Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage:
https://semarakilmu.com.my/journals/index.php/fluid_mechanics_thermal_sciences/index
ISSN: 2289-7879

Unsteady Flow of Hybrid Nanofluids Subjected to a Stretching/Shrinking Sheet with Heat Generation

Noorina Abdul Rahman[1], Najiyah Safwa Khashi'ie[1,*], Iskandar Waini[2], Khairum Hamzah[2], Mohd Afzanizam Mohd Rosli[1], Ioan Pop[3]

1 Fakulti Teknologi dan Kejuruteraan Mekanikal, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100 Durian Tunggal, Melaka, Malaysia
2 Fakulti Teknologi dan Kejuruteraan Industri dan Pembuatan, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100 Durian Tunggal, Melaka, Malaysia
3 Department of Mathematics, Babeş-Bolyai University, R-400084 Cluj-Napoca, Romania

ARTICLE INFO

Article history:
Received 4 November 2023
Received in revised form 26 March 2024
Accepted 5 April 2024
Available online 30 April 2024

Keywords:
Hybrid nanofluid; heat generation; heat transfer; multiple solutions; unsteady flow

ABSTRACT

This work highlights the thermal progress and flow characteristics of the various hybrid nanofluids (graphene-alumina/water and copper-alumina/water) flow over a stretching/shrinking sheet with heat generation and suction effects using numerical approach. This study is important in identifying the nanofluids and physical parameters which beneficial in the increment of the flow and thermal progresses. The control model (partial differential equations) is established based on the boundary layer assumptions and then transformed into a set of ordinary (similar) differential equations. A numerical solver in the MATLAB software called the bvp4c solver is used to compute the solutions by first transforming the reduced ODEs. There is an increase in velocity profile and a decrease in thermal rate with the increased suction parameter. It is observed that between the two hybrid nanofluids, the Cu-Al_2O_3/H_2O hybrid nanofluid has a larger thermal rate and skin friction coefficient compared to the Graphene-Al_2O_3/H_2O, which makes Cu-Al_2O3/H_2O a good option for the industrial cooling processes.

1. Introduction

There is no doubt that with recent advances in information technology, energy demand is at an all-time high. The quest for alternative energy sources, coupled with the steady decline in fossil fuel reserves, has presented a challenge in terms of innovation and the preservation of the planet's ecosystem [1]. The concept of nanotechnology has been in discourse since Richard Feynman's speech and later inspired K. Eric Drexler, leading to the emergence of nanotechnology as a distinct field of study. With the progress in nanotechnology, nanofluids include nanoparticles with sizes less than 100 nm, constituting metallic or non-metallic particles within conventional fluids. Additionally, research has given rise to a subcategory known as hybrid nanofluid, which is formed by dispersing nanoparticles into a base/working fluid, with the addition of additives to facilitate dispersion [2].

* *Corresponding author.*
E-mail address: najiyah@utem.edu.my

https://doi.org/10.37934/arfmts.116.2.5974

Greater stability is achieved through surface modification and the formation of strong cluster bonds between nanoparticles. Anoop *et al.,* [3] conducted a study on convective heat transfer properties in tube flow experiencing constant heat flux, revealing that the nanofluids exhibited significantly higher heat transfer properties compared to the base fluid alone. Additional studies explored the characteristics of hybrid nanofluids in various scenarios and experiments. Sheikholeslami and Sadoughi [4] presented a study investigating the impact of heat transfer when CuO/water nanofluids are disturbed by magnetic fields, resulting in reduced convection performance. Furthermore, Sheikholeslami and Shehzad [5] employed Fe_3O_4/water nanofluid in a porous cavity subjected to a non-uniform magnetic field to obtain temperature gradients, considering an array of shape factors and external magnetic fields. Conversely, Duangthongsuk and Wongwises [6] utilized TiO_2/water hybrid nanofluid to determine thermal conductivity under temperature constraints. The application of nanofluids extends to various industries, as demonstrated by Jacobsen *et al.,* [7], who discussed spray drying using electrohydrodynamic (EHD) in the food industry. Furthermore, applications were found in chemicals and composites, oil and gas, building materials, temperature control for automobiles, and electronic devices [8-17]. Beyond that, numerous papers related to the hybrid nanofluids have been thoroughly numerically examined [18-22].

In an unsteady flow, the quantity of liquid flowing per second varies, indicating a non-constant rate. Numerous studies have explored this phenomenon, with one notable research project conducted by Sreedevi *et al.,* [23]. Their investigation delved into the analysis of unsteady magnetohydrodynamic heat and mass transfer, focusing on the combination of carbon nanotubes and silver nanoparticles interacting with suction and exhibiting slip effects due to chemical reactions. Paul *et al.,* [24] examined the flow velocities, temperature distribution, and concentration profiles in Casson hybrid nanofluids while traversing an unsteady vertical rotating cone. To address the problem's high non-linearity, the study employed MATLAB's bvp4c method. The research delves into investigating the influence of magnetic parameters, radiation parameters, thermophoresis, Brownian motion, and chemical reactions in the context of Casson hybrid nanofluid flow over an unsteady vertical rotating cone. Khan *et al.,* [25] proposed a research study on the heat transfer characteristics of the hybrid nanofluid Cu-Al_2O_3/water passing through a radially shrinking and stretching surface. Continuing with the literature review, Waini *et al.,* [26] presented a paper on the unsteady flow of hybrid nanofluid Al_2O_3/water over a stagnation point on a permeable rigid surface. An insightful treatise was penned by Bilal *et al.,* [27] analysing the unsteady magnetohydrodynamic flow of CNT-Fe_3O_4/water within a porous horizontal channel undergoing dilation. Transitioning to research conducted by Jayadevamurthy *et al.,* [28], the study utilized the flow between rotating disks to determine bioconvective effects within hybrid nanofluids. Additionally, it is crucial to acknowledge the significance of the study conducted by Zainal *et al.,* [29], which focused on the unsteady electromagnetohydrodynamic (EMHD) flow with the application of boundary layer theory over a stretching and shrinking plate for the hybrid nanofluid Cu-Al_2O_3/water. Zainal *et al.,* [30] extended this study with another paper on three-dimensional stationary unsteady MHD convection of the same set of mixed nanofluids.

