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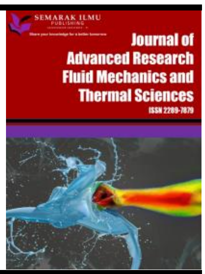

Newtonian Heating on Casson Mixed Convection Transport by Ternary Hybrid Nanofluids over Vertical Stretching Sheet: Numerical Study

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ABSTRACT

Numerous physical characteristics occurring from the structure and micro-motions of fluid nanoparticles can be examined using the governing models of nanofluids. Also, It can describe the demeanor of a vast field of actual fluids. This achieved a significant turn in the enhancement of heat transfer that improves manufacturing and engineering modernization. Depending on that, the assumptions that form our aims are numerically examining energy transport and heat transfer via Casson ternary hybrid nanofluids that contain the states of combined convection flow over a vertical stretching sheet. Also, Newtonian heating boundary conditions and magnetohydrodynamics (MHD) effects are investigated. Mathematical models (governing equations) that emulate and construct the conduct of these boosted ternary hybrid nanofluids are formed by developing the Tiwari and Das models. These governing equations are converted to partial differential equations (PDEs) employing appropriate substitutions. An accurate and efficient method called the Keller box numerical method is executed to get outcomes to the physical model. These numerical calculations are found and presented by employing MATLAB codes, to acquire tables of numerical outcomes, and graphic exhibits, which provide the impacts of important parameters on the physical quantities supporting heat transfer. Furthermore, the new results, in the Newtonian fluid case, were compared with prior literature, which were in excellent agreement. The studied problem parameters are examined at a Casson parameter domain of 1 to 5, a mixed convection parameter domain of -1 to 3, a conjugate parameter of 0.2 to 1, a magnetic parameter domain of 0.1 to 5, and a nanoparticle volume fraction parameter of 0.001 to 0.008. The numerical consequences deduced that the local skin friction rises when the conjugate and mixed convection parameters are increased. But the opposite case happens when the effects are obtained for the nanoparticle volume fraction, magnetic, and Casson parameters. Also, the Nusselt number rises with growing nanoparticle volume fraction and Casson parameters.

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1. Introduction

In recent years, one of the important fields in fluid dynamics sciences is the physical impacts of nanoparticles, which play an important role in the enhancement of heat transfer in less time, due to the development of technologies and industries. Nanofluids presently play a primary role in numerous technical and scientific usefulness as a result of passages in trendy technology, which have enhanced our understanding of the procedures related to heat transfer for suitable cooling. In considerable industrial operations, some fluids are mainly used in convective heat transfer operations. Furthermore, to enhance these operations and because of low thermal conductivities for used fluids, it was introduced the Concept of a combination of the base fluids with the suspended definite nanoparticles as nanofluid Choi and Eastman [1]. Buongiorno [2], and Tiwari and Das [3] studied the convection heat transfer in nanofluids, and nanofluids inside a two-sided lid-driven differentially heated square cavity, respectively. the convection boundary layer flow of laminar fluid flow which around the stretching sheet and vertical stretching sheet in the nanofluid has been studied by Khan and Pop [4], Makinde and Aziz [5], Kuznetsov and Nield [6], Kausar *et al.,* [7]. Swalmeh *et al.,* [8,9], Yaseen *et al.,* [10], and Alwawi *et al,* [11] investigated the influences and impacts of micropolar nanofluid on the physical quantities around solid sphere and horizontal cylinder. Also, the heat transfer convection in Casson nanofluid considered by Alwawi *et al.,* [12-14]. To improve enhancement efficiency for the nanofluid heat transfer, hybrid nanofluids have appeared that are composed of two different figures of nanoparticles suspended in the based fluids. Consequently, hybrid nanofluid has great heat transfer features that don't exist in the nanofluid combinations. Suresh *et al.,* [15] searched the outstanding heat transfer markers and rheological conduct along with thermophysical profiles. Additionally, many investigators have issued different modern interesting research articles regarding the boundary layer flow in a hybrid nanofluid [16-20]. There are ternary hybrid nanofluid simulation models continually used to study the demeanor of nanofluids. The ternary hybrid nanofluids are considered an extension of nanofluid effects, which are characterized by their ability to efficiently heat transfer [21-31].

Merkin [32] has revealed that there are four standard heating methods pinpointing the ambient temperature allocations, such as constant wall temperature (CWT), constant surface heat flux (CHF), and conjugate conditions, where heat is provided and controlled via limited of thickness and heat capacity for a utilized bounding surface [33,34]. The interface temperature is not knowledge a priori but relies on the nanofluids' thermal conductivity of the suggested characteristics of the system. The proportionate between the bounding surface heat transfers with local surface temperature is known as limited heat capacity, which is typically named Newtonian heating boundary condition (NH) [35]. In numerical modeling problems of convective boundary layer flow, the boundary conditions that are typically applied are the temperature and fluid velocity at the beginning and end of the flow. Therefore, the boundary conditions must be determined when constructing the governing equations to adjust the numerical solutions. There are many numerical studies, as Jha and Samaila [36], Wahid *et al.,* [37], and Rehman *et al.,* [38]. In fluid mechanics studies, when free convection and forced convection are combined, they form mixed convection. This results from the induced by both an exterior force and density of the fluid contrast that is generated by temperature inclines. the assisting flow is naturally constructed when the forced flow and free convection are in the identical orientation, which is the mixed convection first type, but the mixed convection second type is called opposing flow when the free convection and forced convection are in opposing orientations [39-41].

