei	700-8000			

#### Table 3

Uncertainty analysis			
Parameter	Uncertainty (%)	Parameter	Uncertainty (%)
Mass flow rates	4.1	Heat transfer coefficients	4.3
Reynolds number	4.5	Nusselt number	4.3
Rate of heat transfer	4.3	Dean number	4.5

#### 4. Results and Discussion

Mass flow rates of tube side hot water and shell side cold water are varied from 0.02 kg/s to 0.1 kg/s. All heat exchangers are analysed by examining cold water temperature difference ( $\Delta T_{sc}$ ), tube side, shell side heat transfer coefficients ( $h_i$  and  $h_o$ ) and tube side, shell side heat Nusselt No. (Nu<sub>i</sub> and Nu<sub>o</sub>). After proposing mathematical correlations of Nu<sub>i</sub> and Nu<sub>o</sub> comparisons are done with results of existing researchers.

# 4.1 Coldwater Temperature Difference (ΔT<sub>sc</sub>, °C)

In all five heat exchangers, tube side, shell side volumes are fixed and shell side volume is more than tube side volume. Also coil geometry is varying as straight helical coil, conical coil and spiral coils. Hence it is necessary to see how much rise is taking place in temperature of cold water, flowing through the shell. Variation in shell side cold water temperature difference  $\Delta T_{sc}$  is plotted against mass flow rate of shell cold water,  $m_{sc}$  in Figure 3(a) to Figure 3(d) for laminar and turbulent flow regimes of coil hot water. Highest  $\Delta T_{sc}$  are obtained for  $m_{sc} = 0.02$  kg/s and decreases as mass flow rate increases to 0.1 kg/s. Also, mass flow rates of coil hot water,  $m_{ch}$  decreased from turbulent to laminar,  $\Delta T_{sc}$  is found to be decreased. Highest  $\Delta T_{sc}$  is obtained for  $m_{ch} = 0.1$  kg/s and  $m_{sc} = 0.02$  kg/s. It is observed that, as  $m_{sc}$  increases from 0.02 to 0.1 kg/s, exit temperature of cold water is decreased however rate of heat transfer increased. The rise in heat transfer is governed by the rise in heat capacity of cold water,  $(mCp)_{sc}$ . Highest  $\Delta T_{sc}$  is found for  $\Theta = 30^{\circ}$  CCHE. For  $\Theta = 0^{\circ}$  HE,  $\Delta T_{sc}$  is found close to  $\Theta = 30^{\circ}$  CCHE followed by  $\Theta = 50^{\circ}$  and 70° CCHEs. Lowest  $\Delta T_{sc}$  is obtained for  $\Theta = 90^{\circ}$  SHCHE. For  $\Theta = 90^{\circ}$  SHCHE, coil diameter is very close to inner shell whereas for  $\Theta = 30^{\circ}$  CCHE, coil top and bottom diameters are very close to inner and outer shell respectively. Due to this, the average diameter of conical coils lies at the centre of shell resulting in better contact of shell fluid with coil surface.



Fig. 3. Variation in temperature of shell cold water,  $\Delta T_{sc}$  against shell cold water mass flow rate,  $m_{sc}$ 

# 4.2 Rate of Heat Transfer, Qave (W)

 $Q_{ave}$  is average of heat rejected by hot water and heat absorbed by cold water.  $Q_{ave}$  is plotted against shell cold water mass flow rate,  $m_{sc}$ . Figure 4 shows that as  $m_{sc}$  increases in the range of 0.02 to 0.1 kg/s,  $Q_{ave}$  increases. Maximum  $Q_{ave}$  is obtained for coil hot water mass flow in turbulent regime. Maximum  $Q_{ave}$  is obtained for conical and spiral coils compared to straight helical coils. Among conical and spiral coils, maximum  $Q_{ave}$  is obtained for  $\Theta = 30^{\circ}$  CCHE. In case of  $\Theta = 30^{\circ}$  CCHE, as coil diameter decreases from base towards apex maximum variation of curvature is taking place from 0.048 to 0.14. In addition to this conical geometry and maximum slant edge length of 0.29 m is making maximum contact of shell water with coil surface. This aids in better heat transfer performance. This does not happen with other coils. Further in this study Wilson plot method is used to obtain values of  $h_i$  and  $h_o$ . These values are entered in the formulation of overall heat transfer coefficient, U and using the same LMTD, again rate of heat transfer is calculated. This rate of heat transfer is labelled as  $Q_{wp}$ . Then (( $Q_{ave} - Q_{wp}/Q_{ave}$ ).100) is obtained to check the relevance of values of  $h_i$  and  $h_o$  obtained from Wilson plots. It is found that for selected values of  $h_i$  and  $h_o$ , (( $Q_{ave}$  -  $Q_{wp}/Q_{ave}$ ).100) is found to be within 7%. These values of  $h_i$  and  $h_o$  are further used to obtain  $Nu_i$  and  $Nu_o$ .



