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The Effect of Viscous Dissipation and Joule Heating on Poiseuille Flow of Hyperbolic Tangent Fluid

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ABSTRACT

In this paper, we showed the computer solutions for the Flow of Poiseuille of (MHD) hyperbolic tangent impossible-to-compress fluid Amid a pair of parallel slabs that are sloped at an angle $\varnothing$ utilizing a uniformly porous medium. Heat transfer attempts and slip boundary conditions are taken into account. In the energy equation, radiation, joule heating, and viscous dissipation are also accounted for. It is presented in a cartesian coordinate system that supplies the model of the governing equations of the hyperbolic tangent fluid flow. The equations of velocity and Temperature are solved numerically by using "ND solver built". The effects of different factors such that Hartmann number, Darcy number, slip parameter, Grashof number, Brinkman number, radiation parameter and others on velocity and temperature profiles are explored and presented in the Mathematica program. It is noticed that the profiles of velocity and Temperature are an increasing function of Darcy number, slip parameter, Grashof number, inclination angle, pressure gradient, Brinkman number, radiation parameter, and index-power parameter while they are a decreasing function of Hartmann and Weissenberg numbers. The research intends to improve our understanding of the thermal behavior of hyperbolic tangent fluids in practical applications by studying the combined effects of viscous dissipation and Joule heating on the Poiseuille flow. The potential ramifications of this phenomenon extend to multiple domains, encompassing engineering, materials science, and fluid dynamics.

1. Introduction

Examining and differentiating between neutral and non-neutral fluids is a subject of considerable curiosity. Scientists are highly interested in this research subject because of the wide range of applications for neutral and non-neutral fluids, such as fiber technology, along the same lines as pharmaceutical physiology, food goods, wire coating, and the creation of crystals. The majority of substances with low molecular weight and Newtonian fluid qualities are shown by a variety of compounds, such as natural and artificial liquids, solutions containing light molecules of chemical salts, and metals melting. In the case of these particular solutions, the correlation between the rate of shear and shear tension is a straight line. In recent years, there has been an increasing

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acknowledgement that numerous industrial substances, particularly those with multiple phases (such as dispersions, polymeric melts, and solutions (both synthetic and natural), as well as various forms of suspension and slurries, emulsions, and foams, for simple shear, you shouldn't rely on the Newtonian assumption that the two quantities are linearly related. Diverse names are given to these fluids, including non-Newtonian and nonlinear fluids. Commonly, three categories classify non-Newtonian fluids: stationary, rheological, and transient. Nevertheless, these categorizations are frequently not delineated or distinct. Fluids demonstrating a composite of characteristics from multiple sets above are referred to as intricate fluids; this term can be applied to non-neutral fluids in general. Consequently, a universal constitutive equation encapsulating all non-neutral fluids' behavior does not exist. Many models have been suggested in the academic literature to represent various flow conditions and varieties of non-neutral fluids. It is one of the most popular types of viscoelastic non-neutral fluids and is used in many fields of engineering and industry. For these reasons, hyperbolic tangent fluid Flow in many geometries has been the focus of substantial investigation [1-4].

One of the fundamental flows is the Poiseuille flow of different non-neutral fluids Fluid dynamics experts have been drawn to this phenomenon because to its wide-ranging applications in engineering, industry, and medicine, that Poiseuille law can be used to calculate the blood flow rate in arteries and veins which is important for understanding blood pressure and cardiovascular health [5-8].

Materials, in their broadest definition, are domains that refer to either different materials or many states of the same material. For instance, paper, foams, textile felt, cardboard, bone, composite materials, sandstone, concrete, sea ice, and soils. Natural materials such as wood and stone frequently circulate fluids via porous media, fundamental to many societal and natural processes. In particular, Fluid movement across a porous media causes several technological challenges. Among these are the extraction of hydrocarbons from permeable rocks, the dispersal of pollutants in fluid-soaked soils, and the use of filters and other separation techniques. In the paper industry, the way single-phase and multiphase fluids move through porous media is relevant to the manufacturing process and the development of new products. As a result, numerous researchers have looked at different scenarios involving fluids passing through porous materials.

A lot of research has been done on porous media. Some of them have given analytical formulas about the velocity distributions of several generalized Newtonian fluids in a porous medium bounded by two parallel platelets that remain still [9]. In another problem, they have explored the relationship between viscosity and pressure in Carreau fluid within a porous medium [10-13].

Practical applications of fluid dynamics research in electrically conductive fluids subjected to magnetic fields include engineering, medical technology, and advanced industries. An extensive numerical study was undertaken to examine the flow properties of a micropolar fluid. Considering the advantageous possibilities of a magnetic field [14]. An investigation was conducted on the phenomena of MHD flow, which involves the physical effects caused by a surface that may be stretched in a fluid that conducts electricity [15]. A thorough examination was conducted on the fascinating phenomenon of two-dimensional magnetized Flow, specifically focusing on the thermal transfer mechanism of a granular fluid that contains nanoparticles [16]. Analytically, the characteristics of the flow problem have been observed. Additional research showcasing the relationship between fluid behavior and the magnetic field is presented in previous studies [17-21].

Thermal radiation's wide-ranging applications in sectors including paper manufacturing, space technology, gas turbines, and nuclear reactors have elevated its significance in the Study of heat transfer to an unprecedented degree. A multitude of researchers performed flow analyses by incorporating thermal radiation effects and employing diverse geometries. An investigation has been conducted on the alteration of Flow characteristics of a magnetized nanofluid that vary with time using Heat emitted as electromagnetic waves by Ishtiaq *et al.,* [22]. A study has been conducted on the impact of Heat emitted as electromagnetic waves on the movement of a magnetic fluid that contains nanoparticles within a porous medium. enclosure by Alipour *et al.,* [23].

Understanding the thermal energy transmission rate and thermal distribution map is crucial. Usually, convective thermal is considered in cases where the liquid has a high viscosity or when the flow velocity is significant. The Brinkman number is typically used to characterize the effect of viscous dissipation. Extensive research has been conducted to explore the influence of viscous dissipation on heat transfer in channel flow, as indicated in various theoretical and experimental investigations [24-27].

Slip boundary condition has been proposed by Neto *et al.,* [28]. A study has been conducted on the impact of thermal radiation on the movement of a magnetic fluid that contains nanoparticles contained within a permeable structure [29]. Consequently, researchers utilized slip boundary conditions to analyze flow issues in a variety of configurations. For bounded domains, it has been demonstrated that stable flows of incompressible fluids with a viscosity of grade three can exist and be unique, regardless of slip or no-slip boundary conditions by Ellahi *et al.,* [30]. Analyze with precision! The power of hybrid techniques and pseudo-spectral collection. Discover the secrets of Eyring-Powell fluid in MHD. Slip circumstances and transport of thermal revealed [31-40].

