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Time-Depending Flow of Ternary Hybrid Nanofluid Past a Stretching Sheet with Suction and Magnetohydrodynamic (MHD) Effects

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ABSTRACT

This research examines the laminar magnetohydrodynamic (MHD) flow of a mixture of three different nanoparticles, known as a ternary hybrid nanofluid, over a permeable stretching sheet. In this analysis, we are considering a permeable stretching sheet that is decelerating, with unsteadiness parameter $\beta \leq 0$. The governing equations are turned into similarity equations by utilizing appropriate similarity transformations. The MATLAB software is then employed to program the code, utilizing the bvp4c function. The skin friction and heat transfer coefficients plots, along with velocity and temperature profiles, are delivered for various values of the suction, unsteadiness, magnet, and nanoparticle volume fraction parameters. According to the numerical findings, both unsteadiness and suction parameters play roles in boosting the heat transfer rate. Nevertheless, the heat transfer rate is reduced by the augmentation of magnetic parameter.

1. Introduction

Nowadays, the need for enhanced thermal conductivity and heat transfer has prompted researchers to develop new fluids to meet this demand. This is because conventional fluids like water, ethylene glycol, ethanol, and glycerine, widely used in applications such as heating and cooling systems, power generation, and chemical processes, have inherent limitations in heat transfer properties. As a result, this has stimulated the advancement of ternary hybrid nanofluids, comprising a stable combination of three distinct types of nanoparticles in a base fluid. These innovative fluids may exhibit unique thermal and fluidic properties that have sparked widespread interest among researchers when compared to both single and hybrid nanofluids. In this context, understanding and harnessing the potential of ternary hybrid nanofluids have become pivotal for addressing challenges and optimizing performance in various heat transfer applications. In their experiment, Ramadhan *et*

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al., [1] noted that the ternary hybrid nanofluid, comprising alumina (Al_2O_3), titania (TiO_2), and silica (SiO_2) in ethylene glycol, achieved its maximum thermal conductivity at a volume concentration of 0.3%. Dezfulizadeh *et al.,* [2] carried out a test on the viscous dynamics and thermal diffusivity of a ternary hybrid nanofluid consisting of Cu-SiO_2-MWCNT particles dispersed in water. In addition, they established practical relationships in this particular situation. Also, numerical studies ternary hybrid nanofluids also were done by a number of researchers. A study was carried out by Ishak *et al.,* [3] on a ternary hybrid nanofluid containing gyrotactic microorganisms over a horizontally stretching/shrinking plate, it was observed that ternary hybrid nanofluid (water/Ag-Al_2O_3-Cu) exhibits superior heat transfer rate performance compared to nanofluids (water/Al_2O_3) and hybrid nanofluids (water/Ag-Al_2O_3). Thermal performance of rotating flow of Al_2O_3-SiC-MWCNT/water with radiation and fluid dissipation due to viscosity was examined by Sarangi *et al.,* [4]. Meanwhile, Animasaun *et al.,* [5] explored the effect of magnetic flux density and heat source/sink on the flow of Ag-Al_2O_3-Al/water at the stagnation point over a sheet subjected to convective heating. Subsequently, Mahmood *et al.,* [6] studied the flow of Cu-Fe_2O_4-SiO_2/water at the stagnation point around a heated stretching/shrinking cylinder, considering the combined influence of suction and a heat source. Nanofluids have extensive applications as coolants, lubricants, and in other practical uses such as in heat transfer, solar energy, heat exchangers, the oil and gas industry, cooling technology, and thermal energy storage systems [7]. The attention of experts has been drawn to ternary hybrid nanofluids, with insights taken from the previous studies [8-11].

Flow that varies with time is termed unsteady flow, while flow unaffected by time is referred to as steady flow. Engineers and researchers generally prefer steady flow as it allows for better control compared to unsteady flow. However, the impact of unsteadiness on flow cannot be ignored. Therefore, researchers must incorporate the effects of unsteady flow in their operations. McCroskey [12] states that certain systems require perform time-dependent motion in order to carry out their fundamental tasks. For instance, start-up procedures entail the shift from one steady flow to another, as well as the recurring movements of the operational substance. In a study conducted by Kebede *et al.,* [13] on the time-dependent flow of Williamson nanofluid, it was revealed that the velocity and temperature gradients exhibit decreasing trends as functions of the unsteadiness parameter within the boundary layer. Madhukesh *et al.,* [9] examined the effect of the unsteadiness parameter on the flow of a hybrid nanofluid on a stretching sheet, taking into account the influence of thermal radiation. The researchers observed a decrease in the velocity and temperature gradients as the unsteadiness parameter increases. Meanwhile, Shoaib *et al.,* [14] investigated the mass and heat transfer characteristics of unsteady hybrid nanofluid flow over a stretching surface. Their research showed that a higher fluid temperature and a steeper velocity gradient are the outcomes of a higher unsteadiness parameter. Kumar *et al.,* [15] conducted a study on unsteady MHD oscillatory flow over a vertical permeable stretching plate in a viscous, incompressible fluid. The study revealed an increasing trend in fluid velocity over time.

There have been a great number of investigations carried out to explore the impact of magnetohydrodynamics (MHD) on diverse fluid flow scenarios, driven by its widespread applications in scientific and engineering domains, including fusion reactors, optical fibre filters, crystal growth, metal casting, optical grafting, plastic sheet stretching, and metallurgical processes. Khashi'ie *et al.,* [16] noted that MHD is employed in certain industrial processes to control and manipulate the flow of conductive fluids, resulting in heightened efficiency in heat transmission and mixing. In the metallurgical context, the use of magnetohydrodynamics involves the cooling of drawn (stretched) strips in a quiescent fluid. The quality of the final products is intricately linked to the cooling rate. Consequently, MHD fluid, with its capacity to regulate cooling rates, can serve as the quiescent fluid [17]. Samat *et al.,* [18] conducted a numerical study on the MHD effect within carbon nanotubes nanofluid over a moving surface. Their findings revealed a direct correlation between the magnetic field intensity and the heat transfer rate, as well as skin friction. These outcomes align with the conclusions of Motozawa *et al.,* [19], suggesting that the presence of a magnetic parameter enhances the heat transfer rate of the fluid. Contrastingly, Shateyi and Muzara [20] delved into the unsteady MHD Williamson fluid flow over a stretching sheet. Their research showed that increasing the magnetic parameter reduces Williamson fluid heat transfer, raising fluid temperature. In a study by Muntazir *et al.,* [21], the unsteady MHD nanofluid flow around a permeable linearly stretching sheet was examined, taking into account the effects of thermal radiation and viscous dissipation. Their findings unveiled the thickness of the thermal boundary layer increased as the magnetic parameter increased. The authors also discovered that Cu nanofluid exhibits a superior heat transfer rate compared to Al_2O_3 nanofluid. Wahid *et al.,* [22] observed an augmentation in the heat transfer rate of hybrid nanofluid over an inclined shrinking surface in the presence of a stronger magnetic field. Zainal *et al.,* [23] scrutinized MHD flow past a stretching/shrinking wedge, taking into account the influence of heat generation or absorption in a hybrid nanofluid. The findings demonstrate an augmentation in the heat transmission rate with an increase in the magnetic parameter. Asghar and Ying [24] observed that with an increase in the magnetic parameter, the velocity of MHD hybrid nanofluid flow over a rotating stretching/shrinking sheet increase, while the temperature decreases.

