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Effects of Suction/Injection on Stagnation Point Flow over a Nonlinearly Stretching/Shrinking Sheet in a Carbon Nanotubes

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ABSTRACT

This research will explore the issue of stagnation point flow in carbon nanotubes with suction/injection impacts by a nonlinear stretching/shrinking sheet. By practising a similarity transformation, the governing partial differential equations (PDEs) are converted to a scheme of nonlinear ordinary differential equations (ODEs). Then, settled numerically with applying a *bvp4c* solver in Matlab. Two types of carbon nanotubes (CNTs) are used which are SWCNTs (single-walled) and MWCNTs (multi-walled) and the base fluid used is water. In the form of graphs, the impact of the velocity, temperature, skin friction and numbers of Nusselts parameter is researched and displayed and interpreted physically. It is found that if only suction rises, the range of solutions will increase.

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1. Introduction

The flow due to stretching sheet is a major issue in fluid mechanics because of its enormous applications in many assembly types in sector, for instance, extraction of polymer sheets, drawing of wire, paper development and many more. The first investigation on the boundary layer flow through a linear stretching sheet was via Crane [1]. Meanwhile, Lok *et al.*, [2] analysed stagnation point flow by a linear shrinking surface with MHD. After that, many researchers interested to extend their work [3-7]. Most of the literature available is about studying the boundary layer flow considering linear surface. We should mention that stretching isn't always linear, several authors have also researched the issue of nonlinear stretching sheet.

Vajravelu [8] studied heat transfer for nonlinear stretching surface. Cortell [9] continued the paper by [8] where two distinct forms of thermal boundary conditions are considered on the sheet.

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Prasad *et al.*, [10] studied the heat transfer of convection by a nonlinear stretching surface with changeable fluid characteristics. Besides paper mentioned, there is also many other researchers who interested to study on nonlinear stretching/shrinking sheet [11-17].

As many study related to nanofluid, and also due to its electrical and mechanical characteristics, CNTs also demonstrate outstanding outcomes. Thus, Choi *et al.*, [18] researched oil-based CNTs' heat conductivity. CNTs is an allotrope of carbon, tube-shaped material and made of carbon. They consist of SWCNT and MWCNTs. Compared to the various nanoparticles with a similar fraction of volume [19-20], CNTs give a greater thermal property. This will enhance both heat transfer of convection and base fluids' thermic conductivity. Since then, numerous researchers come across the benefits of CNTs and investigated various boundary layer problem on CNTs [21-26].

Despite the literature that is often quoted, no study has been made for stagnation point flow by a nonlinear stretching/shrinking surface with CNTs alongside suction/injection. To do so, we extend Malvandi *et al.*, [27] paper which they studied on stagnation point flow of porous nonlinear stretching/shrinking surface.

2. Methodology

Consider an incompressible steady flow concerning stretching/shrinking sheet in CNTs alongside suction/injection. The velocity of free stream and velocity of sheet are presumed to differ nonlinearly from a steady point of stagnation, which complement to $U_w(x) = ax^n$ and $U_\infty(x) = bx^n$, respectively, where a and b are constants. Both SWCNTs and MWCNTs are used with water base fluid. The boundary layer equations can be addressed as follows [28]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_\infty \frac{dU_\infty}{dx} + \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} \quad (3)$$

and the boundary conditions are

$$u = U_w, v = V_w, T = T_w \text{ at } y = 0 \quad (4)$$

$$u \rightarrow U_\infty, T \rightarrow T_\infty \text{ as } y \rightarrow \infty$$

It should be mention that V_w is the mass transfer velocity, and μ_{nf} , α_{nf} , ρ_{nf} are the viscosity, thermal diffusivity and density of the nanofluid, respectively, that Oztop and Abu-Nada [29] offer