The efficiency of non-Newtonian fluids can be predicted using several models, one of which is the Casson model, and it has since established its efficiency by predicting the behaviour of shear-thinning fluids, such as fluids that have high concentrations. human blood, Fruit juice, ketchup, honey, etc.

review work illustrates a survey of various numerical techniques appropriate to Casson fluid by examining different kinds of research articles, which were investigated by Verma and Mondal [42]. Khan *et al.,* [43] presented an examination of an analytical-numerical of heat and mass transmission characteristics of steady, non-Newtonian Casson fluid over a stretching surface. El-Sayed *et al.,* [44] and Yasin *et al.,* [45] investigated mixed convection in Casson nanofluid around horizontal cylinder and solid sphere. heat dissipation, heat radiation, and magnetohydrodynamics in heat and mass transfer in the dusty Casson fluid flow on a permeable stretching sheet is discussed by Roy and Saha [46], and Awan *et al.,* [47]. On the other hand, there are a lot studies of magnetohydrodynamics fields in convection boundary layer flow [48-53].

Motivated by the above-published studies examined, this consideration focuses on supplying the research gap by investigating the effects of Casson ternary hybrid nanofluids under magnetohydrodynamics fields. Relying on that, mixed convection flow across a vertical stretching sheet, in ternary hybrid nanofluids under a magnetic field, is the main topic of this investigation. The Newtonian heating initial profiles were also considered. Additionally, the ternary hybrid nanoparticles, such as Aluminium oxide, Copper oxide, and Graphene oxide/Titanium oxide are incorporated, and water is utilized as the base fluid. The constructed governing equations for the studied problem are changed into a system of partial differential equations (PDEs) by employing the proper similarity transformations. Consequently, these PDEs can be numerically solved via the Keller box method. The influences of the ruling parameters are depicted, graphically and tabularly, here in terms of the local skin friction and local Nusselt number, as well as velocity and temperature profiles. Analyses of the behaviours between two types of ternary hybrid nanoparticles are presented. To check the accuracy of the numerical approach, that was worked, preferably trial computations for $\vartheta(0)$, $\vartheta'(0)$ are carried out for distinct results of Pr. So, these current outcomes were compared with the published results by Salleh *et al.,* [54], and displayed in Table 2. They are seen to be in excellent agreement.

More particularly, the existing study can be outlined through the next questions

i. How are the governing equations of the problem of magnetohydrodynamic (MHD) combined (mixed) convection flow on a vertical stretching sheet in Casson ternary hybrid nanofluids models constructed?
ii. How can a mathematical formulation be employed for the problem?
iii. How do the MHD Casson ternary hybrid nanofluid models compare with the earlier examined mixed heat and mass transfer flow issues?
iv. How do we discuss the numerical outcomes that can be acquired from the impacts studied parameters on the heat transfer and physical quantities?
v. How does the heat transfer examine the used nanoparticles with the based fluid beneath the influence of assumed parameters?

2. Problem Formulation and Equations

In this consideration, the Casson ternary hybrid nanofluids, that were Installed from Al_2O_3/CuO/GO or TiO_2, as nanoparticles and suspended in water as a based fluid were investigated. Additionally, the incompressible magnetohydrodynamics (MHD) mixed convection boundary flow on a vertical stretching sheet was taken into account. The state of Newtonian heating is laid on the wall as boundary conditions. The case of the viscous dissipation influence was to be disregarded. As presented in Figure 1, the effect of magnetic field and electrical conductivity B_0, as well as Casson and nanoparticle volume fraction is contained in the momentum equation. Moreover, T_∞, T_w are ambient and wall temperature. As well as U_∞, and U_w are called external and wall velocity, respectively. The coordinate geometry is determined in such a manner that the ternary hybrid nanofluid flows in the *x*-orientation, which is calculated on the horizontal direction of the vertical stretching sheet. Likewise, the *y*-orientation is measured in length perpendicular to the supposed vertical stretching sheet.

