

Investigation of the Effect of Mutual Diffusion on Hydrodynamic Parameters under Fluid Displacement

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ARTICLE INFO	ABSTRACT
Article history: Received 21 December 2023 Received in revised form 8 May 2024 Accepted 20 May 2024 Available online 15 June 2024	Formation of stagnant zones in a reservoir is mainly determined by differences in the flow velocity of fluids in heterogeneous micro and macro porous media. Additional resistance forces to fluid flow, which are of a different nature, result in a decrease in flow velocity. Forces arising from the diffusion of fluids with different physical-chemical characteristics and acting in different directions are one such class of the above-mentioned drag forces. An increase in the degree of fluid mineralization is capable of causing changes in the flow of a continuous diffusion layer. The mathematical solution of the problems considered in the study was solved using the "Matlab" program using the Finite Difference Method. The paper presents the results of displacement studies in porous media under the assumption that the flow characteristics of displacing and displaced fluids can be controlled. Theoretical and experimental substantiations showing the effect of diffusion process in fluids on flow properties have been carried out in the displacement process. The
Diffusion; porous medium; displacement; concentration; fluid flow	theoretical solution and modelling of concentration change in flow of mutually soluble dispersed solutions were presented. The effect of diffusion on the flow velocity in the process of displacement of mutually soluble liquid mixtures has been evaluated.

1. Introduction

Control of the basic physical-chemical characteristics of water used in the waterflood system in oilfield development is one of the actual issues of hydrocarbon production. Inconsistency between injected and displaced fluids in terms of their physical and chemical properties leads to a number of complications, such as injected water breakthrough through highly permeable channels in oil-saturated reservoirs, fractured displacement front, reduced injection rates, etc. as shown in previous studies [1-4].

It is well known that mutual fluid displacement in porous media is almost never complete. A part of displaced fluid remains in pores and areas are formed where both fluids are simultaneously stagnant zones. Revealing regularities of formation of such zones in porous media and the question

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of their influence on flow process is a rather complicated problem. The conducted researches showed that mutual displacement of liquids in porous medium is accompanied by periodical change of flow rate, the results of which are summarized in the studies by Van Duijn *et al.*, [5], Zaccardi *et al.*, [6], Wen *et al.*, [7], Zalc *et al.*, [8], and Gunawan *et al.*, [9]. These changes are oscillatory in nature and largely depend on the physical and chemical characteristics of the displaced and displacing fluid, as shown by Khalid *et al.*, [10], and Matveenko and Kirsanov [11]. As a fluid flows through a porous medium, it continuously interacts with the diffusion boundary layer. A continuous diffusion layer can cause changes in the flow velocity with increasing salt concentration. Due to diffusion processes in the boundary layer a layer of density centers is continuously formed which causes a change in the percolation rate of the fluid [11]. This hypothesis is consistent with the mechanism of optimal layer formation (diffusion layer) in the form of "semi-micelles". Layer saturation is accompanied by migration of salts. This phase boundary shift is accompanied by a change in pressure gradient, resulting in fluctuating displacement flow rate and filtration velocity.

Studies have shown that this process in porous media can be governed by the addition of salts to the displacing fluid. Some authors note that increase of salinity of solution leads to increase of density of diffusion layer and with increase of salinity ions are distributed in diffusion layer, as can be seen in the research results by Verhallen *et al.*, [12], Goldobin and Brilliantov [13], and Sahri *et al.*, [14]. Such regulation may be possible by selecting ratios of salt concentrations in compressible and compressible liquids.

In particular, seawater used as a displacing agent in the flooding system of offshore hydrocarbon fields differs from formation water in its physical and chemical properties, which may result in nonequilibrium processes during displacement, fractality of the displacement front, outstripping water breakthrough through channels with high permeability [12,13]. The density incompatibility of fluids also greatly affects the displacement pattern in the filtration process. It is assumed that water of a certain density injecting into the reservoir can increase the injectivity of injection wells and increase the reservoir sweep efficiency [14].

