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Eyring-Powell Fluid Flow Past a Shrinking Sheet: Effect of Magnetohydrodynamic (MHD) and Joule Heating

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ABSTRACT

This paper examines the effect of magnetohydrodynamic (MHD) on Eyring-Powell fluid flow over a shrinking sheet. The sheet is permeable and it is shrunk with power-law velocity. By employing the suitable similarity transformations, the governing equations are transformed into the similarity equations, and MATLAB software is used to program the code with the aid of the bvp4c function. Results reveal that the magnetic and the suction parameters raise both the velocity and temperature gradients, which consequently increases the skin friction and the heat transfer coefficients. However, these physical quantities are reduced with the Eyring-Powell fluid parameter. The domain of the solutions is affected by the rise of the magnetic and the Eyring-Powell fluid parameters. From the stability analysis, the second solution is unstable while the first solution is stable over time.

1. Introduction

Most fluids that engineers and scientists work with, like water, air, and oils, can be thought of as Newtonian beneath certain conditions of attraction. However, the assumption of Newtonian behavior is frequently inappropriate, necessitating the demonstration of a rather progressive and intricate non-Newtonian reaction. These situations occur in the chemical preparation and plastics handling industries and are also seen in applications like oil and biological streams as well as in the mining industry. This makes the recreation of the non-Newtonian liquid stream phenomenon important to the business. Owing to the suitability of these fluids for industrial applications including power engineering, food engineering, and petroleum production, non-Newtonian fluid analysis is still of significant interest to researchers [1]. One of the non-Newtonian fluids, Eyring-Powell fluids, has certain benefits over the other models [2,3]. According to a few studies, this model appears to be extremely truthful and reliable in calculating the fluid time scale in different polymer sizes [4-6].

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Roşca and Pop [7] and Jalil *et al.,* [8] examined the Eyring-Powell fluid flows in a parallel free stream, while Ara *et al.,* [9] and Akbar *et al.,* [10] studied the effect of radiation and MHD, respectively. Additionally, Ghadikolaei *et al.,* [11] examined an unsteady flow in stretching channel. Besides, Fatunmbi and Adeosun [12] reported the flow along a vertical Riga plate. Aljabali *et al.,* [13] considered the temperature-dependent viscosity effects. Moreover, Iqbal *et al.,* [14] studied the Eyring-Powell fluid flow over a bidirectionally stretched surface. They found that the radial velocity is reduced while the transverse velocity is enhanced with increasing values of the magnetic field, Hall effect, Ion slip, and material parameters.

The magnetohydrodynamic (MHD) explains how magnetic fields and fluid flow interact with one another. The liquid metals, strong electrolytes, and hot ionised gases are among the electrically conducting, non-magnetic fluids for which MHD flows are explored. The study of MHD flow is crucial and has many uses in the fields of engineering and technology. For example, such flows can be found in design cooling systems, pumps, flow metres, electric motors, MHD generators, and other devices [1]. Noor and Hashim [15] investigated the MHD flow and heat transmission near an implanted permeable shrinking sheet. The MHD flow caused by an exponentially stretched sheet was covered by Ishak [16]. Turkyilmazoglu [17] investigated the characteristics of heat transmission in MHD flow caused by a rotating disk that is contracting. Using the KKL model, Sheikholeslami *et al.,* [18] reported the nanofluid flow with the MHD effect. Rashidi *et al.,* [19] examined natural convection in MHD flow on a flat plate. Besides, Khan *et al.,* [20,21] investigated the effects of MHD flow over a spinning disk and a curved stretching surface. Other than that, research on MHD can also be found in the previous studies [22-30].

Theoretically, the transformation of electrical energy into thermal state energy, causes the generation of heat from resistive losses referring to the joule heating effect. In other words, Joule heating is the process by which heat is generated on a conductor using an electric current. Joule heating is frequently used in a variety of electrical and electronic devices as well as in industrial processes, including resistance ovens, electric heaters, and food cooked on iron [31]. The effects of Joule heating on the MHD Burgers fluid flow over stretching sheet were explored by Hayat *et al.,* [32]. Khashi'ie *et al.,* [33] respectively examined the impact of Joule heating in Cu-Al_2O_3/water along a contracting cylinder. Chamkha *et al.,* [34] scrutinised the Joule heating effect between two parallel plates containing hybrid nanofluid. Moreover, numerous researchers have conducted more studies on the Joule heating effect as reported in the previous studies [35-39].