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}, \rho_{nf} = (1 - \phi)\rho_f + \phi\rho_{CNT},$$

$$(\rho C_p)_{nf} = (1 - \varphi)(\rho C_p)_f + \varphi(\rho C_p)_{CNT}, \quad \frac{k_{nf}}{k_f} = \frac{1 - \varphi + 2\varphi \frac{k_{CNT}}{k_{CNT} - k_f} \ln \frac{k_{CNT} + k_f}{2k_f}}{1 - \varphi + 2\varphi \frac{k_f}{k_{CNT} - k_f} \ln \frac{k_{CNT} + k_f}{2k_f}} \quad (5)$$

where φ is the CNTs volume fraction, $(\rho C_p)_{nf}$ and k_{nf} are the capacity of heat and thermal conductivity of nanofluid, while $(\rho C_p)_{CNT}$, k_{CNT} and ρ_{CNT} are the capacity of heat, thermal conductivity and density of CNTs, respectively, and ρ_f and k_f are the density and thermal conductivity of the fluid. The use of the term for k_{nf}/k_f were taken from Xue [30] where the Maxwell's theory model considers the effects of CNTs space distribution on thermal conductivity.

By introducing the following variables of similarity, we also search similarity solution for Eq. (1)-(3) with boundary conditions (4)

$$\eta = \left(\frac{(n+1)b}{2v_f} \right)^{1/2} y x^{\frac{n-1}{2}}, \quad \psi = \left(\frac{2bv_f}{n+1} \right)^{1/2} x^{\frac{n+1}{2}} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (6)$$

where η is the variable of similarity and ψ is the function of stream described as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, which comply with Eq. (1) identically. Using Eq. (6), Eq. (2)-(3) can be reduced to these ODEs

$$\frac{1}{(1-\varphi)^{2.5}(1-\varphi+\varphi\rho_{CNT}/\rho_f)} f'''' + ff'' + \beta(1 - f'^2) = 0 \quad (7)$$

$$\frac{1}{Pr} \left[\frac{k_{nf}/k_f}{1-\varphi+\varphi(\rho C_p)_{CNT}/(\rho C_p)_f} \right] \theta'' + f\theta' = 0 \quad (8)$$

Thus, subject to boundary conditions (4) we have

$$f(0) = S, f'(0) = \varepsilon, \theta(0) = 1$$

$$f'(\eta) \rightarrow 1, \theta(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty. \quad (9)$$

where $\beta = \frac{2n}{n+1}$ is the nonlinear parameter which varies from 1 to 2 as n grows from unity to infinity, as stated in Malvandi *et al.*, [27], S is suction/injection parameter, Pr is the Prandtl number and ε is the stretching/shrinking parameter given by

$$S = \frac{-V_w}{\left(\frac{bv_f(n+1)}{2} \right)^{\frac{1}{2}} x^{\frac{n-1}{2}}}, \quad Pr = \frac{v_f}{\alpha_f}, \quad \varepsilon = \frac{a}{b} \quad (10)$$

which for stretching is when $\varepsilon > 0$ and shrinking is when $\varepsilon < 0$, while suction when $S > 0$ and injection when $S < 0$.

Physical interest's quantities in this research are the coefficient of skin friction C_f and the local Nusselt number Nu_x , identified as

$$C_f = \frac{\tau_w}{\rho_f U_\infty^2}, \quad Nu_x = \frac{xq_w}{k_f(T_w - T_\infty)} \quad (11)$$

in which the surface shear stress τ_w and the surface heat flux q_w are likely given as

$$\tau_w = \mu_{nf} \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k_{nf} \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (12)$$

with μ_{nf} is the viscosity of the nanofluids and k_{nf} is the thermal conductivity of the nanofluids. The quantities of physical interest that we acquire following transformation are

$$C_f Re_x^{1/2} = \frac{1}{(1-\varphi)^{2.5}} \sqrt{\frac{1}{2-\beta}} f''(0), \quad (13)$$

$$\frac{Nu_x}{Re_x^{1/2}} = -\frac{k_{nf}}{k_f} \sqrt{\frac{1}{2-\beta}} \theta'(0), \quad (14)$$

where $Re_x = U_\infty x / \nu_f$ is the local Reynolds number.

3. Results

Eq. (7)-(8) are numerically solved by applying the bvp4c package in Matlab, in conjunction with boundary conditions (9). Following to Oztop and Abu-Nada [28], we have acknowledged the selection of φ ($0 \leq \varphi \leq 0.2$), where $\varphi = 0$ is regular fluid with $Pr = 6.2$. (water). The thermophysical properties of the base fluid and the CNTs are indexed as in Table 1.

