



A Thermal Conduction Comparative Study Between the FDM and SPH Methods with A Proposed C++ Home Code

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ABSTRACT

The heat transfer phenomenon is modeled by the Finite Difference Method (FDM) and the Smoothed Particle Hydrodynamic (SPH) approach. The numerical approach under investigation may be used to solve many complex problems of applied mechanics. The Finite Element Method (FEM) is generally used for the Lagrangian description, and the FDM is used for the Eulerian report. However, the SPH method, which is better than other approaches to solve some problems, may be used in many aspects. Numerical details on the SPH method are discussed in this paper, with a focus on its application on the heat equation. A simple two-dimensional heat conduction problem is simulated by using the SPH approximation procedure and the newly constructed quartic smoothing function. Besides, a comparison is made between both techniques. Finally, C++ code is proposed for SPH and FDM methods.

1. Introduction

Smoothed particle hydrodynamics (SPH) is a free mesh, and Lagrangian particle approach to model fluid flows. SPH was first employed in modelling the astrophysical problems (polytropes) in three-dimensional open spaces. The application of SPH in such issues was successful due to the similarity of the movement of those particles to that of gas or a liquid [1].

The SPH method has become a good alternative due to the difficulties encountered in modelling some complex problems by the traditional grid-based numerical methods such as the finite element methods (FEM), the finite difference method (FDM), or the finite volume methods (FVM) [2-7]. The main advantages of the SPH method may be encountered in the review of Benz and Monaghan [8].

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Another advantage of the SPH method is that the use of mesh is not necessary to compute the spatial derivatives, which is not the case for the particle-in-cell (PIC) method. The particles are able to move in the space, carry all the computational information, and thus form the computational domain to solve the governing equations [9].

The SPH method is today employed in modelling various problems, such as the collapse and the formation of galaxies, supernova, single and multiple detonations of white dwarfs, binary stars and stellar collisions, the coalescence of black holes with neutron stars, and others. The SPH method is also applied in computational fluid or solid mechanics; this is why it is called smoothed particle mechanics [10].

Some examples of the different problems of mechanics that may be solved by the SPH method, we cite the ice and cohesive grains [11], the heat and mass transfer [12], shallow water [13], fluid-structure interactions [14], incompressible flows [15, 16], multiphase flows [17-26], flow through porous media [27], magneto-hydrodynamics, gravity currents, elastic flow, weakly compressible smoothed particle hydrodynamics method to solve the internal flow problems involving fluid-solid conjugate heat transfer [28]. Ng *et al.*, [29] assessed the accuracies of using the popular dummy particle methods, i.e. (a) the Adami Approach (AA) and (b) the higher-order mirror + Moving Least Square (MMLS) method in predicting the total wall heat transfer rate. Other researchers treated other problems, as detailed in references [30-34].

2. Formulation and Equations

The following key details are considered in the SPH method to obtain the previous mentioned objectives. In the case where the domain is not in the form of particles, an arbitrary distribution of particles may be set to describe the field. In this case, no particle connectivity is required (i.e., Meshfree).

For the approximation of the field function, the integral representation approach that is called the kernel approximation may be employed. Then, the so-called particle approximation (PA) is required, and it is done at every time step. It consists of approximate the kernel approximation by using particles. It is achieved by replacing the integration in the integral representation of the field function and its derivatives with summations over all the corresponding values at the neighbouring particles in a local domain called the support domain (Compact support). Since the particle approximation is needed at every time step, the utilization of particles depends on their current local distribution (Adaptive).

Furthermore, PA is applied for all terms related to field functions in the PDEs, resulting thus in a set of ODEs in the discretized form with respect to time only (Lagrangian).

To obtain a fast time stepping and to get the time history of all the field variables for all the particles (Dynamic), the ODEs are solved using an explicit integration algorithm.

The SPH procedure and the Gaussian kernel for smoothing function are applied to simulate a simple two-dimensional heat conduction problem. The heat conduction equation is a parabolic (PDE), given by

$$\rho C \frac{\partial T}{\partial t} = \nabla \cdot (\bar{k} \nabla T) + Q \quad (1)$$

where ρ , C , k , and Q are constant during the simulation.