Therefore, the main strength of this numerical study is to examine the flow and thermal characteristics of Eyring-Powell fluid flow past a nonlinearly shrinking sheet where the sheet is shrunk with power-law velocity. Most importantly, the study of dual solutions on the flow of the Eyring-Powell with the MHD and Joule heating has not been considered. Moreover, this present paper reported on the critical values of the physical parameters, and the stability analysis of the dual solutions are also conducted.

2. Methodology

The physical model of the Eyring-Powell fluid flow towards a shrinking sheet is described in Figure 1. The velocity of the surface is denoted by $u_w(x) = ax^{1/3}$ with $a > 0$. Furthermore, $T_w(x) = T_\infty + T_0 x^{2/3}$ implies the prescribed surface temperature with the reference T_0 and the ambient T_∞ temperatures, respectively. The magnetic field $B(x) = B_0 x^{-1/3}$ where B_0 is constant magnetic strength [40]. Also, the Joule heating effect and the radiative heat flux $q_r = -(4\sigma^*/3k^*)(\partial T^4/\partial y)$ where $T^4 \cong 4T_\infty^3 T - 3T_\infty^4$

with σ^* and k^* signifies the Stefan-Boltzmann and the Rosseland mean absorption coefficients, are employed in the energy equation [41].

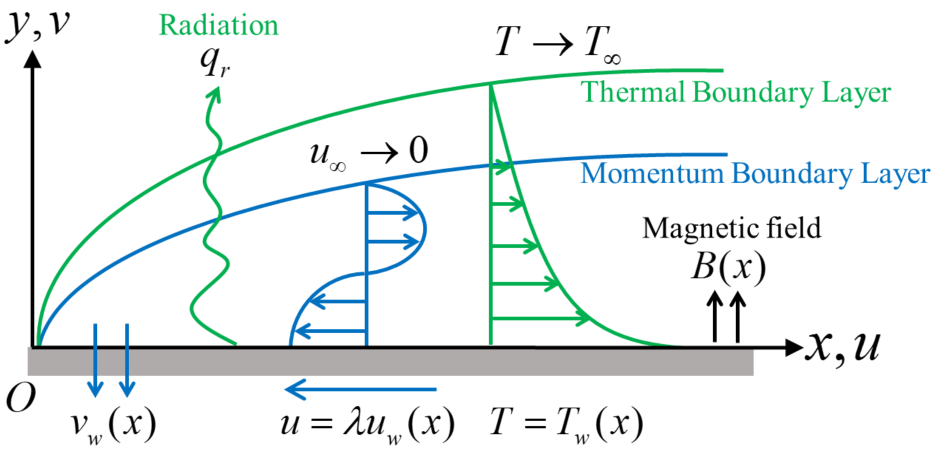

Fig. 1. Schematic model

Accordingly, the governing equations are [7,9,10]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + \frac{1}{\beta\delta}\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{2\beta\delta^3}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} - \sigma B^2 u, \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \left(k + \frac{16\sigma^* T_\infty^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2} + \sigma B^2 u^2, \tag{3}$$

subject to

$$\begin{aligned} &u = \lambda u_w(x),\ v = v_w(x),\ T = T_w(x) \ \text{ at } \ y = 0; \\ &u \to 0,\ T \to T_\infty \ \text{ as } \ y \to \infty, \end{aligned} \tag{4}$$

where T is the temperature, and (u, v) be the velocity components in the (x, y) direction. Besides, β and δ are fluid parameters of the Eyring-Powell model, σ is the electric conductivity, k is the thermal conductivity, ρ is the fluid density, and ρC_p is the heat capacity.

The similarity solutions are [7]

$$\psi = \sqrt{a\nu}\, x^{2/3} f(\eta),\quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\quad \eta = \frac{y}{x^{1/3}}\sqrt{\frac{a}{\nu}}. \tag{5}$$

