

MHD Stagnation Point Flow in Nanofluid Over Shrinking Surface Using Buongiorno's Model: A Stability Analysis

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ABSTRACT

An analysis has been performed using the Buongiorno model on the nanofluid steady 2D stagnation point flow magnetohydrodynamic (MHD) over the shrinking surface to test its stability. Transforming the governing partial equations into a set of ordinary differential equation (ODE) and solved the equations numerically. In this paper, the impact of Brownian motion and thermophoresis has been considered and can be seen in ODE. The physical quantities of interest such as skin friction, local Nusselt number, local Sherwood number as well as the velocity and temperature profiles are acquired by numerical findings for some values of governing parameters such as ϵ , M , Pr , Le , Nb and Nt . Results show that duality of solutions exist for certain values $\epsilon < -1$ while unique solution exist when $\epsilon > -1$. On the other hand, as the parameter of M increased, the gradient of velocity increased, the rate of transmission heat and mass improved. Throughout the analysis, it demonstrates a linearly stable first solution in comparison to linearly unstable second solution.

Keywords:

MHD; stagnation point flow; nanofluid;
Buongiorno's model; stability analysis

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1. Introduction

Numbers of researchers and scientists are interested in studying the effect of stagnation point flow because of its numerous applications in industrial sectors such as, aviation flows, process of hydrodynamics and etc. Hiemenz [1] was the first on stagnation point flow famously known as Heimenz flow. The energy equation is included by Eckert [2] and the precise heat transfer solution was found. The work of extension [1] were regarded by Chiam [3] by taking equally velocities of stretching and shrinking. In addition, the impact of the stagnation point flow effect has been investigated in nanofluid flow. The nanofluid word is implemented by Choi [4], describing the fluid that can enhance the transfer rate of heat. Two models were suggested by Tiwari and Das [5] and Buongiorno [6] in the solution of nanofluid problem in the fluid boundary and heat transfer. The

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model on Tiwari and Das analyze the behavior of nanofluid by taking consideration on the solid volume fraction while Buongiorno focused on the analysis of thermophoresis and Brownian motion on the characteristics of heat transmission. Bachok *et al.*, [7] have studied the problem in stretching/shrinking case using Tiwari and Das model. It discovered that there is a single solution in the stretching sheet while there is a double solution in the shrinking sheet. The effects of the mass fraction parameter were investigated by three different kinds of nanoparticles. Noor *et al.*, [8] have deliberate nanofluid over a shrinking sheet. While Shomali *et al.*, [9] had considered the problem of unsteady flow in nanofluid. Another study has also regarded the effect of magnetohydrodynamics (MHD) boundary layer since this provides excellent support for the fluid flow. Rahman *et al.*, [10] extended problem from [7]. Then, Yazdi *et al.*, [11] studied on radiation effect on two dimensional MHD stagnation point flow. Besides, the problem in the non-linear porous shrinking sheet have been solved by Junoh *et al.*, [12]. Meanwhile, Soid *et al.*, [13] focused on micropolar fluid with slip boundary. There are a lot of work been done by other researchers using Tiwari and Das model (Bukhta *et al.*, [14], Ibrahim *et al.*, [15], Yasin *et al.*, [16]).

Throughout the work using the Buongiorno model, researchers have determined the impact of movement and thermophoresis on liquid flow. Anwar *et al.*, [17] have considered the effects of radiation on MHD over exponentially stretching sheet. Later, Zaimi *et al.*, [18] reported the results on unsteady flow in shrinking cylinder. While, Mansur *et al.*, [19] have extended the problem from Ibrahim *et al.*, [20] by adding the suction effect. The viscoelastic nanofluid problem with heat radiation over permeable stretching / shrinking sheet has been conducted by Jusoh *et al.*, [21]. Najib *et al.*, [22] have originated the idea by adding the slip effect to their problem. Kamal *et al.*, [23] then continue with chemical reaction effect over a permeable surface. Bakar *et al.*, [24] then did their research over stretching/shrinking cylinder. Motivated by the work from Chen *et al.*, [25], Ismail *et al.*, [26] extended their work with the presence of viscous dissipation.

This study seeks to extend the highlight work from the stagnation point flow of MHD to the stretching sheet [20] and stagnation point flow of nanofluid [7] to the nanofluid stagnation point flow of the MHD over shrinking surface using the Buongiorno model with the stability analysis which previously not taken into account.