Transport mediums play a crucial role in this study, with the shapes and materials of mediums such as porous spheres or shrinking sheets significantly influencing the mathematical data. In this paper, the focus is on a stretching/shrinking sheet as the studied medium. In the studies by Goud *et al.,* [31], Bejawada *et al.,* [32], Reddy *et al.,* [33], Goud [34], various porous medium surface was used as the transport mechanism for the respective fluid flow. An interesting study by Anuar and Bachok [35], employing a deformable sheet, revealed double solutions for an unsteady flow in Cu-Al_2O_3/water hybrid nanofluid. Building on previous the research, Anuar *et al.,* [36] published a numerical computational study on Cu-Al_2O_3/water hybrid nanofluid's flow via a deformable sheet experiencing slip. Additional references can be found in Khashi'ie *et al.,* [37]. Mahabaleshwar *et al.,* [38] presented a study on the momentum and impact of thermal radiation on the hybrid nanofluid Al_2O_3-Cu/water past a shrinking/stretching sheet. Khashi'ie *et al.,* [39] found numerical solutions for fluid motion past a deformable surface with controlled heat influx using the hybrid nanofluid Al_2O_3-Cu/water. Some honourable mentions on different transport medium can also be stated such in the case of Waini *et al.,* [40]. The stagnation point on the stretching/shrinking cylinder was examined using mixed nanofluids Cu/water and Al_2O_3/water. In reference to previous works, Mukhopadhyay [41] discussed the laminar boundary layer of a viscous incompressible fluid, exploring the relationship between heat transfer on a stretching porous cylinder and investigating mixed convection. Consequently, buoyancy-aided flow increases fluid velocity and vice versa, revealing a notable result of bifurcation occurring in shrinking areas of the medium. Similar research was undertaken by Hamid *et al.,* [42] and Usman *et al.,* [43], focusing on MoS_2 nanofluid along a stretching surface with varying heat conductivity, and water and ethylene-glycol-based copper nanoparticles for squeezing porous cylinders. Additionally, Waqas *et al.,* [44] published an article on magnetohydrodynamic flow over a vertical stretching cylinder, employing engine oil as the base fluid and a combination of multi-walled carbon nanotubes-(MWCNT-MoS4) and single-walled carbon nanotubes-(SWCNT-Ag) as hybrid nanoparticles of choice. This research provides insights into how the porosity variable slows down fluid flow in relation to a larger temperature ratio. Elsaid *et al.,* [45] developed Finite Element Method (FEM) mathematical models for hybrid nanofluid Al_2O_3 and Cu in a tangent fluid, and Al_2O_3 as a singular nanofluid. The results were fascinating, showing that the diffusion of momentum in the stretching of the cylinder was quicker in nanofluid compared to hybrid nanofluid; however, the hybrid nanofluid exhibited better thermal performance. To delve deeper into the concept of the boundary layer, McLeod and Rajagopal [46] conducted a comprehensive study on the unique solution for a stretching wall, demonstrating that the equations generated did not have a second solution. Another notable research by Devi and Devi [47] provides a distinct comparison of heat transfer characteristics between nanofluid and hybrid nanofluid. This study examines the effects of the Lorentz force over a stretching surface in heating, presenting numerical solutions. The data obtained on the enhancement of mechanical properties primarily reveals a rise in cooling efficiency by 18-28% with varying percentages of nanoparticles used. An examination was done on stagnation points on the stretching/shrinking cylinder with hybrid nanofluids Cu/water and Al_2O_3/water.

Heat generation is a crucial part of the heat transfer performance of hybrid nanofluids. Numerous studies have investigated the relationship between heat generation and hybrid nanoparticles. Jusoh *et al.,* [48] addressed the impacts of heat transfer of nanofluids, including MHD properties and heat generation. Devi and Devi [49] subsequently analysed a parametric study using the hybrid nanofluid Cu-Al_2O_3/water to investigate thermal characteristics over a stretching plate. A study by Joshi *et al.,* [50] investigated the impact of magnetohydrodynamic (MHD) flow on a bidirectional porous stretchable sheet experiencing volumetric heat generation using the hybrid nanofluid SWCNT + Ag + water. In the same year, Masood *et al.,* [51] discussed the heat generation effect on the stagnation point of TiO_2-H_2O flow. Furthermore, Abbas *et al.,* [52] provided a study utilizing the boundary layer flow for Cu-Al_2O_3/water hybrid nanofluid over a curved stretching sheet with variable changes, including a magnetic field, slip factor, radiation, and, most importantly, heat generation. Other than that, in a study done by Murugesan and Kumar [53] published on the impacts of heat generation for radiative hybrid nanofluid Ag-Al_2O_3/water for MHD flow via a stretching porous medium sheet were numerically analysed. Subsequently, Amala and Mahanthesh [54] published a study on the nonlinear convective flow of hybrid nanofluid Al_2O_3/water and Cu-Al_2O_3 over a rotating plate with Hall current experiencing heat absorption. Hayat *et al.,* [55] produced a journal on the similar topic of discussion to examine the effects of MHD with heat generation and nanofluid volume fraction on a rotating stretching surface. Shoaib *et al.,* [56] also studied the flow features of MHD hybrid nanofluid focusing on its heat transfer properties under two parallel plates. Another unique study was conducted by Al-Hossainy and Eid [57] to investigate heat generation in solar cell cooling systems with the deployment of hybrid nanofluids ZnO/PG+ water set on a double tube. The discussion clearly showed that heat generation plays a significant role in the performance of solar cells.

The research conducted in this study is a culmination of extensive investigation, drawing significant inspiration in addressing the challenges associated with unsteady flow and heat generation. The focus is primarily on hybrid nanofluid, aiming to bridge existing knowledge gaps in this specific domain. The chosen approach, as proposed by Devi and Devi, is considered the most promising course of action for advancing our understanding in this field. The uniqueness of this study lies in its utilization of two distinct mediums: a hybrid nanofluid (Cu−Al_2O_3/water) and a nanofluid (Graphene-Al_2O_3/water). By exploring the dynamics of flow over a permeable stretching sheet with heat generation, the research aims to contribute novel insights that go beyond the existing literature. One notable aspect of novelty is the application of the bvp4c solver in MATLAB, employed to derive solutions for the ordinary differential equations governing the system. This method is chosen for its effectiveness in providing accurate solutions to complex mathematical models, enhancing the reliability of the results obtained. In the event of encountering dual visible solutions, the research incorporates stability analysis to discern the most favourable outcome. This analytical approach adds a layer of robustness to the findings, ensuring that the proposed solutions are not only accurate but also stable under different conditions. The practical significance of this work extends to its potential applications in various fields. Understanding the behaviour of hybrid nanofluid and their impact on unsteady flows with heat generation holds promise for advancements in thermal management systems, energy efficiency, and related technologies. The outcomes of this study are anticipated to provide valuable insights for engineers, researchers, and practitioners, offering a foundation for optimizing heat transfer processes in real-world applications. Moreover, the comprehensive analysis presented in the form of tables and discussions on generated figures enhances the clarity and applicability of the findings. The rigorous research undertaken in this study positions it as a unique contribution to the existing body of knowledge, and the fact that it has yet to be explored by other scholars emphasizes its originality and potential to shape future research in the field. This study marks a pioneering advancement in the field, paving the way for numerous real-world applications and capturing the interest of both scholars and industry stakeholders eager to harness these groundbreaking insights for improved efficiency and innovation in various industrial sectors. The practical applicability of this model spans a wide range of fields, including reactor engineering, biomedical devices, environmental engineering, renewable energy, aerospace and spacecraft design, microfluidics, and lab-on-a-chip devices. It holds the potential to transform industrial processes by optimizing heat transfer and fluid dynamics, ultimately resulting in heightened efficiency and cost-effectiveness. The study's multidisciplinary applications position it as a foundational element for future progress in numerous industries.

2. Mathematical Formulation

Consider hybrid nanofluids with unsteady flow (graphene-alumina/water and copper-alumina/water) past a stretching/shrinking sheet as presented in Figure 1. The boundary layer and energy equations are modeled based on the physical assumptions

i. The velocity of the deformable sheet is represented by $u_w = cx/(1-\alpha t);\;\; c>0$ such that t refers to time, α is the representative for the unsteady flow such that $\alpha<0$ decelerates the outer flow, $\alpha=0$ for the steady flow and $\alpha>0$ expedites the flow [58].

ii. T_w and T_∞ represent the surface and far-field temperatures, respectively.

iii. The imposed heat generation effect is mathematically expressed as $Q_1 = Q_0 x/(1-\alpha t)$ where Q_1 and Q_0 is the heat generation factor and constant, respectively [59].