Fig. 4. Plot of rate of heat transfer, Qave against shell cold water mass flow rate, msc

# 4.3 Shell Side Heat Transfer Coefficients, h<sub>o</sub> (W/m<sup>2</sup>°C)

Shell side heat transfer coefficients, ho are obtained using Wilson plot method discussed in section 2. Values of  $h_0$  are obtained according to formulae given in Eq. (2) to Eq. (8). In this study  $\Theta$ is varied from 90° to 0°. As a result of this contact of shell fluid with coil surface is varied. Hence the shell side heat transfer coefficient ho is compared for all heat exchangers. Effect of shell cold water mass flow rates,  $m_{sc}$  over  $h_0$  is illustrated in Figure 5. As  $m_{sc}$  increases,  $h_0$  is found to increase. Maximum  $h_0$  is obtained at  $m_{sc} = 0.1$  kg/s for all heat exchangers. Highest  $h_0$  is obtained for  $\Theta=30^\circ$ CCHE and for  $\Theta$  = 90° SHCHE, lowest h<sub>o</sub> is obtained. For  $\Theta$  = 90° SHCHE, coil diameter is 0.07 m and it is close to the inner shell diameter. This is causing less contact of shell water with the coil surface. But it has the highest  $h_i$  due to highest curvature ratio. On the contrary, for  $\Theta$  = 30° CCHE, coil diameter decreases from D<sub>cb</sub> = 0.35 m to D<sub>ct</sub> = 0.07 m. As a result, maximum portion of the shell is occupied by the coil having a maximum slant edge length of 0.29 m with average diameter (D<sub>ave</sub>) at the centre of the shell. This causes effective contact of shell fluid with coil. For  $\Theta = 0^{\circ}$ , 50° and 70° HEs  $D_{ave}$  is near to centre showing better  $h_0$  than  $\Theta = 90^\circ$  SHCHE. In addition to this  $h_0$  is plotted against m<sub>ch</sub> and shown in Figure 6. Figure 6(c) and Figure 6(d) indicates that for lower Reynolds no. of shell side water (m<sub>sc</sub> = 0.02 and 0.05 kg/s), when m<sub>ch</sub> rises from 0.02 to 0.05 kg/s (laminar to transition regime) ho is found to decrease for conical spiral and straight helical coils and in the turbulent regimes of hot water ( $m_{ch}$  = 0.7 and 0.1 kg/s),  $h_0$  is further increased. Also, for shell side water with higher Reynolds no. (m<sub>sc</sub> = 0.07 and 0.1 kg/s), increment in m<sub>ch</sub> from 0.02 to 0.05 kg/s (laminar to transition regime)  $h_0$  is found to decrease for  $\Theta$ =30° and 0° HEs. And when  $m_{ch}$  increases beyond 0.05 kg/s (turbulent regime) h<sub>o</sub> is increased. Specifically, for Θ=30°CCHE sagging in the curve is found at  $m_{ch} = 0.05$ kg/s and it is predominant when  $m_{sc} = 0.02$  kg/s. But for remaining heat exchangers, at higher  $m_{sc}$ ,  $h_0$  increases directly as  $m_{ch}$  increases from 0.02 to 0.1 kg/s.



