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Thermocapillary Convection in a Deformable Ferrofluid Layer

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ABSTRACT

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A linear stability evaluation is conducted to explore the effect on the onset of Marangoni-Bénard convection in a ferrofluid layer system. The system is heated from below with treatment of both the lower and upper boundaries to completely insulate the temperature disturbance. The eigenvalue problem is solved by using regular perturbation technique to obtain the critical number of Marangoni and also the critical number of Rayleigh. It is observed that the increase in the value Crispation, the magnetic number of Rayleigh and also the magnetic number will destabilize the system while the increasing number of Bonds will delay the convection.

Keywords:

Deformable; Marangoni; Ferrofluid

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1. Introduction

The initial idea of combination between buoyancy force with surface tension forces initiated by Nield [1]. Yang [2] studied the combination of Bénard and Marangoni convection that focusing on the thickness of the plate. The mixed of Rayleigh-Bénard with Marangoni-Bénard in the presence of temperature-dependent viscosity has been done by Skarda and Mccaughan [3]. Marangoni-Bénard and Rayleigh-Bénard convection in a ferrofluid layer system with magnetic field had been examined by Hennenberg *et al.*, [4].

Ferrofluid previously used by the NASA as a rocket fuel and it also used in electrical appliance such as speaker. Kaiser and Miskolczy [5] stated that the ferrofluid is a unique fluid that contains a small particle of magnetic. Finlayson [6] analyzed the instability of convection in the ferrofluid layer system. After that, Stiles and Kagan [7] extended the research with additional strong magnetic field in the ferrofluid layer system. Stationary convection of the ferrofluid in a viscoelastic has been studied by Laroze *et al.*, [8]. The porous layer of the ferrofluid system in the existence of a vertical magnetic field in Brinkman-Benard-Marangoni convection has been examined by Shivakumara *et al.*, [9]. Shivakumara *et al.*, [10] examined the impact of coriolis force and magnetic field dependent (MFD).

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Another research of convection in ferrofluid with the effect of MFD viscosity has been done by Prakash *et al.*, [11].

An attempted to study the stability of the deformable surface with the boundaries are considered isothermal in a numerical way has been done by Benguria and Depassier [12]. Pérez-García and Carneiro [13] also examined the instability of Bénard-Marangoni convection in a deformable surface with a restriction of Prandtl value in order to study the crispation number and bond number. Double-diffusive studied with a deformable effect has been done by McCaughan and Bedir [14]. Another study related to the deformable surface has been done by Garcia-Ybarra *et al.*, [15] with consideration of buoyancy or without of buoyancy. Combination of Marangoni with Couple-Rossenweig in ferrofluid layer system with the upper deformable free boundary has been examined by Hennenberg *et al.*, [16]. The study of deformable effect in Marangoni convection with the combined effect of the magnetic field has been done by Hashim and Arifin [17]. In the presence of internal heating, Mokhtar and Hamid [18] also researched the deformable impact of binary fluid layer convection in Bénard-Marangoni.

In this study, we are focusing on the effect of the deformable boundary layer in a ferrofluid layer system on the onset of Marangoni-Bénard convection. We thought the lower boundary would be rigid while the upper boundary would be deemed a free deformable boundary. In this system, the primary things we observed are surface tension and thermal buoyancy. Then, using regular perturbation technique to obtain the equation of critical Marangoni and critical thermal Rayleigh.

2. Methodology

We thought of a horizontal layer of ferrofluid heated from below as shown in Figure 1 with thickness d . The lower boundary ($z = 0$) are considered to be rigid while the upper boundary ($z = d$) is set to be deformable free surface. Both boundaries are retained at constant but the temperature of the lower boundary, T_l are higher compared to the upper boundary, T_u .