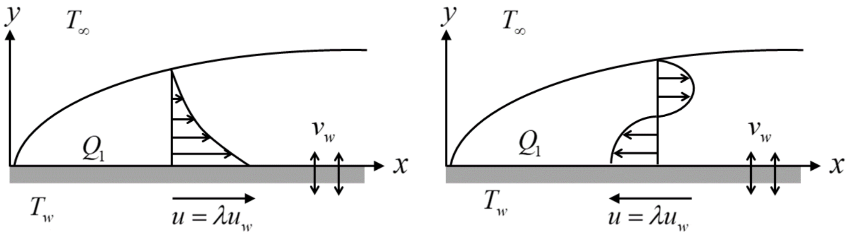


Fig. 1. Coordinate system of the physical problem for stretching (left) and shrinking (right) cases

Hence, the mathematical model in a two-dimensional system which renders this physical problem are given as [58,59]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_{hnf}}\frac{\partial p}{\partial x}+\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_{hnf}}\frac{\partial p}{\partial y}+\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 v}{\partial y^2}, \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{hnf}}{\left(\rho C_p\right)_{hnf}}\frac{\partial^2 T}{\partial y^2}+\frac{Q_1}{\left(\rho C_p\right)_{hnf}}\left(T-T_\infty\right), \tag{4}$$

$$\begin{aligned}&u=\lambda u_w, \quad v=v_w, \quad T=T_w \quad \text{when} \quad y=0\\ &u\to 0, \quad v\to 0, \quad T\to T_\infty \quad \text{as} \quad y\to\infty\end{aligned} \tag{5}$$

By considering u,v as the velocities while T as the temperature of the hybrid nanofluid, the similarity transformation is given by

$$u=\frac{cx}{1-\alpha t}f'(\eta), \; v=-\sqrt{\frac{c\nu_f}{1-\alpha t}}f(\eta), \; \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \; \eta=\sqrt{\frac{c}{\nu_f}}\frac{y}{\sqrt{1-\alpha t}}. \tag{6}$$

Eq. (6) is substituted into Eq. (2) to Eq. (5) to form a set of reduced ordinary (similarity) differential equations which has also been discussed in the previous studies [58,59]

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}f'''+ff''-f'^2-S\left(f'+\frac{1}{2}\eta f''\right)=0, \tag{7}$$

$$\frac{1}{\Pr}\frac{k_{hnf}/k_f}{\left(\rho C_p\right)_{hnf}/\left(\rho C_p\right)_f}\theta''-\left(\frac{1}{2}S\eta-f\right)\theta'+\frac{Q}{\left(\rho C_p\right)_{hnf}/\left(\rho C_p\right)_f}\theta=0, \tag{8}$$

$$\begin{aligned}&f(0)=B,\quad f'(0)=\lambda,\quad \theta(0)=1,\\ &f'(\eta)\to 0,\quad \theta(\eta)\to 0,\quad \text{as } \eta\to\infty\end{aligned} \tag{9}$$

where the parameters in Eq. (7) to Eq. (9) are defined as

i. unsteadiness parameter $(S=\alpha/c)$; $S>0$ for unsteadiness accelerating flow, $S<0$ for unsteadiness decelerating flow and $S=0$ for the steady flow case,
ii. stretching/shrinking parameter (λ); $\lambda<0$ for the shrinking case, $\lambda=0$ for the static sheet and $\lambda>0$ for the stretching case,
iii. Mass flux parameter (B), $B<0$ for the injection process, $B=0$ for the impermeable surface and $B>0$ for the suction process,
iv. heat generation parameter $\left(Q=Q_0/\left(\rho C_p\right)_f c\right)$, $Q<0$ for the heat absorption process and $Q>0$ for the heat generation process, and
v. Prandtl number $\left(\Pr=\left(C_p\mu\right)_f/k_f\right)$

Table 1 and Table 2 display the experimentally validated correlations of properties for hybrid nanofluids and properties for the copper, graphene and water [60,61].

Table 1
General correlations of hybrid nanofluid [60]

Properties	Nanofluid
Thermal Conductivity	$k_{hnf}=\left[\dfrac{\left(\dfrac{\phi_1k_1+\phi_2k_2}{\phi_{hnf}}\right)-2\phi_{hnf}k_f+2\left(\phi_1k_1+\phi_2k_2\right)+2k_f}{\left(\dfrac{\phi_1k_1+\phi_2k_2}{\phi_{hnf}}\right)+\phi_{hnf}k_f-\left(\phi_1k_1+\phi_2k_2\right)+2k_f}\right]k_f$
Heat Capacity	$\left(\rho C_p\right)_{hnf}=\phi_1\left(\rho C_p\right)_{s1}+\phi_2\left(\rho C_p\right)_{s2}+\left(1-\phi_{hnf}\right)\left(\rho C_p\right)_f$
Density	$\rho_{hff}=\phi_1\rho_{s1}+\phi_2\rho_{s2}+\left(1-\phi_{hff}\right)\rho_f$
Dynamic Viscosity	$\mu_{hnf}=\dfrac{\mu_f}{\left(1-\phi_{hnf}\right)^{2.5}};\quad \phi_{hnf}=\phi_1+\phi_2$

Table 2
Properties of the water and nanoparticles (copper, graphene, and alumina) [61]

Properties	Base fluid	Nanoparticles		
	Water	Copper	Graphene	Alumina
ρ (kg/m^3)	997.1	8933	2200	3970
C_p (J/kgK)	4179	385	790	765
k (W/mK)	0.613	400	5000	40

Prandtl number, (Pr)	6.2	-	-	-

The definition of skin friction and local Nusselt number are.

$$C_f = \frac{\mu_{hnf}}{\rho_f u_w^2}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad Nu_x = -\frac{x k_{hnf}}{k_f\left(T_w - T_\infty\right)}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{10}$$

Substituting Eq. (6) into Eq. (10),

$$\begin{aligned} \mathrm{Re}_x^{1/2} C_f &= \frac{\mu_{hnf}}{\mu_f} f''(0), \\ \mathrm{Re}_x^{-1/2} Nu_x &= -\frac{k_{hnf}}{k_f}\theta'(0), \end{aligned} \tag{11}$$

where the local Reynolds number is $\mathrm{Re}_x = u_w x / \nu_f$. Table 3 and Table 4 display the numerically validated correlations of skin friction coefficient and local Nusselt number for Cu-Al_2O_3/H_2O hybrid nanofluid [58].

3. Numerical Method

The bvp4c solver in the Matlab software is the potent tool for computing the nonlinear ODEs, such as the simplified boundary layer equations. Through the calling syntax sol = bvp4c (@OdeBVP, @OdeBC, solinit, options), the 3-stage Lobatto IIIa is ingrained and implemented into this solver. The results are obtained by computing Eq. (7) to Eq. (9) using the properties given in Table 1 and Table 2. However, the similarity equations in Eq. (7) to Eq. (9) need to be coded into the bvp4c language such that

$$f = y(1), \quad f' = y(2), \quad f'' = y(3), \quad \theta = y(4), \qquad \theta' = y(5). \tag{12}$$

Hence, the coded equations and boundary condition for Eq. (7) to Eq. (9) are.

$$\begin{aligned} f''' &= \frac{\rho_{hnf}/\rho_f}{\mu_{hnf}/\mu_f}\left(f'^2 - ff'' + S\left(f' + \frac{1}{2}\eta f''\right)\right), \\ &= \frac{\rho_{hnf}/\rho_f}{\mu_{hnf}/\mu_f}\left(y(2)y(2) - y(1)y(3) + S\left(y(2) + \frac{1}{2}\eta y(3)\right)\right), \end{aligned} \tag{13}$$

$$\begin{aligned} \theta'' &= \mathrm{Pr}\frac{(\rho C_p)_{hnf}/(\rho C_p)_f}{k_{hnf}/k_f}\left(\left(\frac{1}{2}S\eta - f\right)\theta' - \frac{Q}{(\rho C_p)_{hnf}/(\rho C_p)_f}\theta\right), \\ &= \mathrm{Pr}\frac{(\rho C_p)_{hnf}/(\rho C_p)_f}{k_{hnf}/k_f}\left(\left(\frac{1}{2}Sx - y(1)\right)y(5) - \frac{Q}{(\rho C_p)_{hnf}/(\rho C_p)_f}y(4)\right) \end{aligned} \tag{14}$$

$$ya(1)-B,\quad ya(2)-\lambda,\quad ya(4)-1,\quad yb(2),\quad yb(4) \tag{15}$$

where ya and yb are the corresponding bvp4c code language for the conditions. Eq. (7) and Eq. (8) are coded in @OdeBVP while Eq. (9) in @OdeBC. Two solutions are possible if the solinit function contains two starting guesses. This problem necessitates two sets of estimates that are coupled with $\eta_\infty = 20$. The trial and error processes are required in providing guesses for the dual solutions where the first that fulfils Eq. (9) is denoted as the first solution.