Then,

$$u = \frac{\partial \psi}{\partial y} = a x^{1/3} f'(\eta), \quad v = -\frac{\partial \psi}{\partial x} = -\sqrt{a\nu}\, x^{-1/3} \left(\frac{2}{3} f(\eta) - \frac{1}{3} \eta f'(\eta) \right), \tag{6}$$

and

$$v_w(x) = -\frac{2}{3} \sqrt{a\nu}\, x^{-1/3} S, \tag{7}$$

where $f(0) = S$ is the constant mass flux parameter. Here, $S = 0$ and $S > 0$ denote the impermeable and the suction cases, respectively. Then, the similarity equations are

$$\left(1 + \beta_1 - \beta_1 \delta_1 \left(f'' \right)^2 \right) f''' + \frac{2}{3} f f'' - \frac{1}{3} f'^2 - M f' = 0, \tag{8}$$

$$\frac{1}{\Pr} \left(1 + \frac{4}{3} R \right) \theta'' + \frac{2}{3} \left(f \theta' - f' \theta \right) + Ec M f'^2 = 0, \tag{9}$$

subject to

$$\begin{aligned} & f(0) = S, \ \ f'(0) = \lambda, \ \ \theta(0) = 1; \\ & f'(\eta) \to 0, \ \ \theta(\eta) \to 0 \ \text{ as } \ \eta \to \infty, \end{aligned} \tag{10}$$

with $\lambda = 0$ (static sheet), $\lambda = 1$ (stretching sheet) and $\lambda = -1$ (shrinking sheet). Moreover, the Eyring-Powell fluid parameters β_1 and δ_1, the Eckert number Ec, the thermal radiation parameter R, the magnetic parameter M, and the Prandtl number $\Pr$, are defined by

$$\beta_1 = \frac{1}{\mu \beta \delta}, \ \ \delta_1 = \frac{a^3}{2\delta^2 \nu}, \ \ Ec = \frac{a^2}{T_0 C_p}, \ R = \frac{4\sigma^* T_\infty^3}{k k^*}, \ \ M = \frac{\sigma}{\rho a} B_0^2, \ \Pr = \frac{\mu C_p}{k}. \tag{11}$$

The physical quantities of interest are

$$C_f = \frac{\tau_w}{\rho u_w^2(x)}, \qquad Nu_x = \frac{x q_w}{k \left(T_w - T_\infty \right)}, \tag{12}$$

where τ_w (wall shear stress) and q_w (heat flux) are

$$\tau_w = \left(\left(\mu + \frac{1}{\beta\delta} \right) \frac{\partial u}{\partial y} - \frac{1}{6\beta\delta^3} \left(\frac{\partial u}{\partial y} \right)^3 \right)_{y=0}, \qquad q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} + \left(q_r \right)_{y=0}. \tag{13}$$

Thus, one obtains

$$C_f \,\mathrm{Re}_x^{1/2} = f''(0) + \beta_1 f''(0) - \frac{1}{3}\beta_1\delta_1 f''^3(0), \quad Nu_x \,\mathrm{Re}_x^{-1/2} = -\left(1 + \frac{4}{3}R\right)\theta'(0). \tag{14}$$

with $C_f \,\mathrm{Re}_x^{1/2}$ indicate the skin friction coefficient and $Nu_x \,\mathrm{Re}_x^{-1/2}$ is the local Nusselt number where $\mathrm{Re}_x = u_w(x)x/\nu$ is the local Reynolds number.

3. Stability Analysis

The execution of the stability analysis is done in this section following Merkin [42] and Weidman *et al.,* [43]. In this regard, the new variables with the dimensionless time variable Γ are introduced as follows [7]

$$\psi = \sqrt{a\nu}\,x^{2/3} f(\eta,\Gamma), \;\; \theta(\eta,\Gamma) = \frac{T - T_\infty}{T_w - T_\infty}, \;\; \eta = \frac{y}{x^{1/3}}\sqrt{\frac{a}{\nu}}, \;\; \Gamma = \frac{a}{x^{2/3}}t\,. \tag{15}$$

So that

$$\begin{aligned} u &= a x^{1/3}\frac{\partial f}{\partial \eta}(\eta,\Gamma), \\ v &= -\sqrt{a\nu}\,x^{-1/3}\left(\frac{2}{3}f(\eta,\Gamma) - \frac{1}{3}\eta\frac{\partial f}{\partial \eta}(\eta,\Gamma) - \frac{2}{3}\Gamma\frac{\partial f}{\partial \Gamma}(\eta,\Gamma)\right). \end{aligned} \tag{16}$$