The heat conduction equation for particle i is given as

$$\rho_i C_i \left(\frac{\partial T}{\partial t} \right)_i = \nabla \cdot (\bar{k} \nabla T_i) + Q_i \quad (2)$$

A SPH equation for multi-dimensional heat conduction is derived as follows [35]

$$\rho_i C_i \left(\frac{\partial T}{\partial t} \right)_i = \bar{k} \sum_{j=1}^N T_j W_{ij} \frac{m_j}{\rho_j} + Q_i \quad (3)$$

$$W_{ij} = (T_i - T_j, h) = W(R_{ij}, h) = \frac{1}{e^{h(\pi x)^2}} \quad (4)$$

where $R_{ij} = \frac{T_i - T_j}{h}$ (5)

The distribution of SPH particles that are used in this simulation is shown in Figure 1 and 2. A total of 25 particles is used in the first case. However, 81 particles are employed for the second one. Nine particles are located on each boundary, and the rest of the particles are located inside the square domain. The boundary particles are imposed with boundary values in the temperature evaluation process. The smoothing length is given by

$$h = \Delta x + \Delta y \quad (6)$$

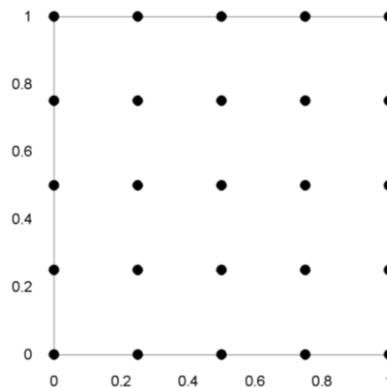


Fig. 1. Domain discretization by SPH method for 5 particles

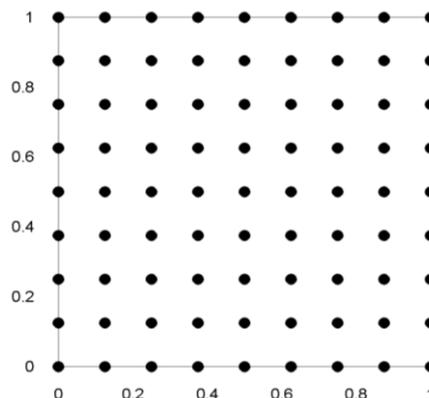


Fig. 2. Domain discretization by SPH method for 9 particles

In addition, the time integration scheme is given by

$$\Delta t = \min\left(\frac{h_i}{c}\right) \quad (7)$$

The explicit time integration schemes are subject to the CFL condition for stability [35]. Figure 3 shows the algorithm of SPH method.

For the FDM method, the geometry of the two-dimensional simulation is set to be a square. The material properties used are $k = 1$, $\rho = 1$, $C = 1$ and $Q = 0$. The initial temperature is 0°C and the temperature on the right and left boundaries are simply set as 10°C and 0°C respectively. The bottom and top boundaries are adiabatic.

The algorithm for the FDM method is given by Figure 4.