Also, the validation of the present model for $Re_x^{1/2}C_f$ and $Re_x^{-1/2}Nu_x$ is presented in Table 3 and Table 4, respectively. The validation process with data by Waini *et al.,* [58] under the influence of unsteadiness parameter and other stated values are in good agreement which affirm the validity of the present numerical calculation. From these tables, it also can be highlighted that as the unsteadiness parameter enhances from $S=0$ to $S=-5$, the values of $Re_x^{1/2}C_f$ reduces while $Re_x^{-1/2}Nu_x$ increases. For further reference and benchmarking, the present data for $Re_x^{1/2}C_f$ and $Re_x^{-1/2}Nu_x$ are enclosed within the specific values of the physical factors as presented in Table 5.

Table 3
Validation of $Re_x^{1/2}C_f$ with Waini *et al.,* [58] when $\phi_1 = 0.2$ (copper), $\phi_2 = 0$ (alumina), $Q=0$, $B=2.1$, $\lambda=-1$ and various S [58]

S	Present		Waini *et al.,* [58]	
	First Solution	Second Solution	First Solution	Second Solution
0	2.528970	0.586548	2.528984	0.584729
-0.2	2.462145	-0.048071	2.462145	-0.048071
-0.4	2.395251	-0.473472	2.395252	-0.473472
-0.6	2.328304	-0.840776	2.328304	-0.840776
-1	2.194247	-1.491281	2.194247	-1.491281
-3	1.521192	-4.144746	1.521197	-4.144746
-5	0.844435	-6.431507	0.844435	-6.431507

Table 4
Validation of $Re_x^{-1/2}Nu_x$ with Waini *et al.,* [58] when $\phi_1 = 0.2$ (copper), $\phi_2 = 0$ (alumina), $Q=0$, $B=2.1$, $\lambda=-1$ and various S [58]

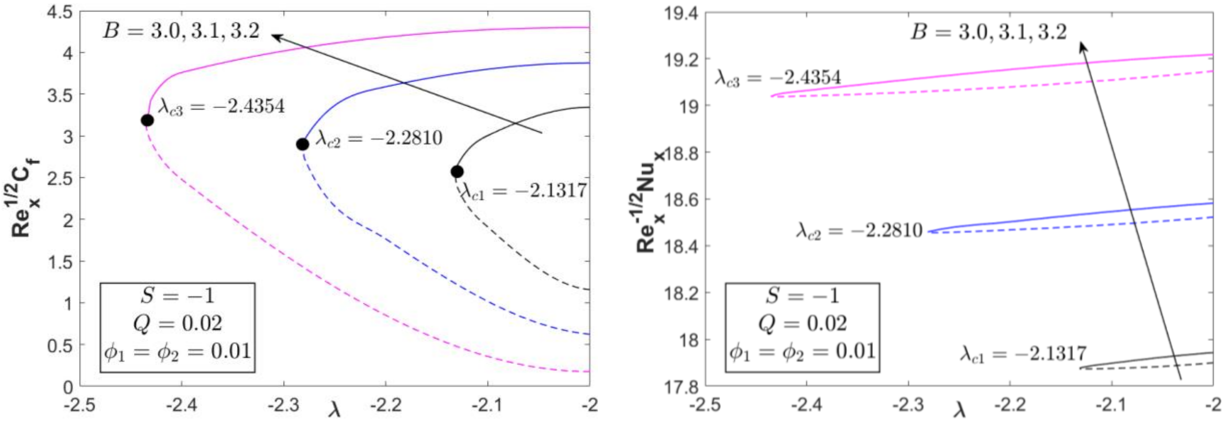
S	Present		Waini *et al.,* [58]	
	First Solution	Second Solution	First Solution	Second Solution
0	6.822453	6.693244	6.822454	6.693105
-0.2	6.875796	6.716536	6.875796	6.716536
-0.4	6.927417	6.755470	6.927417	6.755470
-0.6	6.977507	6.797695	6.977507	6.797695
-1	7.073680	6.884548	7.073680	6.884548
-3	7.497151	7.296176	7.497151	7.296176
-5	7.858446	7.657801	7.858446	7.657801

Table 5
Numerical data for $f''(0)$, $-\theta'(0)$, $Re_x^{1/2}C_f$, and $Re_x^{-1/2}Nu_x$ with different parameters

S	B	λ	Q	ϕ_1	ϕ_2	$f''(0)$	$Re_x^{1/2}C_f$	$-\theta'(0)$	$Re_x^{-1/2}Nu_x$
0	0	0.5	0	0	0	-0.353553	-0.353553	1.252249	1.252249
				0	0.005	-0.353954	-0.358418	1.241106	1.258978
				0.005	0	-0.358268	-0.362786	1.239140	1.257735
				0.005	0.01	-0.358809	-0.372625	1.216756	1.271700
				0.01	0.01	-0.36311	-0.381916	1.204173	1.277273
0	2	-1	0	0.01	0.01	1.295105	1.362196	11.154772	11.831925
	2.5					2.145232	2.256364	14.185105	15.046215
						[0.491072]	[0.516512]	[14.138735]	[14.997030]
0	2.5	-1	0.02	0.01	0.01	2.145232	2.256364	14.176463	15.037049
						[0.491072]	[0.516512]	[14.129939]	[14.987700]
-1	2.5	-1	0.02	0.01	0.01	1.865217	1.961842	14.383952	15.257134
						[-1.285456]	[-1.352048]	[14.304092]	[15.172426]

4. Results and Discussion

The explanations of the flow and thermal behaviors for graphene-alumina water and copper-alumina/water are discussed and presented in Figure 2 to Figure 11 for the variation of suction parameter B, stretching/shrinking parameter λ and heat generation parameter Q. Based on the study done by Waini *et al.,* [58]: Pr = 6.2 (water), decelerating unsteadiness parameter $S=-1$ and volume fraction of hybrid nanofluid particles, $\phi_1=\phi_2=0.01$. The graphical results for $Re_x^{1/2}C_f$ and $Re_x^{-1/2}Nu_x$ are presented in Figure 2 to Figure 5 for both hybrid nanofluids, respectively with various B and Q parameters. The critical values (first and second solutions separator) are clearly presented in Figure 2 to Figure 5 and denoted as λ_c. It is apparent that the critical values for the copper-alumina/water $(\lambda_c=-2.2745,-2.4336,-2.5982)$ are slightly greater than the graphene-alumina water $(\lambda_c=-2.1317,-2.2810,-2.4354)$ for each value of B and Q which implies the significance of the copper-alumina combination in delaying the boundary layer separation. However, there is no change for the critical values when the heat generation is imposed $(Q=0,0.02)$ as can be seen in Figure 2(b) and Figure 3 for the graphene-alumina/water and Figure 4(b) and Figure 5 for the copper-alumina/water. In addition, the graph of $Re_x^{1/2}C_f$ is not provided for $Q=0$ since it is similar to the graphs of $Re_x^{1/2}C_f$ with $Q=0.02$. The suction effect is seen to enhance $Re_x^{1/2}C_f$ as well as $Re_x^{-1/2}Nu_x$ of both hybrid nanofluids. The physical explanation is as B increases, there will be an increase in the shear stress on the working space and this will indirectly push the fluid into an unoccupied space in the surface forcing a rise in temperature to force the fluid to flow. The rise in temperature increases the heat transfer rate causing expansion on the boundary layer.