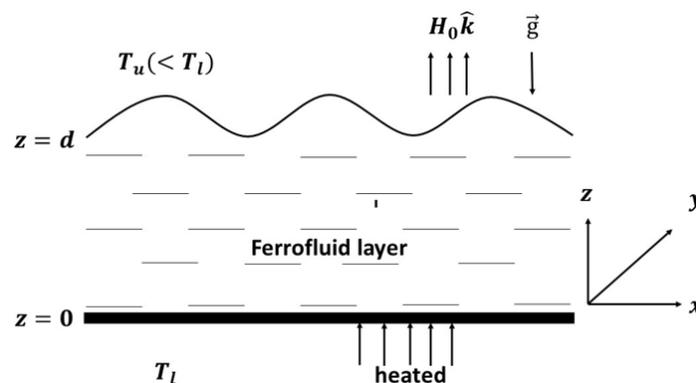


Fig. 1. Physical configuration

The density of the fluid, ρ and also the surface tension, σ are given by

$$\sigma = \sigma_0 - \sigma_T(T - T_0), \tag{1}$$

$$\rho = \rho_0[1 - \alpha_t(T - T_0)]. \tag{2}$$

ρ and σ are considered to linearly with temperature. T_0 , ρ_0 , σ_0 , σ_T and α_t are reference value of temperature, density, surface tension, rate of change of the surface tension at the temperature T and the thermal expansion. Referring to Finlayson [6], the governing equations are

$$\nabla \cdot \vec{q} = 0, \tag{3}$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right] = -\nabla p + \rho \vec{g} + \nabla \cdot (\vec{H} \vec{B}) + \mu \nabla^2 \vec{q}, \tag{4}$$

$$\left[\rho_0 C_{V,H} - \mu_0 \vec{H} \cdot \left(\frac{\partial \vec{M}}{\partial T} \right)_{V,H} \right] \times \frac{DT}{Dt} + \mu_0 T \left(\frac{\partial \vec{M}}{\partial T} \right)_{V,H} \cdot \frac{D\vec{H}}{Dt} = k_1 \nabla^2 T. \tag{5}$$

Here $\vec{q} = (u, v, w)$, μ , μ_0 , p , k_1 , $C_{V,H}$ and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ denote the velocity vector, dynamic viscosity, magnetic permeability of vacuum, pressure, thermal conductivity, specific of magnetic field and heat capacity at constant volume per unit mass and is the Laplacian operator. Based on Finlayson [6] the Maxwell's equation and equation of magnetization are as follows

$$\nabla \cdot \vec{B} = 0, \quad \nabla \times \vec{H} = 0, \tag{6}$$

$$\vec{B} = \mu_0 (\vec{M} + \vec{H}), \tag{7}$$

$$\vec{M} = \frac{\vec{H}}{H} [M_0 + \chi(H - H_0) - K(T - T_0)], \tag{8}$$

where \vec{B} , \vec{H} and \vec{M} denote the magnetic induction, magnetic field density and magnetization. $\chi = \left(\frac{\partial M}{\partial H} \right)_{H_0, T_0}$ and $K = \left(\frac{\partial M}{\partial H} \right)_{H_0, T_0}$ are the magnetic susceptibility, and pyromagnetic co-efficient, $M_0 = M(H_0, T_0)$, $H = |\vec{H}|$ and $M = |\vec{M}|$. The basic state that is quiescent is given by

$$\rho = \rho_b(z), \quad p = p_b(z), \quad \vec{q}_b = 0, \quad T = T_b(z), \quad \vec{M} = \vec{M}_b(z), \quad \vec{H} = \vec{H}_b(z), \tag{9}$$

$$T_b(z) = T_0 - \left(\frac{\Delta T}{d} \right) z, \tag{10}$$

$$\vec{H}_b(z) = \left[H_0 - \frac{K}{1 + \chi} \left(\frac{\Delta T}{d} z \right) \right] \hat{k}, \tag{11}$$

$$\vec{M}_b(z) = \left[M_0 + \frac{K}{1 + \chi} \left(\frac{\Delta T}{d} z \right) \right] \hat{k}, \tag{12}$$

where subscript b stands for the basic state while \hat{k} shows the z -direction in unit vector. The perturbation of the basic state in order to study the basic state are in the following forms