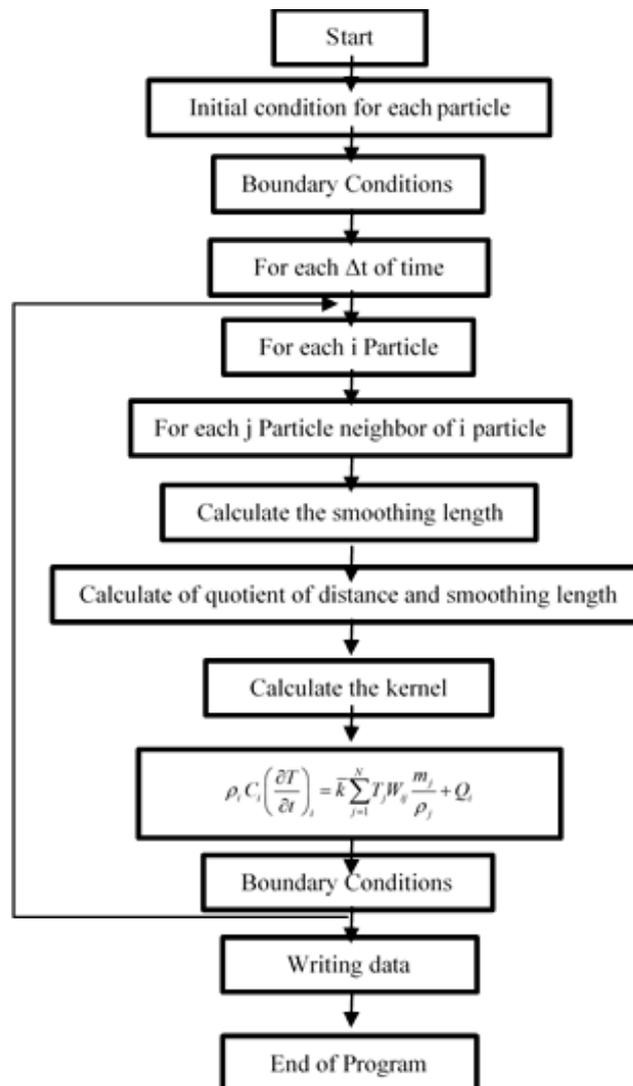


Fig. 3. SPH method algorithm

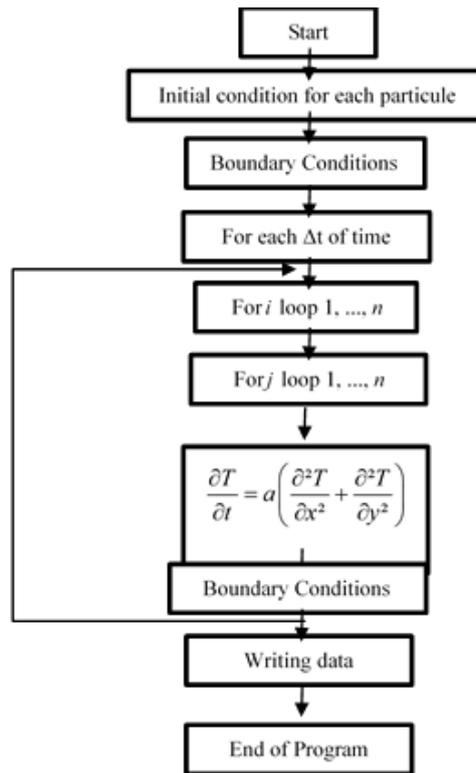


Fig. 4. Algorithm for the FDM method

3. Results and Discussion

3.1 Case No. 1

The first case consists of five particles of the SPH method versus five nodes of the FDM method. Figure 5 and 6 present the simulation of heat distribution that is obtained by the SPH and FDM methods, respectively. The temperature between the limits is almost the same for both methods.

In Figure 7, a good agreement between the FDM & SPH methods is observed; each particle has the same value of the offset node.