Hence, this study extends the work conducted by Ishak [25] to investigate a ternary hybrid nanofluid, taking into account the suction effect. As far as the authors are aware, the current body of research does not address the issue of unsteady flow across a permeable stretching sheet involving ternary hybrid nanofluid. Thus, the authors are motivated to conduct a numerical investigation to study the flow characteristics and heat transfer of the unsteady flow across a permeable stretching sheet using a ternary hybrid nanofluid (Al_2O_3-Cu-TiO_2/water) with suction and magnetic field effect. Notably, the effects of wall mass suction and magnetohydrodynamics on the unsteady flow of ternary hybrid nanofluid have not been considered before. Furthermore, it effectively communicates that the study provides valuable insights for engineers and researchers regarding the parameters that can be manipulated to either enhance or reduce heat transfer rates.

2. Methodology

Consider a two-dimensional unsteady flow of an electrically conducting ternary hybrid nanofluid past a permeable stretching sheet with wall mass suction. Figure 1 depicts the flow configuration considered in this study where Cartesian coordinates are employed. The wall mass suction velocity is assumed to be $v_w(x,t)$ and the velocity of stretching sheet is $u_w(x,t) = cx/(1-\alpha t)$, where c is a positive constant, t is time, and α is the unsteadiness parameter of this problem. Additionally, the wall temperature, T_w and ambient temperature, T_∞ are treated as constants. Simultaneously, a uniform magnetic field is exerted in the positive y-direction, perpendicular to the surface. The induced magnetic field is considered insignificant compared to the applied magnetic field and is so ignored. The ternary hybrid nanofluid is formed through the dispersion of Al_2O_3, Cu, and TiO_2 nanoparticles within a water medium. Assuming thermal balance and no slip between the base fluid and nanoparticles, it is also assumed that the ternary hybrid nanofluid is incompressible and laminar.

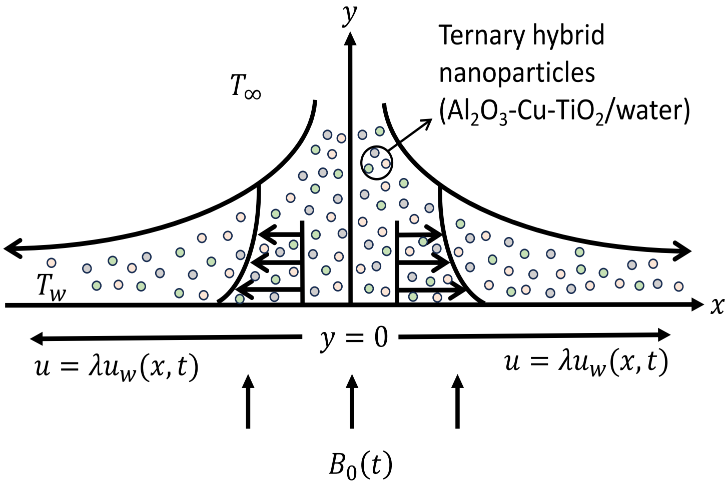

Fig. 1. Two-dimensional unsteady flow configuration

The governing equations for the flow problem are [25,26]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{thnf}}{\rho_{thnf}}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{thnf}}{\rho_{thnf}}B_0^2(t)u \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{thnf}}{(\rho C_p)_{thnf}}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Accompanied with boundary conditions

$$\begin{aligned} &u = \lambda u_w(x, t),\ v = v_w(x, t),\ T = T_w, \text{ at } y = 0; \\ &u \to 0,\ T \to T_\infty \text{ as } y \to \infty \end{aligned} \tag{4}$$

where u and v are the components of velocity in the $x -$ and $y -$ directions, accordingly. T is the ternary hybrid nanofluid temperature, μ is the dynamic viscosity, ρ is the fluid density, k is the thermal conductivity and ρC_p is the heat capacity. Besides, B_0 is the uniform magnetic field, σ is the electrical conductivity and the subscript $thnf$ in the equations represents ternary hybrid nanofluid.

The thermophysical correlation of hybrid nanofluid and ternary hybrid nanofluid is presented in Table 1. In Table 1, φ_1, φ_2 and φ_3 represent Al_2O_3, Cu and TiO_2 nanoparticles volume fraction, respectively. The subscripts f, nf, hnf and $thnf$ correspond to the base fluid, nanofluid, hybrid nanofluid and ternary hybrid nanofluid, respectively. Meanwhile the thermophysical properties for the base fluid (water), aluminium oxide (Al_2O_3), copper (Cu) and titanium oxide (TiO_2) utilized in this study is presented in Table 2.