Thus, there is a need for more detailed mutual displacement studies of fluids with different physical and chemical properties, studies on mutual displacement in porous media have been carried out. In these studies, the effect of mutual displacement of fluids in a porous medium on the nature of flow rate changes was investigated. It was supposed that changes here can depend on physicochemical properties of displacing and displaced fluids.

2. Experiments

In experimental studies performed on a porous medium model, the filtration process was carried out with mutual displacement of seawater and seawater with different salt content (NaCl). The model of porous medium was a mixture of silica sand and silica dust with water permeability of $0.10 \ \mu m^2$, saturation of the porous medium with fluid was carried out from a tank connected to the thermostatically controlled porous medium (Figure 1).



Fig. 1. Experimental setup: 1 - vacuum line; 2 - porous medium; 3 - manometer; 4 - PVT cell; 5 - ultra-thermostat; 6 - pressure measuring device; 7 – monitor

In an experimental study, the process by mutual displacement of seawater and seawater at different salt - *NaCl* concentrations were simulated in a porous medium model. Different changes in diffusion in the mutual displacement of liquids have occurred with changes in concentration. It was seen that the diffusion concentration in the displacing and displaced liquids and the direction of percolation changes according to the diffusion direction, as shown in the study by Tey *et al.*, [15]. As studies show, the mutual displacement of fluids in a porous medium is accompanied by periodic changes in fluid flow rate. These changes depend on the physical and chemical properties of the displacing and displaced fluids [16]. In the course of the study, the rate of fluid flow rate reduction from a porous medium and the diffusion direction have been observed to vary depending on fluid salinity. The results obtained are shown in Figure 2 to Figure 9. The analysis of experimental results has shown that an important condition is the change in flow rate depending on diffusion direction. Depending on the flow direction, some of the results of the studies carried out in the porous medium were as follows.



Fig. 2. Seawater displacement with "seawater +1% *NaCl* " solution

Rate of flow drop is

$$Q/t = (1.85 - 1.79)/180 = 0.00033 \cdot 10^7 m^3 / s^2$$



Fig. 3. Displacement of "seawater +1% NaCl " solution with pure seawater

Rate of flow drop

$$Q/t = (2.9 - 2.2)/180 = 0.004 \cdot 10^7 \ m^3 \ s^2$$
⁽²⁾



"seawater +2% NaCl " solution

Rate of flow drop

$$Q/t = (2.25 - 2.18)/180 = 0.00039 \cdot 10^7 \ m^3 \ / \ s^2$$
(3)

(1)



"seawater +1% *NaCl* " solution

Rate of flow drop

 $Q/t = (2.28 - 2)/180 = 0.0016 \cdot 10^7 m^3 / s^2$ 1,85 1,8 1,75 Flow rate, Q \cdot 6 \cdot 10⁷, m³/s 1,7 1,65 1,6 1,55 1,5 1,45 1,4 1,35 0 50 150 100 200 250 Time t/60, sec Fig. 6. Displacement of "seawater +2% NaCl" solution with "seawater +3% NaCl " solution

Rate of flow drop

$$Q/t = (1.8 - 1.62)/180 = 0.001 \cdot 10^7 \ m^3 \ / \ s^2$$
(5)

(4)



"seawater +2% NaCl " solution

Rate of flow drop is

$$Q/t = (2.02 - 1.85)/120 = 0.0014 \cdot 10^7 m^3 / s^2$$



"seawater +4% *NaCl* " solution

Rate of flow drop

$$Q/t = (2.31 - 2)/182 = 0.0017 \cdot 10^7 \, m^3 \, s^2 \tag{7}$$

(6)



+3% NaCl " solution

Rate of flow drop

$$Q/t = (2.31 - 1.92)/180 = 0.002 \cdot 10^7 \ m^3 \ / \ s^2$$
(8)

The results show that it is possible to reverse the diffusion process by adjusting the salt concentration in the displacement, thereby influencing the flow parameters in the porous medium. In this case, the regulation of the displacement process in the porous medium is considered to be an important factor [17].