$$\vec{q} = \vec{q}', \quad p = p_b(z) + p', \quad \vec{H} = \vec{H}_b(z) + \vec{H}', \quad T = T_b(z) + T', \quad \vec{M} = \vec{M}_b(z) + \vec{M}', \tag{13}$$

where the primed quantity denotes perturbed variables. Substituted Eq. (13) into Eq. (7) and by using Eq. (8) yields

$$H_x + M_x = \left(1 + \frac{M_0}{H_0}\right) H_x, \quad (14)$$

$$H_y + M_y = \left(1 + \frac{M_0}{H_0}\right) H_y, \quad (15)$$

$$H_z + M_z = (1 + \chi)H_z - KT. \quad (16)$$

The normal mode expansion is assumed in the following form

$$\{w, T, \phi\} = \{W(z), \theta(z), \phi(z)\} e^{i(l_1x + l_2y)}, \quad (17)$$

where l_2 and l_1 are the wave number in y and x directions. Substituting Eq. (13) into momentum equation, Eq. (4), energy equation, Eq. (5) and also Maxwell equation, Eq. (6). Eliminate the pressure from the momentum equation, Eq. (4) by using curl identity. Used Eq. (14)-(17) and used the following setting

$$W^* = \frac{d}{\nu} W, \quad \phi^* = \frac{(1 + \chi)\kappa}{K\beta\nu d^2} \phi, \quad \theta^* = \frac{\kappa}{\beta\nu d} \theta, \quad (x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right), \quad (18)$$

where $\beta = \frac{\Delta T}{d}$, $\kappa = \frac{\kappa_1}{\rho_0 c_0}$ is the thermal diffusivity and $\nu = \frac{\mu}{\rho_0}$ is the kinematic viscosity. Then we obtained the following equation

$$(D^2 - a^2)^2 W = Rt \ a^2 \theta - Rm \ a^2 (D\phi - \theta), \quad (19)$$

$$(D^2 - a^2)\theta = -(1 - M_2)W, \quad (20)$$

$$(D^2 - M_3 a^2)\phi = D\theta, \quad (21)$$

where $D = d/dz$, $Rm = Rt \ M_1 = \frac{\mu_0 K^2 \beta^2 d^4}{1 + \chi}$, $a = \sqrt{l_1^2 + l_2^2}$, $M_1 = \frac{\mu_0 K^2 \beta}{1 + \chi}$, $M_2 = \frac{\mu_0 T_a K^2}{1 + \chi}$, $M_3 = \frac{1 + M_0/H_0}{1 + \chi}$, $Rt = \frac{\alpha_t g \beta d^4}{\nu \kappa}$.

Here, W , ϕ and θ are the amplitude of vertical component velocity, magnetic potential and temperature respectively. Rm is the magnetic Rayleigh number, M_1 is the magnetic number, a is the wave number, M_2 is the magnetic parameter, Rt is the thermal Rayleigh number, and M_3 is the nonlinearity of ferrofluid parameter. Based on Finlayson [6], value M_2 are assumed to be zero since the value is too small which is 10^{-6} .

We set the lower boundary in this research to be rigid while the upper boundary is set to be deformable surface. It is presumed that both are completely insulated to any disturbance of temperature. The boundary conditions are as follows

$$W = DW = D\theta = \phi = 0 \text{ at } z = 0 \quad (22)$$

$$W = D\theta = D\phi = (D^2 + \alpha^2)W + Ma \alpha^2(\theta - E) = Cr(D^2 - 3\alpha^2)DW - (Bo + \alpha^2)\alpha^2 E \quad (23)$$

$$= 0 \text{ at } z = 1,$$

where $Ma = \frac{\sigma_T \Delta T d}{\mu \kappa}$, $Cr = \frac{\mu \kappa}{\sigma d}$, $Bo = \frac{\Delta \rho g d^2}{\sigma}$. Here Ma , Cr and Bo is the Marangoni, crispation and bond number. In order to solve the eigenvalue problem of the Eq. (19)-(21) with the boundary conditions (22) and (23), we used regular perturbation method by taking variables W , ϕ and θ in the form as follows

$$(W, \phi, \theta) = (W_0, \phi_0, \theta_0) + \alpha^2(W_1, \phi_1, \theta_1) + \dots \quad (24)$$