Table 1
Thermophysical properties of ternary hybrid nanofluid [27-29]

Property	Ternary hybrid nanofluid	Hybrid nanofluid
Dynamic viscosity	$\mu_{thnf} = \frac{\mu_{nf}}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}(1-\varphi_3)^{2.5}}$.	$\mu_{hnf} = \frac{\mu_{nf}}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}$.
Density	$\rho_{thnf} = (1-\varphi_3)\{(1-\varphi_2)[(1-\varphi_1)\rho_f + \varphi_1\rho_{s1}] + \varphi_2\rho_{s2}\} + \varphi_3\rho_{s3}$	$\rho_{hnf} = (1-\varphi_2)[(1-\varphi_1)\rho_f + \varphi_1\rho_{s1}] + \varphi_2\rho_{s2}$
Thermal conductivity	$\frac{k_{thnf}}{k_{hnf}} = \frac{k_{s3}+2k_{hnf}-2\varphi_3(k_{hnf}-k_{s3})}{k_{s3}+2k_{hnf}+\varphi_3(k_{hnf}-k_{s3})}$, where $\frac{k_{hnf}}{k_{nf}} = \frac{k_{s2}+2k_{nf}-2\varphi_2(k_{nf}-k_{s2})}{k_{s2}+2k_{nf}+\varphi_2(k_{nf}-k_{s2})}$, where $\frac{k_{nf}}{k_f} = \frac{k_{s1}+2k_f-2\varphi_1(k_f-k_{s1})}{k_{s1}+2k_f+\varphi_1(k_f-k_{s1})}$.	$\frac{k_{hnf}}{k_{nf}} = \frac{k_{s2}+2k_{nf}-2\varphi_2(k_{nf}-k_{s2})}{k_{s2}+2k_{nf}+\varphi_2(k_{nf}-k_{s2})}$, where $\frac{k_{nf}}{k_f} = \frac{k_{s1}+2k_f-2\varphi_1(k_f-k_{s1})}{k_{s1}+2k_f+\varphi_1(k_f-k_{s1})}$.
Heat capacity	$(\rho C_p)_{thnf} = (1-\varphi_3)\{(1-\varphi_2)[(1-\varphi_1)(\rho C_p)_f + \varphi_1(\rho C_p)_{s1}] + \varphi_2(\rho C_p)_{s2}\} + \varphi_3(\rho C_p)_{s3}$	$(\rho C_p)_{hnf} = (1-\varphi_2)[(1-\varphi_1)(\rho C_p)_f + \varphi_1(\rho C_p)_{s1}] + \varphi_2(\rho C_p)_{s2}$
Electrical conductivity	$\frac{\sigma_{thnf}}{\sigma_{hnf}} = \frac{\sigma_{s3}+2\sigma_{hnf}-2\varphi_3(\sigma_{hnf}-\sigma_{s3})}{\sigma_{s3}+2\sigma_{hnf}+\varphi_3(\sigma_{hnf}-\sigma_{s3})}$, where $\frac{\sigma_{hnf}}{\sigma_{nf}} = \frac{\sigma_{s2}+2\sigma_{nf}-2\varphi_2(\sigma_{nf}-\sigma_{s2})}{\sigma_{s2}+2\sigma_{nf}+\varphi_2(\sigma_{nf}-\sigma_{s2})}$, where $\frac{\sigma_{nf}}{\sigma_f} = \frac{\sigma_{s1}+2\sigma_f-2\varphi_1(\sigma_f-1)}{\sigma_2+2\sigma_f+\varphi_1(\sigma_f-\sigma_2)}$.	$\frac{\sigma_{hnf}}{\sigma_{nf}} = \frac{\sigma_{s2}+2\sigma_{nf}-2\varphi_2(\sigma_{nf}-\sigma_{s2})}{\sigma_{s2}+2\sigma_{nf}+\varphi_2(\sigma_{nf}-\sigma_{s2})}$, where $\frac{\sigma_{nf}}{\sigma_f} = \frac{\sigma_{s1}+2\sigma_f-2\varphi_1(\sigma_f-1)}{\sigma_2+2\sigma_f+\varphi_1(\sigma_f-\sigma_2)}$.

Table 2
Thermophysical properties of utilized nanoparticles and water [26,28]

Property	Al_2O_3	Cu	TiO_2	Water
$C_p(J/kgK)$	765	385	686.5	4179
$\rho(kg/m^3)$	3970	8933	4250	997.1
$k(W/mK)$	40	400	8.9538	0.613
$\sigma(\Omega\text{m})^{-1}$	35×10^6	5.96×10^7	1.0×10^{-18}	5.5×10^{-6}
Prandtl Number, Pr				6.2

Since the governing equations are in partial differential form, then the equations are reduced to system of ordinary differential equations using similarity transformation. Further, to obtain similarity solutions of Eq. (1) to Eq. (4), the unsteady magnetic field B_0 is in the form $B_0 = \frac{B}{\sqrt{1-\alpha t}}$, where B is a constant. The similarity variables used in this study are adopted from Tie-Gang *et al.,* [30] and Ishak [25]

$$\psi = x\sqrt{\frac{c\nu_f}{1-\alpha t}}f(\eta),\ \eta = y\sqrt{\frac{c}{\nu_f(1-\alpha t)}},\ \theta = \frac{T-T_\infty}{T_w-T_\infty} \tag{5}$$

Then,

$$u = \frac{\partial\psi}{\partial y} = \frac{cx}{1-\alpha t}f'(\eta),\ v = -\frac{\partial\psi}{\partial x} = -\sqrt{\frac{c\nu_f}{1-\alpha t}}f(\eta) \text{ and } v_w(x,t) = -\sqrt{\frac{c\nu_f}{1-\alpha t}}S. \tag{6}$$

where $f(0) = S$ is the constant wall mass transfer parameter. $S = 0$ and $S < 0$ denote the impermeable sheet and injection case, respectively. But in this study, only suction case is considered where $S > 0$. The similarity equations are obtained as follows:

$$\frac{\mu_{thnf}/\mu_f}{\rho_{thnf}/\rho_f} f''' + ff'' - f'^2 - \beta\left(f' + \frac{\eta}{2}f''\right) - \frac{\sigma_{thnf}}{\rho_{thnf}} Mf' = 0 \tag{7}$$

$$\frac{1}{Pr}\frac{k_{thnf}/k_f}{(\rho C_p)_{thnf}/(\rho C_p)_f}\theta'' + f\theta' - \beta\frac{\eta}{2}\theta' = 0 \tag{8}$$

with the boundary conditions

$$f(0) = S, f'(0) = \lambda, \theta(0) = 1, \text{ at } \eta = 0;$$
$$f'(\eta) \to 0, \theta(\eta) \to 0, \text{ as } \eta \to \infty \tag{9}$$

where prime denotes the differentiation with respect to η. Here, the parameter λ, which signifies the velocity ratio, takes on different values: $\lambda = 0$ corresponds to the static sheet, $\lambda < 0$ indicates the shrinking sheet, and $\lambda > 0$ indicates the stretching sheet. At the same time, $\beta = \frac{\alpha}{c}$ serves as the unsteadiness parameter, where $\beta \leq 0$ is assumed to indicate a decelerating stretching sheet in this present study. Meanwhile, M is the magnetic parameter where $M = \frac{\sigma_f}{\rho_f}\frac{B^2}{a}$. It is worth mentioning that when $M = 0$ and $\varphi_1 = \varphi_2 = \varphi_3 = 0$, the current problem simplifies to the case studied by Tie-Gang *et al.,* [30] Additionally, when $\beta = 0$ (indicating steady-state flow) and $\varphi_1 = \varphi_2 = \varphi_3 = 0$, the problem under consideration reduces to the one investigated by Liu [31].