3. Theoretical Background Research

In order to show some features of the microscopic view of the displacement process in which miscible liquids are involved, consider first the displacement of one liquid from a circular rectilinear capillary by another liquid miscible with it. If the viscosities of the two liquids are the same and the densities are not the same, then the speed distribution across the tube will depend on the properties of the liquids.

As shown in research by Collins [18], for slow steady flow the velocity at a distance from the pipe axis is equal

$$\upsilon(r) = 2\overline{\upsilon} \left(1 - \frac{r^2}{a^2} \right) \tag{9}$$

Here, *a* is the radius of the tube, \overline{v} is the average velocity across the cross section. Thus, at the wall the liquid does not move at all, while at the tube's axis the liquid's velocity is maximum.

If at the moment t=0 the displacing liquid concentration distribution is described by the function C(x,r), where x the coordinate is counted along the axis, then due to convection at the moment t it will already be described by the function

$$C = C(x - \upsilon t, r) \tag{10}$$

where v is defined by Eq. (1). Consequently, as a result of convection alone, the displacing liquid is dispersed into the displaced liquid. As the liquids mix, dispersion of the displacing liquid is also due to diffusion. To derive the equation describing the concentration change due to convection and diffusion, we use the law of conservation of mass of the displacing liquid.

As the liquids mix, the distribution of the displacing liquid in the displaced one is also due to diffusion. To derive the equation describing the concentration change due to convection and diffusion, we use the law of mass conservation of the displaced liquid.

Consider *a* test volume inside the tube, located between cylinders of radius *r* and $r + \Delta r$ and cross-sections *x* and $x + \Delta x$. Equating the mass of liquid flowing into the volume due to diffusion and convection to the incremental mass of this liquid in the test volume itself, we obtain the equation [18].

$$D\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial x^2}\right) = \rho \frac{\partial C}{\partial t} + 2\rho \overline{\nu} \left(1 - \frac{r^2}{a^2}\right) \frac{\partial C}{\partial x}$$
(11)

In most cases, diffusion along the axis can be neglected compared to diffusion in the radial direction (by displacing a low-concentration *NaCl* solution with a highly concentrated solution). Therefore, the term $\frac{\partial^2 C}{\partial x^2}$ can be neglected. Introducing dimensionless variables

$$\bar{x} = \frac{x}{a}; \quad \bar{r} = \frac{r}{a}; \quad \tau = \frac{Dt}{\rho a^2}; \quad \bar{C} = \frac{C}{C_0}$$
 (12)

where C_0 is some characteristic concentration, we obtain the equation

$$\frac{\partial^2 C}{\partial r^2} + \frac{1}{\bar{r}} \frac{\partial C}{\partial \bar{r}} = \frac{\partial C}{\partial \tau} + 2 \frac{a \rho \bar{\nu}}{D} \left(1 - \bar{r}^2 \right) \frac{\partial \bar{C}}{\partial \bar{x}}$$
(13)

As a boundary condition on the pipe wall, we require that

$$\frac{\partial \overline{C}}{\partial r} = 0$$
 at $\overline{r} = 1$ (14)

This means that there is no diffusion through the wall. It is assumed that diffusion in radial direction prevails considerably over convection in axial direction. If in Eq. (11) the term containing $\frac{\partial C}{\partial x}$, is discarded, then the solution of the resulting equation is the function

$$\overline{C} = e^{-\alpha \tau} J_0\left(\sqrt{\alpha r}\right) \tag{15}$$

Here J_0 is a Bessel function of the first kind of zero order. From the boundary condition Eq. (14) we find that

$$J_1\left(\sqrt{\alpha}\right) = 0 \tag{16}$$

The smallest root of this equation is $\sqrt{\alpha} = 3.8$. Therefore, the time it takes for the concentration difference to decrease to 1/e of its original value is

$$\tau_1 = (3.8)^{-2} \text{ or } t_1 = \frac{a^2 \rho}{D} (3.8)^{-2}$$
 (17)