By substituting Eq. (24) into Eq. (19)-(21) and also the Eq. (22)-(23).

$$D^4W_0 + \alpha^2 D^4W_1 - 2\alpha^2(D^2W_0 + \alpha^2 D^2W_1 + \alpha^4) + \alpha^4(W_0 + \alpha^2W_1) + Rm \alpha^2(D\phi_0 + \alpha^2 D\phi_1 - \theta_0 - \alpha^2\theta_1) - Rt \alpha^2(\theta_0 + \alpha^2\theta_1) \quad (25)$$

$$D^2\theta_0 + \alpha^2 D^2\theta_1 - \alpha^2(\theta_0 + \alpha^2\theta_1) + (1 - M_2)(W_0 + \alpha^2 W_1) \quad (26)$$

$$D^2\phi_0 + \alpha^2 D^2\phi_1 - \alpha M_3(\phi_0 + \alpha^2\phi_1) - D\theta_0 - \alpha^2 D\theta_1 \quad (27)$$

with the boundary conditions

$$(W_0 + \alpha^2W_1) = D(W_0 + \alpha^2W_1) = (\phi_0 + \alpha^2\phi_1) = D(\theta_0 + \alpha^2\theta_1) = 0 \text{ at } z = 0 \quad (28)$$

$$(W_0 + \alpha^2W_1) = 0,$$

$$D(\phi_0 + \alpha^2\phi_1) = 0,$$

$$D(\theta_0 + \alpha^2\theta_1) = 0, \quad (29)$$

$$D^2W_0 + \alpha^2 D^2W_1 + \alpha^2(W_0 + \alpha^2W_1) + Ma \alpha^2(\theta_0 + \alpha^2\theta_1 - E) = 0,$$

$$Cr(D^3W_0 + \alpha^2 D^3W_1 - 3\alpha^2(DW_0 + \alpha^2DW_1)) - (\alpha^2 + Bo)\alpha^2 E = 0 \text{ at } z = 1$$

First, we will collect all the zero th order term of the Eq. (25)-(27) and boundary conditions (28)-(29) to get the following

$$D^4W_0 = 0, \quad (30)$$

$$D^2\theta_0 + W_0 = 0, \quad (31)$$

$$D^2\phi_0 - D\theta_0 = 0, \quad (32)$$

the boundaries are set as follow

$$W_0 = DW_0 = \phi_0 = D\theta_0 = 0, \quad \text{at } z = 0 \quad (33)$$

$$W_0 = D^2W_0 = CrD^3W_0 = D\phi_0 = D\theta_0 = 0 \text{ at } z = 1. \quad (34)$$

Eq. (30)-(32) and the boundary conditions (33) and (34) are then solve by using MAPLE to get the following solutions

$$W_0 = 0, \quad \theta_0 = 1, \quad \phi_0 = 0. \quad (35)$$

By substituting (35) into Eq. (25)-(27) and boundary conditions (28)-(29), we are then will get the first order equations as follows

$$D^4W_1 - Rt - Rm = 0, \quad (36)$$

$$D^2\theta_1 - 1 + W_1 = 0, \quad (37)$$

$$D^2\phi_1 - D\theta_1 = 0, \quad (38)$$

boundary conditions for $z = 0$

$$W_1 = DW_1 = \phi_1 = D\theta_1 = 0, \quad (39)$$

and $z = 1$

$$W_1 = D\phi_1 = D\theta_1 = CrD^3W_1 - Bo \left(\frac{D^2W_1}{Ma} + 1 \right) = 0. \quad (40)$$

Then the Eq. (36)-(38) and boundary conditions (39)-(40) are solved using MAPLE software. After performing a regular perturbation method, it revealed that M_3 does not contribute to the stability of the Marangoni-Bénard convection in ferrofluid system. The same finding also reported by Nanjundappa *et al.*, [19]. The solution of the system will produce equations of critical Marangoni number, Ma_c , and critical thermal rayleigh number, Rt_c in terms of Rm , Cr , M_1 , Bo and M_3 .