(a) (b)

Fig. 2. (a) Skin friction coefficient, and (b) heat transfer rate over λ with heat generation $(Q = 0.02)$ and various suction parameter for graphene-alumina/water

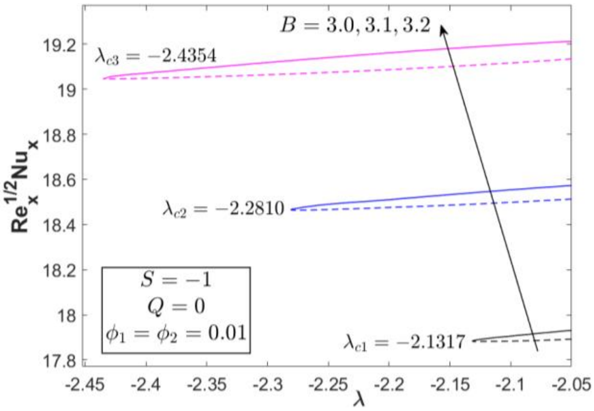
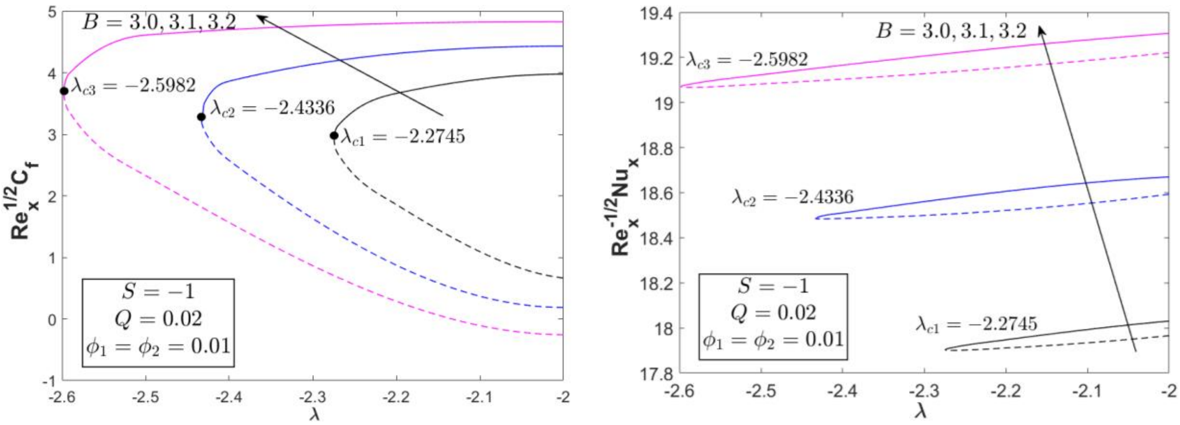


Fig. 3. Heat transfer rate over λ without heat generation $(Q = 0)$ and various suction parameter for graphene-alumina/water



(a) (b)

Fig. 4. (a) Skin friction coefficient, and (b) heat transfer rate over λ with heat generation $(Q = 0.02)$ and various suction parameter for copper-alumina/water

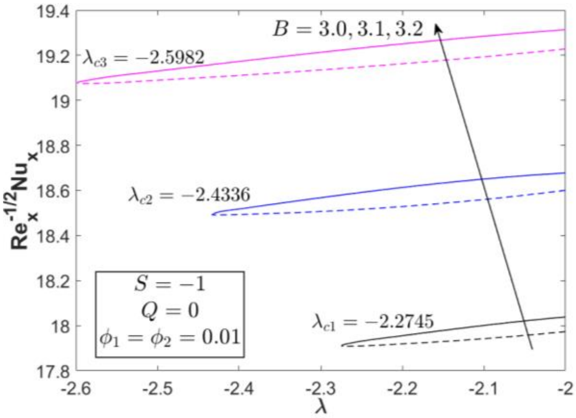


Fig. 5. Heat transfer rate over λ without heat generation $(Q=0)$ and various suction parameter for copper-alumina/water

The trends for the flow and thermal profiles of copper-alumina/water are shown in Figure 6 and Figure 7 with different values of suction for both shrinking $(\lambda=-2)$ and stretching $(\lambda=2)$ cases, respectively. The profiles fulfill the boundary condition in Eq. (9) implies the legitimate model. Further, the velocity distribution (first solution) in Figure 6(a) expands with the increment of B, but the temperature distribution (Figure 6(b)) shows an adverse result where the temperature is a decreasing function of B. The addition of the suction parameters at varying rate results in an increase in the velocity profile with the first solution (realizable solution) and a decrease in velocity profile for the second solution as the surface is experiencing shrinking motion, $\lambda=-2$. The increasing suction decreases the fluid velocity inside the boundary layer and in consequence increases the velocity gradient at the surface. Interestingly, the velocity distribution for the stretching case has a different flow pattern as compared to the shrinking case.

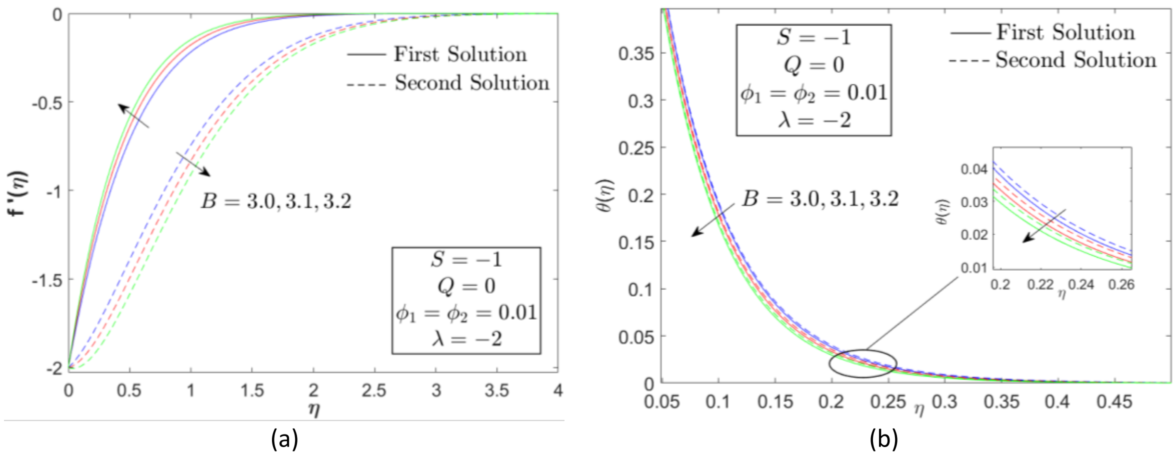
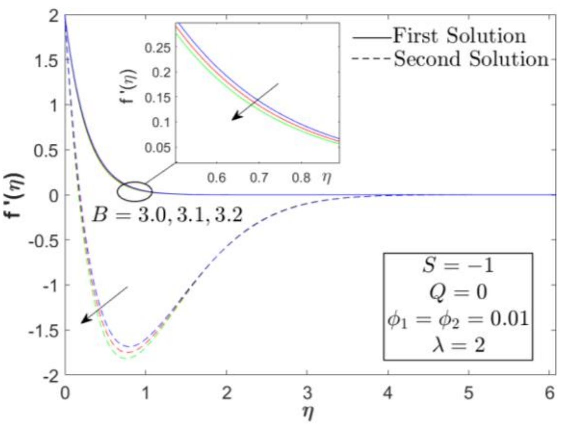
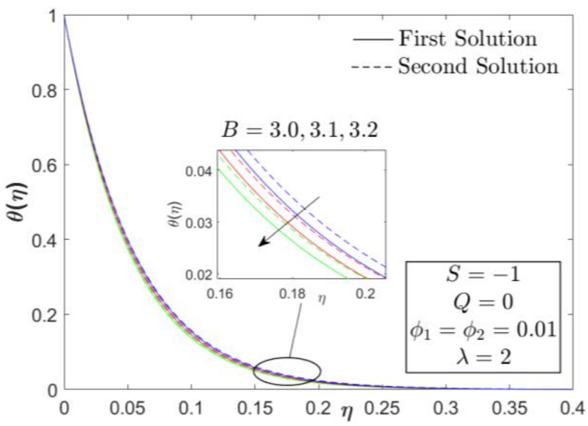