The objective of this paper is to conduct a theoretical investigation into how Dissipation of viscous fluid and joule heating. Magnetohydrodynamics (MHD) can induce hyperbolic tangent behavior in the Poiseuille flow of a fluid as it passes through a permeable material. Inside of two parallel, inclined platelets. The slip boundary conditions and the impacts of heat transmission have been duly considered. There are six distinct sections in this document. The mathematical formulation of the problem is formulated in section (2). The system of the governing equations of energy and motion is elaborated upon in Section 3. The constitutive equations for the shear stress of a hyperbolic tangent fluid on display in sec. (4). The approach utilized to resolve the issue is detailed in sec (5), along with an analysis of the numerical outcomes. In section 6, the conclusion to the evaluated results is presented. Numerical outcomes of the utilized parameters and variables are generated by the Mathematica software.

2. Mathematical Model of the Problem

Consider an incompressible MHD hyperbolic tangent fluid between two parallel surfaces inclined at an angle ϕ of infinity with respect to the horizontal via a permeable material medium Amidst $y = \pm h$. Assuming no platelets move., The smooth progression is owing to the constant positive x-direction Gradient of pressure (see Figure 1). Also, in this situation, the Temperature of the top and bottom platelets is kept constant T_1 and T_0 respectively.

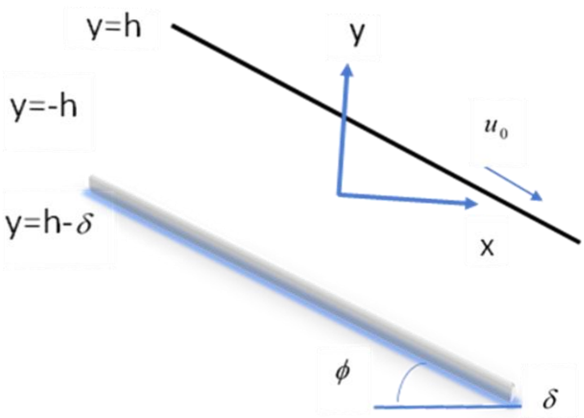

Fig. 1. Physical model

The velocity field $\vec{V} = (u(y), 0, 0)$ is established between parallel surfaces to facilitate unidirectional steady motion. In accordance with the criteria for continuity in mathematical expression. In the context of this velocity distribution, let us consider the fluid to possess electrical conductivity and an ongoing transverse magnetic field to be present within the channel A_0. Additionally, the field of induced magnetic fields is disregarded due to the diminished magnetic force field. The Reynolds numbers. As a result, a transformation $\vec{A} = (0, A_0, 0)$ is applied to the overall vector that represents the magnetic field. Moreover, the absence of an electric field is assumed. Such way, it is

$$\vec{S} = \sigma(\vec{D} + \vec{V} \times \vec{A}) \tag{1}$$

$\vec{S}$ is the current Vector, σ The term "lit conductance " refers to the fluid's ability to conduct electricity., fam and $\vec{D}$ is the field of electricity. In this case, Eq. (1) yields:

$$\vec{S} \times \vec{A} = (-\sigma A_0{}^2 \vec{u}, 0, 0) \tag{2}$$

The X-axis, which passes through the separation (2h) amidst the lowest platelet and the highest platelet of the channels, indicates the flow direction.

3. The Problem-Governing Equations System

Within the fixed frame, the fluid dynamics are regulated by the subsequent equations:

Continuity equation

$$\frac{\partial \overline{u}}{\partial \overline{x}} = 0 \tag{3}$$

$$0 = -\frac{\partial \overline{P}}{\partial \overline{X}} + \frac{\partial}{\partial \overline{Y}} \tau_{\overline{XY}} - \sigma A_0^2 \overline{u} - \frac{\mu_0}{k_0} \overline{u} + \rho g \beta (T - T_0) \sin\phi \tag{4}$$

$$\rho c_p\left(\frac{\partial \overline{T}}{\partial \overline{t}}+\overline{u}\,\frac{\partial \overline{T}}{\partial \overline{X}}\right)=k\left(\frac{\partial^2 \overline{T}}{\partial \overline{Y}^2}\right)+\tau_{\overline{XY}}\,\frac{\partial \overline{u}}{\partial \overline{Y}}+\sigma A_0^{\,2}(\vec{u})^2-\frac{\partial}{\partial \overline{Y}}\overline{qr} \tag{5}$$

In which $\frac{\partial}{\partial \overline{Y}}\overline{qr}=4\lambda^2(T_0-T)$

Adopting the boundary criteria for slip

$$\overline{u}-\overline{\beta}\left(\frac{\partial \overline{u}}{\partial \overline{Y}}\right)=0 \quad \text{at } \overline{Y}=(-h)$$
$$\overline{u}+\overline{\beta}\left(\frac{\partial \overline{u}}{\partial \overline{Y}}\right)=0 \quad \text{at } \overline{Y}=(h) \tag{6}$$

$$T=T_0 \quad \text{at } \overline{Y}=(-h)$$
$$T=T_1 \quad \text{at } \overline{Y}=(h) \tag{7}$$

In this study, we have the steady Flow. The quantities that are not dimensional given by

$$u=\frac{\overline{u}}{u_0},\ x=\frac{\overline{X}}{h},\ y=\frac{\overline{Y}}{h},\ p=\frac{h\overline{P}}{\mu_0 u_0},\ \mathrm{We}=\frac{\Gamma u_0}{h},\ \mathrm{Dr}=\frac{h}{\sqrt{k_0}}=\frac{1}{Da},\ t=\frac{\overline{t}u_0}{h},$$
$$\mathrm{Re}=\frac{\rho h u_0}{\mu_0},\ M=\sqrt{\frac{\sigma}{\mu_0}}A_0 h,\ \theta=\frac{T-T_0}{T_1-T_0},\ \mathrm{Nr}=\frac{2\lambda h}{\sqrt{k_1}},\ \rho_r=\frac{\mu_0 c\rho}{k_1},$$
$$\mathrm{Gr}=\frac{\beta g h^2\rho(T_1-T_0)}{u_0\mu_0},\ \tau_{\overline{XY}}=\overline{\tau_{YX}}=\frac{\mu_0 u_0}{h}\tau_{XY},\ \dot{y}=\frac{h}{u_0}\overline{\dot{y}},\ \mathrm{Ec}=\frac{u_0^{\,2}}{c\rho(T_1-T_0)},$$
$$\mathrm{Br}=\rho r\mathrm{Ec}=\frac{\mu_0 u_0^{\,2}}{k_1(T_1-T_0)},\ \beta=\frac{\overline{\beta}}{h} \tag{8}$$

