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Comparison of Basic Iterative Methods Used to Solve of Heat and Fluid Flow Problems

Abdulbaset S Alkrbash[1], Alhadi A Abosbaia[1], Abdulhafid M Elfaghi[2,*]

[1] Mechanical and Industrial Department, Faculty of Engineering, University of Zawia, Libya
[2] Faculty of Mechanical and Manufacturing Engineering, Universiti Tun Hussein Onn Malaysia, Batu Pahat, 86400 Johor, Malaysia

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ABSTRACT

In this research, a comparison of the convergence rate of different basic methods is made in order to solve the Poisson equation, which is similar to some of the resulting equations in computational fluid mechanics. The Jacobi method, the point Gauss-Seidel method, the successive over-relaxation method (SOR), the line Gauss method (TDMA), the ADI method and Strongly Implicit (SIP) method are among the iterative approaches investigated, which are then compared to find the most optimal method. The selection criteria included the number of iterations and the time needed to reach convergence. In both selection criteria, the SIP approach has been shown to be the most efficient.

1. Introduction

Computational Fluid Dynamics (CFD) is a computer simulation method used to characterize the flow of fluids under a certain geometric or boundary condition [1,2]. The performance criterion is how well the numerical simulation results are matched to the experimental results performed under certain conditions [3,4]. The use of classic iterative solvers such as point Jacobi and point Gauss-Seidel (GS), (GS is sometimes known as method of successive relaxation) despite their simplicity, may prove rather expensive even though successive over-relaxation provides a way of accelerating the convergence. If an increase in precision is needed (say by a factor of 4) we need to cut the spatial interval by half. This would require in a 2D problem 4 times as many spatial points (i.e., 4 times as many computations per iteration) and to make things worse it also requires 4 times as many iterations. Therefore, the total cost of the calculations would increase more than 16 times [5]. Therefore, increased accuracy comes at very high cost. This is unacceptable scaling, and other methods should be sought.

The iterative approximation of fixed points is critical in solving a wide range of problems encountered in various research areas. It is very interesting in computer mathematics to know that iteration processes have converged faster, with the smallest error of the required solution. Many

* *Corresponding author.*
E-mail address: abdulhafid@uthm.edu.my

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researchers are interested in developing fixed point schemes that provide better approximations to analytical solutions of given problems [6,7]. There has been a great deal of interest in finding a solution to the problem of second-order boundary values in differential equations in recent years. Particular emphasis is placed on determining the existence and approximation of the solution of two- or three-point boundary value problems [8,9]. Mann and his colleagues also generalised the variational iteration method for proper treatment of the boundary value problem in [10]. Kafri *et al.,* [11] recently presented a new approach for obtaining a numerical solution to Troesch's nonlinear boundary value problem by utilising Green's functions and manipulating fixed-point iterations such as Picard's and Kranoselkii-schemes. Mann's To approximate the solution of a two-point boundary value problem, we propose a fixed-point iteration method like the Mann iteration process in this paper. The method converts a given boundary value problem into an algorithm, which is then implemented in Maple.

Many iteration algorithms have recently been introduced with the claim of faster convergence rate and qualitative features such as convergence, rate of convergence, stability, and data dependence, in line with the expansion of application fields of iteration algorithms [12,13].

The numerical analysis of linear and nonlinear fractional differential equations remains an important task. Various attempts have been made to develop new methods for determining analytic and approximate solutions to a wide range of fractional differential equations [14]. This investigation has focused on improving the convergence of iterative solvers with the aim of eventually incorporating efficient schemes into current research problems undertaken by the authors in turbulent flow in fluid flow and heat transfer applications. It is hoped that this investigation will review and adopt methods and choices to improve convergence to acceptable levels.

2. Basic Iterative Methods

To test the performance of the Jacobi method, a 2D steady state problem was carried out, as well as study the effect of the mesh. To study the convergence time needed for the Jacobi method to reach convergence for different grids, two different of meshes with the same number of nodes are chosen. The dimensions of the first grid contain of 40000 nodes with mesh of 200×200, whereas the second grid has the same nodes with different mesh of 400×100, as for the convergence criteria 5×10^{-4} was chosen. Figure 1 shows the plate with boundary conditions.