Employing the unsteady flow as follows

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + \frac{1}{\beta\delta}\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{2\beta\delta^3}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} - \sigma B^2 u\,, \tag{17}$$

$$\rho C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \left(k + \frac{16\sigma^* T_\infty^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2} + \sigma B^2 u^2\,, \tag{18}$$

while Eq. (1) remains unchanged. Then, one obtains

$$\begin{aligned} &\left(1 + \beta_1 - \beta_1\delta_1\left(\frac{\partial^2 f}{\partial \eta^2}\right)^2\right)\frac{\partial^3 f}{\partial \eta^3} \\ &\quad + \frac{2}{3}f\frac{\partial^2 f}{\partial \eta^2} - \frac{1}{3}\left(\frac{\partial f}{\partial \eta}\right)^2 - M\frac{\partial f}{\partial \eta} - \frac{\partial^2 f}{\partial \eta\partial \Gamma} - \frac{2}{3}\Gamma\left(\frac{\partial f}{\partial \Gamma}\frac{\partial^2 f}{\partial \eta^2} - \frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \eta\partial \Gamma}\right) = 0, \end{aligned} \tag{19}$$

$$\frac{1}{\Pr}\left(1+\frac{4}{3}R\right)\frac{\partial^2\theta}{\partial\eta^2} + \frac{2}{3}\left(f\frac{\partial\theta}{\partial\eta}-\frac{\partial f}{\partial\eta}\theta\right)+EcM\left(\frac{\partial f}{\partial\eta}\right)^2-\frac{\partial\theta}{\partial\Gamma}-\frac{2}{3}\Gamma\left(\frac{\partial f}{\partial\Gamma}\frac{\partial\theta}{\partial\eta}-\frac{\partial f}{\partial\eta}\frac{\partial\theta}{\partial\Gamma}\right)=0, \tag{20}$$

subject to

$$\begin{aligned}&f(0,\Gamma)-\Gamma\frac{\partial f}{\partial\Gamma}(0,\Gamma)=S,\ \frac{\partial f}{\partial\eta}(0,\Gamma)=\lambda,\ \theta(0,\Gamma)=1;\\&\frac{\partial f}{\partial\eta}(\eta,\Gamma)\to 0,\ \theta(\eta,\Gamma)\to 0\ \text{ as }\ \eta\to\infty\end{aligned} \tag{21}$$

The perturbation functions are [43]

$$f(\eta,\Gamma)=f_0(\eta)+e^{-\alpha\Gamma}F(\eta,\Gamma),\ \theta(\eta,\Gamma)=\theta_0(\eta)+e^{-\alpha\Gamma}H(\eta,\Gamma), \tag{22}$$

where $F(\eta,\Gamma)$ and $H(\eta,\Gamma)$ are arbitrary functions and α denotes the eigenvalue. By setting $\Gamma=0$, then $F(\eta,\Gamma)=F_0(\eta)$ and $G(\eta,\Gamma)=G_0(\eta)$. Using Eq. (22), one gets

$$\left(1+\beta_1\right)F_0''' -\beta_1\delta_1\left(f_0''\right)^2F_0''' -2\beta_1\delta_1 f_0'''f_0''F_0'' +\frac{2}{3}\left(f_0F_0''+f_0''F_0\right)-\frac{2}{3}f_0'F_0'-MF_0'+\alpha F_0'=0, \tag{23}$$

$$\frac{1}{\Pr}\left(1+\frac{4}{3}R\right)H_0''+\frac{2}{3}\left(f_0H_0'+\theta_0'F_0\right)-\frac{2}{3}\left(f_0'H_0+\theta_0F_0'\right)+2EcMf_0'F_0'+\alpha H_0=0, \tag{24}$$

subject to:

$$\begin{aligned}&F_0(0)=0,\ F_0'(0)=0,\ H_0(0)=0;\\&F_0'(\eta)\to 0,\ H_0(\eta)\to 0\ \text{ as }\ \eta\to\infty.\end{aligned} \tag{25}$$

Here, $F_0'(\eta)\to 0$ as $\eta\to\infty$ is changed to $F_0''(0)=1$ to determine the values of α in Eq. (23) and Eq. (24) [44-46].

4. Results and Discussion

The solutions to Eq. (8) to Eq. (10) are obtained using the bvp4c function [47]. The effects of various physical characteristics are then investigated and displayed in both graphical and tabular formats. The reliability of the present model is validated by providing a comparison value of $f''(0)$ for several values of *S* as shown in Table 1. For limiting cases, results are in agreement with those reported by Waini *et al.,* [48], Cortell [49], and Ferdows *et al.,* [50]. This supports the validity, accuracy, and precision of the current numerical outcomes. Meanwhile, Table 2 is provided to show the variation of $\mathrm{Re}_x^{1/2}C_f$ and $\mathrm{Re}_x^{-1/2}Nu_x$ towards M and β_1 for shrinking case ($\lambda=-1$).