2. Formulation of Problem

In the occurrence of magnetic field, we contemplate the two-dimensional of nanofluid stagnation point flow at the area $y > 0$ towards a shrinking surface located at $y = 0$ with fixed point $x = 0$. Furthermore, we assume that the wall does not have a slip condition. The flow is kept at a constant temperature, T_w and concentration, ϕ_w . Assume that the velocity of shrinking and ambient fluid velocity (free stream) are $u_w = ax$ and $U_\infty = bx$, respectively. Note that $a < 0$ is due to the shrinking surface, while b is a positive constant. Using these assumptions, the mathematical modelling equations are (see [7] and [20])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_\infty \frac{\partial U_\infty}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial y^2} \right) + \frac{\sigma B_0}{\rho_f} (U_\infty - u) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial y^2} \right) + \tau \left[D_B \left(\frac{\partial T}{\partial y} \frac{\partial \phi}{\partial y} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] \quad (3)$$

$$u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = D_B \left(\frac{\partial^2 \phi}{\partial y^2} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

together with the initial and boundary conditions (IBCs),

$$u = u_w(x), v = 0, T = T_w, \phi = \phi_w \text{ at } y = 0$$

$$u \rightarrow U_\infty, T \rightarrow T_\infty, \phi \rightarrow \phi_\infty \text{ at } y \rightarrow \infty \quad (5)$$

where u and v are the velocity along the x and y axes, T is the temperature of nanofluid, σ is conductance of electrical fluid, B_0 represents the shrinking sheet magnetic field we applied to, ρ_f for base fluid density, ν the kinematics viscosity, α is the fluid thermal diffusiveness where $\alpha = \frac{k}{(\rho c)_f}$.

Next, τ is the proportion between the nanoparticle effectiveness heat capacity and fluid heat capacity, ϕ is the concentration of the nanoparticle, D_T and D_B are the diffusion of thermophoresis and Brownian dispersion, respectively. Introducing the following similarity transformations for Eq. (1)-(4) subjected to the conditions Eq. (5)

$$\eta = \sqrt{\frac{b}{v}} y, \psi = \sqrt{vb} x f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, h(\eta) = \frac{\phi - \phi_\infty}{\phi_w - \phi_\infty} \quad (6)$$

where h is concentration, η and ψ are the similarity variable and stream function, respectively which define $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. By using this defining parameter, Eq. (1) satisfied. The remaining governing equations, Eq. (2)-(4) are transformed to a set of ODE as follows

$$f''' + f f'' - (f')^2 + M(1 - f') + 1 = 0 \quad (7)$$

$$\theta'' + Pr N b h' \theta' + Pr f \theta' + Pr N t (\theta')^2 = 0 \quad (8)$$

$$h'' + Le h' f + \frac{Nt}{Nb} \theta'' = 0 \quad (9)$$

subjected to the IBCs, Eq. (5) now is

$$f(0) = 0, f'(0) = \varepsilon, \theta(0) = 1, h(0) = 1$$

$$f'(\eta) \rightarrow 1, \theta(\eta) \rightarrow 0, h(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (10)$$

From Eq. (7)-(9), the prime denotes the differentiation with respect to η . While, six governing parameters are defined such as Pr is Prandtl number where $Pr = \frac{\nu}{\alpha}$, ε is the velocity ratio parameter where $\varepsilon = \frac{a}{b}$, M is the magnetic parameter where $M = \frac{\sigma B_0}{\rho_f b}$, Nb is Brownian motion parameter where $Nb = \frac{\tau D_B (\phi_w - \phi_\infty)}{\nu}$, Nt is thermophoresis parameter where $Nt = \frac{\tau D_T (T_w - T_\infty)}{\nu T_\infty}$ and Le is the Lewis number parameter where $Le = \frac{\nu}{D_B}$.

In this problem, our physical interests are the skin friction coefficient, c_f , local Nusselt number, Nu_x and local Sherwood number, Sh_x which given by [7]

$$c_f = \frac{\tau_w}{\rho U_\infty^2}, Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, Sh_x = \frac{xh_w}{D_B(\phi_w - \phi_\infty)} \quad (11)$$

Besides, the skin friction τ_w , heat flux wall, q_w and mass stream h_m are as follows

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}, h_m = -D_B \left(\frac{\partial \phi}{\partial y} \right)_{y=0} \quad (12)$$

Again, applying the Eq. (6) into Eq. (12), the reduced skin friction coefficient, local Nusselt number and local Sherwood number are

$$f''(0) = c_f Re_x^{1/2}, \quad -\theta'(0) = Nu_x Re_x^{-1/2}, \quad -h'(0) = Sh_x Re_x^{-1/2} \quad (13)$$

where $Re_x = \frac{U_\infty x}{\nu}$ is the local Reynold number.