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x which are precisely specified as follows:

$$C_f = \frac{\tau_w}{\rho_f u_w^2}, Nu_x = \frac{xq_w}{k_f(T_w - T_\infty)} \tag{10}$$

where the wall shear stress τ_w and heat flux from the surface q_w are given by

$$\tau_w = \mu_{thnf}\left(\frac{\partial u}{\partial y}\right)_{y=0} \text{ and } q_w = -k_{thnf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{11}$$

By employing the similarity variables (5) and substituting (11) in (10), the following are obtained

$$C_f Re_x^{1/2} = \frac{\mu_{thnf}}{\mu_f} f''(0) \text{ and } Nu_x Re_x^{-1/2} = -\frac{k_{thnf}}{k_f}\theta'(0) \tag{12}$$

where the local Reynolds number is $Re_x = u_w x/\nu_f$.

3. Results and Discussion

The nonlinear ordinary differential equations Eq. (7) and Eq. (8) along with the boundary conditions in (9) are solved using the bvp4c solver in MATLAB. Bvp4c stands for the boundary value problem solver that utilizes finite difference code and is developed by Shampine *et al.,* [32]. This solver implements the three stages of Lobatto IIIa formula which is a collocation formula that has fourth-order accuracy. Since it is an iteration technique, its effectiveness in obtaining a solution depends on the ability to provide the algorithm with a good initial guess. In order to solve this boundary value issue, it is important to initially convert the equations into a system of first order ordinary differential equations.

To validate the developed mathematical model, we compare the numerical results of the current model with those obtained in a prior investigation, covering various cases as presented in Table 3 to Table 5. Based on the data from Table 3, when examining a regular fluid and setting $M = S = \beta = 0$, while $\lambda = 1$, it is evident that the current findings (which are $-\theta(0)$ values) are consistent with the values obtained by Gorla and Sidawi [33], Waini *et al.,* [26], and Priyadharshini *et al.,* [34]. Meanwhile, Table 4 shows $f''(0)$ and $-\theta(0)$ values for the current and previous study from Waini *et al.,* [35] for Al_2O_3/water nanofluid with varied volume fractions φ_1. The current findings align well with the values reported by Waini *et al.,* [35]. The validation process also involved the use of Al_2O_3-Cu/water nanofluid with various values of φ_2 while keeping $Pr = 6.135, M = S = \beta = 0$ and $\lambda = 1$. Values of $Re^{1/2}C_f$ from present study were compared with results from Devi and Devi [29] and Rasool *et al.,* [36]. A favourable agreement with the data obtained by the aforementioned authors is demonstrated by the comparison.

Table 3
$-\theta(0)$ values for regular fluid ($\varphi_1 = \varphi_2 = \varphi_3 = 0$) when $M = S = \beta = 0$ and $\lambda = 1$

Pr	Gorla and Sidawi [33]	Waini *et al.,* [26]	Priyadharshini *et al.,* [34]	Present Results
2	0.9114	0.911353	0.9113	0.911358
6.13	-	1.759682	-	1.759685
7	1.8954	1.8954	1.8954	1.895403
20	3.3539	3.353902	3.3539	3.353904

Table 4
$f''(0)$ and $-\theta(0)$ for Al_2O_3/water nanofluid ($\varphi_2 = \varphi_3 = 0$) when $M = S = \beta = 0, \lambda = 1$ (stretching case) and $Pr = 6.2$

φ_1	Waini *et al.,* [35]		Present Results	
	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0.05	-1.00538	1.62246	-1.00538	1.62246
0.10	-0.99877	1.49170	-0.99877	1.49170
0.15	-0.98184	1.37543	-0.98185	1.37542
0.20	-0.95592	1.27118	-0.95593	1.27117

Table 5
$Re^{1/2}C_f$ values for Al_2O_3-Cu/water when $Pr = 6.135, M = S = \beta = 0, \lambda = 1$ (stretching case) and $\varphi_1 = 0.1$

φ_2	$Re^{1/2}C_f$		
	Devi and Devi [29]	Rasool *et al.,* [36]	Present Results
0.005	-1.327310	-1.325862	-1.327086
0.02	-1.409683	-1.404648	-1.409471
0.04	-1.520894	-1.511257	-1.520693
0.06	-1.634279	-1.620177	-1.634082

It should be noted that negative values of skin friction in Figure 2, Figure 4 and Figure 6 indicate that the stretching sheet applies a force on the fluid. Figure 2 and Figure 3 show the variations of $Re_x^{1/2}C_f$ and $Re_x^{-1/2}Nu_x$ for different values of M and φ when $S = 2.2, \beta = -1$ and $\lambda = 1$. Figure 2 demonstrates that the absolute values of $Re_x^{1/2}C_f$ rise as both the magnetic parameter and volume percentage increase. This finding is consistent with finding reported by Tiwari and Das [37], which indicates an increase in volume fraction results an increase in skin friction coefficient values. In addition, a heightened Lorentz force is produced when the magnetic parameter is increased, which leads to heightened skin friction. Nevertheless, as depicted in Figure 3, the concurrent impact of an elevated magnetic parameter and volume fraction contributes to diminishing rate of heat transfer. This observation aligns with the findings of Rafique *et al.,* [27], where an escalation in the volume percentage of nanoparticles corresponds to an increase in fluid viscosity, introducing greater resistance to fluid motion. Consequently, the fluid's efficacy in heat transfer through convection diminishes, leading to a direct decline in the local Nusselt number.

