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# Numerical Treatment on a Chaos Model of Fluid Flow Using New Iterative Method

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**ABSTRACT**

This article treats analytically and numerically to the three dimensional Rössler system. The governing equations of the problem are derived from traditional Lorentz system of fluid mechanics to nonlinear ordinary differential equations (ODEs) for modelling. A semi-analytic solution is developed by using New Iterative Method (NIM) whereas the numerical solution is presented by Runge-Kutta order four (RK4) scheme. A comparative study of the analytical and numerical solutions are made. The results confirm clearly that the two methods coincide closely for both the chaotic and non-chaotic cases of the determined system. This observation would be helpful to apply NIM on nonlinear problems of fluid flow with different fluid parameters in future.

## 1. Introduction

Many investigations rely heavily on experimental techniques; however, the development of computer hardware and software for analysing fluid systems over the last few decades has advanced our understanding of fluid flow. Fluid behaviour in complex geometries can be investigated through the use of computational fluid dynamics (CFD) techniques. As a result, chaotic dynamics has sparked a lot of interest in mixing equipment and other process equipment both experimentally and numerically [1]. In chaos theory and fluid dynamics, chaotic mixing is a process by which flow tracers develop into complex fractals under the action of a fluid flow. Such flows have long been studied analytically, because of various mathematical simplifications that are possible and because of greater tractability in computational fluid dynamics. Fluid systems are nonlinear, they are also spatially distributed, and the underlying dynamical equations are infinite-dimensional. Chaos theory, on the other hand, concerns finite dimensional nonlinear systems. In fact, much of the modern interest in chaos theory was precipitated by Edward Lorenz's modelling of Rayleigh-Bénard convection, and several early experiments testing the applicability of chaos theory involved fluid systems [2]. The fluid flow is characterized by an exponential growth of fluid filaments. Chaos arises from deterministic

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systems. The nonlinearity of the system comes mainly from the convective motion of the fluid flow. The fluid motion can develop unpredictable, yet observable behaviour with the proper choice of the initial conditions [3]. Chaos may be incompletely but accurately described fluid flow problems as the ability of simple nonlinear systems to produce complicated behaviour. The chaotic behaviour and sensitive dependence upon initial conditions of solutions of differential equations provides a mechanism for understanding turbulence. Turbulence in fluids is almost always chaotic. It's also dissipative, random, and multiscale in time and space [4].

Continuous chaos has, under the name of deterministic nonperiodic flow, been first described by E.N. Lorenz in a model of turbulence. The Rössler system which directly generates a similar flow and forms only a single spiral, it is considered as a model of a nonlinear Lorenz model of fluid flow [5]. The ordinary differential equations of the determined system are an approximation to a system of partial differential equations describing finite amplitude convection in a fluid layer heated from below [6]. The Lorenz equation consists of three coupled ordinary differential equations (ODEs) which contain two nonlinear terms, whereas the Rössler system consists of only a single nonlinear term. The objective of this study is not a qualitative analysis of the rich dynamic behaviour of the noted fluid flow problem, but to present an accurate solution to the Rössler system using the New Iterative Method (NIM). Recently, Goh *et al.,* applied modified version of Variational Iteration Method (VIM) to the Rössler system [7]. The approximate analytical solution of Rössler system has been presented by Chowdhury *et al.,* [8] using multistage Homotopy Perturbation Method (MHPM).

In recent years, the semi-analytic solution procedure New Iterative Method (NIM) has been widely applied for conducting numerical study in various nonlinear problems. Daftardar-Gejji and Jafari [9] envisioned the New Iterative Method in order to solve fractional, ordinary and partial differential equations. Later, quite a few studies based on the New Iterative Method (NIM) was conducted when solving Nonlinear Integro-Differential Equations [10], Fokker-Planck Equation [11], Klein-Gordon equations [12], nonlinear functional equations [13], Falkner-Skan Equation [14], Jeffery-Hamel flow problem [15], Chemical kinetics equations [16, 17], Lake pollution model [18] and many other problems. This method is also referred as the Daftardar-Jafari method (DJM) or Iterative method (IM). To the best of our knowledge the NIM is applied first time to the Rössler system. NIM is very straightforward, and computer friendly as imaginary number is not involved in the calculation. At the same time, it provides fast convergent explicit series solutions without any transformation, linearization, and discretization.