2.1 Solution for Stability Analysis

Since the numerical findings from bv4pc indicate that for certain values of ε there exist two solutions which are first solution and second solution. An analysis has to been set up to identify which of these solutions are stable. To initiate a stability analysis, [27] the unsteady state flow case must be included as it is the step to study the temporal stability of duality of solutions. Consider the unsteady form by adding $\partial u/\partial t$, $\partial T/\partial t$, $\partial \phi/\partial t$ to each of Eq. (2)-(4), respectively and introduce a new time dimensionless parameter, τ . Then, we have

$$\eta = \sqrt{\frac{b}{\nu}} y, \psi = \sqrt{b} x f(\eta, \tau), \theta(\eta, \tau) = \frac{T - T_\infty}{T_w - T_\infty}, h(\eta, \tau) = \frac{\phi - \phi_\infty}{\phi_w - \phi_\infty}, \tau = bt \quad (14)$$

Thus Eq. (2)-(4) become

$$\frac{\partial^3 f}{\partial \eta^3} - \frac{\partial^2 f}{\partial \eta \partial \tau} + f \frac{\partial^2 f}{\partial \eta^2} - \left(\frac{\partial f}{\partial \eta} \right)^2 + M \left(1 - \frac{\partial f}{\partial \eta} \right) + 1 = 0 \quad (15)$$

$$\frac{\partial^2 \theta}{\partial \eta^2} - Pr \frac{\partial \theta}{\partial \tau} + Pr f \frac{\partial \theta}{\partial \eta} + Pr N b \frac{\partial \theta}{\partial \eta} \frac{\partial h}{\partial \eta} + Pr N t \left(\frac{\partial \theta}{\partial \eta} \right)^2 = 0 \quad (16)$$

$$\frac{\partial^2 h}{\partial \eta^2} - Le \frac{\partial h}{\partial \tau} + Le f \frac{\partial h}{\partial \eta} + \frac{Nt}{Nb} \frac{\partial^2 \theta}{\partial \eta^2} = 0 \quad (17)$$

subjected to boundary conditions (BCs),

$$f(0, \tau) = 0, \frac{\partial f}{\partial \eta}(0, \tau) = \varepsilon, \theta(0, \tau) = 1, h(0, \tau) = 1$$

$$\frac{\partial f}{\partial \eta}(\infty, \tau) \rightarrow 1, \theta(\infty, \tau) \rightarrow 0, h(\infty, \tau) \rightarrow 0 \quad (18)$$

We consider some small perturbation (Merkin [28]) where $f(\eta) = f_0(\eta)$, $\theta(\eta) = \theta_0(\eta)$ and $\phi(\eta) = \phi_0(\eta)$ which satisfying Eq. (15)-(17) such as

$$\begin{aligned}
 f(\eta, \tau) &= f_0(\eta) + e^{-\gamma\tau} F(\eta, \tau) \\
 \theta(\eta, \tau) &= \theta_0(\eta) + e^{-\gamma\tau} G(\eta, \tau) \\
 h(\eta, \tau) &= h_0(\eta) + e^{-\gamma\tau} H(\eta, \tau)
 \end{aligned}
 \tag{19}$$

where $F(\eta, \tau)$, $G(\eta, \tau)$ and $H(\eta, \tau)$ are the small relative to $f_0(\eta)$, $\theta_0(\eta)$ and $h_0(\eta)$, respectively while γ is an unknown parameter of eigenvalue. This consideration is taken due to check the steady flow solution stability. Next, differentiate Eq. (19) and equate them to Eq. (15)-(18). By setting $\tau = 0$, $F = F_0$, $G = G_0$ and $H = H_0$, we obtain the following linearized equations

$$F_0''' + f_0 F_0'' + f_0'' F_0 + (\gamma - 2f_0' - M)F_0' = 0 \tag{20}$$

$$G_0'' + (Pr f_0 + Pr N b h_0' + 2Nt Pr \theta_0')G_0' + Pr \gamma G_0 + (Pr F_0 + Pr N b H_0')\theta_0' = 0 \tag{21}$$

$$H_0'' + Le \gamma H_0 + Le f_0 H_0' + Le F_0 h_0' + \frac{Nt}{Nb} G_0'' = 0 \tag{22}$$

correspond to the BCs,

$$F_0'(0) = 0, F_0(0) = 0, G_0(0) = 0, H_0(0) = 0$$

$$F_0'(\infty) \rightarrow 0, G_0(\infty) \rightarrow 0, H_0(\infty) \rightarrow 0 \tag{23}$$

To fulfil steady flow solution stability, the least eigenvalue γ needs to be determined. If $\gamma < 0$, it will lead to the unstable flow. According to Harris *et al.*, [29], the range of possible eigenvalues can be dictated if one of the boundary conditions on $F_0'(\eta)$, $G_0(\eta)$ or $H_0(\eta)$ is relaxing. In our problem, we choose the condition on which we relax is, $F_0'(\infty) \rightarrow 0$ when $\eta \rightarrow \infty$ and for fix eigenvalues γ , we solved Eq. (20)-(22) subject to boundary conditions Eq. (23) by changing boundary condition with the new one which is $F_0''(0) = 1$.