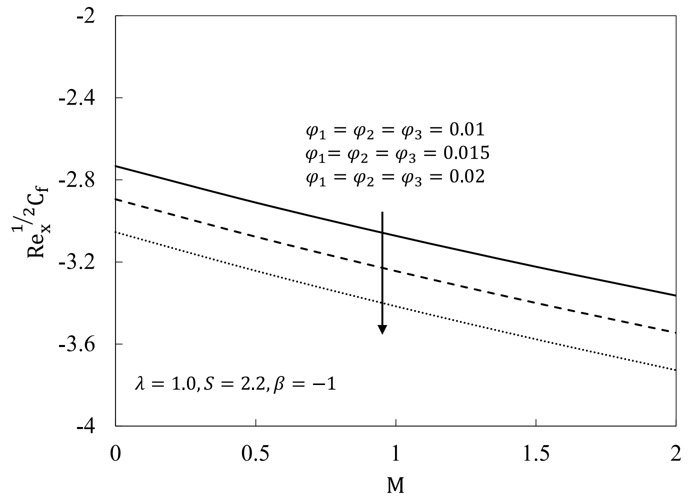

Fig. 2. Variations of $Re_x^{1/2} C_f$ against M and φ

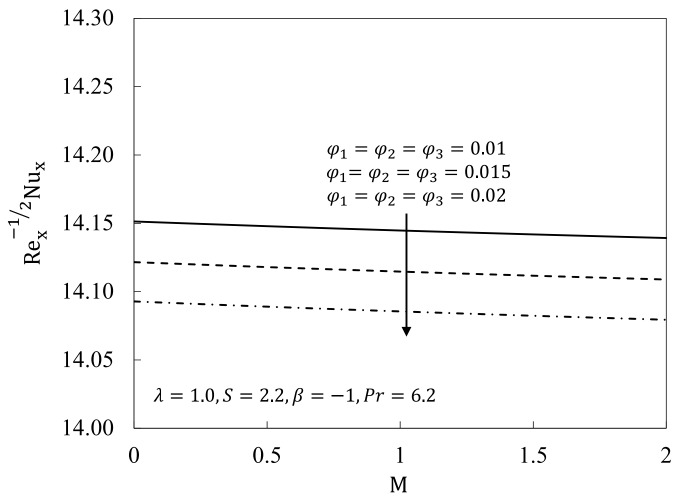

Fig. 3. Variations of $Re_x^{-1/2} Nu_x$ against M and φ

Derived from Figure 4, skin friction exhibits a declining pattern with the escalation of unsteady and stretching parameters. In practical terms, the consequence of the elongation parameter increase is the expansion of the surface area exposed to fluid flow. Consequently, when the fluid has to traverse a larger surface, its velocity decreases, thereby reducing the surface shear stress. The influence of the unsteady and stretching parameters on the heat transmission is seen in Figure 5. The heat transfer rate is found to positively correlate with the stretching and unsteadiness parameters. This is due to the fact that an increase in surface area results in a decrease in fluid flow velocity. This phenomenon results in an elevated viscosity on the surface, hence enhancing the efficiency of heat transfer.

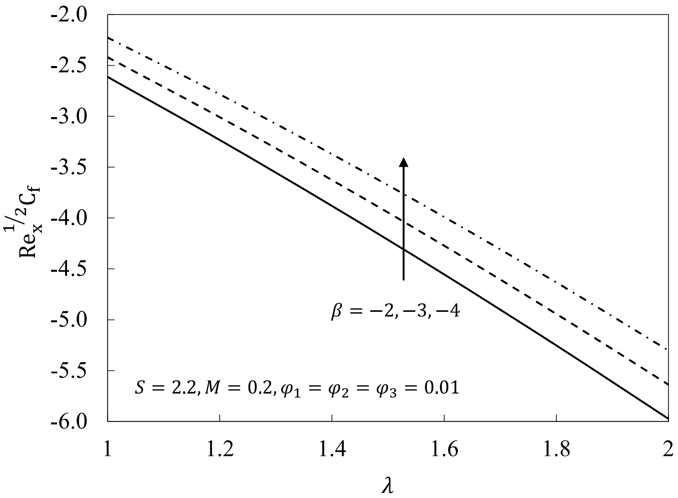

Fig. 4. Variations of $Re_x^{1/2}C_f$ against λ and β

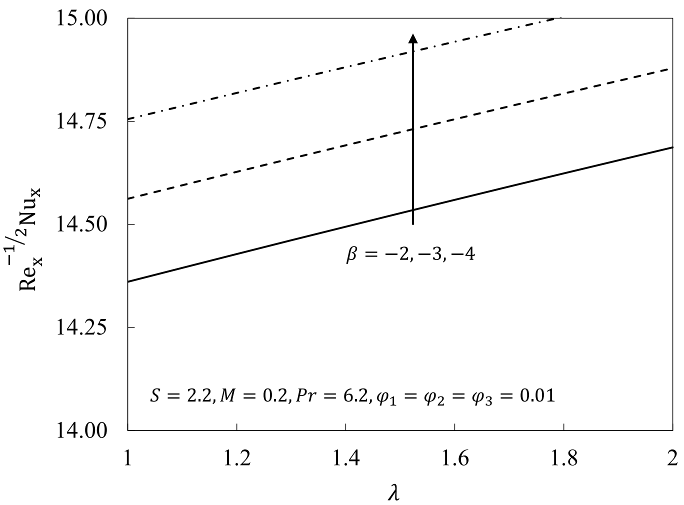

Fig. 5. Variations of $Re_x^{-1/2}Nu_x$ against λ and β

Figure 6 illustrates the impact of suction on $Re_x^{1/2}C_f$. An increase in suction strength may lead to an increase in the value of $\left|Re_x^{1/2}C_f\right|$ for ternary hybrid nanofluid. When suction is applied to the stretching sheet, the fluid will be pulled closer to the sheet, resulting in the thinning of the boundary layer, as depicted in Figure 8. The reduction in the thickness of the boundary layer leads to an increase in surface shear stress, resulting in an increase in skin friction. Meanwhile, Figure 7 illustrates how an increase in suction parameter values can result in a corresponding increase in heat transfer rate. As a result of the fluid approaching the stretching plate due to the suction, the thickness of the thermal boundary layer decreases. An increased local Nusselt number can be attributed to the enhanced heat transmission from the sheet to the fluid facilitated by the thinner thermal boundary layer, as seen in Figure 9. It is demonstrated by Figure 9 that, as the suction strength increases, the thickness of the temperature boundary layer is reduced. Furthermore, it is evident from Figure 8 and Figure 9 that the far-field boundary criteria were met asymptotically.