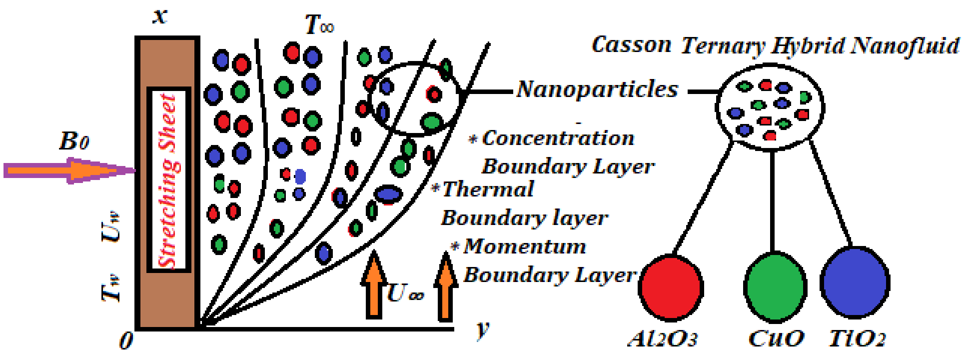

Fig. 1. Physical model and coordinates system

Concerning the above assumptions, and referring to Casson impacts, ternary hybrid nanofluids model, and magnetic influences, the two dimensional governing equations, i.e. continuity, momentum, and thermal equations, can be appointed as [43,55,56]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial x}=u_e\frac{\partial u_e}{dx}+\frac{\mu_{thnf}}{\rho_{THNF}}\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}+(\rho\beta)_{THNF}g(T-T_\infty)-\frac{\sigma_{THNF}}{\rho_{THNF}}B_0^2(u-u_e), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{THNF}}{(\rho C_P)_{THNF}}\frac{\partial^2 T}{\partial y^2}\,, \tag{3}$$

(u, v), T, β_0^2, h_s are indicated to velocity coordinates which rely on the *x* and *y* orientations, temperature, and magnetic field strength, respectively. β is denoted by the Casson parameter. The physical quantities k_{THNF}, β_{THNF}, $(\rho c_p)_{THNF}$ are kinematic viscosity, thermal expansion coefficient, and heat capacity of the ternary hybrid nanofluid. μ_{THNF} and σ_{THnf} are dynamic viscosity, electrical conductivity of ternary hybrid nanofluid. These physical quantities for the ternary hybrid nanofluid are presented in as [55]:

$$(\mu)_{HTnf}\;=\;\frac{\mu_f}{(1-\chi_{Al2O3})^{2.5}(1-\chi_{CuO})^{2.5}(1-\chi_{TiO2})^{2.5}},$$

$$(\rho)_{HTnf}=(1-\chi_{Al2O3})[(1-\chi_{CuO})\,[(1-\chi_{TiO2})\rho_{water}+\chi_{TiO2}\rho_{\chi_{TiO2}}]+\chi_{CuO}\rho_{cuo}]+\chi_{Al2O3}\rho_{Al2O3},$$

$$(\rho c_p)_{HTnf}=(1-\chi_{Al2O3})[(1-\chi_{CuO})\,[(1-\chi_{TiO2})(\rho c_p)_{water}+\chi_{TiO2}(\rho c_p)_{\chi_{TiO2}}]+\chi_{CuO}(\rho c_p)_{cuo}]+\chi_{Al2O3}(\rho c_p)_{Al2O3},$$

$$\frac{k_{THNF}}{k_{HNF}}=\frac{(k_{Al2O3}+2\,k_{HNF})-2\,\chi_{TiO2}\,(k_{HNF}-k_{Al2O3})}{(k_{Al2O3}+2\,k_{HNF})+\chi_{TiO2}\,(k_{HNF}-k_{Al2O3})}, \tag{4}$$

$\frac{k_{HNF}}{k_{NF}} = \frac{(k_{CuO}+2\,k_{NF})-2\,\chi_{CuO}(k_{nf}-k_{CuO})}{(k_{CuO}+2\,k_{NF})+\chi_{CuO}(k_{NF}-k_{CuO})}, \frac{k_{NF}}{k_{water}} = \frac{(k_{TiO2}+2\,k_{water})-2\chi_{TiO2}(k_{water}-k_{TiO2})}{(k_{TiO2}+2\,k_{water})+\chi_{TiO2}(k_{water}-k_{TiO2})},$

$$\sigma_{HTNF} = \left[1 + \frac{3\left((\sigma_{Al2O3}/\sigma_{HNF})-1\right)\chi_{Al2O3}}{((\sigma_{Al2O3}/\sigma_{HNF})+2)-\chi_{Al2O3}\left((\sigma_{Al2O3}/\sigma_{Hnf})-1\right)}\right]\sigma_{HNF},$$

$$\sigma_{HNF} = \left[1 + \frac{3\left((\sigma_{CuO}/\sigma_{NF})-1\right)\chi_{CuO}}{((\sigma_{CuO}/\sigma_{NF})+2)-\chi_{CuO}\left((\sigma_{CuO}/\sigma_{NF})-1\right)}\right]\sigma_{NF},$$

$$\sigma_{NF} = \left[1 + \frac{3\left((\sigma_{TiO2}/\sigma_{water})-1\right)\chi_{TiO2}}{\left((\sigma_{TiO2}/\sigma_{f})+2\right)-\chi_{TiO2}\left((\sigma_{TiO2}/\sigma_{f})-1\right)}\right]\sigma_{water},$$