3. Results

Results for this problem are revealed in this section. The numerical results are found by transforming Eq. (7)-(9) together with the boundary condition (10) using the coding in MATLAB software. Figure 1-16 illustrate the governing parameters effects on skin friction, heat transfer and mass transfer coefficients, as well as the profiles for the velocity, temperature and concentration. The impact of Pr and Le are also been analyzed and discussed.

We compared our present numerical results with those stated by Kamal *et al.*, [23] for the case when $Cr = 0$ and $Sr=1$. As we can see through Table 1 the comparisons have been found very good and provide us a high confidence to report further numerical outcomes.

3.1 Analysis on Skin Friction and Heat transfer

Three dissimilar values of M together with selected values of ε with variation of the $f''(0)$, $-\theta'(0)$ and $-h'(0)$ are presented in Figure 1, when $Pr = 2$, $Le = 2$, and $Nb = Nt = 0.1$. In our paper, we chose three values of M to be investigate such as M=0, 0.2, 0.4 which M=0 implies that the fluid has no magnetic effect. From our calculation, it is possible to obtain dual solution. There exist a

critical value of ε , namely ε_c which denotes that it is in shrinking surface. Based on the graph these graphs, a unique solution exist when $\varepsilon > -1$, the dual solution starts to appear when $\varepsilon_c < \varepsilon < -1$ and no solution found when $\varepsilon < \varepsilon_c$. Bachok *et al.*, [7] stated that the first solution is stable which means realizable compared to the second solution which is unstable. We observed for $M = 0, 0.2, 0.4$ and the critical values are $\varepsilon_c = -1.24658, -1.42403, -1.60446$ respectively. Clearly, as ε increased, the reduce skin friction coefficient decrease. Perhaps it gives a higher value when the magnetic parameter, M increased. This is because the Lorentz force in the boundary layer increase indicates that the force retarded the flow and reduce the flow motion. As we note, the range of the duality of solution widen with the increment of M . An opposite trait for temperature and concentration can be seen as the ε increased, the values of heat transfer coefficient and mass transfer coefficient increased which correspond to the increment of M . The larger the value of M will involved the delaying of the boundary layer separation.

3.2 Analysis on Effect of Thermophoresis, Nt and Brownian Motion, Nb

Variation of $Nu_x Re_x^{-\frac{1}{2}}$ and $Sh_x Re_x^{-\frac{1}{2}}$ with Nt for different M are plot in Figure 2. As the value of Nu_x and Sh_x decreasing, the values of Nt increase together with the values of M . Here, we can state that Nt is a decreasing magnetic parameter function. The thermophoresis force works in nanofluids against the gradient of the temperature and changes nanoparticles from the transition state (hot to cold). Through the graph we can observed that the rise of M tends to decrease the gradients of temperature in the boundary layer and hence reduce the Nusselt number.

On the other hand, the effect of Nb parameter with different value of M can be seen in Figure 3. The local Nu_x reduce as the numbers of Nb increases along the increment of M . We can remark that Nu_x is a Brownian motion parameter reducing function. This is because the Nb tends to interchange the nanoparticles from elevated level to low [30]. Likewise, the Sh_x is increase as the value of the Nb increase. We can observe that the value of M decrease at first, and increase when $Nb=0.1$. This is may be because of the slower collision occur between the nanoparticles in the fluid. This collision will move the nanoparticle away from the surface and we can see that the increment of the Brownian parameter does not gives any significant on boundary layer thickness.

Table 1

Comparison for the values of $c_f Re_x^{\frac{1}{2}}$, $Nu_x Re_x^{-\frac{1}{2}}$ and $Sh_x Re_x^{-\frac{1}{2}}$ for different values of $\varepsilon, M = Cr = 0$ of Kamal *et al.*, (2019) with $Pr = 7, Nb = Nt = 0.1, Le = Sr = 1$

M	ε	Kamal <i>et al.</i> , (2019)			Present results		
		$c_f Re_x^{\frac{1}{2}}$	$Nu_x Re_x^{-\frac{1}{2}}$	$Sh_x Re_x^{-\frac{1}{2}}$	$c_f Re_x^{\frac{1}{2}}$	$Nu_x Re_x^{-\frac{1}{2}}$	$Sh_x Re_x^{-\frac{1}{2}}$
0	-1	1.328817	0.016105	0.440494	1.328817	0.016105	0.440494
	-0.5	1.495670	0.347080	0.455138	1.49567	0.34708	0.455138
	0.5	0.713295	1.211774	-0.019361	0.713295	1.211774	-0.01936
1	-1				2.42996	0.12772	0.55021
	-0.5				2.12019	0.46904	0.41917
	0.5				0.86962	1.22965	-0.02134