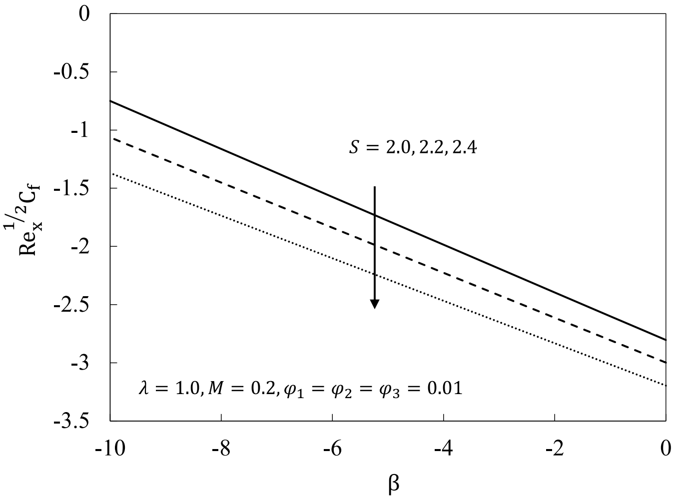

Fig. 6. Variations of $Re_x^{1/2}C_f$ against β and S

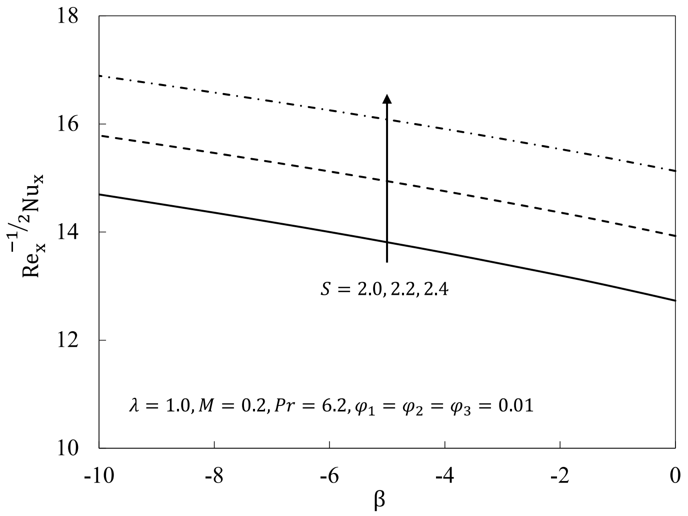

Fig. 7. Variations of $Re_x^{-1/2}Nu_x$ against β and S

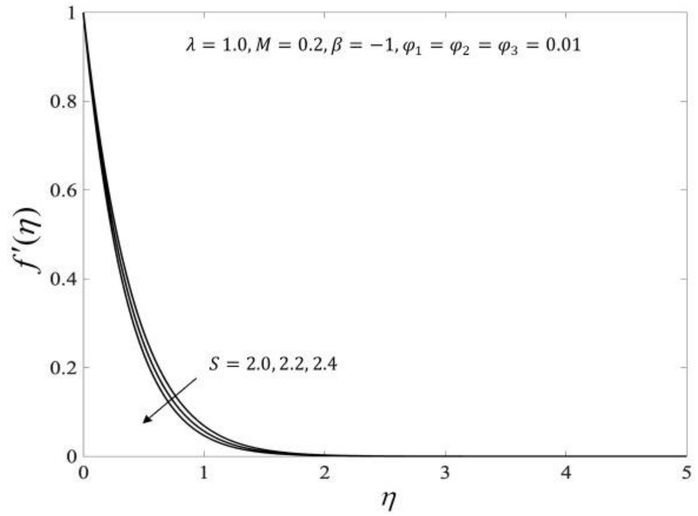

Fig. 8. Velocity distribution for different values of S

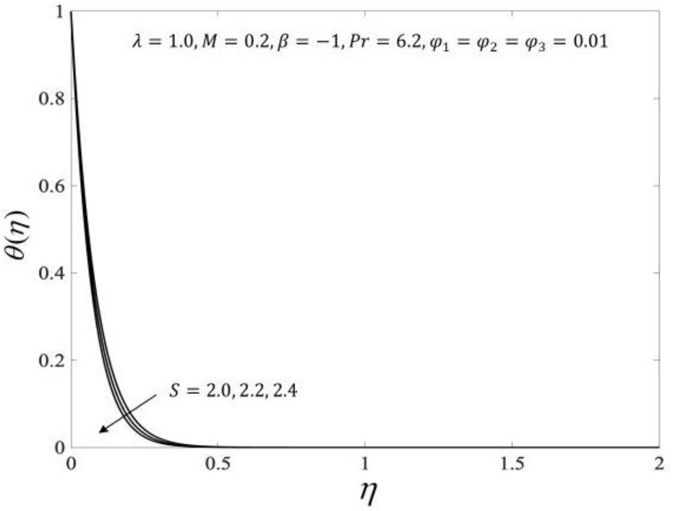

Fig. 9. Temperature distribution for different values of S

4. Conclusions

An unsteady Al_2O_3-Cu-TiO_2 ternary hybrid nanofluid flow past a permeable stretching sheet with suction and magnetohydrodynamic (MHD) effects was considered. The permeable stretching sheet is assumed to stretch linearly**.** The findings are as follows

i. A decrease in the heat transfer rate is brought about by higher values of the volume fraction of ternary hybrid nanoparticles with respect to the magnetic parameter, while an increase in skin friction is observed with higher values.
ii. The ternary hybrid Al_2O_3-Cu-TiO_2 nanofluid has greater heat transfer rate when $\varphi_1 = \varphi_2 = \varphi_3 = 0.1$ as compared to $\varphi_1 = \varphi_2 = \varphi_3 = 0.15$ and 0.2.
iii. The suction parameter and unsteadiness parameter are the manipulating parameter that can be manipulated to control the heat transfer rate.
iv. Both the skin friction coefficient values and heat transfer rate increase with an increase in the suction parameter.

Note that the final outcomes disclosed in this study pertain solely to the devised model and the specific ternary hybrid nanofluid under consideration, which consists of alumina, copper, and titania with water. Nevertheless, researchers have the opportunity to expand upon this study by exploring alternative combinations of ternary hybrid nanofluids or adjusting physical parameters to attain their desired outcomes. In the future, we plan to expand the current study by incorporating thermal radiation and viscous dissipation effects.