3. Results

In the present paper, we tend to study the impact of deformable of a ferrofluid layer system on the Marangoni-Bénard convection. The lower boundary is rigid with deformable surface of the upper free boundary. The parameter Rm , M_1 , M_3 Bo and Cr are considered as a function of critical Marangoni number, Ma_c and also thermal Rayleigh number, Rt_c . The equations of both Ma_c and Rt_c are obtained through regular perturbation method. The behaviour of the parameters are shown in Figure 2 until Figure 5.

Figure 2 represents the effect of Bo on Ma_c for different values of Rm with Rt is set to be 10 and Cr is equal to 0.001. Based on the figure, the increment of Bo lead to the rise in Ma_c and it delays the convection of the system while the increasing of Rm compresses the value of Ma_c and hasten the convection of ferrofluid.

Different values of M_1 on Ma_c as a function of Cr when the value of Rt and Bo is set to be one are demonstrated in Figure 3. The figure shows the increasing of Cr and also M_1 will cause to the reduce of Ma_c . The reason behind the behaviour of Cr parameter is that based on the formulation of Cr it is directly proportional to κ while it is inversely proportional to σ . The increasing of temperature will lead the decreasing of surface tension, σ while the simultaneous increasing of κ and decreasing of σ enhance the heat transfer (Mokhtar and Hamid [18]).

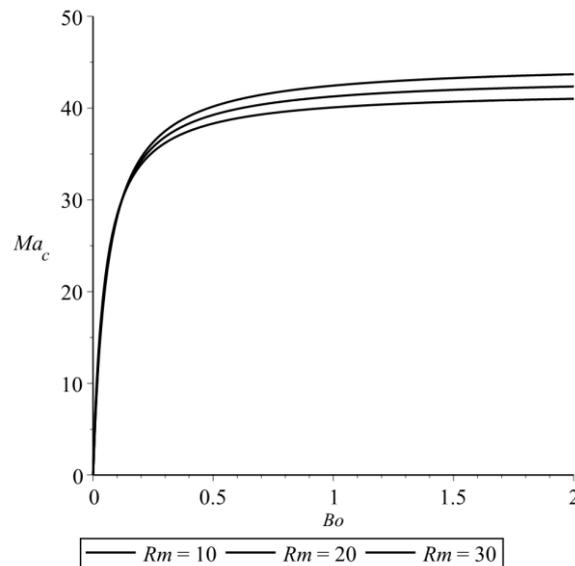


Fig. 2. Impact of Rm on Ma_c against Bo .

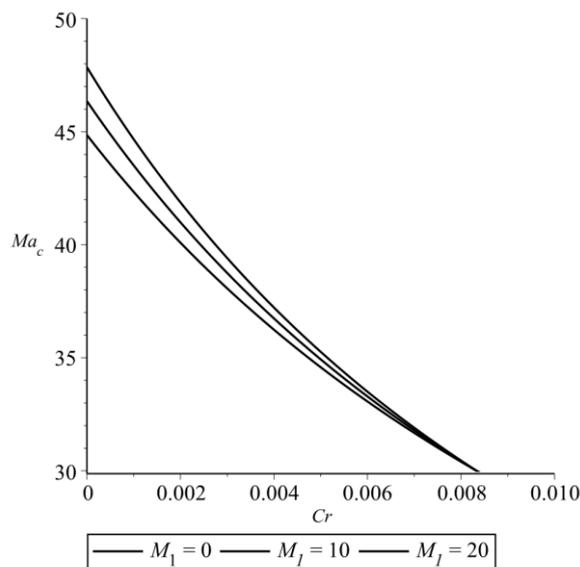


Fig. 3. Ma_c versus Cr for different value of M_1

The effect of Bo on Ma_c for a selected value of M_1 is presented in Figure 4. Based on Figure 4, as the value of Bo elevated, the value of Ma_c also increase as in Figure 2. It is because the increasing of Bo value leads to reducing the surface tension, σ hence it delays the convection as explained by Mokhtar and Hamid [18]. Meanwhile, the increase of M_1 will lessen the Ma_c and hasten the ferroconvection.