Table 1
 Thermophysical properties of CNTs [31]

Physical properties	Base fluids	Nanoparticle	
		SWCNT	MWCNT
ρ (kg/m^3)	997	2600	1600
c_p (J/kgK)	4179	425	796
k (W/mK)	0.613	6600	3000

Figure 1 illustrate the $f''(0)$ and $-\theta'(0)$ graphs for ε and φ , where $\varphi = 0, 0.1$ and 0.2 for water-SWCNTs at $S = 0.5$ and $\beta = 2$. There exist dual solutions when $\varepsilon_c < \varepsilon \leq -1$, unique solution when $\varepsilon > -1$ and no solutions when $\varepsilon < \varepsilon_c < 0$ (ε_c is the critical value). Figure 2 show the $f''(0)$ and $-\theta'(0)$ graphs for ε and S , where $S = -0.5, 0$ and 0.5 for water-SWCNTs when $\beta = 1.5$ and $\varphi = 0.1$. As parameter S increases, it also increases the skin friction and heat loss from the surface. Hence, the suction slows the separation of the boundary layer while the injection speeds it up.

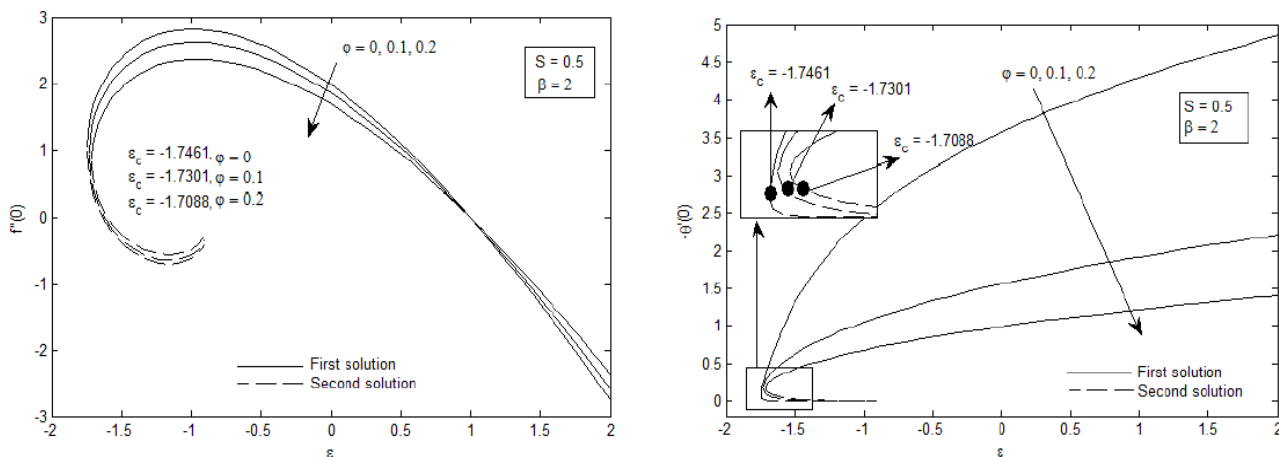


Fig. 1. $f''(0)$ and $-\theta'(0)$ graphs for φ and ε with water-SWCNTs, respectively