### *1.1 Mathematical Model of Rössler System*

Solving nonlinear dynamical systems that has chaotic behaviour is considered a crucial issue. This is because the chaotic systems can be so complicated that cannot be solved analytically, so these systems should be treated using distinctive procedures. In this paper we have implemented NIM to solve the differential equations of chaotic Rössler system. The differential equations of the determined system are in the following form [8]

$$\begin{cases} \frac{d}{dt}X(t) = -Y(t) - Z(t), \\ \frac{d}{dt}Y(t) = X(t) + aY(t), \\ \frac{d}{dt}Z(t) = b + X(t)Z(t) - cZ(t), \end{cases} \tag{1}$$

where $X,Y,Z$ are the time dependent variables and are positive parameters. Rössler used these equations to explain the kinetics of chemical reactions in a stirred tank [19]. We notice that this nonlinear autonomous system has only a single nonlinear term $X(t)Z(t)$. We have studied the non-chaotic and chaotic cases of the system through different parameters of $c$. The first two parameters, $a$ and $b$ have been fixed at $0.2$. The initial conditions used are $X(0)=2, Y(0)=3, Z(0)=2$, [7].

## 2. Methodology

Let us, consider a class of first-order nonlinear ordinary differential equations in general

$$\begin{cases} L\mu_1 + \varsigma_1(t,\mu_1,\mu_2,\ldots,\mu_m) = g_1(t), \\ L\mu_2 + \varsigma_2(t,\mu_1,\mu_2,\ldots,\mu_m) = g_2(t), \\ \vdots \\ L\mu_m + \varsigma_m(t,\mu_1,\mu_2,\ldots,\mu_m) = g_m(t). \end{cases} \tag{2}$$

where, $L \equiv \frac{d}{dt}$ is a linear operator and the initial conditions are as follows

$$\mu_1(t_0) = a_1, \mu_2(t_0) = a_2, \ldots, \mu_m(t_0) = a_m. \tag{3}$$

The inverse linear operator $L^{-1}(.) \equiv \int_{t_0}^{t} (.)dt$ in the system with respect to the initial conditions.

$$\mu_1 + N_1(\mu_1,\mu_2,\ldots,\mu_m) = f_1, \tag{4}$$

$$\mu_2 + N_2(\mu_1,\mu_2,\ldots,\mu_m) = f_2, \tag{5}$$

$$\vdots$$

$$\mu_m + N_m(\mu_1,\mu_2,\ldots,\mu_m) = f_m. \tag{6}$$

where $N_1, N_2, \ldots, N_m$ are nonlinear operators and $f_1, f_2, \ldots, f_m$ are functions of $t$.

Therefore, according to NIM the approximate solutions of (4)-(6) can be expressed as

$$\mu_1 = \mu_{1,0} + \mu_{1,1} + \cdots = \sum_{i=0}^{\infty} \mu_{1,i}, \tag{7}$$

$$\mu_2 = \mu_{2,0} + \mu_{2,1} + \cdots = \sum_{i=0}^{\infty} \mu_{2,i}, \tag{8}$$

$$\vdots$$

$$\mu_m = \mu_{m,0} + \mu_{m,1} + \cdots = \sum_{i=0}^{\infty} \mu_{m,i}, \tag{9}$$

and the components $\mu_{ij}, (i = 1,2,\ldots,m\,;\, j = 0,2,\ldots,n)$ can be calculated by following way

$$\begin{cases} \mu_{1,0} = f_1, \\ \mu_{1,1} = N_1(\mu_{1,0}), \\ \mu_{1,2} = N_1(\mu_{1,0} + \mu_{1,1}) - N_1(\mu_{1,0}), \\ \quad \vdots \\ \mu_{1,n} = N_1(\sum_{i=0}^{n} \mu_{1,i}) - N_1(\sum_{i=0}^{n-1} \mu_{1,i}), n = 1,2,.. \end{cases} \tag{10}$$