Fig. 6. (a) Velocity, and (b) temperature of Cu-Al_2O_3/H_2O for the shrinking case



(a)



(b)

Fig. 7. (a) Velocity, and (b) temperature of Cu-Al_2O_3/H_2O for the stretching case

5. Conclusions

The numerical investigation of the unsteady flow of hybrid nanofluids (graphene-alumina/water and copper-alumina/water) with heat transfer past a stretching/shrinking sheet with the suction and heat generation parameters has several conclusions as follows

i. Dual solutions are attainable for both cases (stretching and shrinking surfaces) within a specific range of the governing parameters (heat generation, unsteadiness and suction parameters).
ii. Suction parameter is essential in delaying the boundary layer separation based on the extension of the critical value with the increment of this parameter.
iii. The copper-alumina/water has higher critical values than the graphene-alumina/water which implies the significance of the copper-alumina/water in controlling the boundary layer separation.
iv. The addition of heat generation parameter gives no effect in extending/reducing the critical values which means that this parameter has no impact on the skin friction coefficient. However, the inclusion of this parameter only slightly increases the heat transfer for both hybrid nanofluids.
v. Suction parameter has a significant effect on the velocity and temperature distributions. The velocity profile increases with the increment of the suction parameter while the reduction in temperature profile is found with the increasing of suction parameter. This phenomenon can be attributed to the rise in shear stress within the working space, resulting in the fluid being pushed into unoccupied areas on the surface.

Acknowledgement
We acknowledge Universiti Teknikal Malaysia Melaka and Ministry of the Higher Education (MoHE-Malaysia) for the grant FRGS/1/2021/STG06/UTEM/03/1.

References
[1] Ahmed, Mahmoud Salem. "Nanofluid: new fluids by nanotechnology." In *Thermophysical properties of Complex materials*. London, UK: IntechOpen, 2019.
[2] Srinivas, V., Ch. V. K. N. S. N. Moorthy, V. Dedeepya, P. V. Manikanta, and V. Satish. "Nanofluids with CNTs for automotive applications." *Heat and Mass Transfer* 52 (2016): 701-712. https://doi.org/10.1007/s00231-015-1588-1

[3] Anoop, K. B., T. Sundararajan, and Sarit K. Das. "Effect of particle size on the convective heat transfer in nanofluid in the developing region." *International Journal of Heat and Mass Transfer* 52, no. 9-10 (2009): 2189-2195. https://doi.org/10.1016/j.ijheatmasstransfer.2007.11.063

[4] Sheikholeslami, Mohsen, and M. K. Sadoughi. "Simulation of CuO-water nanofluid heat transfer enhancement in presence of melting surface." *International Journal of Heat and Mass Transfer* 116 (2018): 909-919. https://doi.org/10.1016/j.ijheatmasstransfer.2017.09.086

[5] Sheikholeslami, M., and S. A. Shehzad. "Numerical analysis of Fe_3O_4-H_2O nanofluid flow in permeable media under the effect of external magnetic source." *International Journal of Heat and Mass Transfer* 118 (2018): 182-192. https://doi.org/10.1016/j.ijheatmasstransfer.2017.10.113

[6] Duangthongsuk, Weerapun, and Somchai Wongwises. "Measurement of temperature-dependent thermal conductivity and viscosity of TiO_2-water nanofluids." *Experimental Thermal and Fluid Science* 33, no. 4 (2009): 706-714. https://doi.org/10.1016/j.expthermflusci.2009.01.005

[7] Jacobsen, Charlotte, Pedro J. García-Moreno, Ana C. Mendes, Ramona V. Mateiu, and Ioannis S. Chronakis. "Use of electrohydrodynamic processing for encapsulation of sensitive bioactive compounds and applications in food." *Annual Review of Food Science and Technology* 9 (2018): 525-549. https://doi.org/10.1146/annurev-food-030117-012348

[8] Anandika, Rajeev, V. Puneeth, S. Manjunatha, and Ali J. Chamkha. "Thermal optimisation through multilayer convective flow of CuO-MWCNT hybrid nanofluid in a composite porous annulus." *International Journal of Ambient Energy* 43, no. 1 (2022): 6463-6473. https://doi.org/10.1080/01430750.2021.2023044

[9] Nejad, Shahin Mohammad, Rajat Srivastava, Francesco Maria Bellussi, Hernán Chávez Thielemann, Pietro Asinari, and Matteo Fasano. "Nanoscale thermal properties of carbon nanotubes/epoxy composites by atomistic simulations." *International Journal of Thermal Sciences* 159 (2021): 106588. https://doi.org/10.1016/j.ijthermalsci.2020.106588

[10] Ahmed, Waqar, Salim Newaz Kazi, Zaira Zaman Chowdhury, Mohd Rafie Bin Johan, Naveed Akram, M. A. Mujtaba, M. Gul, and Cheen Sean Oon. "Experimental investigation of convective heat transfer growth on ZnO@TiO_2/DW binary composites/hybrid nanofluids in a circular heat exchanger." *Journal of Thermal Analysis and Calorimetry* 143 (2021): 879-898. https://doi.org/10.1007/s10973-020-09363-x

[11] Kumar, Vivek, Jahar Sarkar, and Wei-Mon Yan. "Thermal-hydraulic behavior of lotus like structured rGO-ZnO composite dispersed hybrid nanofluid in mini channel heat sink." *International Journal of Thermal Sciences* 164 (2021): 106886. https://doi.org/10.1016/j.ijthermalsci.2021.106886

[12] Gawusu, Sidique, and Xiaobing Zhang. "Hydrodynamics analysis of Taylor flow in oil and gas pipelines under constant heat flux." *Heat and Mass Transfer* 57, no. 3 (2021): 515-527. https://doi.org/10.1007/s00231-020-02965-z

[13] Mahmoudpour, Maziar, Peyman Pourafshary, Babak Moradi, Mohammad Reza Rasaei, and Kamran Hassani. "Reduction of residual oil in oil-wet carbonate formations by application of hybrid smart water/silica nanofluid enhanced oil recovery method." In *Offshore Technology Conference Asia*, p. D041S040R003. OTC, 2022. https://doi.org/10.4043/31488-MS

[14] Mohamed, Yasser Mahmoud A., and Mohamed F. Mady. "Silicon-metal hybrid nanoparticles as nanofluid scale inhibitors in oil/gas applications." In *Silicon-Based Hybrid Nanoparticles*, pp. 353-361. Elsevier, 2022. https://doi.org/10.1016/B978-0-12-824007-6.00014-9

[15] Khan, Rafid Jahangir, Md Zubayer Hossain Bhuiyan, and Dewan Hasan Ahmed. "Investigation of heat transfer of a building wall in the presence of phase change material (PCM)." *Energy and Built Environment* 1, no. 2 (2020): 199-206. https://doi.org/10.1016/j.enbenv.2020.01.002

[16] Hafeez, Muhammad Bilal, Rohul Amin, Kottakkaran Sooppy Nisar, Wasim Jamshed, Abdel-Haleem Abdel-Aty, and M. Motawi Khashan. "Heat transfer enhancement through nanofluids with applications in automobile radiator." *Case Studies in Thermal Engineering* 27 (2021): 101192. https://doi.org/10.1016/j.csite.2021.101192