Table 1
Values of $f''(0)$ for different S when $M=\beta_1=\delta_1=0$ and $\lambda=1$ (stretching case)

S	Waini *et al.,* [48]	Cortell [49]	Ferdows [50]	Present Result
0.5	-0.873643	-0.873627	-0.873643	-0.873643
0	-0.677648	-0.677647	-0.677648	-0.677648
-0.5	-0.518869	-0.518869	-0.518869	-0.518869

Table 2
Values of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ when $\delta_1=0.1, S=2.4, Ec=0.1, R=2, \Pr=7,$ and $\lambda=-1$ (shrinking case)

M	β_1	$\mathrm{Re}_x^{1/2} C_f$		$\mathrm{Re}_x^{-1/2} Nu_x$	
		First Solution	Second Solution	First Solution	Second Solution
0	0.1	1.005976	0.859916	7.929873	7.461377
0.01		1.046092	0.826692	8.009216	7.244228
0.02		1.076254	0.804147	8.062001	6.994229
0.01	0	1.153161	0.767147	8.347821	6.813277
	0.05	1.107409	0.785428	8.201309	7.021389
	0.1	1.046092	0.826692	8.009216	7.244228

Figure 2 and Figure 3 show the impact of suction, S and the magnetic, M parameters on $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$. Note that the solutions are generated with less suction strength as M value increases, where the critical points occur at $S_{c1}=2.3821$ $(M=0)$, $S_{c2}=2.3589$ $(M=0.01)$ and $S_{c3}=2.3354$ $(M=0.02)$. Besides, the values of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ are boosted by the imposition of M. Physically, the fluid motion is effectively opposed by the increasing Lorentz force that results from the magnetic field. While the flow is contracting, the suction helps to stabilise the unconfined vorticity, which tends to simultaneously raise the skin friction coefficient. Additionally, the magnetic parameter increases the thermal rate, and aids in the fluid's movement while also pushing the hot particles in the direction of the plate.

Contradictly, the effect of Eyring-Powell fluid parameter β_1 reduces the values of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ as shown in Figure 4 and Figure 5. In addition, more suction strength is required for the existence of the dual solutions, where the solutions are terminated at $S_{c1}=2.2498$ $(\beta_1=0)$, $S_{c2}=2.3050$ $(\beta_1=0.05)$ and $S_{c3}=2.3589$ $(\beta_1=0.1)$. Meanwhile, the velocity and the temperature for several values of Eyring-Powell fluid parameter β_1 are portrayed in Figure 6 and Figure 7. Note that the far-field boundary conditions were satisfied asymptotically. It can be seen that the thicknesses of the momentum and the thermal boundary layers are increased with an increase in β_1. From the physical insight, these behaviours are caused by the fluid viscosity becoming less viscous, and consequently, the fluid velocity is decreased for larger value of β_1. Similarly, higher temperature is also noticed with larger values of β_1.