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References
[1] Ramadhan, Anwar Ilmar, Wan Hamzah Azmi, and Rizalman Mamat. "Experimental investigation of thermo-physical properties of tri-hybrid nanoparticles in water-ethylene glycol mixture." *Walailak Journal of Science and Technology (WJST)* 18, no. 8 (2021): 9335-15. https://doi.org/10.48048/wjst.2021.9335
[2] Dezfulizadeh, Amin, Alireza Aghaei, Ali Hassani Joshaghani, and Mohammad Mahdi Najafizadeh. "An experimental study on dynamic viscosity and thermal conductivity of water-Cu-SiO_2-MWCNT ternary hybrid nanofluid and the

development of practical correlations." *Powder Technology* 389 (2021): 215-234. https://doi.org/10.1016/j.powtec.2021.05.029

[3] Ishak, Siti Shuhada, Mohd Rijal Ilias, and Seripah Awang Kechil. "Ternary Hybrid Nanofluids Containing Gyrotactic Microorganisms with Magnetohydrodynamics Effect over a Shrinking/Stretching of the Horizontal Plate." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 109, no. 2 (2023): 210-230. https://doi.org/10.37934/arfmts.109.2.210230

[4] Sarangi, M. K., D. N. Thatoi, M. K. Nayak, J. Prakash, K. Ramesh, and M. Azam. "Rotational flow and thermal behavior of ternary hybrid nanomaterials at small and high Prandtl numbers." *International Communications in Heat and Mass Transfer* 138 (2022): 106337. https://doi.org/10.1016/j.icheatmasstransfer.2022.106337

[5] Animasaun, I. L., Se-Jin Yook, Taseer Muhammad, and Alphonsa Mathew. "Dynamics of ternary-hybrid nanofluid subject to magnetic flux density and heat source or sink on a convectively heated surface." *Surfaces and Interfaces* 28 (2022): 101654. https://doi.org/10.1016/j.surfin.2021.101654

[6] Mahmood, Zafar, N. Ameer Ahammad, Sharifah E. Alhazmi, Umar Khan, and Mutasem Z. Bani-Fwaz. "Ternary hybrid nanofluid near a stretching/shrinking sheet with heat generation/absorption and velocity slip on unsteady stagnation point flow." *International Journal of Modern Physics B* 36, no. 29 (2022): 2250209. https://doi.org/10.1142/S0217979222502095

[7] Zafar, Mudasar, Hamzah Sakidin, Mikhail Sheremet, Iskandar B. Dzulkarnain, Abida Hussain, Roslinda Nazar, Javed Akbar Khan et al. "Recent development and future prospective of tiwari and das mathematical model in nanofluid flow for different geometries: a review." *Processes* 11, no. 3 (2023): 834. https://doi.org/10.3390/pr11030834

[8] Alharbi, Khalid Abdulkhaliq M., Ahmed El-Sayed Ahmed, Maawiya Ould Sidi, Nandalur Ameer Ahammad, Abdullah Mohamed, Mohammed A. El-Shorbagy, Muhammad Bilal, and Riadh Marzouki. "Computational valuation of Darcy ternary-hybrid nanofluid flow across an extending cylinder with induction effects." *Micromachines* 13, no. 4 (2022): 588. https://doi.org/10.3390/mi13040588

[9] Madhukesh, Javali Kotresh, Ioannis E. Sarris, Ballajja Chandrappa Prasannakumara, and Amal Abdulrahman. "Investigation of thermal performance of ternary hybrid nanofluid flow in a permeable inclined cylinder/plate." *Energies* 16, no. 6 (2023): 2630. https://doi.org/10.3390/en16062630

[10] Sarfraz, Mahnoor, Masood Khan, A. Al-Zubaidi, and S. Saleem. "Tribology-informed analysis of convective energy transfer in ternary hybrid nanofluids on inclined porous surfaces." *Tribology International* 188 (2023): 108860. https://doi.org/10.1016/j.triboint.2023.108860

[11] Sarfraz, Mahnoor, Masood Khan, A. Al-Zubaidi, and S. Saleem. "Enhancing energy transport in Homann stagnation-point flow over a spiraling disk with ternary hybrid nanofluids." *Case Studies in Thermal Engineering* 49 (2023): 103134. https://doi.org/10.1016/j.csite.2023.103134

[12] McCroskey, William J. "The 1976 freeman scholar lecture: some current research in unsteady fluid dynamics." *Journal of Fluids Engineering* 99, no. 1 (1977): 8-39. https://doi.org/10.1115/1.3448570

[13] Kebede, Tesfaye, Eshetu Haile, Gurju Awgichew, and Tadesse Walelign. "Heat and mass transfer in unsteady boundary layer flow of Williamson nanofluids." *Journal of Applied Mathematics* 2020 (2020): 1-13. https://doi.org/10.1155/2020/1890972

[14] Shoaib, Muhammad, Marwan Abukhaled, Muhammad Asif Zahoor Raja, Muhammad Abdul Rehman Khan, Muhammad Tauseef Sabir, Kottakkaran Sooppy Nisar, and Iqra Iltaf. "Heat and mass transfer analysis for unsteady three-dimensional flow of hybrid nanofluid over a stretching surface using supervised neural networks." *Frontiers in Physics* 10 (2022): 949907. https://doi.org/10.3389/fphy.2022.949907

[15] Kumar, Vasa Vijaya, Mamidi Narsimha Raja Shekar, and Shankar Goud Bejawada. "Heat and Mass Transfer Significance on MHD Flow over a Vertical Porous Plate in the Presence of Chemical Reaction and Heat Generation." *CFD Letters* 16, no. 5 (2024): 9-20. https://doi.org/10.37934/cfdl.16.5.920

[16] Khashi'ie, Najiyah Safwa, Iskandar Waini, Abdul Rahman Mohd Kasim, Nurul Amira Zainal, Anuar Ishak, and Ioan Pop. "Magnetohydrodynamic and viscous dissipation effects on radiative heat transfer of non-Newtonian fluid flow past a nonlinearly shrinking sheet: Reiner-Philippoff model." *Alexandria Engineering Journal* 61, no. 10 (2022): 7605-7617. https://doi.org/10.1016/j.aej.2022.01.014

[17] Van Gorder, Robert, and K. Vajravelu. "Multiple solutions for hydromagnetic flow of a second grade fluid over a stretching or shrinking sheet." *Quarterly of Applied Mathematics* 69, no. 3 (2011): 405-424. https://doi.org/10.1090/S0033-569X-2011-01211-1