**Fig. 5.** Plots of  $h_o$  vs.  $m_{sc}$  for (a)  $m_{ch}$  = 0.1 kg/s, (b)  $m_{ch}$  = 0.07, kg/s, (c)  $m_{ch}$  = 0.05 kg/s, (d)  $m_{ch}$  = 0.02 kg/s



**Fig. 6.** Plots shell side heat transfer coefficients,  $h_o$  with respect to variation of coil side mass flow rate,  $m_{ch}$  for (a)  $m_{sc} = 0.1$  kg/s, (b)  $m_{sc} = 0.07$  kg/s, (c)  $m_{sc} = 0.05$  kg/s, (d)  $m_{sc} = 0.02$  kg/s

Looking at the formulation of U,  $h_i$  and  $h_o$ , if  $h_i$  is very much higher than  $h_o$  then small changes in  $h_o$  makes maximum effect on U compared to  $h_i$ . In CCHE  $h_i$  is very high compared to  $h_o$  and due to conical geometry increment in  $h_o$  is more compare to SHCHE. Further using  $D_{eq}$  as the characteristic dimension in the formulation shell side Reynolds number,  $Re_o$  and shell side Nusselt number,  $Nu_o$ 

are obtained. Plot of Nu<sub>o</sub> against Re<sub>o</sub> is shown in Figure 7. As expected Nu<sub>o</sub> is found to increase, when Re<sub>o</sub> increases. In case of coiled heat exchangers for selected range of parameters mathematical correlations to obtain Nu<sub>o</sub> are not found. Hence correlation to obtain Nu<sub>o</sub> is proposed here. In this study shell dimensions are varied to accommodate conical coils, spiral coil and straight helical coil which results in variation in contact of shell fluid with coil surface. To consider its effect, ratio of height of heat exchanger, H<sub>s</sub> to shell diameter D<sub>so</sub> is considered as a non-dimensional parameter in the derivation of Nu<sub>o</sub> correlation. To develop the correlation procedure discussed by Kern [54] is followed considering Re<sub>o</sub>, Pr<sub>o</sub> and H<sub>s</sub>/D<sub>so</sub> as non-dimensional parameters. Linear plot of the same is shown in Figure 7(b) in which R<sup>2</sup>value is equal to 0.93. This correlation is given as Eq. (18).

$$Nu_{o} = 16.55 \cdot Re_{o}^{0.55} \cdot Pr_{0}^{0.4} \cdot \left(\frac{H_{s}}{D_{so}}\right)^{0.096} \text{ for } 50 \ll Re_{0} \ll 400, \ 5 \ll Pr_{o} \ll 6$$
(18)

Comparison between  $Nu_0$  obtained from experimentation and  $Nu_0$  predicted using Eq. (18) is shown in Figure 7(c). Maximum 14 % variation is found.



Fig. 7. (a) Plot of  $Nu_o$  vs  $Re_o$ , (b) Plot for  $Nu_o$  equation (Eq. (18)), (c) Comparison of  $Nu_o$ 

# 4.4 Tube Side Heat Transfer Coefficients, h<sub>i</sub> (W/m<sup>2</sup>°C)

To study the effect of variation of curvature ratio, CR in SHC, conical and spiral coils tube side heat transfer coefficients,  $h_i$  are discussed. In SHC, curvature ratio, CR remains constant along the axis whereas CR varies in CC and spiral coils. To understand the effect of variation of curvature ratio,  $h_i$  is discussed here.  $h_i$  is obtained using Wilson plot method (Eq. (2) to Eq. (6)). Figure 8 shows variation of  $h_i$  with hot water mass flow rate,  $m_{ch}$ . As expected, increase in  $m_{ch}$  causes increase in  $h_i$ . Highest  $h_i$  is found for  $\Theta$ =90°SHCHE which is having maximum curvature ratio. Also, lowest  $h_i$  is obtained for  $\Theta = 30^{\circ}$ CCHE which is having lowest CR<sub>ave</sub> as 0.048. Thus, curvature ratio increases, h<sub>i</sub> increases. This is in agreement with the results of researchers. In addition to this tube side Nusselt numbers (Nu<sub>i</sub>) are calculated. Plots of Nu<sub>i</sub> with respect to tube side Reynolds number, Re<sub>i</sub> and tube side Dean number, De<sub>i</sub> are shown in Figure 9. It is found that, as Re<sub>i</sub> and De<sub>i</sub> increases, Nu<sub>i</sub> also increases. To check the relevance of Nu<sub>i</sub>, it is decided to compare Nu<sub>i</sub> obtained for every heat exchanger with Nu<sub>i</sub> predicted from the correlations suggested by Schmidt [23] (Eq. (12)), Roger and Mayhew [13] (Eq. (9)), Mori and Nakayama [24] (Eq. (13)), Roger and Mayhew [13] and Hewitt *et al.*, [26] (Eq. (14)) and Seban and McLaughlin [30] (Eq. (16)).