$$\begin{cases} \mu_{2,0} = f_2, \\ \mu_{2,1} = N_2(\mu_{2,0}), \\ \mu_{2,2} = N_2(\mu_{2,0} + \mu_{2,1}) - N_2(\mu_{2,0}) \\ \quad \vdots \\ \mu_{2,n} = N_2(\sum_{i=0}^{n} \mu_{2,i}) - N_2(\sum_{i=0}^{n-1} \mu_{2,i}), n = 1,2,.. \end{cases} \tag{11}$$

In the similar manner the remaining components can be calculated

$$\begin{cases} \mu_{m,0} = f_m, \\ \mu_{m,1} = N_m(\mu_{m,0}), \\ \mu_{m,2} = N_m(\mu_{m,0} + \mu_{m,1}) - N_m(\mu_{m,0}), \\ \quad \vdots \\ \mu_{m,n} = N_m(\sum_{i=0}^{n} \mu_{m,i}) - N_m(\sum_{i=0}^{n-1} \mu_{m,i}), n = 1,2,.. \end{cases} \tag{12}$$

Then, in view of (10)-(12), the n-terms approximate solutions are

$$\mu_1 = f_1 + \sum_{i=1}^{n} \mu_{1,i}, \tag{13}$$

$$\mu_2 = f_2 + \sum_{i=1}^{n} \mu_{2,i}, \tag{14}$$

$$\vdots$$

$$\mu_m = f_m + \sum_{i=1}^{n} \mu_{m,i}. \tag{15}$$

## 3. Solution by NIM

We first integrate the system of equations (1) from $0$ to $t$ on the Rössler system, employing the initial conditions $X(0) = 2, Y(0) = 3, Z(0) = 2$; we obtain

$$\begin{cases} X(t) = 2 - \int_0^t Y(t)\,dt - \int_0^t Z(t)\,dt, \\ Y(t) = 3 + \int_0^t X(t)\,dt + \int_0^t aY(t)\,dt, \\ Z(t) = 2 + bt + \int_0^t X(t)Z(t)\,dt - \int_0^t cZ(t)\,dt. \end{cases} \tag{16}$$

Let us, take the initial approximations be

$$\begin{cases} X_0(t) = 2, \\ Y_0(t) = 3, \\ Z_0(t) = bt + 2. \end{cases} \tag{17}$$

According to the solution procedure of NIM the successive approximate are

$$\begin{cases} X_1(t) = N_1(X_0) = -5t - \frac{1}{2}bt^2, \\ Y_1(t) = N_2(Y_0) = 3at + 2t, \\ Z_1(t) = N_3(Z_0) = bt^2 + 4t - \frac{1}{2}cbt^2 - 2ct. \end{cases} \tag{18}$$

$$\begin{cases} X_2(t) = N_1(X_1 + X_0) - N_1(X_0) = -\frac{(3a+2)t^2}{2} - \frac{\left(b - \frac{1}{2}cb\right)t^3}{3} - \frac{(4-2c+b)t^2}{2} + \frac{bt^2}{2}, \\ Y_2(t) = N_2(Y_1 + Y_0) - N_2(Y_0) = -\frac{5t^2}{2} - \frac{bt^3}{6} + \frac{a(3a+2)t^2}{2}, \\ Z_2(t) = N_3(Z_1 + Z_0) - N_3(Z_0) = -\frac{b(b - \frac{1}{2}cb)t^5}{10} + \frac{\left(-5b + \frac{5cb}{2} - \frac{b(4-2c+b)}{2}\right)t^4}{4} \\ \qquad + \frac{(-bc - 4b + 10c - 20)t^3}{3} + \frac{(-2 - 4c + 2b)t^2}{2} - \frac{c(b - \frac{1}{2}cb)t^3}{3} \\ \qquad - \frac{c(4-2c+b)t^2}{2} - bt^2 + \frac{cbt^2}{2}, \end{cases} \tag{19}$$