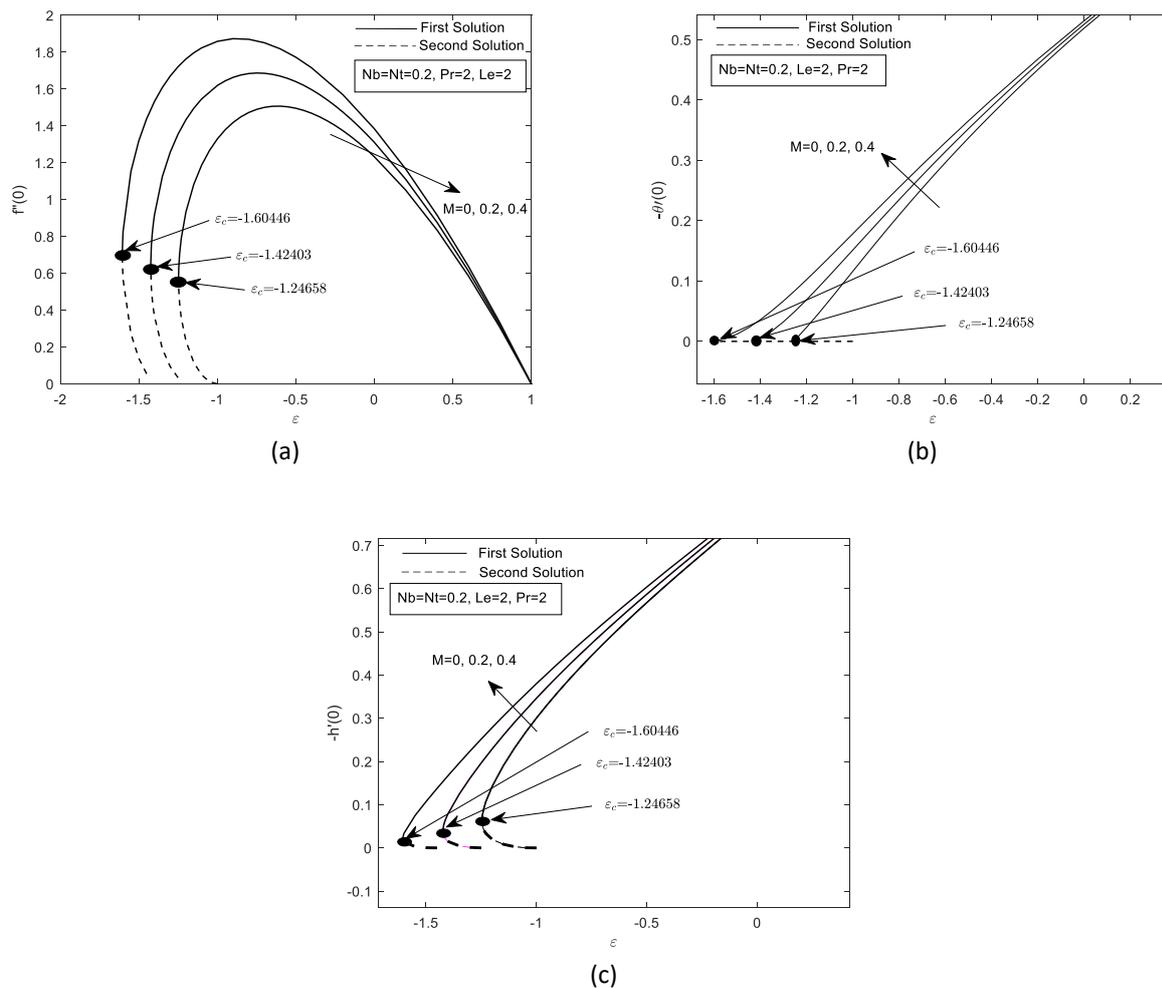


Fig. 1. Skin friction coefficient, $f''(0)$, heat transfer coefficient, $-\theta'(0)$ and mass transfer coefficient, $-h'(0)$ with ϵ for various values of M