4. The Equation Constituting the Hyperbolic Tangent Shear Stress

The form of the constitutive equations for hyperbolic tangent is given by [4]

$$\overline{\tau}=(n_\infty+(n_0+n_\infty)\tanh(\Gamma\overline{\dot{y}})^n)\overline{\dot{y}} \tag{9}$$

$$\overline{\dot{y}}=\sqrt{\frac{1}{2}\sum_i\sum_j\overline{\dot{y}}_{ij}\overline{\dot{y}}_{ji}}=\sqrt{\frac{1}{2}\Pi} \tag{10}$$

where Π denotes the second fundamental value of the strain rate tensor, n_0 is the zero-share rate viscosity, n_∞ is the infinity share rate viscosity, $\overline{\tau}$ is stress tensor extra. Eq. (9) is taken into account in the constitutive equation. The rationale supporting which $n_\infty=0$ and for the purpose of composing:

$$\bar{\tau} = (n_\infty + (n_0 + n_\infty)\tanh(\Gamma \overline{\dot{y}})^n)\overline{\dot{y}} \tag{11}$$

which was written by

$$\bar{\tau} = (n_0 + (1 + n(\Gamma \overline{\dot{y}} - 1))\overline{\dot{y}} \tag{12}$$

That is, if n=1 or $\bar{\Gamma} = 0$ we simplified it to a Newtonian fluid model, which shows how shear stress is made up $\bar{\tau}$ can be written as

$$\overline{\tau_{\overline{XY}}} = (n_0 + (1 + n(\Gamma \overline{\dot{y}} - 1))\frac{\partial \bar{u}}{\partial Y}$$
$$\frac{\partial \bar{u}}{\partial \bar{Y}} = \overline{\dot{y}} \tag{13}$$

By replacing Eq. (8) in Eq. (3) through Eq. (13), the following result is obtained

$$\frac{\partial u}{\partial x} = 0 \tag{14}$$

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} t_{xy} - (M^2 + \frac{1}{Da^2})u + Gr\theta\sin\phi \tag{15}$$

$$\frac{\partial u}{\partial y} = \dot{y}$$
$$t_{xy} = (1-n)\frac{\partial u}{\partial y} + nwe(\frac{\partial u}{\partial y})^2 \tag{16}$$

$$\mathrm{Re\,pr}(\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x}) = \frac{\partial^2 \theta}{\partial y^2} + Brt_{xy}\frac{\partial u}{\partial y} + \mathrm{M^2Bru^2} + Nr^2\theta \tag{17}$$

Given the specified boundary conditions

$$\left.\begin{aligned} u - \beta\frac{\partial \mathrm{u}}{\partial \mathrm{y}} = 0,\ \text{at}\ (\mathrm{y} = -1) \\ u + \beta\frac{\partial \mathrm{u}}{\partial \mathrm{y}} = 0,\ \text{at}\ (\mathrm{y} = 1) \end{aligned}\right] \tag{18}$$

$$\left.\begin{aligned} \theta = 0,\ \text{at}\ (\mathrm{y} = -1) \\ \theta = 1,\ \text{at}\ (\mathrm{y} = 1) \end{aligned}\right] \tag{19}$$

By considering the Reynolds number to be modest and less than one, we obtain the following equation by substituting Eq. (15) into Eq. (17)

$$0=\frac{\partial^2\theta}{\partial y^2}+\mathrm{Br}((1-\mathrm{n})(\frac{\partial u}{\partial y})^2+\mathrm{n\ we}(\frac{\partial u}{\partial y})^3)+M^2\mathrm{Br}u^2+Nr^2\theta \tag{20}$$

In the Poiseuille flow, we are assumed that the pressure gradient between parallel platelets is constant; thus, Eq. (15) can be taken the following formula

$$0=G+\frac{\partial^2 u}{\partial y^2}((1-\mathrm{n})+2\mathrm{n\ we}(\frac{\partial u}{\partial y}))-(M^2+\frac{1}{D^2a})u+Gr\theta\sin\phi \tag{21}$$

5. Solution Methodology and Discussion of the Results

We solve Eq. (20) and Eq. (21) using a numerical technique, taking into account the boundary conditions (18) and (19). The ND-solve command in Mathematica is a powerful tool for numerical analysis. It allows for efficient error minimization and reduces the computational time required for each evaluation. It selects a suitable algorithm for solving the problem. In addition, this technique allows for the avoidance of complex solutions and enables direct visualization of graphical results. This section's focus is on the consequences of the parameters that are cleverly hidden within the code, such as $\mathrm{M},\beta,\mathrm{Dr},\mathrm{G},\mathrm{We},\mathrm{n},\mathrm{Br}\ \ \mathrm{Nr},\mathrm{Gr},\phi$ the dispersion of speed and thermal characteristics for the Poiseuille flow of tangent of hyperbolic nature fluid due to the consequences of falling and radiation conditions.

5.1 Features Related to Velocity

Figure 2 to Figure 11 Get ready to witness the mighty impact of parameters! $\mathrm{M},\mathrm{Dr},\beta,\mathrm{G},\mathrm{We},\mathrm{n},\mathrm{Br},\mathrm{Gr},\ \mathrm{Nr},\phi$ Unleash the Power of Axial Velocity with Poiseuille Flow. From Figure 2, we can analyze that when M it increases, the velocity decreases in this Flow, which is due to the fact that the parameter relies on the Lorentz force to be present in other words, an agent that opposes the Flow is this force. This results in a decrease in the velocity of the fluid, as shown in the studies by Sheikholeslami *et al.,* [25], and Ramesh [27].

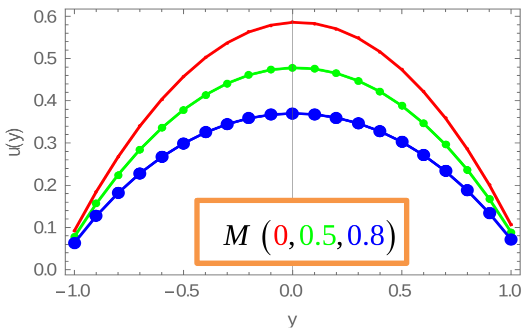

Fig. 2. Implications for M on velocity $n=0.5, G=0.5, we=0.01, \beta=0.1, Da=3,$ $Gr=0.5, \phi=0.15, Nr=0.5, Br=1.7$

Figure 3 shows that flow velocity increases with. This indicates that near the channel middle, the velocity exhibits an upward trend while transitioning from a porous to a non-porous medium. This is physically justifiable since a more permeable porous material provides less fluid flow resistance, increasing fluid velocity.