Hence the 6-iterations approximate series solutions are as follows

$$\begin{cases} X(t) \approx \sum_{i=0}^{6} X_i(t), \\ Y(t) \approx \sum_{i=0}^{6} Y_i(t), \\ Z(t) \approx \sum_{i=0}^{6} Z_i(t). \end{cases} \tag{20}$$

## 4. Result Discussion

The solutions of NIM have been found out by using the Maple software and we have also used the RK4 algorithm which is Maple's built-in programmes to produce the reference solutions. In all of the calculations in this study, the Maple environment variable Digits, which controls the number of significant digits, is set to 16. The numeric precisions of RK4 are found out by considering the constant step size $\Delta t = 0.001$. We have compared the behaviour of the NIM and RK4 solutions for the following non chaotic and chaotic cases.

### *4.1 Non-Chaotic Case*

We have considered the non-chaotic solutions of the system (1) when $a = 0.2$, $b = 0.2$, $c = 2.3$. The 6-iterations NIM series solutions to the system (1) are obtained as follows

$$
\begin{aligned}
X(t) &= \sum_{i=0}^{6} X_i(t) \\
&= -3.674309229864786 \times 10^{-22} t^{28} - 1.020562941859239 \times 10^{-20} t^{27} \\
&\quad +1.275887709072233 \times 10^{-17} t^{26} + 1.044665663575200 \times 10^{-15} t^{25} - 9.093827787977629 \times 10^{-14} t^{24} \\
&\quad -1.540190882291652 \times 10^{-11} t^{23} - 7.085502523014309 \times 10^{-10} t^{22} - 9.119991522994238 \times 10^{-9} t^{21} \\
&\quad +1.766206745954104 \times 10^{-7} t^{20} + 3.896408676419895 \times 10^{-6} t^{19} - 0.00002218171425248279 t^{18} \\
&\quad -0.0003701865756216659 t^{17} + 0.001035416724692603 t^{16} + 0.01224355889842623 t^{15} \\
&\quad -0.01179519004815254 t^{14} - 0.08950847483811985 t^{13} + 0.04312552591546148 t^{12} \\
&\quad +0.3198958231461701 t^{11} - 0.1431859974417988 t^{10} - 0.8436547609126985 t^{9} \\
&\quad +0.08969023908730183 t^{8} + 1.269381961507936 t^{7} - 0.009537388888888878 t^{6} \\
&\quad -1.604431 t^{5} + 0.02216666666666668 t^{4} + 2.393333333333334 t^{3} - 1.1 t^{2} - 5t + 2,
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
Y(t) &= \sum_{i=0}^{6} Y_i(t) \\
&= 1.361655773420479 \times 10^{-14} t^{18} + 9.854983660130718 \times 10^{-13} t^{17} \\
&\quad -2.463946168745276 \times 10^{-10} t^{16} - 3.206123775364847 \times 10^{-8} t^{15} - 1.099847429356359 \times 10^{-6} t^{14} \\
&\quad -3.298220066970104 \times 10^{-7} t^{13} + 0.0003780267809042811 t^{12} - 0.0004703324065055317 t^{11} \\
&\quad -0.02110619479717814 t^{10} + 0.1295719543650794 t^{8} - 0.006163313492063504 t^{7} \\
&\quad +0.01492930434303350 t^{9} - 0.2664938444444446 t^{6} + 0.02733466666666670 t^{5} \\
&\quad +0.5725333333333335 t^{4} - 0.516 t^{3} - 2.24 t^{2} + 2.6t + 3,
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
Z(t) &= \sum_{i=0}^{6} Z_i(t) \\
&= 2.717661607890919 \times 10^{-26} t^{41} + 9.539943224570151 \times 10^{-24} t^{40} \\
&\quad +4.837094364348754 \times 10^{-22} t^{39} - 4.561312517147778 \times 10^{-20} t^{38} - 7.147984354936831 \times 10^{-18} t^{37} \\
&\quad -3.863815909501305 \times 10^{-16} t^{36} - 8.878817195459058 \times 10^{-15} t^{35} + 1.718234613461412 \times 10^{-14} t^{34} \\
&\quad +4.730800284908109 \times 10^{-12} t^{33} + 5.161143395074361 \times 10^{-11} t^{32} - 9.371113514627284 \times 10^{-10} t^{31} \\
&\quad -1.484826431028027 \times 10^{-8} t^{30} + 1.137062441840452 \times 10^{-7} t^{29} + 5.603521701318846 \times 10^{-35} t^{45} \\
&\quad +5.452229064474890 \times 10^{-33} t^{44} - 2.726718643817844 \times 10^{-30} t^{43} - 4.284003520271432 \times 10^{-28} t^{42} \\
&\quad +1.594625591165876 \times 10^{-6} t^{28} - 7.700096578979901 \times 10^{-6} t^{27} - 0.0000814585958920306 9t^{26} \\
&\quad +0.0002585383311577815 t^{25} + 0.001986895590371094 t^{24} - 0.003725094928936324 t^{23} \\
&\quad -0.01990739728484831 t^{22} + 0.02329405932056232 t^{21} + 0.1109296380229821 t^{20} \\
&\quad -0.08800988503138483 t^{19} - 0.4120838003734502 t^{18} + 0.2491696446504369 t^{17} \\
&\quad +1.175918710992455 t^{16} - 0.5074357948480074 t^{15} - 2.605151241089246 t^{14} \\
&\quad +0.9084358977261154 t^{13} + 4.757966742128333 t^{12} - 1.503186375219782 t^{11} \\
&\quad -7.592904570984835 t^{10} + 1.134109747965168 t^{9} + 8.925583539037701 t^{8} \\
&\quad -0.5983825186904764 t^{7} - 8.794651705555557 t^{6} + 0.02988966666666660 t^{5} \\
&\quad +7.449616666666668 t^{4} + 0.4273333333333333 t^{3} - 4.94 t^{2} - 0.4t + 2.
\end{aligned} \tag{23}
$$