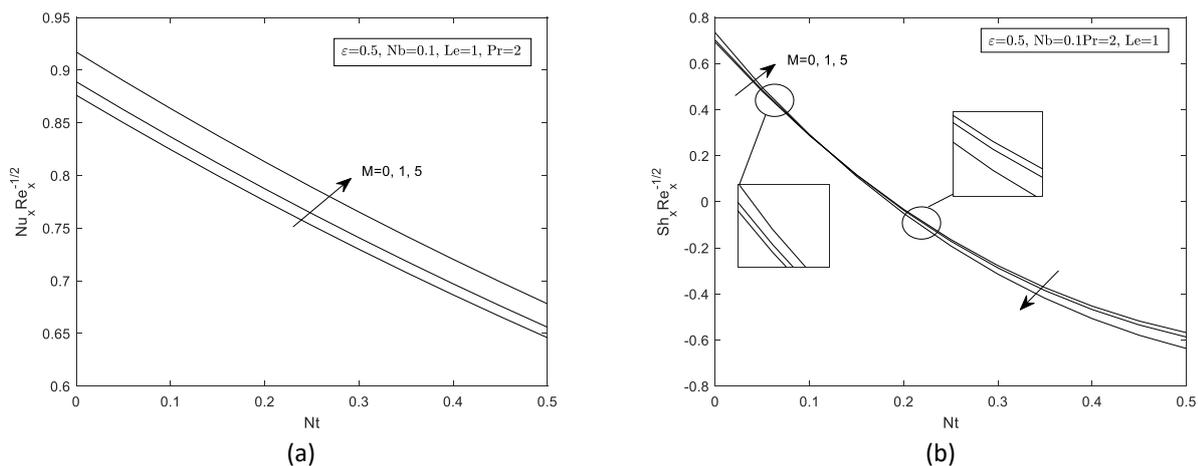


Fig. 2. Variation of $Nu_x Re_x^{-1/2}$ and $Sh_x Re_x^{-1/2}$ with Nt for different M

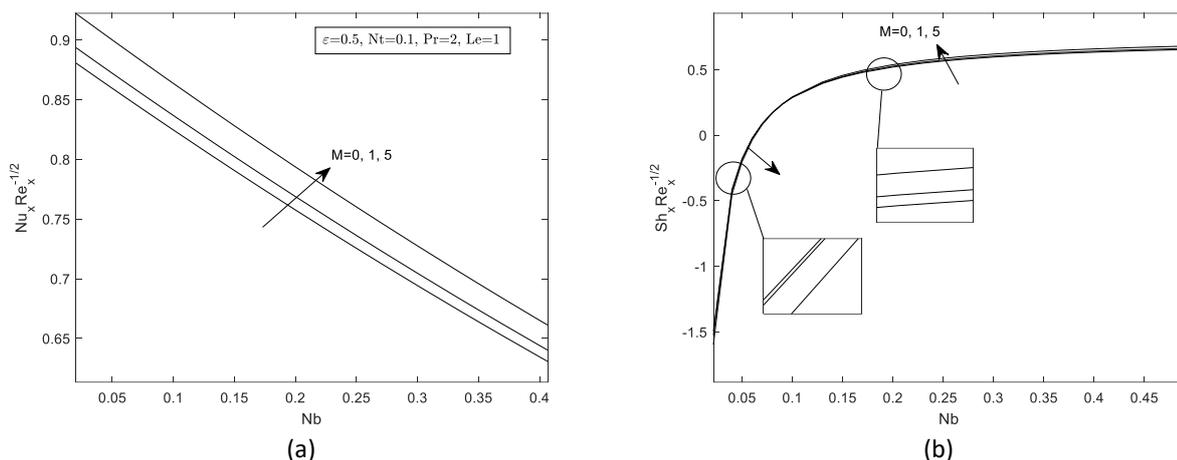
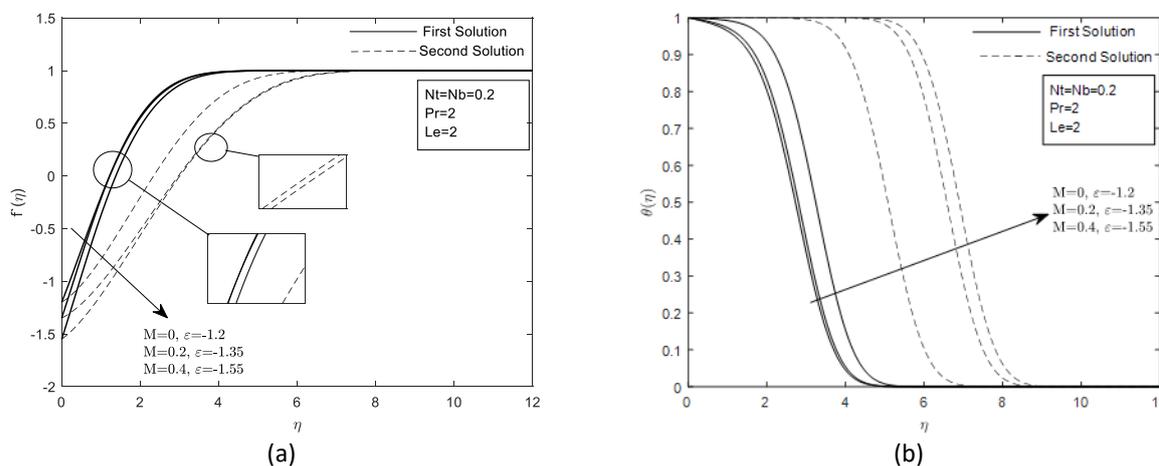