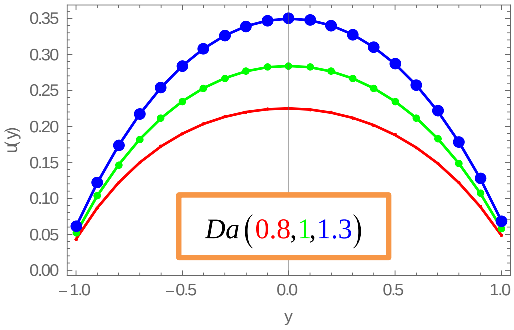

Fig. 3. Implications for Da on velocity $n = 0.5, G = 0.5, we = 0.01, \beta = 0.1, M = 0.5, Gr = 0.5, \phi = 0.15, Nr = 0.50, Br = 1.70$

From Figure 4, it is seen that β More fluid slides near the border, increasing its Thrilled by the impact of velocity by the boundary motion in the Poiseuille flow. The analogous conduct is observed by Ramzan *et al.,* [31], and Ramesh [32].

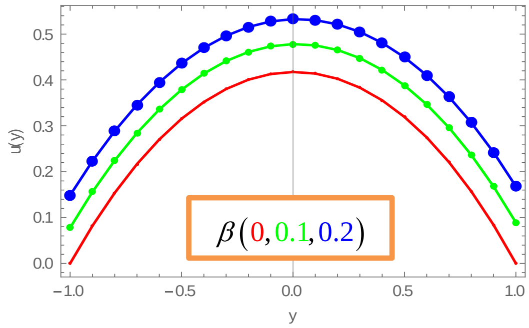

Fig. 4. Implications for β on velocity $n = 0.5, G = 0.5, we = 0.01, M = 0.5, Da = 3, Gr = 0.5, \phi = 0.15, Nr = 0.5, Br = 1.7$

Figure 5 reveals the impact on the velocity. It is evident from this figure that the velocity rises as it increases. This phenomenon has a physical basis, as higher pressure leads to a greater flow of fluid with higher velocity in the channel. Ramesh [27,32] demonstrates a similar pattern. Similar behaviour is noticed for the effects of Br Nr, ϕ on velocity profile and their efforts are plotted in Figure 6, Figure 7, and Figure 8 respectively.

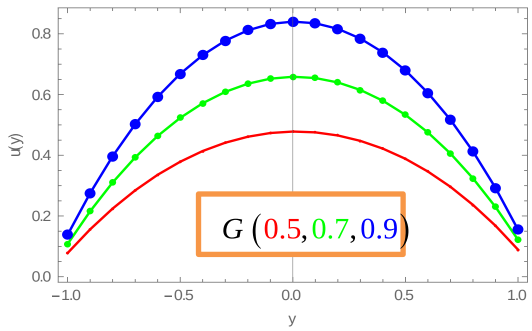

Fig. 5. Implications for G on velocity $n = 0.5, \beta = 0.1, we = 0.01, M = 0.5, Da = 3,$ $Gr = 0.5, \phi = 0.15, Nr = 0.5, Br = 1.7$

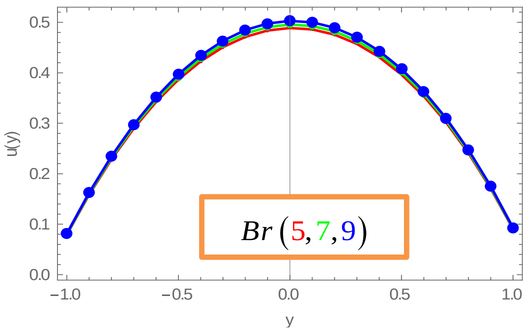

Fig. 6. Implications for Br on velocity $n = 0.5, \beta = 0.1, we = 0.01, M = 0.5, Da = 3,$ $Gr = 0.5, \phi = 0.15, Nr = 0.5, G = 0.5$

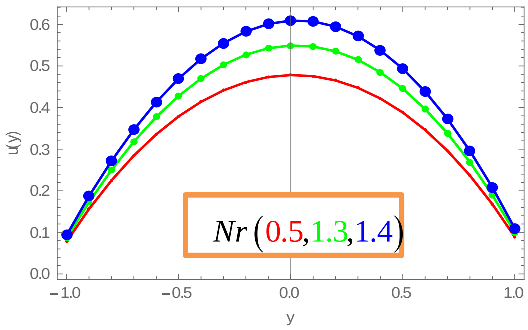

Fig. 7. Implications for Nr on velocity $n = 0.5, \beta = 0.1, we = 0.01, M = 0.5, Da = 3,$ $Gr = 0.5, \phi = 0.15, Br = 1.7, G = 0.5$

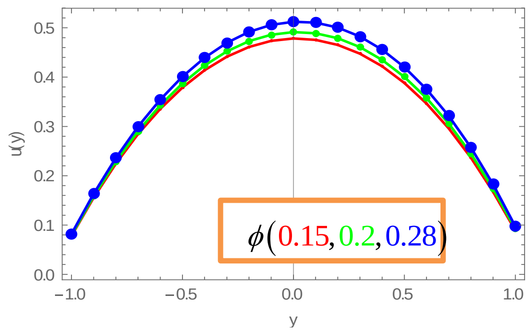

Fig. 8. Implications for ϕ on velocity $n = 0.5, \beta = 0.1, we = 0.01, M = 0.5, Da = 3,$ $Gr = 0.5, Nr = 0.5, Br = 1.7, G = 0.5$

The impress Gr is shown in Figure 9, it is noted from this figure, that an increase Gr leads to an augmentation in the magnitude of fluid's velocity, it is back to the fact that a decreased viscosity results in an increase in velocity. The effects We, n on velocity are shown in Figure 10 and Figure 11 respectively, it is noted that there is a clear enhancement in values of the velocity of a fluid with an increase of these parameters especially on the right side of the channel, that is the velocity of the fluid is more than of Newtonian fluid when (we=0 or n=1).

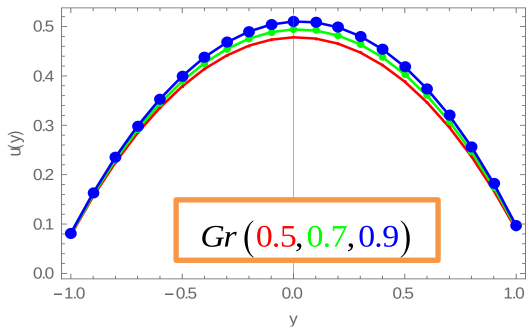

Fig. 9. Implications for Gr on velocity $n=0.5, \beta=0.1, we=0.01, M=0.5, Da=3,$ $\phi=0.15, Nr=0.5, Br=1.7, G=0.5$