Fig. 8. Variation of h<sub>i</sub> against tube side mass flow rate, m<sub>ch</sub>



**Fig. 9.** (a) Plot of Nu<sub>i</sub> vs. Re<sub>i</sub>, (b) Plot of Nu<sub>i</sub> vs. De<sub>i</sub>, (c) and (d) Plots for Nu<sub>i</sub> correlations (Eq. (19) and Eq. (20))

Nu<sub>i</sub> obtained in this study for all heat exchangers are showing agreement with above mentioned researchers. % variation in Nu<sub>i</sub> obtained in this study is 0-5%, 0-17%, 1-18 %, 1- 18 % respectively when compared with Schmidt [23] (Eq. (12)), Mori and Nakayama [24] (Eq. (13)), Rogers and

Mayhew [13] (Eq. (9)) and Seban and McLaughlin [30] (Eq. (16)) respectively. But in comparison with Rogers and Mayhew [13] and Hewitt *et al.*, [26] (Eq. (14)) % variation in Nu<sub>i</sub> is found to be up to 23%.

Further these calculated Nu<sub>i</sub> values are used to suggest two correlations on the basis of Re<sub>i</sub> and De<sub>i</sub>. Effect of variation of curvature ratio in conical coils, spiral coil and straight helical coil is considered simultaneously selecting curvature ratio on the basis of average coil diameter. Also, two separate linear plots are obtained on the basis of Re<sub>i</sub> and De<sub>i</sub>. These plots are shown in Figure 9(c) and Figure 9(d). It shows values of R<sup>2</sup>as 0.989 and 0.986 respectively. First correlation is expressed in Eq. (19) in which Re<sub>i</sub>, Pr<sub>i</sub> and curvature ratio are used as non-dimensional parameters.

 $\begin{aligned} \text{Nu}_{i} &= 0.0512 \text{ . } \text{Re}_{i}^{\ 0.80} \text{ . } \text{Pr}_{i}^{\ 0.4} \text{ . } \text{CR}_{\text{ave}}^{\ 0.18} \\ \text{for } 3700 \ll \text{Re}_{i} \ll 21000, 4 \ll \text{Pr}_{i} \ll 5 \text{ and } 0.04 \ll \text{CR} \ll 0.15 \end{aligned} \tag{19}$ 

Second correlation is proposed using Dei, Pri and CR as:

 $Nu_{i} = 0.0365 . De_{i}^{0.815} . Pr_{i}^{0.4} . CR_{ave}^{-0.3}$ for 700  $\ll$  De<sub>i</sub>  $\ll$  8000, 4  $\ll$  Pr<sub>i</sub>  $\ll$  5 and 0.04  $\ll$  CR  $\ll$  0.15 (20)

In addition to this, Nu<sub>i</sub> obtained from correlations suggested in this study (Eq. (19) and Eq. (20)) are further compared with Nu<sub>i</sub> predicted by Rogers and Mayhew [13] (Eq. (9)), Schmidt [23] (Eq. (12)), Mori and Nakayama [24] (Eq. (13)), Rogers and Mayhew [13] and Hewitt *et al.*, [26] (Eq. (14)) and Seban and McLaughlin [30] (Eq. (16)). Comparative plots for each heat exchanger are shown in Figure 10. Thus, comparison shows that maximum variations in Nu<sub>i</sub> obtained from the Eq. (19) and Eq. (20) are 17 % and 13 % respectively. In a few cases this maximum variation in Nu<sub>i</sub> reaches up to 20% when compared with Rogers and Mayhew [13] and Hewitt *et al.*, [26] (Eq. (14)). Thus, while designing coiled heat exchangers cone coils are selected preferably with an angle around 30°. For the minimum possible coil diameter at the smaller end of cone coil, a bigger diameter of coil is obtained according to space requirements. Using proposed correlations Nusselt numbers, heat transfer coefficients and overall heat transfer coefficients are obtained for coiled heat exchangers. This is used to decide area required to achieve the desired outlet temperature of either of the fluids.