Figure 5 shows the effect of thermal buoyancy convection Rt_c against the surface tension convection Ma_c for various value of M_1 when $Cr = 0.001$ and $Bo = 10$. This figure represents that for different values of M_1 , Rt_c and Ma_c will converged to $Ma_c = 47.66$. The convergence value of this system is lower compared to Nanjundappa *et al.*, [19] which is $Ma_c = 50.3919$ for the case of ferroconvection with the temperature dependent viscosity $B = 2$, thus it is more stable compared to the ferroconvection in a deformable surface.

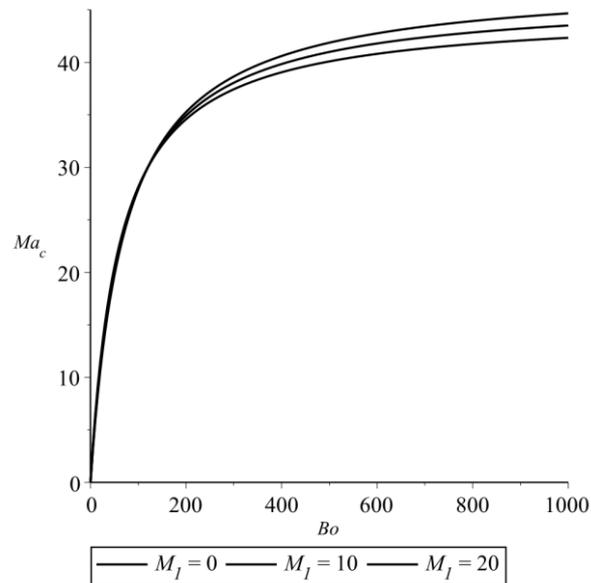


Fig. 4. Ma_c versus Bo for different value of M_1 .

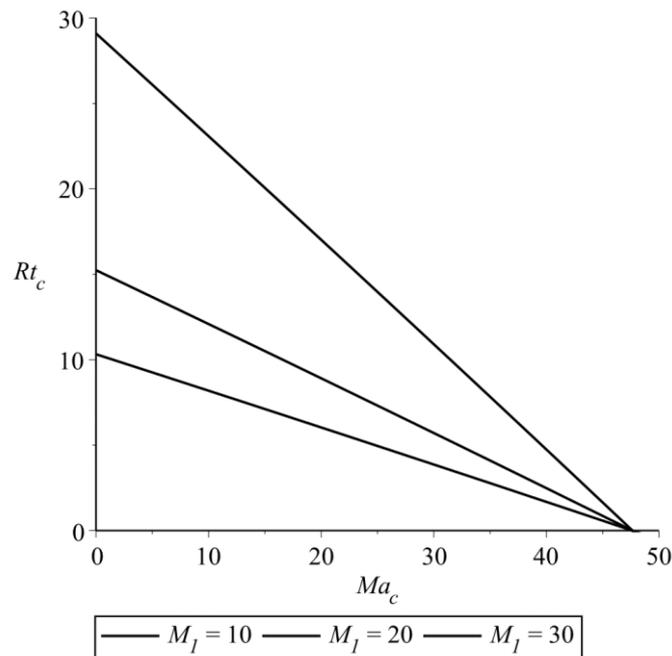


Fig. 5. Rt_c versus Ma_c for different value of M_1

4. Conclusions

The study of the deformable effect in ferrofluid has been done. We can conclude that the increase of Rm , Cr and also M_1 hasten the convection while Bo delays the onset of Marangoni-Bénard convection. We also found that the combination of decreasing Rm and increasing of Bo will stabilize the system while the increasing of both Cr and M_1 value can hasten the convection of ferrofluid. Other than that, the combination of increment Bo value and reducing of M_1 help in stabilizing the system of ferrofluid on the onset of Marangoni-Bénard convection.

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