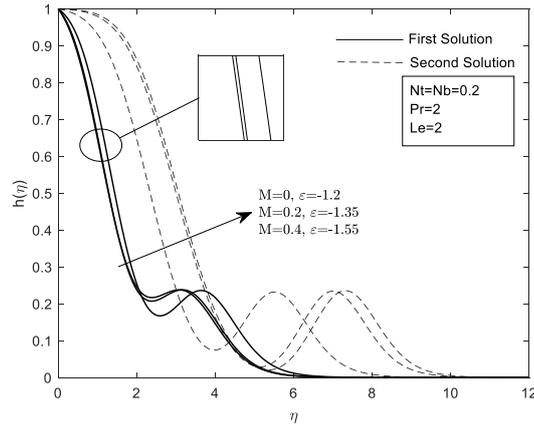
Fig. 3. Variation of $Nu_x Re_x^{-1/2}$ and $Sh_x Re_x^{-1/2}$ with Nb for different values of M

3.3 Analysis on Velocity, Temperature and Concentration Profile

Figure 4 illustrate the graphs of $f'(\eta)$, $\theta(\eta)$ and $h(\eta)$ for different value of M for shrinking surface. These profiles actually validate the result from the above figures which shows that when $\varepsilon_c < \varepsilon < -1$, duality of solutions exist. As the value of M increase, we realize that the separation of boundary layer thickness become fast. Besides, Figure 5 show the $f'(\eta)$, $\theta(\eta)$ and $h(\eta)$ for various ε . Based on these figures, it clearly visible that arising of ε will increase the $f'(\eta)$, $\theta(\eta)$ and $h(\eta)$ for the first solution but likewise for another one. In addition, these figures also fulfilled and satisfied the boundary condition (10). Hence it provide on excellent validity of the results obtained and the duality of the solution.

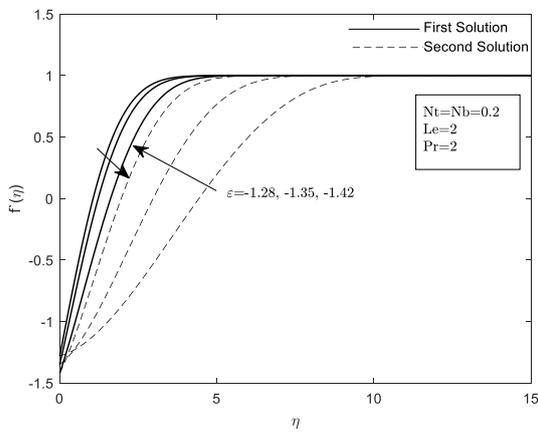
Moreover, the $\theta(\eta)$ for some values of Pr are shown in Figure 6. In this paper we consider three types of Pr which is $Pr = 0.7, 1, 7$ which signify the type of gaseous, water and liquid, respectively. The increment of Pr increase the heat transfer of the fluid. From our assumption maybe because the momentum boundary layer is thicker compared to the thermal boundary layer which enable the fluid to transmit the momentum faster through the liquid. Note that we are unable to predict the changes in thermal boundary layer thickness when the $Pr > 7$. The reason is because the desirable energy for the fluid to flow is slower until certain distance. In Figure 7, we can observe the consequence of Le on $h(\eta)$. Increasing of Le will decrease the concentration profile. Thru definition, it is worth to mention that the increment of Le will slower the mass diffusivity of the fluid hence increases the heat diffusivity. Thus, the boundary layer thickness of concentration profile increases.



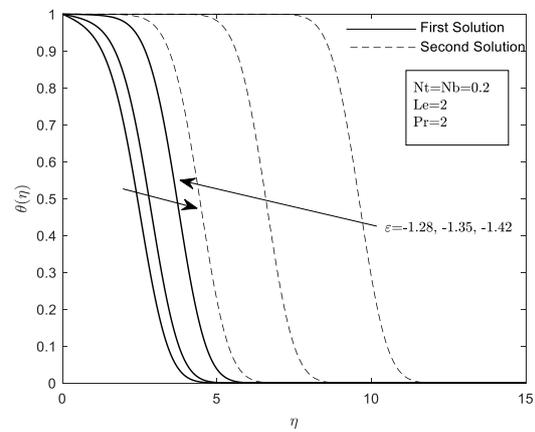


(c)

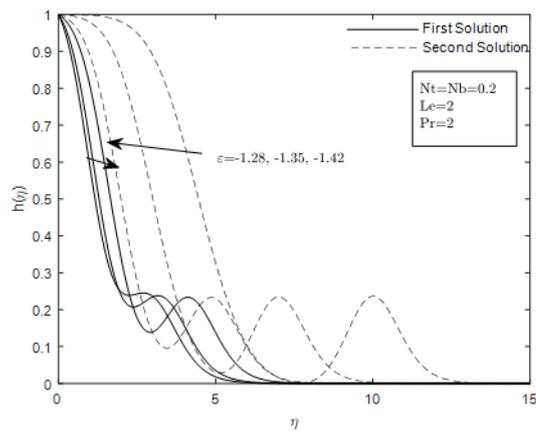
Fig. 4. Velocity profile, $f'(\eta)$, temperature profile, $\theta(\eta)$ and concentration profile, $h(\eta)$ for different values of M for shrinking surface