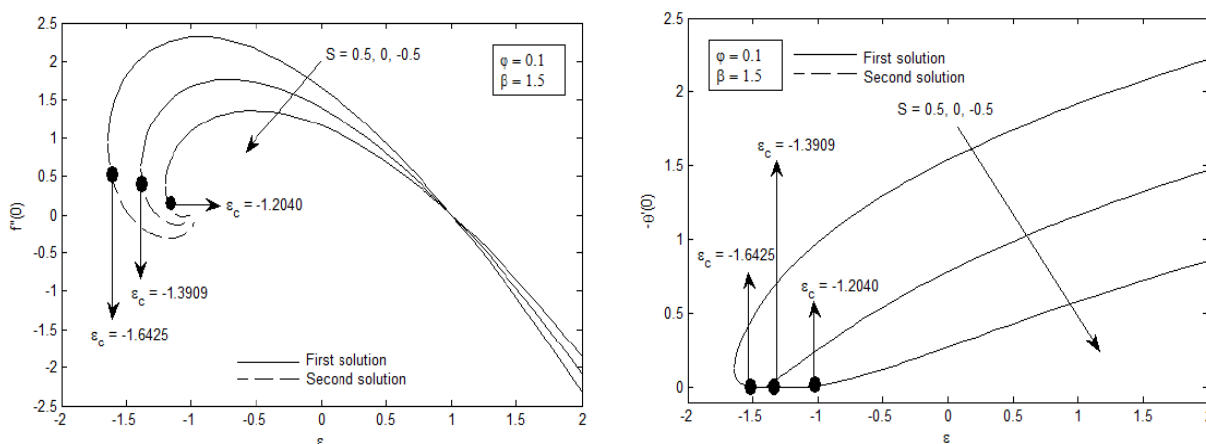


Fig. 2. $f''(0)$ and $-\theta'(0)$ graphs for S and ε with water-SWCNTs, respectively

Figure 3 show the $f''(0)$ and $-\theta'(0)$ graphs for ε and β , where $\beta = 1, 1.5$ and 2 for water-SWCNTs when $S = 0.5$ and $\varphi = 0.1$. It shows that when β increases, the velocity gradients as well as temperature gradients also increases. From both Figure 2 and 3, we can conclude that parameter $S > 0$ (suction) and β widen the range of solutions compared to φ .

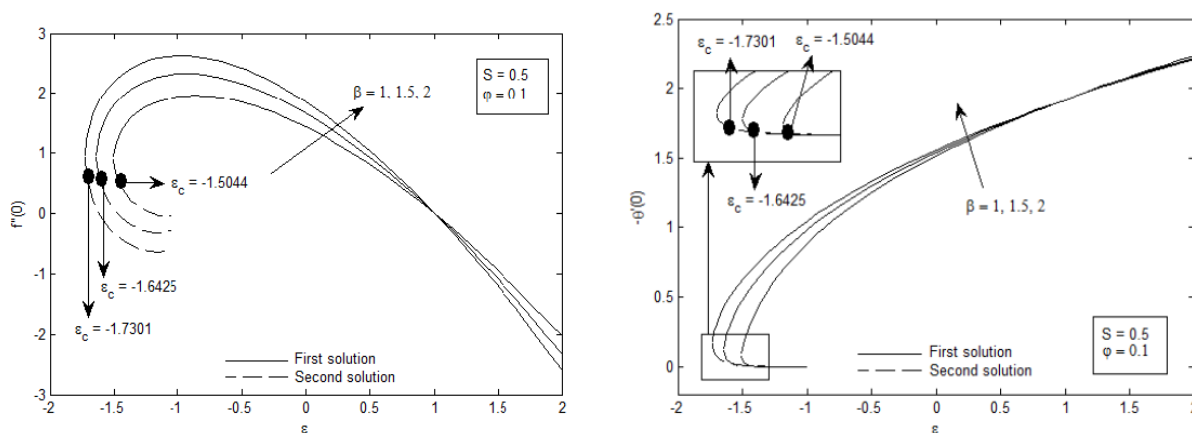


Fig. 3. $f''(0)$ and $-\theta'(0)$ graphs for β and ε with water-SWCNTs, respectively

Figure 4 explain the coefficient of skin friction and the local Nusselt number graphs, as per Eq. (13)-(14) for φ and S which are $S = -0.5, 0$ and 0.5 with $\varepsilon = 0.5$ and $\beta = 1.5$. It is concluded that, when the $S > 0$ (suction) parameter is increasing, the coefficient of skin friction also increasing along with the local Nusselt number. The higher coefficient of skin friction and local Nusselt number are SWCNTs compared to MWCNTs, because their density and thermal conductivity is higher.