On the other hand, if at the initial moment the concentration of the displacing fluid was non-zero over the length l of the tube, the time after which convection will cause this distribution to spread appreciably along the axis is of the order of magnitude.

$$t_2 \approx \frac{2l}{\nu} \tag{18}$$

Therefore, convection will prevail over diffusion if

$$t_2 \ll t_1 \tag{19}$$

or

$$\frac{2l}{\nu} << \frac{a^2 \rho}{D} (3.8)^{-2} \tag{20}$$

Conversely, if

$$\frac{2l}{\nu} << \frac{a^2 \rho}{D} (3.8)^{-2} \tag{21}$$

then diffusion will prevail over convection.

3.1 Concentration Changes During Displacement of Mutually Soluble Solutions

In the most general form, the flow of two mutually soluble liquids with different physico-chemical properties is described by Bagriantsev *et al.*, [19] using a system of differential equations:

$$m\frac{\partial C}{\partial t} + \frac{k}{\mu(C)} [grad \ p - g\rho(C)gradC] - D\Delta C = 0$$

$$div \left[\frac{k}{\mu(C)} (grad \ p - g\rho(C))\right] = 0$$
(22)

where *C* is concentration of the solution at a certain point of the porous medium; *m* - porosity; *k* - permeability; *p* - pressure; $\mu(C)$, $\rho(C)$ - respectively, viscosity and density of the solution, which are functions of concentration. We can write the equation as an identity, assuming that concentration transitions in the process of displacement of different solutions, happen quickly enough [20-23].

$$m\frac{\partial C}{\partial t} + \frac{V(t)}{x^{n}} - \frac{\partial C}{\partial x} - D\frac{1}{x^{n}}\frac{\partial}{\partial x}\left(x^{n}\frac{\partial C}{\partial x}\right) = 0$$
(23)

The initial and boundary conditions for solving the equation, are as follows

$$C|_{t_0=0} = e^x; \qquad C|_{x=0} = 1; \qquad C|_{x=1} = 0$$
 (24)

Here, m = 0.01, v = 0.1, D = 0.1 the change in concentration has been estimated using MatLab software and is shown in Figure 10 and Figure 11.



Fig. 10. Changes in concentration of aqueous solutions in porous media



Fig. 11. Variation of concentration in porous media in the two-dimensional case

3.2 Effect of Diffusion on Fluid Velocity in the Displacement Process

Later, in laboratory experiments, the dependence of speed change on diffusion and intensity was found [16,17]. Figure 12 and Figure 13 show diagrams of intensity and diffusion variations depending on flow direction within investigated conditions.



Fig. 12. Dependence of velocity change on diffusion D and intensity λ (with diffusion in the flow direction)

In the case where diffusion goes in the direction of flow

$$\upsilon(t) = \frac{D}{\lambda} \tag{25}$$

When diffusion happens in the opposite direction to the flow

$$\upsilon(t) = -\frac{D}{\lambda} \tag{26}$$

and the diffusion intensity is opposite to the velocity. These dependencies are accepted under the following conditions

$$D = 0.1 \div 1; \ \lambda = 0.1 \div 0.5$$
 (27)



Fig. 13. Dependence of speed change on diffusion D and intensity λ (when diffusion happens in the opposite direction to the flow)

4. Conclusions

Experimental and evaluation substantiations showing the influence of diffusion processes on fluid flow parameters in the process of displacement in porous media are presented.

In the course of investigations change of rate of fluid flow reduction depending on salt concentration in fluids in relation to diffusion direction is studied.

The possibility of controlling flow parameters in porous media by changing the direction of the diffusion process caused by changes in mineralization of solutions during displacement has been substantiated.

Analysis of the results of the study shows that the rate of change in fluid flow rate, is an important condition of the filtration process, which depends on the diffusion direction.

Graphs of changes in diffusion intensity and direction depending on given conditions are presented.

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