Poisson's Equation, [15-17]

$$\frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2} = S_\phi \tag{1}$$

This equation is a second-order partial differential equation, and in order to apply this equation to the plate, it is transformed into a linear algebraic equation by Taylor series, as seen in Eq. (2) [18, 19]. Of the various existing finite difference formulations, the so-called ''five-point formula'' is the most commonly used. in this representation of the PDE, central differencing which is second-order accurate is utilized. Therefore, model Eq. (2) is approximated as [20, 21]

$$\frac{\emptyset_{i+1,j} - 2\emptyset_{i,j} + \emptyset_{i-1,j}}{(\Delta x)^2} + \frac{\emptyset_{i,j+1} - 2\emptyset_{i,j} + \emptyset_{i,j-1}}{(\Delta y)^2} = S_{i,j} \tag{2}$$

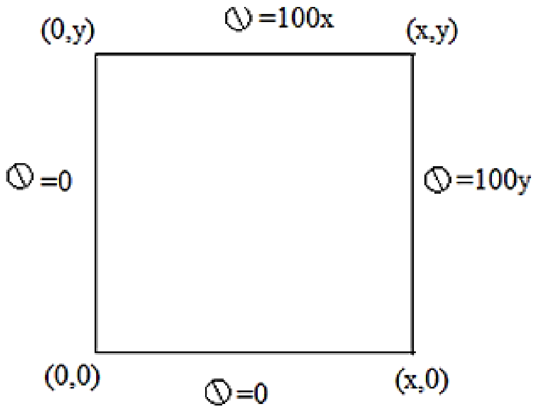

Fig. 1. The plate with boundary conditions

3. Results and Discussion

The results of the iterative methods are presented from the least implicit to the most implicit, with a grid of 200×200 is interlaced to produce 40,000 nodes and another grid of 100×400 are interlaced to produce the same interlacing to know the fastest way to reach the solution and determine the time to show the results. Five basic iterative methods are chosen in this research as follow.

3.1 Jacobi Convergence Duration

As can be seen in Table 1 a mesh of 200×200 need 811.74sec (13.53min) to reach convergence, which is 54.15% less time than the mesh of 400×100, which needed 1770.5sec (29.5min). However, the average time needed per iteration for each grid was 0.00955 iter/sec for the mesh of 200×200 and 0.00981 iter/sec for the mesh of 400×100 grid mesh, which is not very different from each other, with only a 2.6% difference between them.

Table 1
Time needed to reach convergence (Jacobi)

Gird size	No. elements	No. iterations	Time needed (sec)
200×200	40000	~84970	811.7448
400×100	40000	~180500	1770.5

3.2 Point Gauss - Seidel Method

Gauss-Seidel's method is classified as a point iteration method which is similar to the Jacobi method, just this method has three updated values of the same iteration out of five.

Table 2 illustrated that the mesh of 200×200 needed a 296.64sec to reach convergence which is 53.41% less time than the mesh of 400×100 which needs 636.75sec. However, the average time need per iteration for each mesh were 0.007iter/sec and 0.00699iter/sec for selected meshes of 200×200 and 400×100 respectively, which has a very tiny difference between them, in about 0.12%.

Table 2
Time needed to reach convergence (Gauss-Seidel)

Gird size	No. elements	No. iterations	Time needed (sec)
200×200	40000	~42330	296.64
400×100	40000	~90970	636.75

3.3 Line Gauss method (TDMA)

To study the convergence time needed for the Line Gauss (TDMA) method to reach convergence similar grids to the previous methods will be used. From Table 3, it can be seen, a mesh of 400×100 needs 77.11sec (1.29min) to reach convergence which is 52.56% less time than a mesh of 200×200, which 313.82sec (5.23min). However, the average time needed per iteration for each grid is 0.0147iter/sec for the mesh of 400×100 and 0.0148iter/sec for the mesh of 200×200 which has about 0.41% difference between them.

Table 3
Time needed to reach convergence (TDMA)

Gird size	No. elements	No. iterations	Time needed (sec)
200×200	40000	~21270	313.82
400×100	40000	5248	77.11

3.4 ADI method

To study the convergence time needed for the ADI method to reach convergence similar grids to the previous methods were carried out. As can be seen in Table 4 a 400×100 grid needed 212.28sec (7.77min) to reach convergence, which is 54.49% less time than a 200×200 grid that needed 466.45sec (3.54min). However, the average time needed per iteration for each grid was 0.0438iter/sec for the 400×100 grids and 0.0427 iter/sec for the 200×200 grid which 2.55% difference between them.