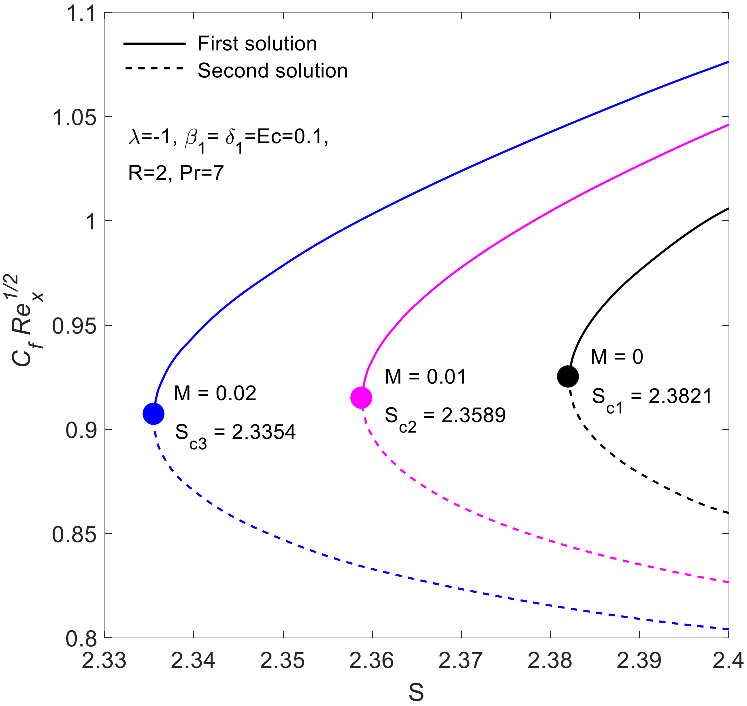

Fig. 2. Variations of $C_f \, \mathrm{Re}_x^{1/2}$ against S and M

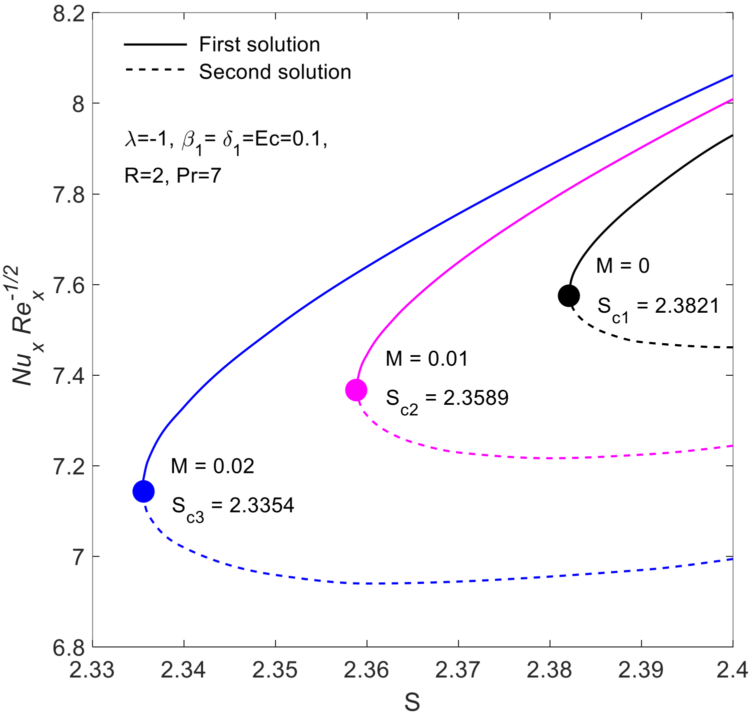

Fig. 3. Variations of $Nu_x \, \mathrm{Re}_x^{-1/2}$ against S and M

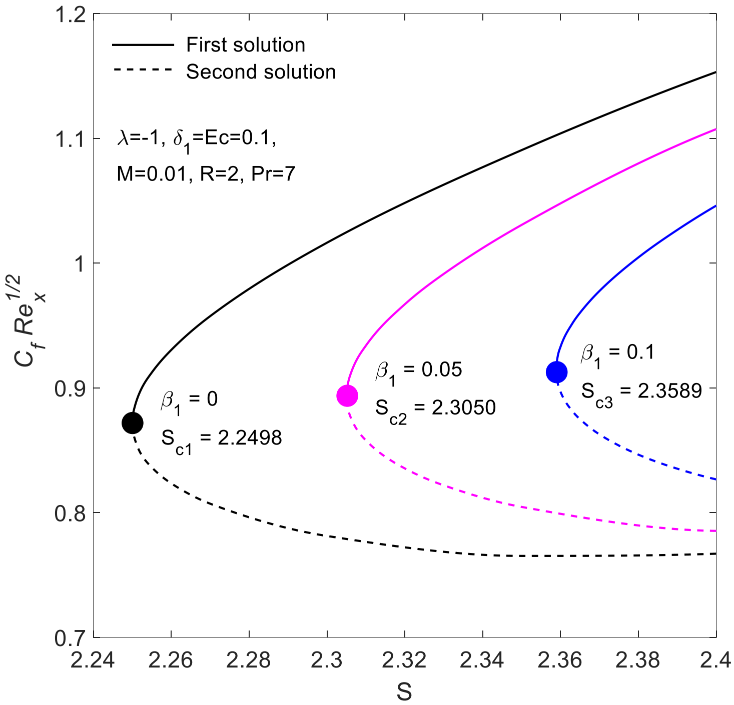

Fig. 4. Variations of $C_f\,\mathrm{Re}_x^{1/2}$ against S and β_1

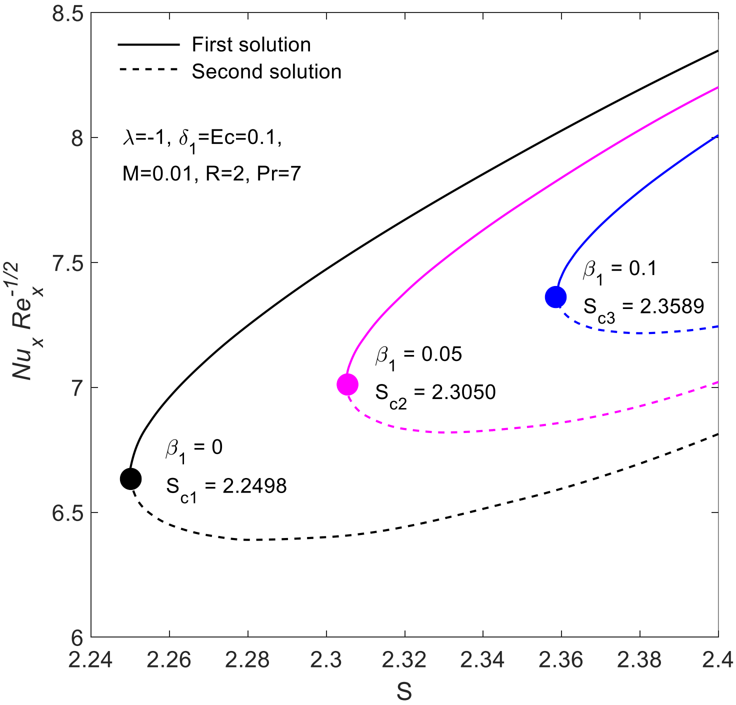

Fig. 5. Variations of $Nu_x\,\mathrm{Re}_x^{-1/2}$ against S and β_1

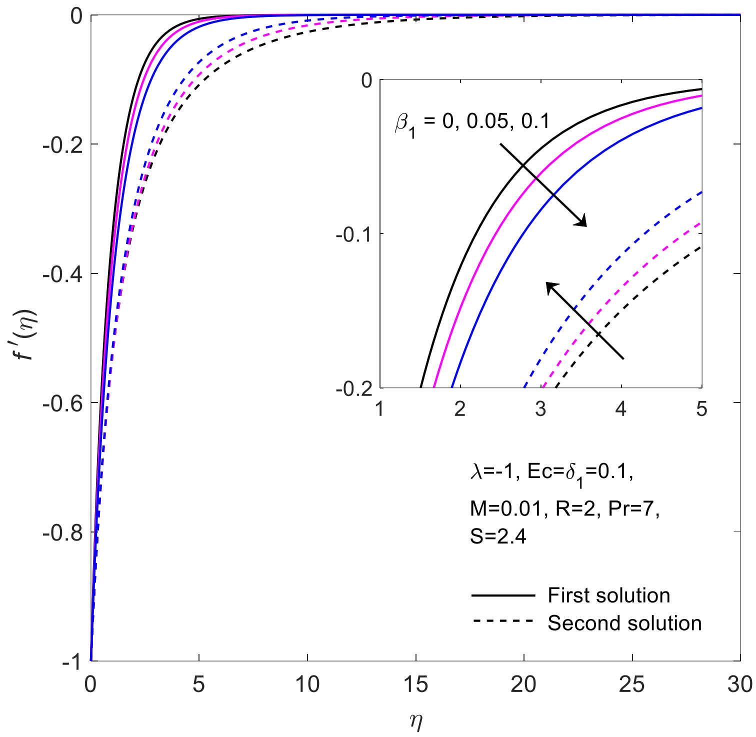

Fig. 6. Velocity profiles $f'(\eta)$ for different values of β_1

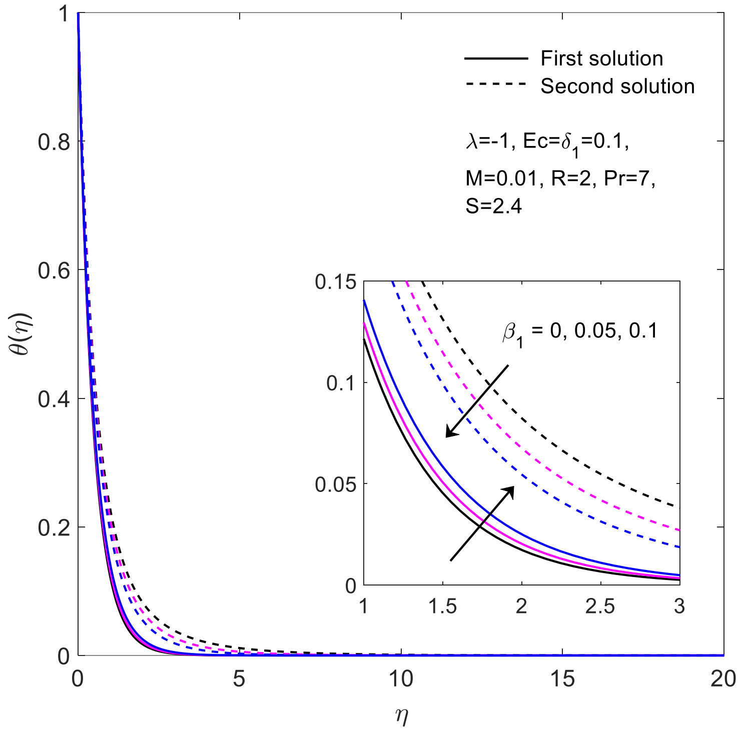

Fig. 7. Temperature profiles $\theta(\eta)$ for different values of β_1