From Table 1, we can find out that the numerical solutions of NIM match with the RK4 solutions to at most 11 places of decimal. The numeric data points obtained from the two methods coincide with each other in the Figure 1. That's why it is quite clear that NIM solutions provide good accuracy in comparison to RK4.

**Table 1**
Differences between 6-iterations NIM with RK4 solutions in Non chaotic case

| $\Delta = \lvert RK4_{0.001} - NIM \rvert$ | | | |
|---|---|---|---|
| t | $\Delta X(t)$ | $\Delta Y(t)$ | $\Delta Z(t)$ |
| 0.05 | 1.871e-11 | 9.65e-13 | 7.041e-12 |
| 0.1 | 2.194e-09 | 2.768e-11 | 1.396e-09 |
| 0.2 | 2.058e-07 | 3.617e-08 | 2.285e-07 |
| 0.3 | 2.026e-06 | 9.633e-07 | 2.878e-06 |
| 0.4 | 6.315e-06 | 7.692e-06 | 6.963e-06 |
| 0.5 | 1.755e-05 | 2.755e-05 | 4.529e-06 |
| 0.6 | 0.0001802 | 2.458e-05 | 0.0004046 |
| 0.7 | 0.001462 | 0.0002582 | 0.004799 |
| 0.8 | 0.007728 | 0.00174 | 0.02984 |
| 0.9 | 0.03054 | 0.006856 | 0.1305 |
| 1 | 0.09866 | 0.02093 | 0.4512 |