[17] Nahal, Ali Mokhtari, Mohammad Hassan Nobakhti, Cyrus Aghanajafi, and Morteza Khayat. "Analysis of combined radiation and convection heat transfer inside a porous medium with heat source electronic devices." *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* 236, no. 8 (2022): 4357-4366. https://doi.org/10.1177/09544062211046791

[18] Maheswari, Chundru, Ravuri Mohana Ramana, G. Balaji Prakash, D. Ramesh, and D. Vijaya Kumar. "Influence of Thermophoresis and Brownian Motion on MHD Hybrid Nanofluid MgO-Ag/H_2O Flow along Moving Slim Needle." *Journal of Advanced Research in Applied Sciences and Engineering Technology* 36, no. 2 (2023): 67-90. https://doi.org/10.37934/araset.36.2.6790

[19] Khashi'Ie, Najiyah Safwa, Khairum Bin Hamzah, Iskandar Waini, Nurul Amira Zainal, Sayed Kushairi Sayed Nordin, Abdul Rahman Mohd Kasim, and Ioan Pop. "Response surface methodology of the unsteady axisymmetric magnetic

hybrid nanofluid flow subject to a shrinking disk." *Journal of Advanced Research in Applied Mechanics* 112, no. 1 (2024): 137-148. https://doi.org/10.37934/aram.112.1.137148
[20] Zainal, Nurul Amira, Iskandar Waini, Najiyah Safwa Khashi'ie, Roslinda Nazar, and Ioan Pop. "MHD Hybrid Nanofluid Flow Past A Stretching/Shrinking Wedge With Heat Generation/Absorption Impact." *CFD Letters* 16, no. 6 (2024): 146-156. https://doi.org/10.37934/cfdl.16.6.146156
[21] Aljabali, Ahlam, Abdul Rahman Mohd Kasim, Najiyah Safwa Khashi'ie, Iskandar Waini, and Noor Amalina Nisa Ariffin. "Eyring-Powell Hybrid Nanofluid with Radiative Heat Flux: Case Over a Shrinking Sheet." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 111, no. 1 (2023): 109-121. https://doi.org/10.37934/arfmts.111.1.109121
[22] Khashi'ie, Najiyah Safwa, Iskandar Waini, Nur Syahirah Wahid, Norihan Md Arifin, and Ioan Pop. "Unsteady separated stagnation point flow due to an EMHD Riga plate with heat generation in hybrid nanofluid." *Chinese Journal of Physics* 81 (2023): 181-192. https://doi.org/10.1016/j.cjph.2022.10.010
[23] Sreedevi, P., P. Sudarsana Reddy, and Ali Chamkha. "Heat and mass transfer analysis of unsteady hybrid nanofluid flow over a stretching sheet with thermal radiation." *SN Applied Sciences* 2, no. 7 (2020): 1222. https://doi.org/10.1007/s42452-020-3011-x
[24] Paul, Ashish, Neelav Sarma, and Bhagyashri Patgiri. "Mixed convection of shear-thinning hybrid nanofluid flow across a radiative unsteady cone with suction and slip effect." *Materials Today Communications* 37 (2023): 107522. https://doi.org/10.1016/j.mtcomm.2023.107522
[25] Khan, Umair, Iskandar Waini, Anuar Ishak, and Ioan Pop. "Unsteady hybrid nanofluid flow over a radially permeable shrinking/stretching surface." *Journal of Molecular Liquids* 331 (2021): 115752. https://doi.org/10.1016/j.molliq.2021.115752
[26] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Unsteady hybrid nanofluid flow on a stagnation point of a permeable rigid surface." *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik* 101, no. 6 (2021): e202000193. https://doi.org/10.1002/zamm.202000193
[27] Bilal, Muhammad, Hamna Arshad, Muhammad Ramzan, Zahir Shah, and Poom Kumam. "Unsteady hybrid-nanofluid flow comprising ferrousoxide and CNTs through porous horizontal channel with dilating/squeezing walls." *Scientific Reports* 11, no. 1 (2021): 12637. https://doi.org/10.1038/s41598-021-91188-1
[28] Jayadevamurthy, Punith Gowda Ramanahalli, Naveen Kumar Rangaswamy, Ballajja Chandrappa Prasannakumara, and Kottakkaran Sooppy Nisar. "Emphasis on unsteady dynamics of bioconvective hybrid nanofluid flow over an upward-downward moving rotating disk." *Numerical Methods for Partial Differential Equations* 40, no. 1 (2024): e22680. https://doi.org/10.1002/num.22680
[29] Zainal, Nurul Amira, Roslinda Nazar, Kohilavani Naganthran, and Ioan Pop. "Unsteady EMHD stagnation point flow over a stretching/shrinking sheet in a hybrid Al_2O_3-Cu/H_2O nanofluid." *International Communications in Heat and Mass Transfer* 123 (2021): 105205. https://doi.org/10.1016/j.icheatmasstransfer.2021.105205
[30] Zainal, Nurul Amira, Roslinda Nazar, Kohilavani Naganthran, and Ioan Pop. "Unsteady MHD mixed convection flow in hybrid nanofluid at three-dimensional stagnation point." *Mathematics* 9, no. 5 (2021): 549. https://doi.org/10.3390/math9050549
[31] Goud, B. Shankar, P. Pramod Kumar, and Bala Siddulu Malga. "Effect of heat source on an unsteady MHD free convection flow of Casson fluid past a vertical oscillating plate in porous medium using finite element analysis." *Partial Differential Equations in Applied Mathematics* 2 (2020): 100015. https://doi.org/10.1016/j.padiff.2020.100015
[32] Bejawada, Shankar Goud, Yanala Dharmendar Reddy, Wasim Jamshed, Kottakkaran Sooppy Nisar, Abdulaziz N. Alharbi, and Ridha Chouikh. "Radiation effect on MHD Casson fluid flow over an inclined non-linear surface with chemical reaction in a Forchheimer porous medium." *Alexandria Engineering Journal* 61, no. 10 (2022): 8207-8220. https://doi.org/10.1016/j.aej.2022.01.043
[33] Reddy, Y. Dharmendar, B. Shankar Goud, Kottakkaran Sooppy Nisar, B. Alshahrani, Mona Mahmoud, and Choonkil Park. "Heat absorption/generation effect on MHD heat transfer fluid flow along a stretching cylinder with a porous medium." *Alexandria Engineering Journal* 64 (2023): 659-666. https://doi.org/10.1016/j.aej.2022.08.049
[34] Goud, B. Shankar. "Heat generation/absorption influence on steady stretched permeable surface on MHD flow of a micropolar fluid through a porous medium in the presence of variable suction/injection." *International Journal of Thermofluids* 7 (2020): 100044. https://doi.org/10.1016/j.ijft.2020.100044
[35] Anuar, Nur Syazana, and Norfifah Bachok. "Double solutions and stability analysis of micropolar hybrid nanofluid with thermal radiation impact on unsteady stagnation point flow." *Mathematics* 9, no. 3 (2021): 276. https://doi.org/10.3390/math9030276
[36] Anuar, Nur Syazana, Norfifah Bachok, and Ioan Pop. "Numerical computation of dusty hybrid nanofluid flow and heat transfer over a deformable sheet with slip effect." *Mathematics* 9, no. 6 (2021): 643. https://doi.org/10.3390/math9060643