The smallest eigenvalues α against S is shown in Figure 8, with the negative eigenvalue designating the second solution while the positive eigenvalue indicating the first solution. This leads to the conclusion that the first solution is consistent and increasingly trustworthy over time, but the second solution behaves in the opposite way.

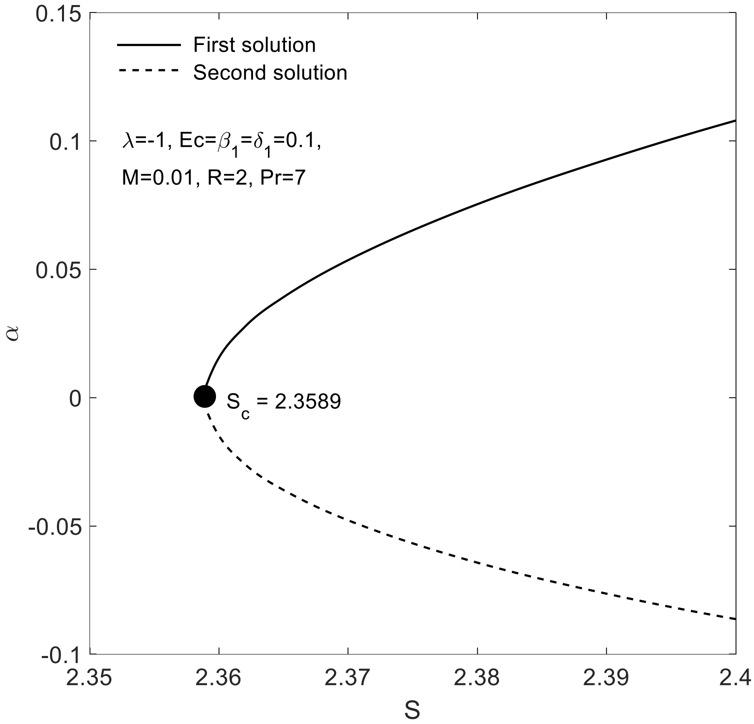

Fig. 8. Smallest eigenvalue α for different values of S

5. Conclusions

An Eyring-Powell fluid flow with the magnetohydrodynamic (MHD) and Joule heating effects was considered. The sheet is assumed to move forward and backward from the slit with the power-law velocity. The findings are as follows

i. Adequate suction strength is needed for the dual solutions to be exists and the domain of the solutions for *S* are expanded with the rise of the magnetic parameter.
ii. The magnetic parameter led to enhance the heat transfer rate and the skin friction since Lorentz force exists to control fluid motion.
iii. Eyring-Powell fluid parameter with a larger value lessening the fluid viscosity and consequently lowers the skin friction and the heat transfer rate.
iv. From the stability analysis, the second solution is unstable while the first solution is stable over time.

For future study, this problem can be extended to different geometries such as cylinder and wedge. Besides, to get more insight towards the fluid behaviour, the inclusion of various thermal conditions can also can be considered, such as, heat flux and Newtonian heating conditions.

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