Substituting the Eq. (4) into Eq. (1) to Eq. (3), then we get:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} = u_e\frac{\partial u_e}{dx} + \frac{\rho_{water}}{\rho_{THNF}}\left[\frac{1}{(1-\chi_{Al2O3})^{2.5}(1-\chi_{CuO})^{2.5}(1-\chi_{TiO2})^{2.5}}\right]\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} + \left(\frac{1}{\rho_{THNF}}(1-\chi_{Al2O3})\left[(1-\chi_{CuO})\left[(1-\chi_{TiO2})+\chi_{TiO2}\frac{\beta_{TiO2}}{\beta_{water}}\right]+\chi_{CuO}\frac{\beta_{CuO}}{\beta_{water}}\right]+\chi_{Al2O3}\frac{\beta_{Al2O3}}{\beta_{water}}\right)\mathrm{g}(T-T_\infty) - \frac{\sigma_{THNF}}{\rho_{THNF}}B_0^2(u_e-u), \tag{6}$$

$$u\frac{\partial T}{\partial x}v\frac{\partial T}{\partial y} = \frac{1}{\Pr}\left(\frac{k_{THNF}/k_{water}}{(1-\chi_{Al2O3})[(1-\chi_{CuO})[(1-\chi_{TiO2})+\chi_{TiO2}\frac{(\rho c_p)_{TiO2}}{(\rho c_p)_{water}}]+\chi_{CuO}\frac{(\rho c_p)_{CuO}}{(\rho c_p)_{water}}]+\chi_{Al2O3}\frac{(\rho c_p)_{Al2O3}}{(\rho c_p)_{water}}}\right)\frac{\partial^2 T}{\partial y^2}, \tag{7}$$

from which subject to Newtonian heating (NH) boundary conditions as [42]:

$$u = u_w = ax,\ v = 0,\ T = -h_s\frac{\partial T(x)}{\partial y}, \quad at\ y = 0$$

$$u \to u_e\ ,\ T \to T_\infty \text{ as } y \to \infty, \tag{8}$$

where χ_{Al2O3}, χ_{CuO}, and χ_{TiO2} are referred to the nanoparticle volume fraction parameters and h_s is heat transfer parameter.

To convert the above dimensional equation to partial differential equations PDEs, we define the following similarity transformation:

$$v = -\frac{\partial\psi}{\partial x}, u = \frac{\partial\psi}{\partial y}$$

$$\psi = (a\,v)^{\frac{1}{2}}\,x\,f(\omega),\ \omega = (a/v)^{\frac{1}{2}}y,\ \theta(\omega) = \frac{T-T_\infty}{T_\infty}, \tag{9}$$

ψ is called the stream function.

Consequently, substituting Eq. (9) into Eq. (5) to Eq. (8), we reach the following PDEs:

$\frac{\rho_{water}}{\rho_{THNF}}\left[\frac{1}{(1-\chi_{Al2O3})^{2.5}(1-\chi_{CuO})^{2.5}(1-\chi_{TiO2})^{2.5}}\right]\left(1+\frac{1}{\beta}\right)f''' + ff'' - (f')^2 + \left(\frac{1}{\rho_{THNF}}(1-\chi_{Al2O3})\right[(1-$

$$\chi_{CuO})\left[(1-\chi_{TiO2})+\chi_{TiO2}\frac{\beta_{TiO2}}{\beta_{water}}\right]+\chi_{CuO}\frac{\beta_{CuO}}{\beta_{water}}\right]+\chi_{Al2O3}\frac{\beta_{Al2O3}}{\beta_{water}}\Big)\lambda\theta-\frac{\sigma_{THNF}}{\rho_{THNF}}\frac{\rho_{water}}{\sigma_{water}}M(1-f')= 0, \quad (10)$$

$$\frac{1}{\mathrm{Pr}}\left(\frac{k_{THNF}/k_{water}}{(1-\chi_{Al2O3})[(1-\chi_{CuO})[(1-\chi_{TiO2})+\chi_{TiO2}\frac{(\rho c_p)_{TiO2}}{(\rho c_p)_{water}}]+\chi_{CuO}\frac{(\rho c_p)_{CuO}}{(\rho c_p)_{water}}]+\chi_{Al2O3}\frac{(\rho c_p)_{Al2O3}}{(\rho c_p)_{water}}}\right)(\theta'')+f\theta'-f'\theta=0, \quad (11)$$

and the boundary conditions:

$$\begin{aligned} &f(0)=0,\quad f'(0)=1, \theta'(0)=-H_s-\theta(0), \quad ,at\ \omega=0 \\ &f'(\omega)\to 1,\ \theta(\omega)\to 0\ \ at\ \ \omega\to\infty \end{aligned}, \quad (12)$$

$\mathrm{Pr}=v_{water}\ /\ k_{water}$, $M=\frac{\sigma_{water}B_0^2\,a^2}{\rho_{water}v_{THNF}}$, $H_s=\left(\frac{v_{water}}{a}\right)^{1/2}h_s$ are called the Prandtl number, magnetic parameter, and the conjugate parameter.

Newtonian heating is governed by a non-dimensional conjugate parameter, ranging from a minimum value of 0 (indicating an insulated wall) to ∞ (representing a wall with a constant temperature). The application of Newtonian heating typically occurs in what is referred to as conjugate convection flow. In this scenario, heat is supplied to the convective fluid through a bounding surface with a finite heat capacity. This configuration is prevalent in various essential engineering systems, exemplified by heat exchangers. In such systems, the interaction between conduction and convection significantly influences the solid tube wall as fluid flows over it.