[37] Khashi'ie, Najiyah Safwa, Norihan Md Arifin, Ioan Pop, Roslinda Nazar, Ezad Hafidz Hafidzuddin, and Nadihah Wahi. "Flow and heat transfer past a permeable power-law deformable plate with orthogonal shear in a hybrid nanofluid." *Alexandria Engineering Journal* 59, no. 3 (2020): 1869-1879. https://doi.org/10.1016/j.aej.2020.05.029
[38] Mahabaleshwar, U. S., A. B. Vishalakshi, and Helge I. Andersson. "Hybrid nanofluid flow past a stretching/shrinking sheet with thermal radiation and mass transpiration." *Chinese Journal of Physics* 75 (2022): 152-168. https://doi.org/10.1016/j.cjph.2021.12.014
[39] Khashi'ie, Najiyah Safwa, Iskandar Waini, Nurul Amira Zainal, Khairum Hamzah, and Abdul Rahman Mohd Kasim. "Hybrid nanofluid flow past a shrinking cylinder with prescribed surface heat flux." *Symmetry* 12, no. 9 (2020): 1493. https://doi.org/10.3390/sym12091493
[40] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Hybrid nanofluid flow towards a stagnation point on a stretching/shrinking cylinder." *Scientific Reports* 10, no. 1 (2020): 9296. https://doi.org/10.1038/s41598-020-66126-2
[41] Mukhopadhyay, Swati. "Mixed convection boundary layer flow along a stretching cylinder in porous medium." *Journal of Petroleum Science and Engineering* 96 (2012): 73-78. https://doi.org/10.1016/j.petrol.2012.08.006
[42] Hamid, Muhammad, Muhammad Usman, Tamour Zubair, Rizwan Ul Haq, and Wei Wang. "Shape effects of MoS_2 nanoparticles on rotating flow of nanofluid along a stretching surface with variable thermal conductivity: A Galerkin approach." *International Journal of Heat and Mass Transfer* 124 (2018): 706-714. https://doi.org/10.1016/j.ijheatmasstransfer.2018.03.108
[43] Usman, Muhammad, Muhammad Hamid, Rizwan Ul Haq, and Wei Wang. "Heat and fluid flow of water and ethylene-glycol based Cu-nanoparticles between two parallel squeezing porous disks: LSGM approach." *International Journal of Heat and Mass Transfer* 123 (2018): 888-895. https://doi.org/10.1016/j.ijheatmasstransfer.2018.03.030
[44] Waqas, Hassan, Syed Muhammad Raza Shah Naqvi, M. S. Alqarni, and Taseer Muhammad. "Thermal transport in magnetized flow of hybrid nanofluids over a vertical stretching cylinder." *Case Studies in Thermal Engineering* 27 (2021): 101219. https://doi.org/10.1016/j.csite.2021.101219
[45] Elsaid, Essam M., and Mohamed S. Abdel-Wahed. "Impact of hybrid nanofluid coolant on the boundary layer behavior over a moving cylinder: numerical case study." *Case Studies in Thermal Engineering* 25 (2021): 100951. https://doi.org/10.1016/j.csite.2021.100951
[46] McLeod, J. B., and K. R. Rajagopal. "On the uniqueness of flow of a Navier-Stokes fluid due to a stretching boundary." *Archive for Rational Mechanics and Analysis* 98 (1987): 385-393. https://doi.org/10.1007/BF00276915
[47] Devi, S. Suriya Uma, and S. P. Anjali Devi. "Numerical investigation of three-dimensional hybrid Cu-Al_2O_3/water nanofluid flow over a stretching sheet with effecting Lorentz force subject to Newtonian heating." *Canadian Journal of Physics* 94, no. 5 (2016): 490-496. https://doi.org/10.1139/cjp-2015-0799
[48] Jusoh, Rahimah, Roslinda Nazar, and Ioan Pop. "Impact of heat generation/absorption on the unsteady magnetohydrodynamic stagnation point flow and heat transfer of nanofluids." *International Journal of Numerical Methods for Heat & Fluid Flow* 30, no. 2 (2020): 557-574. https://doi.org/10.1108/HFF-04-2019-0300
[49] Devi, Suriya Uma, and S. P. Anjali Devi. "Heat transfer enhancement of Cu−Al_2O_3/water hybrid nanofluid flow over a stretching sheet." *Journal of the Nigerian Mathematical Society* 36, no. 2 (2017): 419-433.
[50] Joshi, Navneet, Himanshu Upreti, Alok Kumar Pandey, and Manoj Kumar. "Heat and mass transfer assessment of magnetic hybrid nanofluid flow via bidirectional porous surface with volumetric heat generation." *International Journal of Applied and Computational Mathematics* 7, no. 3 (2021): 64. https://doi.org/10.1007/s40819-021-00999-3
[51] Masood, Sadaf, Muhammad Farooq, and Aisha Anjum. "Influence of heat generation/absorption and stagnation point on polystyrene-TiO_2/H_2O hybrid nanofluid flow." *Scientific Reports* 11, no. 1 (2021): 22381. https://doi.org/10.1038/s41598-021-01747-9
[52] Abbas, Z., M. Naveed, and M. Sajid. "Hydromagnetic slip flow of nanofluid over a curved stretching surface with heat generation and thermal radiation." *Journal of Molecular Liquids* 215 (2016): 756-762. https://doi.org/10.1016/j.molliq.2016.01.012
[53] Murugesan, Thiagarajan, and M. Dinesh Kumar. "Effects of thermal radiation and heat generation on hydromagnetic flow of nanofluid over an exponentially stretching sheet in a porous medium with viscous dissipation." *World Scientific News* 128, no. 2 (2019): 130-147.
[54] Amala, Syawaludin, and B. Mahanthesh. "Hybrid nanofluid flow over a vertical rotating plate in the presence of hall current, nonlinear convection and heat absorption." *Journal of Nanofluids* 7, no. 6 (2018): 1138-1148. https://doi.org/10.1166/jon.2018.1550
[55] Hayat, Tanzila, S. Nadeem, and A. U. Khan. "Numerical analysis of Ag-CuO/water rotating hybrid nanofluid with heat generation and absorption." *Canadian Journal of Physics* 97, no. 6 (2019): 644-650. https://doi.org/10.1139/cjp-2018-0011

[56] Shoaib, Muhammad, Muhammad Asif Zahoor Raja, Muhammad Touseef Sabir, Kottakkaran Sooppy Nisar, Wasim Jamshed, Bassem F. Felemban, and I. S. Yahia. "MHD hybrid nanofluid flow due to rotating disk with heat absorption and thermal slip effects: an application of intelligent computing." *Coatings* 11, no. 12 (2021): 1554. https://doi.org/10.3390/coatings11121554

[57] Al-Hossainy, Ahmed F., and Mohamed R. Eid. "Structure, DFT calculations and heat transfer enhancement in $[ZnO/PG+ H_2O]^C$ hybrid nanofluid flow as a potential solar cell coolant application in a double-tube." *Journal of Materials Science: Materials in Electronics* 31, no. 18 (2020): 15243-15257. https://doi.org/10.1007/s10854-020-04089-w

[58] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Unsteady flow and heat transfer past a stretching/shrinking sheet in a hybrid nanofluid." *International Journal of Heat and Mass Transfer* 136 (2019): 288-297. https://doi.org/10.1016/j.ijheatmasstransfer.2019.02.101

[59] Kumbhakar, Bidyasagar, and Susmay Nandi. "Unsteady MHD radiative-dissipative flow of $Cu\text{-}Al_2O_3/H_2O$ hybrid nanofluid past a stretching sheet with slip and convective conditions: A regression analysis." *Mathematics and Computers in Simulation* 194 (2022): 563-587. https://doi.org/10.1016/j.matcom.2021.12.018

[60] Takabi, Behrouz, and Saeed Salehi. "Augmentation of the heat transfer performance of a sinusoidal corrugated enclosure by employing hybrid nanofluid." *Advances in Mechanical Engineering* 6 (2014): 147059. https://doi.org/10.1155/2014/147059

[61] Oztop, Hakan F., and Eiyad Abu-Nada. "Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids." *International Journal of Heat and Fluid Flow* 29, no. 5 (2008): 1326-1336. https://doi.org/10.1016/j.ijheatfluidflow.2008.04.009