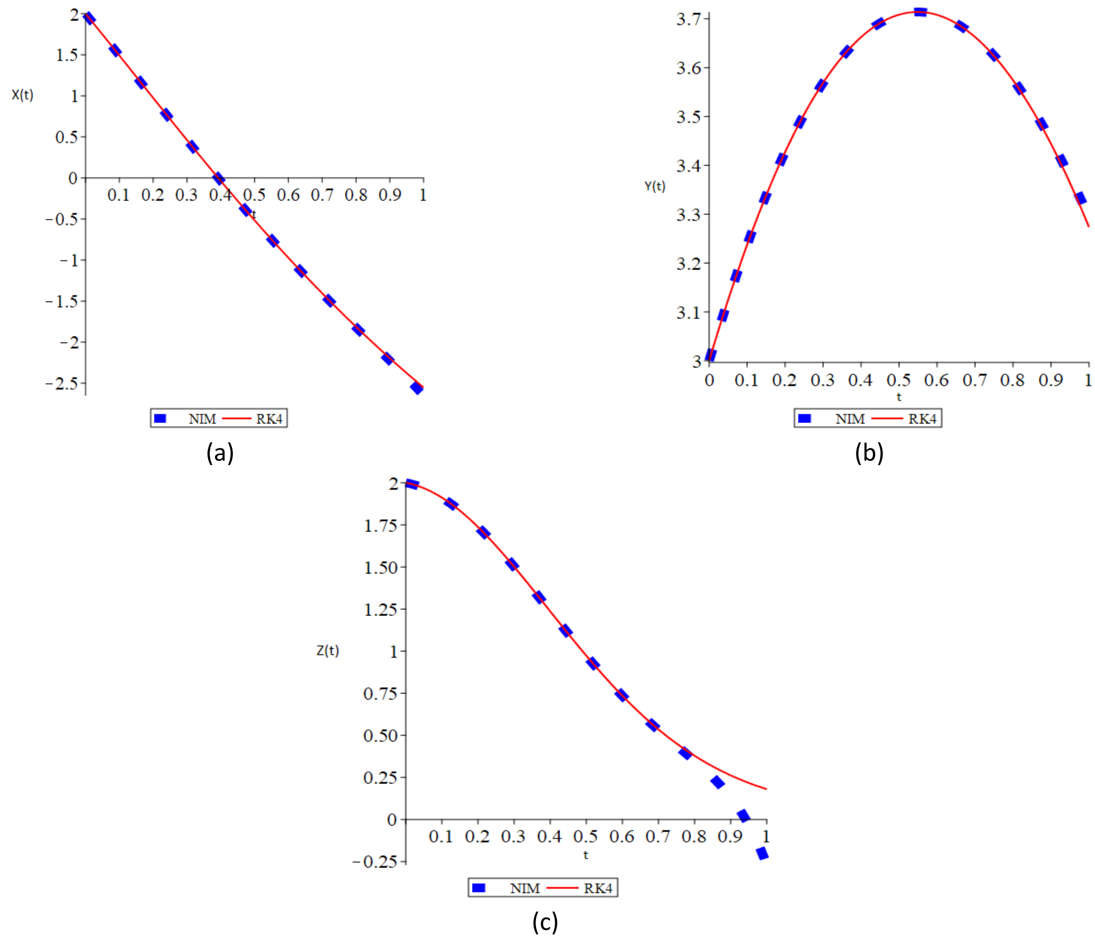



**Fig. 1.** Graphical comparison among NIM (space blue box), RK4 (red line) for(a) $X(t)$, (b) $Y(t)$, (c) $Z(t)$ in Non chaotic case

### *4.2 Chaotic Case*

We have considered the chaotic solutions of the system (1) when a = 0.2, b = 0.2, c = 5.7. The 6-iterations NIM series solutions to the Chaotic Rössler system (1) are obtained as follows

$$\begin{aligned} X(t) &= \sum_{i=0}^{6} X_i(t) \\ &= -1.048523214981262\times 10^{-16}t^{28} - 4.172811762448426\times 10^{-14}t^{27} \\ &\quad -7.114397612103642\times 10^{-12}t^{26} - 6.780078465912944\times 10^{-10}t^{25} - 3.943970260824102\times 10^{-8}t^{24} \\ &\quad -1.427913001466898\times 10^{-6}t^{23} - 0.00003098831518848374t^{22} - 0.0003476808082820207t^{21} \\ &\quad -0.0008574630927043700t^{20} + 0.01391638042184592t^{19} + 0.03177200584411776t^{18} \\ &\quad -0.119971450859285t^{17} - 0.2676354852176616t^{16} + 0.2538331833339068t^{15} \\ &\quad +0.1322321909485544t^{14} - 1.447041069280689t^{13} - 0.0027890496781566t^{12} \\ &\quad +2.166194162609382t^{11} - 1.350760959261464t^{10} + 0.0102543238756620t^{9} \\ &\quad +5.466587217261906t^{8} - 0.635376009126984t^{7} - 2.770053111111112t^{6} \\ &\quad +3.811146666666666t^{5} - 0.972333333333333t^{4} - 2.026666666666666t^{3} + 2.3t^{2} - 5t + 2, \end{aligned} \tag{24}$$