[18] Samat, Nazrul Azlan Abdul, Norfifah Bachok, and Norihan Md Arifin. "Carbon Nanotubes (CNTs) Nanofluids Flow and Heat Transfer under MHD Effect over a Moving Surface." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 103, no. 1 (2023): 165-178. https://doi.org/10.37934/arfmts.103.1.165178

[19] Motozawa, Masaaki, Jia Chang, Tatsuo Sawada, and Yasuo Kawaguchi. "Effect of magnetic field on heat transfer in rectangular duct flow of a magnetic fluid." *Physics Procedia* 9 (2010): 190-193. https://doi.org/10.1016/j.phpro.2010.11.043

[20] Shateyi, Stanford, and Hillary Muzara. "On the numerical analysis of unsteady MHD boundary layer flow of Williamson fluid over a stretching sheet and heat and mass transfers." *Computation* 8, no. 2 (2020): 55. https://doi.org/10.3390/computation8020055
[21] Muntazir, Rana M. A., Muhammad Mushtaq, Shamaila Shahzadi, and Kanwali Jabeen. "MHD nanofluid flow around a permeable stretching sheet with thermal radiation and viscous dissipation." *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* 236, no. 1 (2022): 137-152. https://doi.org/10.1177/09544062211023094
[22] Wahid, Nur Syahirah, Norihan Md Arifin, Najiyah Safwa Khashi'ie, and Ioan Pop. "Mixed convection MHD hybrid nanofluid over a shrinking permeable inclined plate with thermal radiation effect." *Alexandria Engineering Journal* 66 (2023): 769-783. https://doi.org/10.1016/j.aej.2022.10.075
[23] Zainal, Nurul Amira, Iskandar Waini, Najiyah Safwa Khashi'ie, Roslinda Nazar, and Ioan Pop. "MHD Hybrid Nanofluid Flow Past A Stretching/Shrinking Wedge With Heat Generation/Absorption Impact." *CFD Letters* 16, no. 6 (2024): 146-156. https://doi.org/10.37934/cfdl.16.6.146156
[24] Asghar, Adnan, and Teh Yuan Ying. "Three dimensional MHD hybrid nanofluid Flow with rotating stretching/shrinking sheet and Joule heating." *CFD Letters* 13, no. 8 (2021): 1-19. https://doi.org/10.37934/cfdl.13.8.119
[25] Ishak, Anuar. "Unsteady MHD flow and heat transfer over a stretching plate." *Journal of Applied Sciences* 10, no. 18 (2010): 2127-2131. https://doi.org/10.3923/jas.2010.2127.2131
[26] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Unsteady flow and heat transfer past a stretching/shrinking sheet in a hybrid nanofluid." *International Journal of Heat and Mass Transfer* 136 (2019): 288-297. https://doi.org/10.1016/j.ijheatmasstransfer.2019.02.101
[27] Rafique, Khadija, Zafar Mahmood, Umar Khan, Sayed M. Eldin, Mowffaq Oreijah, Kamel Guedri, and Hamiden Abd El-Wahed Khalifa. "Investigation of thermal stratification with velocity slip and variable viscosity on MHD flow of Al_2O_3−Cu−TiO_2/H_2O nanofluid over disk." *Case Studies in Thermal Engineering* 49 (2023): 103292. https://doi.org/10.1016/j.csite.2023.103292
[28] Manjunatha, S., V. Puneeth, B. J. Gireesha, and Ali Chamkha. "Theoretical study of convective heat transfer in ternary nanofluid flowing past a stretching sheet." *Journal of Applied and Computational Mechanics* 8, no. 4 (2022): 1279-1286.
[29] Devi, Suriya Uma, and S. P. Anjali Devi. "Heat transfer enhancement of Cu−Al_2O_3/water hybrid nanofluid flow over a stretching sheet." *Journal of the Nigerian Mathematical Society* 36, no. 2 (2017): 419-433.
[30] Tie-Gang, Fang, Zhang Ji, and Yao Shan-Shan. "Viscous flow over an unsteady shrinking sheet with mass transfer." *Chinese Physics Letters* 26, no. 1 (2009): 014703. https://doi.org/10.1088/0256-307X/26/1/014703
[31] Liu, I-Chung. "A note on heat and mass transfer for a hydromagnetic flow over a stretching sheet." *International Communications in Heat and Mass Transfer* 32, no. 8 (2005): 1075-1084. https://doi.org/10.1016/j.icheatmasstransfer.2005.02.003
[32] Shampine, Lawrence F., Ian Gladwell, and Skip Thompson. *Solving ODEs with matlab*. Cambridge university press, 2003. https://doi.org/10.1017/CBO9780511615542
[33] Gorla, Rama Subba Reddy, and Ibrahim Sidawi. "Free convection on a vertical stretching surface with suction and blowing." *Applied Scientific Research* 52 (1994): 247-257. https://doi.org/10.1007/BF00853952
[34] Priyadharshini, P., M. Vanitha Archana, Nehad Ali Shah, and Mansoor H. Alshehri. "Ternary hybrid nanofluid flow emerging on a symmetrically stretching sheet optimization with machine learning prediction scheme." *Symmetry* 15, no. 6 (2023): 1225. https://doi.org/10.3390/sym15061225
[35] Waini, Iskandar, Farah Nadzirah Jamrus, Abdul Rahman Mohd Kasim, Anuar Ishak, and Ioan Pop. "Homogeneous-Heterogeneous Reactions on Al_2O_3-Cu Hybrid Nanofluid Flow Over a Shrinking Sheet." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 102, no. 1 (2023): 85-97. https://doi.org/10.37934/arfmts.102.1.8597
[36] Rasool, Ghulam, Xinhua Wang, Ubaidullah Yashkun, Liaquat Ali Lund, and Hasan Shahzad. "Numerical treatment of hybrid water based nanofluid flow with effect of dissipation and Joule heating over a shrinking surface: Stability analysis." *Journal of Magnetism and Magnetic Materials* 571 (2023): 170587. https://doi.org/10.1016/j.jmmm.2023.170587
[37] Tiwari, Raj Kamal, and Manab Kumar Das. "Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids." *International Journal of Heat and Mass Transfer* 50, no. 9-10 (2007): 2002-2018. https://doi.org/10.1016/j.ijheatmasstransfer.2006.09.034