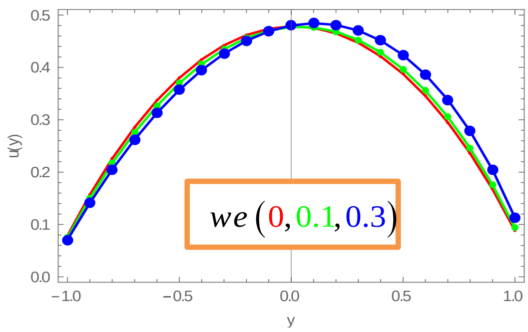

Fig. 10. Implications for we on velocity $n=0.5, \beta=0.1, Gr=0.5, M=0.5, Da=3,$ $\phi=0.15, Nr=0.5, Br=1.7, G=0.5$

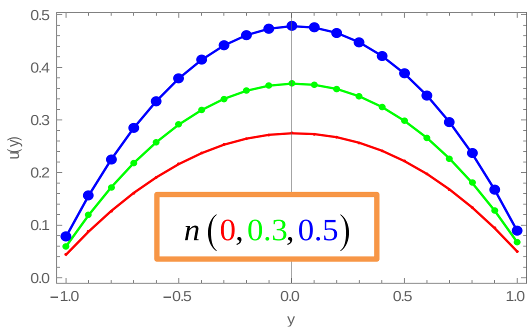

Fig. 11. Implications for n on velocity $we=0.01, \beta=0.1, Gr=0.5, M=0.5, Da=3,$ $\phi=0.15, Nr=0.5, Br=1.7, G=0.5$

It is also found from the above figures, that Speed distribution traces a Forming a curve that is shaped like a parabola path having the highest value in close proximity to the middle of the channel.

5.1.1 Numerical Results of Velocity Distribution Variation

In this section, we will give the computational result of various parameters $(M, Da, we, \beta, G, n, Gr, \phi, Nr, Br)$ on velocity profile by using Proposed method "ND -Solve" command in Mathematica software and shown them in Table 1 to Table 10.

Table 1
The velocity-y connection for different M values

y	M=0.00	M=0.50	M=0.80
0.1	0.582919	0.475548	0.367767
0.2	0.570377	0.465351	0.360017
0.3	0.548078	0.447522	0.346727
0.4	0.515901	0.421903	0.327704
0.5	0.473707	0.388285	0.302675
0.6	0.421338	0.346405	0.271283
0.7	0.358616	0.295948	0.233085
0.8	0.285344	0.236541	0.187548
0.9	0.201307	0.167751	0.134035
1.	0.106275	0.0890894	0.0718064

Table 2
The velocity-y connection for various Da values

y	Da=0.8	Da=1	Da=1.3
0.1	0.22333	0.282051	0.34747
0.2	0.21904	0.276323	0.340187
0.3	0.211897	0.266678	0.327758
0.4	0.20167	0.252896	0.309982
0.5	0.188026	0.234661	0.286572
0.6	0.170521	0.211556	0.257156
0.7	0.148593	0.183053	0.221268
0.8	0.121535	0.148504	0.17834
0.9	0.0884868	0.10713	0.127702
1.	0.0483976	0.0579962	0.0685616

Table 3
The velocity-y connection for different we value

y	We=0	We=0.1	We=0.3
0.1	0.475514	0.476549	0.484396
0.2	0.46509	0.468414	0.481268
0.3	0.447049	0.452545	0.470442
0.4	0.421244	0.428682	0.451438
0.5	0.387478	0.396503	0.423642
0.6	0.3455	0.355607	0.386256
0.7	0.295011	0.305513	0.338238
0.8	0.235655	0.245649	0.278183
0.9	0.167022	0.175334	0.20412
1.	0.0886453	0.0937658	0.113078

Table 4
The velocity-y connection for different β value

y	$\beta = 0$	$\beta = 0.1$	$\beta = 0.2$
0.1	0.414103	0.475548	0.530599
0.2	0.402735	0.465351	0.521373
0.3	0.383297	0.447522	0.504903
0.4	0.355621	0.421903	0.481042
0.5	0.319483	0.388285	0.449592
0.6	0.274604	0.346405	0.410305
0.7	0.220645	0.295948	0.362885
0.8	0.157209	0.236541	0.30698
0.9	0.083836	0.167751	0.242185
1.	-2.91434*10^-16	0.0890894	0.168039

Table 5
The velocity-y connection for different G values

y	G=0.5	G=0.7	G=0.9
0.1	0.475548	0.654512	0.834974
0.2	0.465351	0.640089	0.816352
0.3	0.447522	0.61519	0.784378
0.4	0.421903	0.579621	0.738818
0.5	0.388285	0.533116	0.679345
0.6	0.346405	0.475333	0.605539
0.7	0.295948	0.405859	0.516886
0.8	0.236541	0.324202	0.412772
0.9	0.167751	0.22979	0.292484
1.	0.0890894	0.12197	0.155203

Table 6
The velocity-y connection for different n value

y	n=0	n=0.3	n=0.5
0.1	0.273171	0.366848	0.475548
0.2	0.267146	0.358834	0.465351
0.3	0.256613	0.34484	0.447522
0.4	0.241516	0.324771	0.421903
0.5	0.22178	0.298502	0.388285
0.6	0.197318	0.265878	0.346405
0.7	0.168028	0.226717	0.295948
0.8	0.13379	0.180807	0.236541
0.9	0.0944744	0.127905	0.167751
1.	0.0499318	0.0677375	0.0890894

Table 7
The velocity-y connection for different of Gr value

y	Gr=0.5	Gr=0.7	Gr=0.9
0.1	0.475548	0.491809	0.508372
0.2	0.465351	0.481761	0.498465
0.3	0.447522	0.463764	0.480288
0.4	0.421903	0.437633	0.453626
0.5	0.388285	0.403129	0.418214
0.6	0.346405	0.359962	0.37373
0.7	0.295948	0.307784	0.319797
0.8	0.236541	0.246192	0.255981
0.9	0.167751	0.174723	0.18179
1.	0.0890894	0.0928545	0.0966682

Table 8
The velocity-y connection for different of ϕ value

y	$\phi = 0.15$	$\phi = 0.2$	$\phi = 0.28$
0.1	0.475548	0.48892	0.510435
0.2	0.465351	0.478845	0.500545
0.3	0.447522	0.460879	0.482345
0.4	0.421903	0.434839	0.455617
0.5	0.388285	0.400494	0.420091
0.6	0.346405	0.357556	0.375442
0.7	0.295948	0.305684	0.32129
0.8	0.236541	0.24448	0.257198
0.9	0.167751	0.173487	0.182668
1.	0.0890894	0.092187	0.0971418