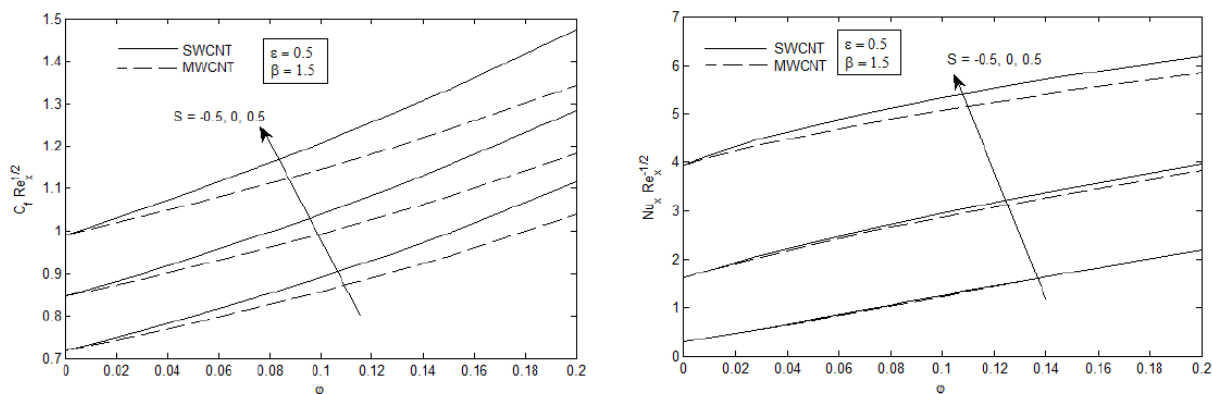


Fig. 4. Coefficient of skin friction and local Nusselt number graphs for φ , S and CNTs, respectively

Figure 5-7 show the outcomes of S , β and CNTs on the profiles of velocity and temperature, respectively. The term for both first and second solutions applies to the curves shown in Figure 1-3 and asymptotically these profiles follow the boundary conditions (9), which then support the existence of dual solutions shown in Figure 1-3.

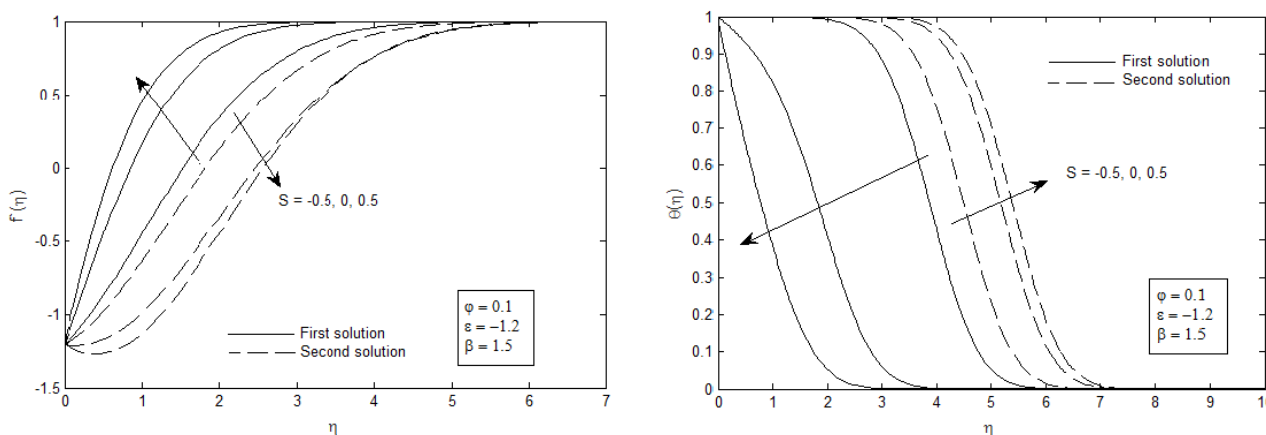


Fig. 5. Profiles of velocity and temperature for S with water-SWCNT, respectively