$$\begin{aligned} Y(t) &= \sum_{i=0}^{6} Y_i(t) \\ &= 2.554516662632130\times 10^{-11}t^{18} + 6.831174541575888\times 10^{-9}t^{17} \\ &\quad +7.090877395190331\times 10^{-7}t^{16} + 0.00003563257062884390t^{15} + 0.0008634609881630721t^{14} \\ &\quad +0.007661802373575506t^{13} - 0.01805598744949498t^{12} - 0.1073589551676488t^{11} \\ &\quad +0.004417548611111129t^{10} + 0.1196903314594356t^{9} - 0.5192647519841271t^{8} \\ &\quad -0.4715307579365080t^{7} + 0.6280744888888888t^{6} - 0.2134986666666666t^{5} \\ &\quad -0.4757999999999997t^{4} + 0.6173333333333334t^{3} - 2.24t^{2} + 2.6t + 3, \end{aligned} \tag{25}$$

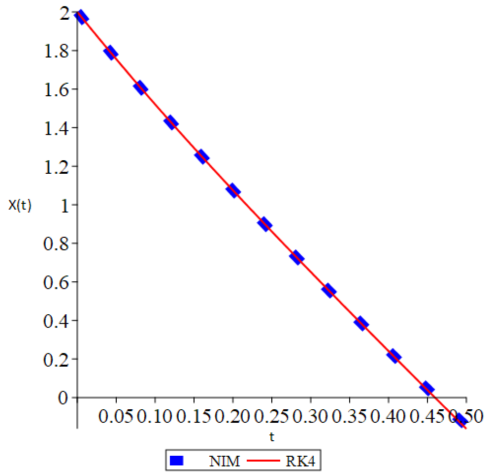
$$\begin{aligned} Z(t) &= \sum_{i=0}^{6} Z_i(t) \\ &= 1.253645879551166\times 10^{-16}t^{41} + 1.075274521405469\times 10^{-14}t^{40} \\ &\quad +6.736861970271979\times 10^{-13}t^{39} + 3.116141036450111\times 10^{-11}t^{38} + 1.060360964696553\times 10^{-9}t^{37} \\ &\quad +2.601355037630130\times 10^{-8}t^{36} + 4.386671801225805\times 10^{-7}t^{35} + 4.530391555788401\times 10^{-6}t^{34} \\ &\quad +0.00001819789099367777t^{33} - 0.0001298391237880344t^{32} - 0.001431826067766996t^{31} \\ &\quad +0.0002636905653790753t^{30} + 0.02395865716501892t^{29} + 2.999886426685395\times 10^{-26}t^{45} \\ &\quad +1.952260093663045\times 10^{-23}t^{44} + 5.795373774312516\times 10^{-21}t^{43} + 1.038727950476994\times 10^{-18}t^{42} \\ &\quad +0.02096550270526169t^{28} - 0.1532428293263898t^{27} - 0.1401577288584000t^{26} \\ &\quad +0.5022851152767470t^{25} - 0.2198098582473085t^{24} - 2.420585917316581t^{23} \\ &\quad +0.6940048921456685t^{22} + 3.856957557801462t^{21} - 5.836777151199948t^{20} \\ &\quad -4.433818399058566t^{19} + 13.90179953500629t^{18} - 2.229775117278036t^{17} \\ &\quad -15.00986262819147t^{16} + 23.49621422150204t^{15} + 6.893777366418208t^{14} \\ &\quad -33.14217408533764t^{13} + 19.85127242043869t^{12} + 20.67444469520864t^{11} \\ &\quad -29.95581031401001t^{10} + 25.34055288106924t^{9} + 33.26021557266865t^{8} \\ &\quad -27.04852854726191t^{7} + 5.983561533333333t^{6} + 16.83381733333333t^{5} \\ &\quad -18.57993333333333t^{4} + 3.272000000000000t^{3} + 8.32t^{2} - 7.2t + 2. \end{aligned} \tag{26}$$