Table 9
The velocity-y connection for disparate of Nr value

y	Nr=0.5	Nr=1.3	Nr=1.4
0.1	0.475548	0.546816	0.60707
0.2	0.465351	0.535416	0.594555
0.3	0.447522	0.514825	0.571531
0.4	0.421903	0.484906	0.537889
0.5	0.388285	0.44551	0.49354
0.6	0.346405	0.396477	0.438419
0.7	0.295948	0.33763	0.372478
0.8	0.236541	0.268779	0.295685
0.9	0.167751	0.189716	0.20802
1.	0.0890894	0.100209	0.109465

Table 10
The velocity-y connection for disparate of Br value

y	Br=5	Br=7	Br=9
0.1	0.48598	0.492792	0.500027
0.2	0.475527	0.482172	0.48923
0.3	0.457238	0.463583	0.470322
0.4	0.43096	0.436876	0.443159
0.5	0.396493	0.401854	0.407548
0.6	0.353583	0.358271	0.363251
0.7	0.301929	0.305835	0.309985
0.8	0.241176	0.244204	0.24742
0.9	0.170918	0.172987	0.175184
1.	0.0906967	0.0917465	0.0928617

5.2 Temperature Features

Figure 12 to Figure 21 ready to demonstrate parameter impacts $\mathrm{M}, \mathrm{Dr}, \beta, \mathrm{We} \,\&\, \mathrm{n}, \mathrm{Br}, \mathrm{Gr}, \phi \,\&\, \mathrm{G}$ $\mathrm{Nr}, \mathrm{G}, \phi$ on the heat distribution for the Poiseuille flow of fluid. These graphs show virtually parabolic temperature profiles with the highest Temperature near the channel center. Obviously, in a viscous fluid flow, the fluid's viscosity converts kinetic Energy from its motion into internal Energy that heats it. Viscous dissipation is partially irreversible. The central channel fluid temperature rises due to viscous dissipation. The impact M on Temperature is shown in Figure 12. From this graph, it is apparent that the Temperature drops as a function of M. Being present in a transverse magnetic field causes an electrically conducting fluid to experience a resistive force known as the Lorentz force.

This force induces resistance in the fluid by augmenting the friction between its layers, resulting in a reduction in its Temperature. The Temperature rises as Da increases, shown by Figure 13, it is quite interesting to note that the transparent medium actually has a higher temperature than the porous medium.

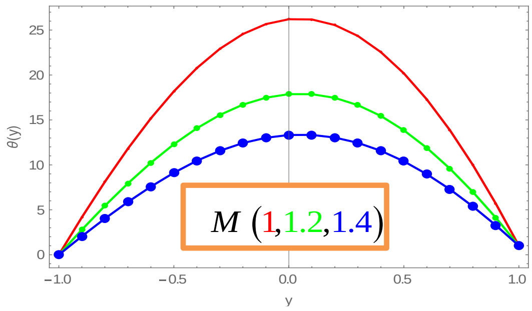

Fig. 12. Implications for M temperature $n=0.5, G=2.3, we=0.01, \beta=0.5, Da=3, Gr=0.5, \phi=0.15, Nr=1, Br=5$

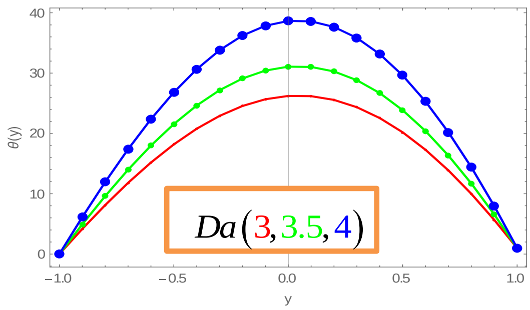

Fig. 13. Implications for Da on warmth $n=0.50, G=2.3, we=0.010, \beta=0.5, M=1, Gr=0.5, \phi=0.15, Nr=1, Br=5$

It has come to our attention from the Figure 14. It is quite amusing how the Temperature seems to have a mind of its own, dropping as β increases. The temperatures observed in the no-slip case are higher than those in the slipcase.

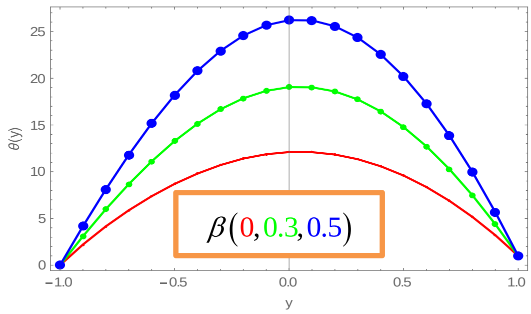

Fig. 14. Implications for β on warmth $n=0.50, G=2.3, we=0.01, M=1.0, Da=3, Gr=0.5, \phi=0.15, Nr=1, Br=5$

The situation has taken an interesting turn with an increase in Br and Nr, as shown in Figure 15 and Figure 16, which is due to the fact that, an increase in values of Br rises the resistance offered by shear in Flow, which consequently increases heat generation as a result of the phenomenon of viscous dissipation effects, raises the fluid's Temperature, thereby increasing the radiation parameter As Nr specifies the proportion of conducting transfer of Heat to transfer of thermal radiation, an increase in Nr values results in a distinct increase in fluid Temperature.

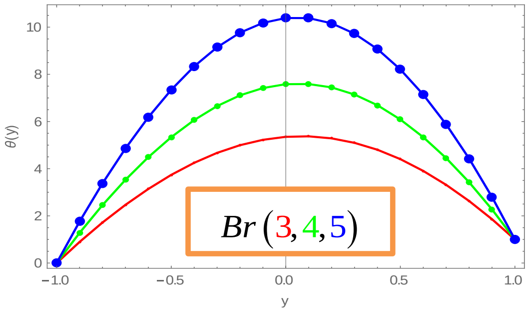

Fig. 15. Implications for Br on warmth $n = 0.5, G = 2.3, we = 0.01, \beta = 0.5, Da = 3, Gr = 0.5, \phi = 0.15, Nr = 1, M = 1.0$

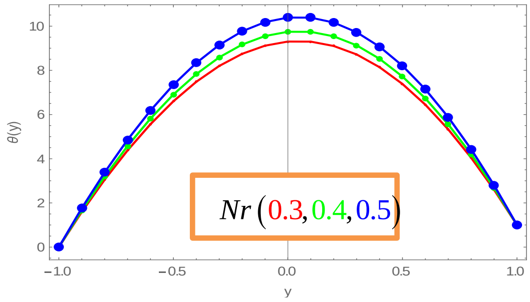

Fig. 16. Implications for Nr on warmth $n = 0.5, G = 2.3, we = 0.01, \beta = 0.5, Da = 3, Gr = 0.5, \phi = 0.15, M = 1, Br = 5$

Figure 17 is designed for the effect of Gr the temperature profile, it is found that an increase in this parameter leads to a strengthening in values of fluid's Heat. It can be inferred that an increase in the value of Gr or any parameter related to buoyancy leads to an increase in wall temperature. This causes the bonds between the fluid particles to weaken, resulting in a decrease in internal friction. Additionally, the force of gravity becomes stronger, leading to a noticeable difference in specific weight between the fluid layers closest to the wall. Parameters ϕ & G exhibit a comparable impact on the Temperature of the fluid which has an increased impress on the fluid's Heat and their effects are plotted in Figure 18 and Figure 19 respectively.