In this consideration, the skin friction coefficient C_f and Nusselt number Nu are engineering interesting, which has been an important needful for examining the happening that occurs through fluids and solid geometries. A mathematical definition for the noted physical quantities is offered as:

$$C_f=\frac{\tau_w}{\rho u_w^2}\ ,Nu=\frac{xq_w}{k\ (T_w-T_\infty)}, \quad (13)$$

Here, τ_w and $q_w$are expressed to be shear stress and heat flux on the plane of the wall and are provided as follows

$$q_w=k_{THNF}\frac{\partial T}{\partial y},\ \tau_w=\mu_{THNF}\left(\frac{\partial u}{\partial y}\right)at\ y=0, \quad (14)$$

So, C_f and Nu are resulted as:

$$C_f=Re^{-\frac{1}{2}}\left[\frac{1}{(1-\chi_{Al2O3})^{2.5}(1-\chi_{CuO})^{2.5}(1-\chi_{TiO2})^{2.5}}\right]\left(1+\frac{1}{\beta}\right)f''(0),\ Nu=Re^{\frac{1}{2}}\frac{k_{THNF}}{k_{water}}\left(1+\frac{1}{\theta(0)}\right), \quad (15)$$

where $Re=\frac{ax^2}{v_f}$ is the local Reynolds number.

3. Numerical Method

The numerical methodology was utilized to complete the numerical outcomes for the studied problem, where an above-considered system of partial differential Eq. (10) to Eq. (12) will be solved by the Keller box efficient method. This method is a suitable scheme to get good agreement comparison outcomes with previously published results. Initially, the higher-order partial differential equations will be reduced to the first order utilizing the finite difference method. The differences equations will be linearized by Newton's technique. Finally, resolved using the block elimination technique. The numerical outcomes are acquired by programming the algorithm of a linear system implemented via MATLAB version. The Thermo-physical characteristics of the studied nanoparticles and based fluid are displayed in Table 1. We define the nanoparticle volume fraction $\chi = \chi_{Al2O3} = \chi_{CuO} = \chi_{TiO2} = \chi_{GO}$.

Table 1
Thermo-physical characteristics of the based fluid and used nanoparticles [57]

Thermo-Physical feature	Al_2O_3	CuO	TiO_2	GO	Water
C_p (J/kg K)	765	540	4250	1800	4179
ρ (kg/m^3)	3970	6510	686.2	717	997.1
K (W/m K)	40	18	8.9538	5000	0.613
σ(S/m)	$3.5x10^7$	$5.96x10^7$	$2.38x10^6$	$6.30x10^7$	$5.5x10^{-6}$
$Bx10^{-5}(K^{-1})$	0.85	0.85	0.9	28	21
Pr	…	…	…	…	6.2

4. Testing Accuracy

To verify the validation and the accuracy of the current results and the efficiency of the numerical used method, we compare our current results with previously published outcomes. Accordingly, after solving the above partial differential equations and using the MATLAB program to extract results in a special case, Newtonian fluid (M = 0, $\beta \to \infty$, χ = 0, λ = 0, and γ =1), Table 2 shows a comparison of the results of the current problem with Salleh *et al.,* [54], where it was found that the current results are in excellent agreement.

Table 2
Comparison of presents results with previous published study (M = 0, $\beta \to \infty$, χ = 0, λ = 0, and γ =1), with several values of Pr

	Salleh *et al.,* [54]		Present	
Pr	$\vartheta(0)$	$\vartheta'(0)$	$\vartheta(0)$	$\vartheta'(0)$
3	6.02577	7.02577	6.02581	7.02579
5	1.76594	2.76594	1.76598	2.76597
7	1.13511	2.13511	1.13532	2.13515
10	0.76531	1.76531	0.76536	1.76537
100	0.16115	1.16115	0.16121	1.16122

5. Results and Discussion

This section provides discussions of the mixed convection boundary layer flow over a vertical stretching sheet, in the presence of ternary hybrid nanofluids. The magnetic and Newtonian heating conducts also are considered. The approximations and transformations employed in the investigation of this problem are summarized in the above mathematical formulation section. On the other hand, the governing equations were transformed and solved via an efficient numerical method which is the Keller-Box method. The effects of the parameters; Casson parameter β, mixed convection parameter λ, conjugate parameter γ, Magnetic parameter M, and volume fraction χ parameter on both temperature profile and velocity distribution for two different Casson ternary hybrid nanofluids (Al_2O_3-Cuo-GO/water and Al_2O_3-Cuo-TiO_2/water) have been investigated and represented graphically in Figure 2 to Figure 11. Figure 2 illustrates the influence of Casson β parameter on the temperature profile. It is seen that the increment of β values leads to decrement in the temperature curves. This is usually referred to as the role of β in raising the viscosity of the formed fluid which causes resistance in the heat exchange process between the heat source and fluid, ending up with lower temperature results.