Fig. 10. Comparative plots of Nu<sub>i</sub> vs. Re<sub>i</sub> for all heat exchangers

#### 5. Conclusion

In comparison with straight helical coils in conical and spiral coils, variation in coil diameter causes variation in curvature ratio and effective contact of shell fluid with coil surface. This is contributing to heat transfer. For designing these heat exchangers values of shell and tube side heat transfer coefficients ( $h_o$  and  $h_i$ ) is necessary and can be obtained from shell and tube side Nusselt number (Nu<sub>o</sub> and Nu<sub>i</sub>) correlations. For cone and spiral coils enough information about this is not available. To analyze this heat exchangers consisting of three cone coil heat exchangers (angle  $\Theta$  = 70°, 50° and 30° CCHE), spiral coil ( $\Theta$  = 0° HE) and straight helical coil ( $\Theta$  = 90° SHCHE) are studied. For tube side flow, Reynolds number, Re<sub>i</sub> and Dean number, De<sub>i</sub> varies from 3700-21000 and 700-8000 respectively.  $h_i$  and  $h_o$  are obtained using Wilson plot method.

- i. Shell side heat transfer coefficient,  $h_o$  is found to increase as shell side mass flow rate is increased. Maximum  $h_o$  is obtained for  $\Theta = 30^\circ$  CCHE. Also, small variation in  $h_o$  causes significant effect over rate of heat transfer. It is achieved with conical coils due to conical geometry. Also, for flow of shell side water with higher Reynolds no. ( $m_{sc} = 0.07$  and 0.1 kg/s), increment in the tube side hot water mass flow rate ( $m_{ch}$ ) from 0.02 to 0.05 kg/s (laminar to transition regime)  $h_o$  is found to decrease for  $\Theta = 30^\circ$  and  $0^\circ$  HEs. And when  $m_{ch}$  increases beyond 0.05 kg/s (turbulent regime)  $h_o$  is increased. Considering new non-dimensional parameter as ratio of height of shell,  $H_s$  and diameter of shell,  $D_{so}$ , correlation to calculate  $Nu_o$  is proposed as:  $Nu_o = 16.55$ .  $Re_o^{0.55}$ . $Pr_o^{0.4}$ . $(H_s/D_{so})^{0.096}$ . A comparison of these values with experimental values shows 0-14 % variation.
- ii.  $h_i$  is found to increase when  $Re_i$  and  $De_i$  are increased. Maximum  $h_i$  is obtained for  $\Theta = 90^\circ$  SHCCHE as it has maximum curvature ratio in the selected range. Further  $Nu_i$  are obtained. These are compared with  $Nu_i$  predicted by other researchers and agreement is

found. % variation in Nu<sub>i</sub> obtained in this study is 0-5 %, 0-17 %, 1-18 %, 1-18 % respectively when compared with Nu<sub>i</sub> predicted by Schmidt [23], Mori and Nakayama [24], Rogers and Mayhew [13] and Seban and McLaughlin [30] respectively. But in comparison with Nu<sub>i</sub> proposed by Rogers and Mayhew [13] % variation in Nu<sub>i</sub> is found to be up to 23%.

- iii. Also, the first correlation is predicted as: Nui = 0.0512. Rei<sup>0.80</sup>.Pri<sup>0.4</sup>.CRave<sup>0.18</sup>. Comparison between Nui obtained from Wilson plot, Nui obtained using this correlation and Nui predicted from correlations suggested by above mentioned researcher's shows maximum variation up to 17 %.
- iv. In addition, the second correlation is predicted as:  $Nu_i = 0.0365$ .  $De_i^{0.815}$ .  $Pr_i^{0.4}$ .  $CR_{ave}^{-0.3}$ . Similarly, comparison of  $Nu_i$  shows maximum variation up to 13 %.
- v. Also, after selecting values of  $h_i$  and  $h_o$  from Wilson plots, to check the correctness, rate of heat transfer ( $Q_{wp}$ ) is obtained using the same values of log mean temperature difference. Variation between  $Q_{ave}$  and  $Q_{wp}$  is found in the range of 0 to 7 %.

Thus, preferably conical coiled heat exchangers can be used. Nusselt numbers are obtained by using proposed correlations in this study and surface area is decided to achieve the desired outlet temperature of one of the fluids.

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### **Conflict of Interest**

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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