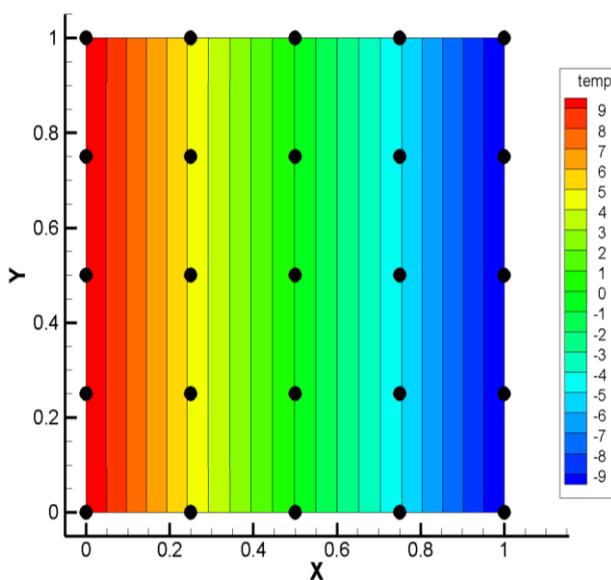


Fig. 5. Distribution of temperature of five particles of SPH method

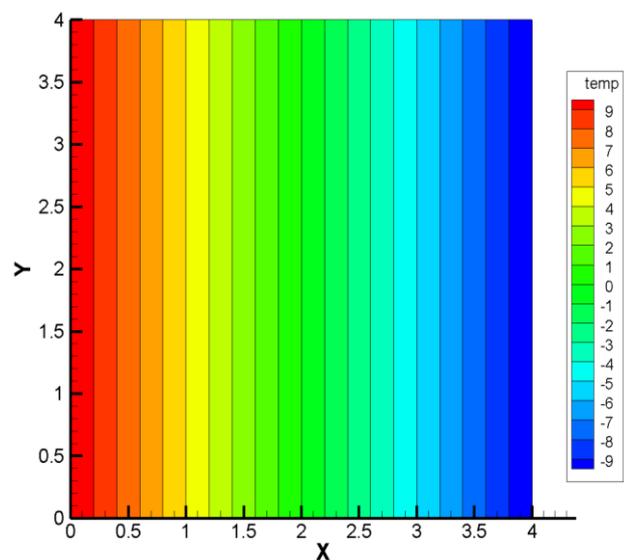


Fig. 6. Distribution of temperature of five nodes of FDM method

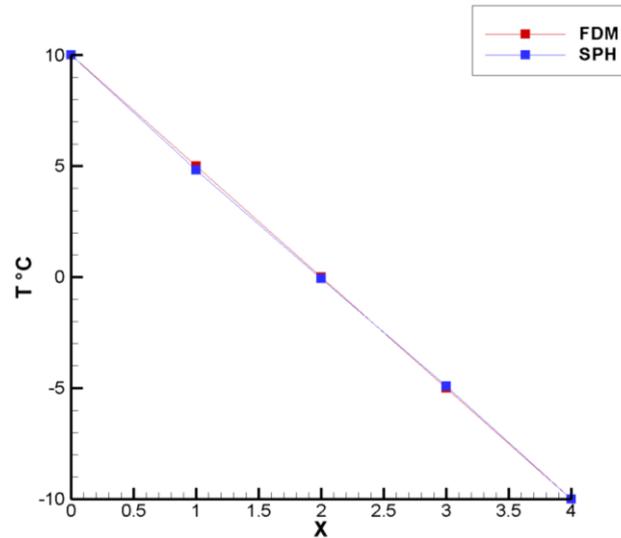


Fig. 7. Comparison between the SPH particles in the middle square with the others of the FDM nodes

3.2 Case No. 2

The second case consists of nine particles for the SPH method versus nine nodes for the FDM method.

Figure 8 and 9 represent, respectively, the distribution of the temperature for both methods SPH and FDM. As clearly observed, nearly the same distribution of temperature is obtained.

However, a small difference between the FDM and SPH methods is remarked, as illustrated in Figure 10. The main reason of the discrepancy is that we've taken the same values of dx and dy in both methods, while in the SPH method the length h is given as mention in Eq. (6). Each particle is approximated to the other values of nodes. The difference is caused by the time step in the SPH method and the number of iterations in space loops of the FDM method.