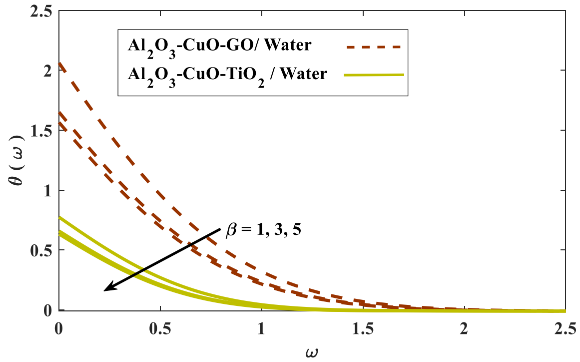

Fig. 2. Temperature profiles ϑ *(ω)* when χ = 0.001, $\gamma = 0.4$, M = 0.2, λ = 2

Figure 3 also shows another negative impact on the temperature as the values of the mixed convection factor λ increase. The rest of the factors have a direct relationship with temperature; in Figure 4, it is clear that as the conjugate factor γ grows up the curves of temperature as well goes up, raising the heat gained by the composed fluid. The magnetic parameter M in this study plays a role in improving the heat profile when its values get bigger as manifested in Figure 5. Figure 6 displays the effect of the volume fraction parameter χ with temperature, here it's noticeable that as the volume fraction of the ternary nanoparticles gets enlarged, the temperature of the entire nanofluid uplifts as a consequence of expanding the surface exposed to the source of heating. besides, a rise in the volume fraction parameter enables heat transport between the external of the nanoparticles wall and the based fluid, which helps boost the boundary layer thickness.

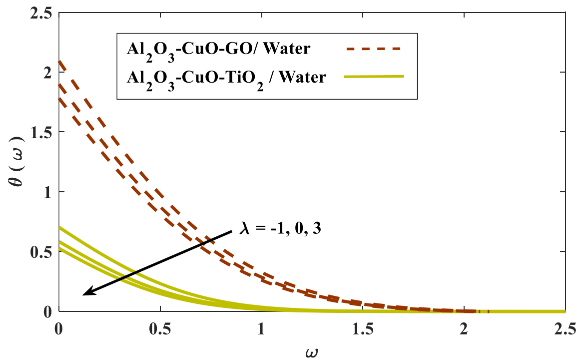

Fig. 3. Temperature profiles ϑ *(ω)* when χ = 0.001, $\gamma = 0.4$, β = 2, M = 0.2

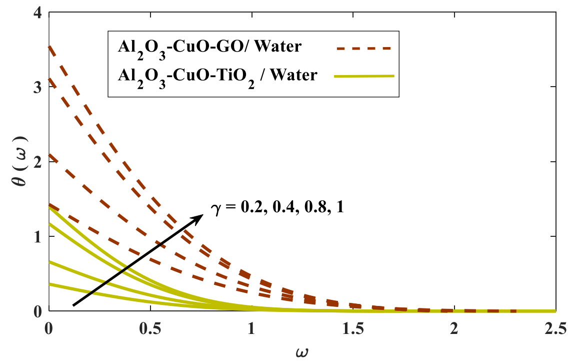

Fig. 4. Temperature profiles ϑ *(ω)* when χ = 0.001, β = 2, M = 0.2, λ = 2

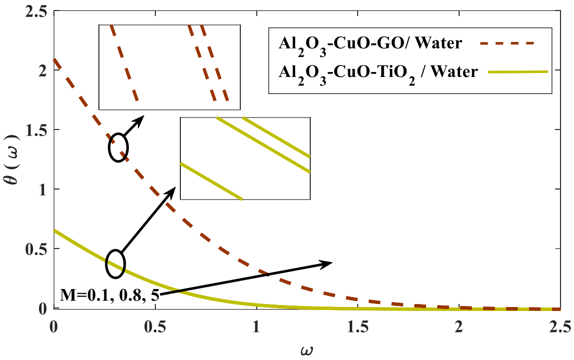

Fig. 5. Temperature profiles ϑ *(ω)* when χ = 0.001, $\gamma = 0.4$, β = 2, λ = 2

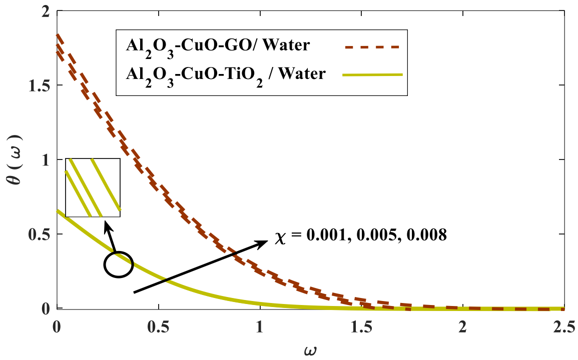

Fig. 6. Temperature profiles ϑ *(ω)* when $\gamma = 0.4$, β = 2, M = 0.2, λ = 2

On the other side, these parameters were checked out on the velocity profile curves, the outcomes plotted out in Figure 7 to Figure 11. We conclude the reverse correlation between the velocity and the Casson parameter *β* from Figure 7. Again, more values of *β* means more viscous fluid which clarifies the deceleration happening to the particles of the nanofluid formed.