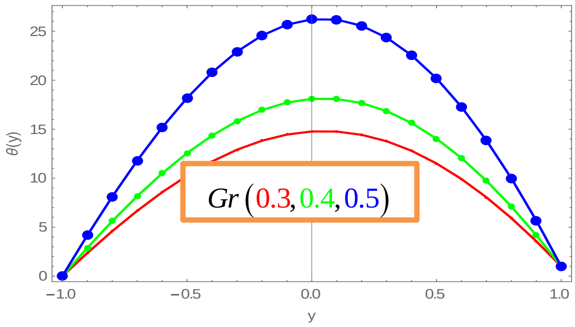

Fig. 17. Implications for Gr on warmth $n = 0.50, G = 2.3, we = 0.010, \beta = 0.5, Da = 3, M = 1, \phi = 0.15, Nr = 1, Br = 5$

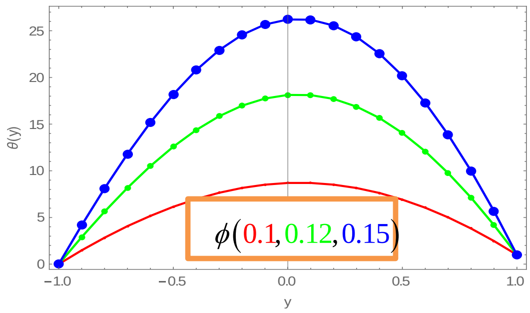

Fig. 18. Implications for ϕ on temperature $n = 0.5, G = 2.3, we = 0.01, \beta = 0.5, Da = 3, Gr = 0.5, M = 1.0, Nr = 1, Br = 5$

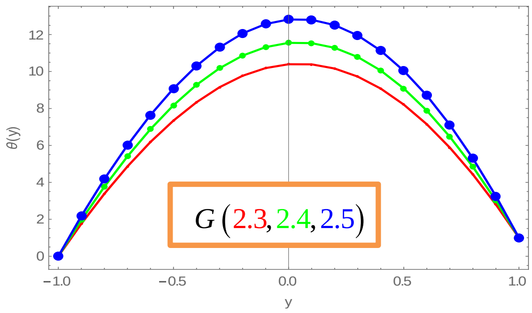

Fig. 19. Implications for G on temperature $n = 0.5, M = 1.0, we = 0.01, \beta = 0.5, Da = 3, Gr = 0.5, \phi = 0.15, Nr = 1, Br = 5$

In Figure 20 and Figure 21 respectively, can be seen the influence of $We \& n$ respectively on the temperature function. We can notice that an increase in these parameters presented a strengthen in values of the Temperature of the fluid, we can say that the Temperature of this fluid is more than that of Newtonian fluid when (n=1 or we=0). The most findings of this study have agreement behavior as seen in previous studies [27,31-33]. It is also found from the above figures, that the function of fluid's Temperature has a parabolic path as the same we have observed of velocity function.

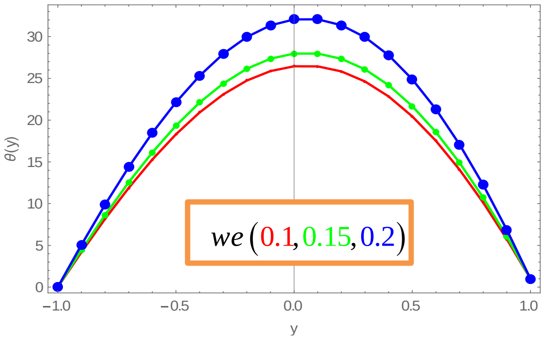

Fig. 20. Implications for we on temperature $n = 0.5, G = 2.3, M = 1.0, \beta = 0.5, Da = 3, Gr = 0.5, \phi = 0.15, Nr = 1, Br = 5$

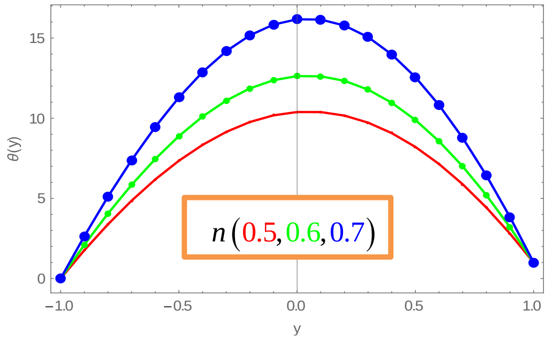

Fig. 21. Implications for n on temperature $M = 1.0, G = 2.3, we = 0.01, \beta = 0.5, Da = 3, Gr = 0.5, \phi = 0.15, Nr = 1, Br = 5$

5.2.1 Numerical Results of Temperature Distribution Variation

In this section, we will give the computational result of various parameters $(M, Da, we, \beta, Gr, \phi, Nr, Br, G, n)$ on temperature profile by using Proposed method "ND -Solve" command in Mathematica software and shown them in Table 11 to Table 20.

Table 11
Temperature-y correlation for various worth of M

y	M=1	M=1.2	M=1.4
0.1	26.1793	17.8681	13.3327
0.2	25.5572	17.461	13.0429
0.3	24.3441	16.6494	12.4533
0.4	22.5476	15.4401	11.5694
0.5	20.1841	13.8465	10.4019
0.6	17.2788	11.8886	8.9663
0.7	13.8647	9.59204	7.28273
0.8	9.98196	6.98792	5.37527
0.9	5.67678	4.11125	3.27119
1.	1.	1.	1.

Table 12
Temperature-y correlation for various worth of Da

y	Da=3	Da=3.5	Da=4
0.1	26.1793	31.0215	38.589
0.2	25.5572	30.2759	37.6517
0.3	24.3441	28.8271	35.8357
0.4	22.5476	26.6837	33.1511
0.5	20.1841	23.8651	29.6217
0.6	17.2788	20.4009	25.2844
0.7	13.8647	16.3306	20.1881
0.8	9.98196	11.7023	14.3934
0.9	5.67678	6.57145	7.97091
1.	1.	1.	1.

Table 13
Temperature-y correlation for various worth of we

y	we=0.1	we=0.15	we=0.2
0.1	26.437	27.9716	32.0778
0.2	25.8318	27.3436	31.3716
0.3	24.628	26.0808	29.9354
0.4	22.8305	24.1866	27.7704
0.5	20.4534	21.6741	24.8888
0.6	17.5204	18.5667	21.3146
0.7	14.064	14.8979	17.0842
0.8	10.1253	10.7111	12.2459
0.9	5.75257	6.05845	6.86052
1.	1.	1.	1.