From Table 2, we notice that the NIM precisions agree with those of RK4 at most 11 decimal places. The numeric data points obtained from the two methods overlap with each other in the Figure 2. That’s why it is again quite clear that NIM solutions provide good accuracy in comparison to RK4.

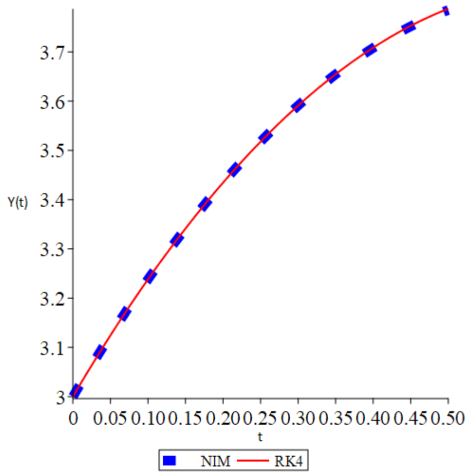
**Table 2**
Differences between 6-iterations NIM with RK4 solutions in chaotic case.

$\Delta = |RK4_{0.001} - NIM|$

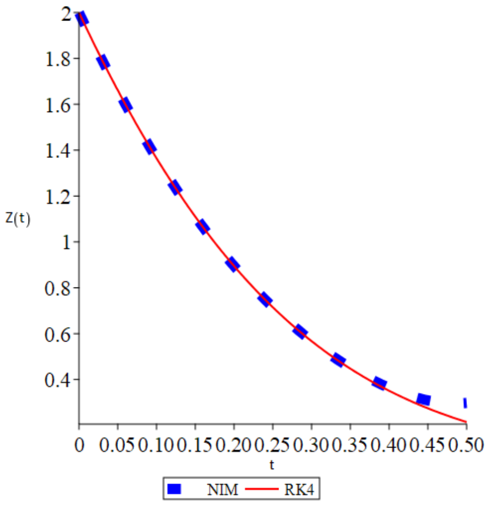
| t | $\Delta X(t)$ | $\Delta Y(t)$ | $\Delta Z(t)$ |
|---|---|---|---|
| 0.05 | 3.212e-10 | 8.903e-11 | 1.202e-09 |
| 0.1 | 5.262e-08 | 1.354e-08 | 2.123e-07 |
| 0.15 | 1.117e-06 | 2.696e-07 | 4.801e-06 |
| 0.2 | 1.015e-05 | 2.317e-06 | 4.612e-05 |
| 0.25 | 5.754e-05 | 1.251e-05 | 0.0002748 |
| 0.3 | 0.0002413 | 5.020e-05 | 0.001205 |
| 0.35 | 0.0008195 | 0.0001638 | 0.004264 |
| 0.4 | 0.002382 | 0.0004592 | 0.01288 |
| 0.45 | 0.006138 | 0.001145 | 0.03442 |
| 0.5 | 0.01437 | 0.002603 | 0.08349 |



(a)



(b)



(c)

**Fig. 2.** Graphical comparison among NIM (space blue box), RK4 (red line) for (a) $X(t)$, (b) $Y(t)$, (c) $Z(t)$ in chaotic case

## 5. Conclusion

We have our own particular niche in the application of chaos theory to fluid dynamics. The results acquired by the New Iterative Method are compared with the conventional Runge–Kutta method (order four) to ascertain its effectiveness. Its results are in very well agreement with RK4. The numerical method RK4 cannot provide us series solutions and also the computational errors are sometimes very high, as it depends on continuous step size. One of NIM's benefits is its ability to provide us with continuous series solutions without any transformation, linearization, and discretization. In this research the performance of NIM can lead to a promising approach for many engineering applications.

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