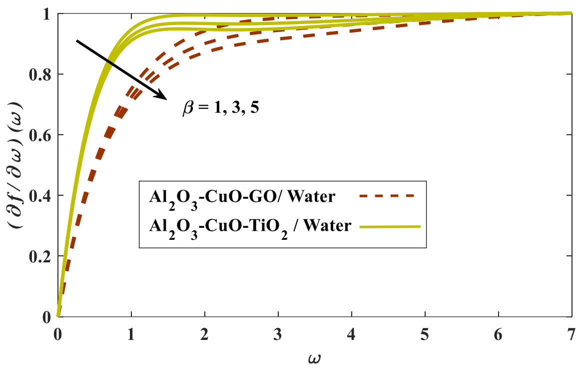

Fig. 7. Velocity profiles (∂ *f/* ∂ *ω) (ω)* when χ = 0.001, $\gamma = 0.4$, M = 0.2, λ =2

In Figure 8 the mixed convection factor *λ* exhibited a direct positive relation with velocity; as the values of *λ* elevate, it enhances the velocity distribution values. The effect of conjugate factor *γ* on the velocity factor examined in Figure 9, it found that *γ* has a direct proportionality with velocity. Magnetic Field factor as expected from Lorentz force for most cases reveals an inverse proportionality with velocity as seen in Figure 10; actually, any conductive fluid under the effect of magnetic and electric field suffers from two different forces, which works to improve the opposition of the fluid to pushing, what mainly affords behind these declines.

Figure 11 explains how augimation on the size of the volume fraction factor reflects on the results of velocity distributions, it's obvious that they have a direct relation; an increment of χ induces the growing up (∂ *f/* ∂ *ω)*. According to all the presented figures, there is a comparison could take place between the two ternary hybrid nanofluids: Al_2O_3-CuO-GO/water and Al_2O_3-CuO-TiO_2/water. it is found that the Al_2O_3-CuO-GO/water has higher temperature profiles than Al_2O_3-CuO-TiO_2/water, but the opposite case happens when we move to the velocity profiles figures, Al_2O_3-CuO-TiO_2/water have got higher velocity distribution curves than Al_2O_3-CuO-GO/water, as shown in all figures due to the different physical properties of the last suspend hybrid nanoparticles (TiO_2 and GO).

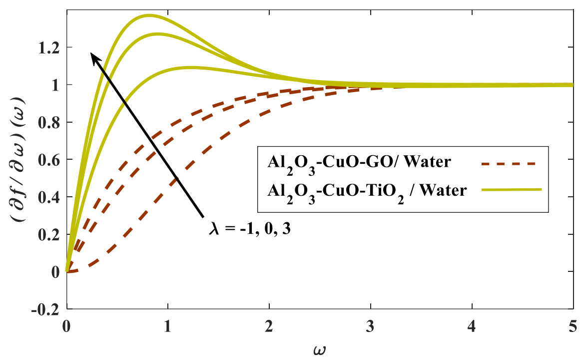

Fig. 8. Velocity profiles (∂ *f*/ ∂ *ω*) (*ω*) when χ = 0.001, $\gamma = 0.4$, β = 2, M = 0.2

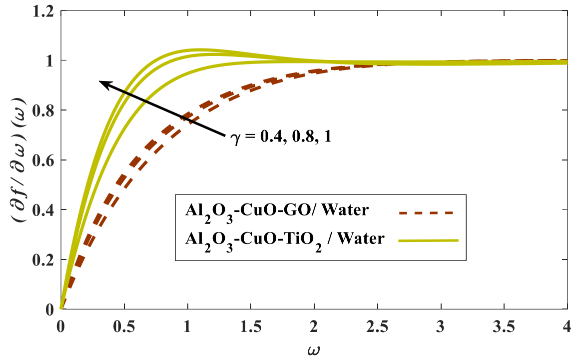

Fig. 9. Velocity profiles (∂ *f*/ ∂ *ω*) (*ω*) when χ = 0.001, β = 2, M = 0.2, λ = 2

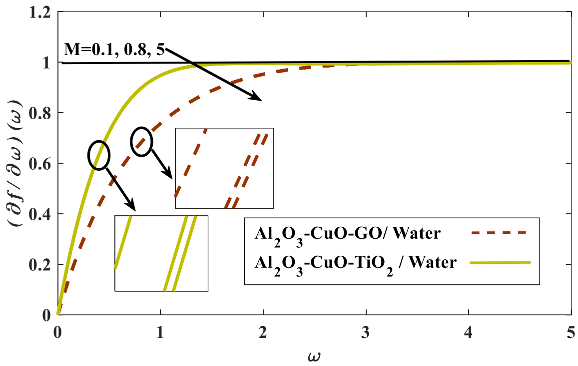

Fig. 10. Velocity profiles (∂ *f*/ ∂ *ω*) (*ω*) when χ = 0.001, $\gamma = 0.4$, β = 2, λ = 2