(a)



(b)



(c)

Fig. 5. Velocity profile, $f'(\eta)$, temperature profile, $\theta(\eta)$ and concentration profile, $h(\eta)$ for different values of ϵ

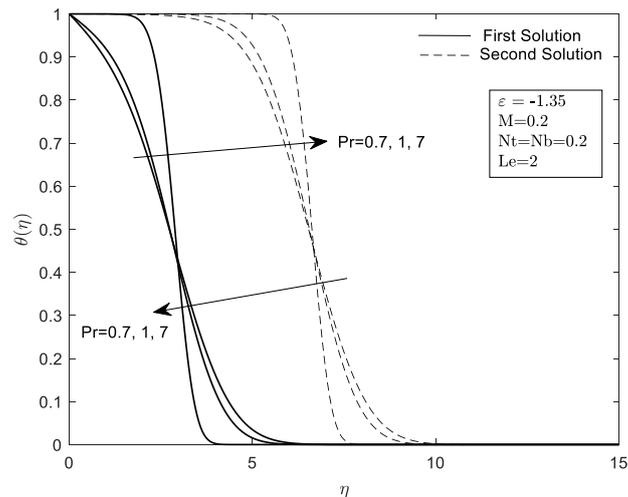


Fig. 6. Effect on Prandtl number, Pr on temperature profile

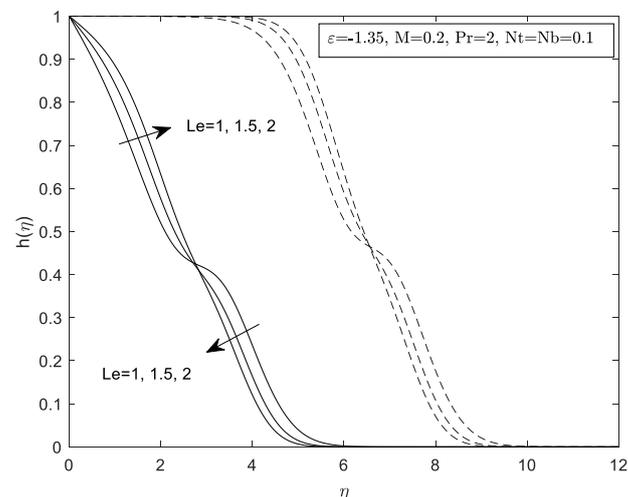


Fig. 7. Effect on Lewis number, Le on concentration profile

3.4 Analysis on Stability Solution

Duality solutions for the above problem has to be analyzed. Therefore, in order to classify the solutions which is physically feasible, a stability analysis is introduced. Substitute Eq. (20) – (22) together with (23) into the stability coding in bvp4c in MATLAB. Choosing the value of ε which is approximate to ε_c is important in order to compute the smallest eigenvalues ($\gamma \rightarrow 0$). Hence, Table 2 are the values from our computation. As we can see, the first solution lies on positive real numbers while opposite sign are shown in second solution. Previous study have stated that the first solution is stable and reliable because in the flow system there are just a slightly disturbance that do not interrupt the separation of boundary layer. With regard to the second solution, we can conclude that it is volatile which means that an early rise in disturbance would disrupt the separation of the boundary layer.

Table 2
Least eigenvalues γ for selected ε

M	epsilon, ε	First solution	Second solution
0	-1.2465	0.008	-0.008
	-1.246	0.0622	-0.0614
	-1.24	0.2121	-0.2036
0.2	-1.4240	0.015	-0.0149
	-1.423	0.0845	-0.0831
	-1.42	0.1681	-0.1629
0.4	-1.6044	0.0222	-0.0221
	-1.604	0.0578	-0.0572
	-1.6	0.1803	-0.1744

4. Conclusions

This research shown the effect of six governing parameters including Pr, Le, M, ε , Nb, and Nt. We can summarized the findings in three main point as below

i. Duality solution

- It obviously demonstrates from the above figures that the duality exists in shrinking surface ($\varepsilon < -1$).
- The range on the shrinking surface are widen and the ε_c indicates the boundary layer separation as the values of M increased.

ii. Skin friction

- The magnetic field, M affected the skin friction.
- When the value of M increased, the surface shear stress increase, the fluid flow is delayed resulting in increased of velocity gradient at the surface.

iii. Heat Transfer and mass transfer

- The local Nusselt number is a reducing function for M.
- Increasing of Nt will decrease the rate of the heat transfer and rate of the mass transfer.
- The local Nusselt number is a decreasing function for Nb.
- As the Nb arise, the rate of the mass transfer increased.

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