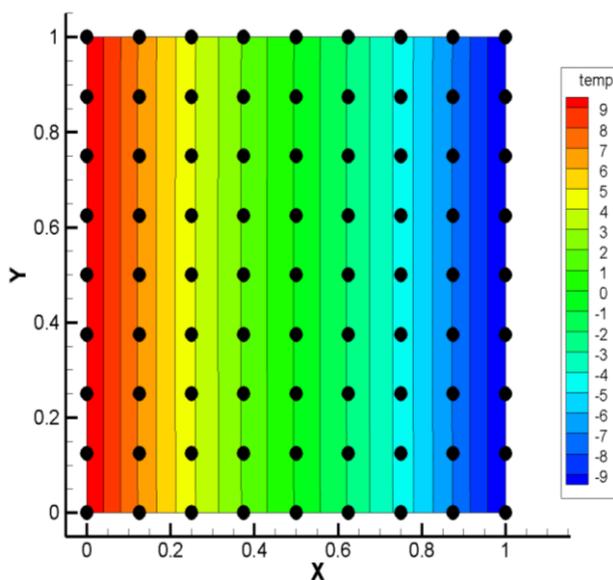


Fig. 8. Distribution of temperature of 9 particles

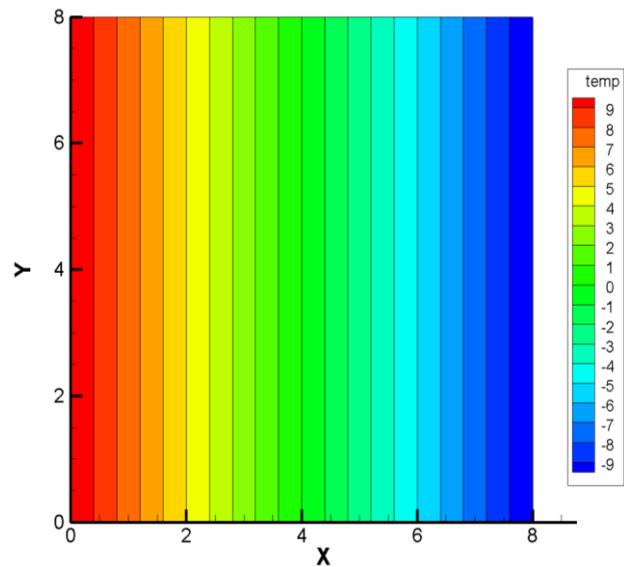


Fig. 9. Distribution of temperature of 9 nodes

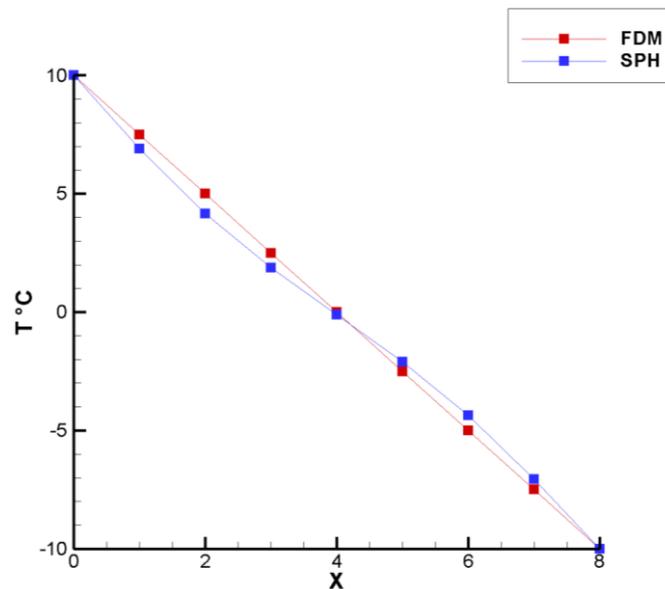


Fig. 10. Compare the SPH particle in the middle square with the other of FDM nodes

4. Conclusions

The FDM was used to study the behaviour of particles (position and temperature) by the SPH method. The numerical details of the used process were discussed with a focus on numerical implementation and interpretation. The newly constructed quartic (Eq. (4)) smoothing function and the SPH approximation procedure were applied to simulate a simple two-dimensional heat conduction problem. Furthermore, a comparison was made between the FDM and SPH techniques.

Encouraging results were obtained, especially for the SPH method. However, some problems have been encountered regarding the computational time for the SPH method, which arrived at 640 seconds for nine particles. Besides that, the algorithm for searching the neighbour particle pushes us to the parallel programming.

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