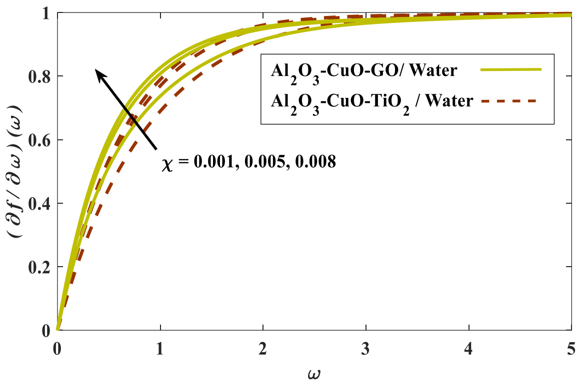

Fig. 11. Velocity profiles (∂ *f*/ ∂ *ω*) (*ω*) when, $\gamma = 0.4$, β =2, M = 0.2, λ =2

Table 3 depicts the extent to which all parameter simulates the local skin friction coefficient of hybrid nanofluids of two types Al_2O_3-CuO-TiO_2 and Al2O3-CuO-GO suspended in water. Local skin friction values augment as conjugate parameter values increase. This rise is due to the correlation of the conjugate parameter with the temperature components of the boundary conditions. On the other hand, it has been marked that increasing the conjugate parameter tends to decline the local Nusselt number. These declines are due to the inverse relationship between the Nusselt number and temperature as shown in the law of the Nusselt number. Besides that, it has been found that boosting the nanoparticle volume fraction coefficient relieves the local skin friction and improves the Nusselt number, for two types of used ternary hybrid nanofluid. Also, concerning a magnetic parameter, we notice that the local skin friction and the Nusselt number decrease significantly when this parameter increases. Explaining that fluid flow is created by a growth in the strength of the magnetic domain, which are determined the convection and then lowers the local skin friction and heat transfer. In the same table, describes the relationship *β* with the skin friction coefficient, and Nusselt number, concurrently. It is seen that the Casson parameter *β* is in backward relation to the skin friction coefficient, but it is straight proportional to the Nusselt number, physically, when the values of *β* increase, the yield stress drops, and thus the skin friction coefficient declines. The influence of going up mixed convection parameter outcomes on the local skin friction and Nusselt number is expounded table. It is clear that the growth of the values of this parameter tends to increase local skin friction and Nusselt number. Raising the external flow fluids improves the fluid motion, which enhances heat transfer and friction. It is remarking that for all values of utilized parameters *β, λ, γ,* M, or *χ*, Al_2O_3-CuO-GO is most prominent in behaviors of *Nu* and C_f.

Table 3
Outcomes of *Nu* and C_f with several values γ, χ, M, β and λ

					Al_2O_3-CuO-TiO_2 Water		Al_2O_3-CuO-GO Water	
γ	χ	β	M	λ	C_f	*Nu*	C_f	*Nu*
0.4	0.001	2	0.6	2	6.3285	1.4896	7.0917	1.5991
0.8					7.7094	1.3321	8.7993	1.4044
1					8.2711	1.2925	9.3558	1.3901
0.4	0.001	2	0.6	2	6.4316	1.4896	7.2241	1.5991
	0.005				6.3880	1.5562	7.1201	1.6623
	0.008				6.3285	1.6259	7.0917	1.6971
0.4	0.01		0.1	2	6.4226	1.6008	7.1838	1.7145
			0.8		6.3990	1.5060	7.1473	1.6617
			5		6.3285	1.4896	7.0917	1.5991
1	0.002	2	0.5	-1	3.0452	1.3101	3.9964	1.4830
				0	7.7094	1.6739	8.5716	1.8024
				3	8.7813	1.7928	10.1732	1.8573
1	0.002	1	0.5	2	29.5272	1.4896	31.8739	1.5991
		3			15.7358	1.8580	16.5721	2.1870
		5			6.3285	1.8965	7.0917	2.5587

6. Conclusions

In this study, we have investigated the demeanor of Casson ternary hybrid nanofluids-based magnetic fields from a vertical stretching sheet created by mixed convection under Newtonian heating impacts. So, the next conclusions are worth noting

i. The magnetic and Casson parameters have an opposite relation with velocity profiles, but the nanoparticle volume fraction, conjugate, and mixed convection parameters have a positive relationship with velocity profiles.
ii. In view of a boost in Casson and mixed convection parameters, the temperature profiles are increased.
iii. The values of C_f rise when the conjugate and mixed convection parameters are increased. also, when the values are increased for the nanoparticle volume fraction, magnetic, and Casson parameters, C_f values are decreased
iv. Nusselt number rises with growing nanoparticle volume fraction and Casson parameters.
v. Generally, the result of this study deduced that the demeanor of Casson ternary hybrid nanofluid flow is greatly impacted by mixed convection, conjugate, Casson, nanoparticle volume fraction, and magnetic parameters. Thus, this investigation is anticipated to count as a scientific insertion into the fluid mechanic field.

vi. In this paper, only Casson mixed convection boundary layers in MHD ternary hybrid nanofluid subject to boundary conditions such as Newtonian heating are evaluated. Consequently, there are a lot of studies that can be examined for forthcoming investigations. For example; considering other effects as micropolar ternary nanofluid. Also, it can detect the heat transfer characteristics of other boundary conditions like constant heat flux, and study the model over a Vertical Truncated Cone.

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