Table 14
Temperature-y correlation for various worth of β

y	$\beta = 0$	$\beta = 0.3$	$\beta = 0.5$
0.1	12.0937	19.0273	26.1793
0.2	11.8385	18.5899	25.5572
0.3	11.3355	17.7308	24.3441
0.4	10.5852	16.4551	22.5476
0.5	9.59041	14.7738	20.1841
0.6	8.35467	12.7036	17.2788
0.7	6.88085	10.2655	13.8647
0.8	5.16904	7.48433	9.98196
0.9	3.2137	4.38681	5.67678
1.	1.	1.	1.

Table 15
Temperature-y correlation for various worth of Gr

y	Gr=0.3	Gr=0.4	Gr=0.5
0.1	14.7633	18.0862	26.1793
0.2	14.4313	17.6695	25.5572
0.3	13.7733	16.8497	24.3441
0.4	12.7943	15.6324	22.5476
0.5	11.5038	14.0295	20.1841
0.6	9.91603	12.0582	17.2788
0.7	8.04917	9.7411	13.8647
0.8	5.9247	7.10512	9.98196
0.9	3.56656	4.18058	5.67678
1.	1.	1.	1.

Table 16
Temperature-y correlation for various worth of ϕ

y	$\phi = 0.1$	$\phi = 0.12$	$\phi = 0.15$
0.1	8.69091	18.1107	26.1793
0.2	8.51215	17.6933	25.5572
0.3	8.15483	16.8723	24.3441
0.4	7.62094	15.6533	22.5476
0.5	6.91421	14.0481	20.1841
0.6	6.03992	12.074	17.2788
0.7	5.00447	9.75355	13.8647
0.8	3.81489	7.11381	9.98196
0.9	2.4781	4.1851	5.67678
1.	1.	1.	1.

Table 17
Temperature-y correlation for various worth of Nr

y	Nr=0.3	Nr=0.4	Nr=0.5
0.1	9.29778	9.74285	10.3847
0.2	9.1034	9.53722	10.1628
0.3	8.71823	9.13026	9.7244
0.4	8.144	8.52386	9.07157
0.5	7.38418	7.72183	8.20862
0.6	6.44376	6.72966	7.1418
0.7	5.32877	5.5541	5.87886
0.8	4.04578	4.20255	4.42847
0.9	2.60112	2.6824	2.7995
1.	1.	1.	1.

Table 18
Temperature-y correlation for various worth of Br

y	Br=3	Br=4	Br=5
0.1	5.37255	7.5912	10.3847
0.2	5.28795	7.44533	10.1628
0.3	5.09754	7.14474	9.7244
0.4	4.80243	6.691	9.07157
0.5	4.40476	6.08723	8.20862
0.6	3.90756	5.33792	7.1418
0.7	3.31454	4.44855	5.87886
0.8	2.62973	3.42519	4.42847
0.9	1.85711	2.27391	2.7995
1.	1.	1.	1.

Table 19
Temperature-y correlation for various worth of G

y	G=2.3	G=2.4	G=2.5
0.1	10.3847	11.5352	12.7981
0.2	10.1628	11.2819	12.5105
0.3	9.7244	10.7866	11.9529
0.4	9.07157	10.0517	11.128
0.5	8.20862	9.08198	10.041
0.6	7.1418	7.88438	8.69988
0.7	5.87886	6.46761	7.1142
0.8	4.42847	4.84142	5.29493
0.9	2.7995	3.01582	3.25338
1.	1.	1.	1.

Table 20
Temperature-y correlation for various worth of n

y	n=0.5	n=0.6	n=0.7
0.1	12.6159	12.6159	16.1551
0.2	12.3347	12.3347	15.7834
0.3	11.7838	11.7838	15.057
0.4	10.9665	10.9665	13.9806
0.5	9.88921	9.88921	12.5639
0.6	8.56127	8.56127	10.8212
0.7	6.99449	6.99449	8.77099
0.8	5.20243	5.20243	6.43474
0.9	3.19954	3.19954	3.83625
1.	1.	1.	1.

6. Conclusion of the Results

A comprehensive analysis of a fundamental flow referred to as Poiseuille flow has been undertaken. The flow takes place in a hyperbolic tangent fluid that is incompressible (MHD).

Amid the two sloped platelets in a parallel configuration with a specific angle. ϕ pass through identical Permeable material. Heat transfer effort and slip boundary condition scheme are taken into consideration by using the influence of heat transfer, radiation, and viscous dissipation in the equation of Energy. The controlling equations of the Flow are modelled in a two-dimensional cartesian coordinate system and solved numerically by using " Nd Solve" of size step by (0.1) after using dimensionless variables and cancelling the inertia force. The impact of several factors on the velocity and profiles of thermal is investigated, and the outcomes are illustrated graphically. The following observations are the main views which are noticed through this problem

i. The velocity profile exhibits a positive correlation with the variables. $(Da, \beta, G, Br, Nr, \phi, Gr, n)$ where it is a decreasing function of M and has vacillating manner for an increase of we .
ii. Because of efforts of the viscous dissipation, the distributions of the velocity of Temperature are proportional in a direct way, thus the parameters $(Da, \beta, G, Br, Nr, \phi, Gr, n, we)$ have an affirmative effect on the temperature properties and the case has conversed for the influence of M on it too.
iii. In the most of figures, we notice that the paths of velocity and Temperature are taken on a parabolic graph with the maximum value near the center of the channel.

7. Application of Present Study

In Poiseuille flow, the outcomes of this analysis may provide significant insights into the influence of Joule heating and viscous dissipation on the fluid's temperature distribution, flow characteristics, and potentially electrical properties. This specific subject matter possesses the capacity to be implemented across various fields, such as materials science, fluid dynamics, and engineering.

8. Upcoming Projects

i. The Study examines the impact of the Couette inflow of a hyperbolic tangent fluid while considering the effects of viscous dissipation and Joule heating.
ii. The Study investigates the effects of viscous dissipation and Joule heating on the impact of generalized Couette inflow of a hyperbolic tangent fluid.
iii. Exploring the implications of fluid viscosity variation on (Poiseuille, Couette & generalized Couette inflow) of Hyperbolic Tangent Fluid, exploring the impact of viscous dissipation and Joule heating.
iv. It is possible to incorporate the impact of porous lining within the walls of the canal into the current study.
v. The study examines the impact of parameters on the Couette and generalized Couette inflow of a hyperbolic tangent fluid. The effects of viscous dissipation, Joule heating, and a porous channel lining are considered.

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