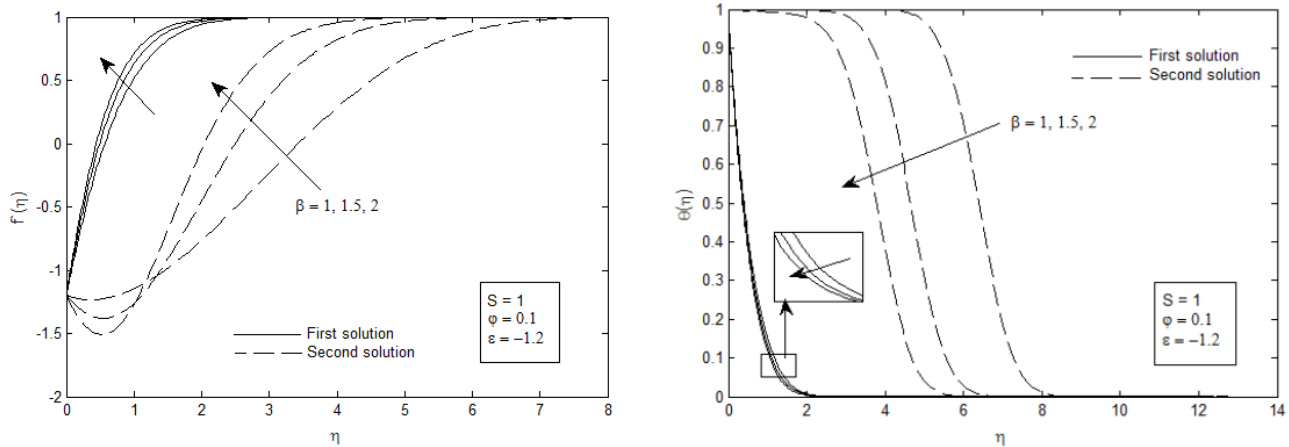


Fig. 6. Profiles of velocity and temperature for β with water-SWCNT, respectively

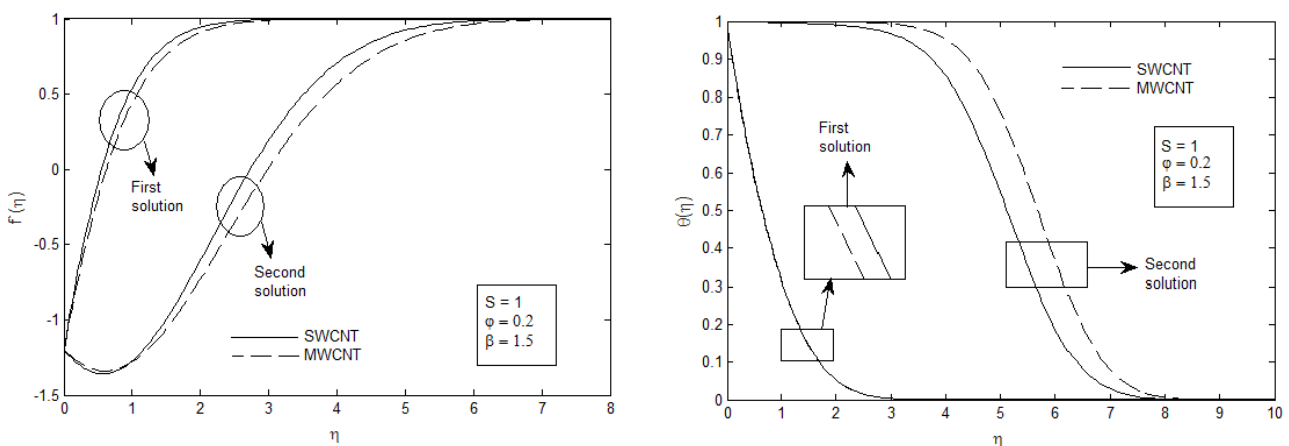


Fig. 7. Profiles of velocity and temperature for CNTs in water base fluid, respectively

4. Conclusions

In this paper, we have investigated theoretically and analysed the φ and S consequences on the stagnation point flow over a stretching/shrinking sheet. The results indicate that

- i. Solutions for a stretching sheet are unique and solutions for a shrinking sheet are dual.
- ii. The range of solutions widen with an increase of S as well as β parameters.
- iii. While, for injection, it decreases the range of solutions.
- iv. As $S > 0$ (suction) increases, the coefficient of skin friction increases too.
- v. The heat transfer also increases with an increase of $S > 0$ (suction) parameter.
- vi. SWCNTs are more effective than MWCNTs in both skin friction and local Nusselt number.

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