Table 4
Time needed to reach convergence (ADI)

Gird size	No. elements	No. iterations	Time needed (sec)
200×200	40000	~10640	466.45
400×100	40000	4969	212.28

3.5 SIP Method Performance

To evaluate the (SIP) method, it is necessary to comparison with the basic iterative methods which have previously studied, namely Linear Gaussian (TDMA) and (ADI). Figure 2 shows this comparison for interlacing with a mesh of 200×200, with standard convergence 5e-4, it should be noticed that a finer grid was used to ensure that the performance difference for each method was clearly displayed. As shown in Figure 2, the performance of the SIP method was significantly more efficient than the Linear Gaussian method (TDMA). while using interlacing with mesh of 200×200, the SIP method required, 709 iterations to reach convergence, while the ADI method and Linear Gaussian (TDMA) method required (10470) repetitions and (21333) repetitions, respectively.

From the results, it can be noticed that the slowest way to reach the solution is the Jacobi method, this regarding the solution in this method, is a point solution method, and it is known that the finite difference system for two-dimensional plates is five points.

While Gauss-Seidel's method is faster than Jacobi's method, this is regarding the number of points that are analyzed and updated is more than Jacobi's method where it uses three points implicit and only two points explicit, and the amount of speed to get to the solution compared to Jacobi method is found in the results that have been clarified, and this method is followed by the TDMA method The solution to this method is linear at the same time, i.e. four implicit points and one explicit points are solved. As for the ADI method, the same idea is the TDMA method, except that the difference in the ADI method is to change the horizontal and vertical direction to produce four and a half points implicit and half an explicit point.

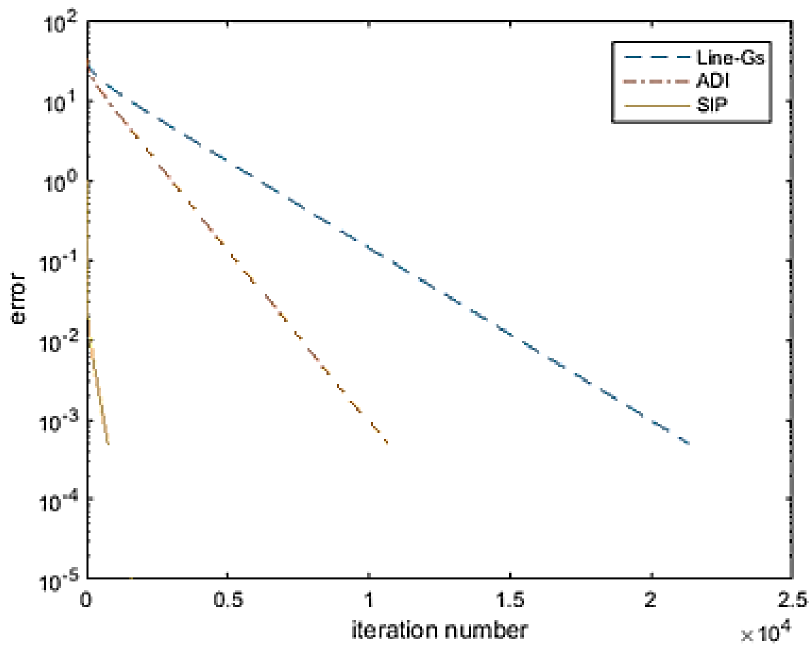

Fig. 2. Number of iterations needed to reach convergence for grid of 200×200 for methods of TDMA, ADI and SIP

4. Conclusions

There are mainly two methods used to solve partial differentiation equations, the direct method and the iterative method. The direct method has no error but has been proven to be too taxing in respect to computing capacity and computing time which is why iterative methods are used instead despite the existence of errors. This study has studied numerous iterative methods and their characteristics using a code written with MATLAB programming code. The iterative methods that were studied included the Jacobi method, point Gauss-Seidel method, Successive Over-Relaxation (SOR) method, line Gauss (TDMA) method, ADI method and Strongly Implicit (SIP) method which were then compared to each other to find the most optimum method. The selection criteria included the number of iterations and the time needed to reach convergence. The SIP method was found to be the most efficient method in both selection criteria.

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