



## Natural Convection Enhancement in the Annuli Between Two Homocentric Cylinders by Using Ethylene Glycol / Water Based Titania Nanofluid

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### ABSTRACT

The natural convection heat transfer in annulate region confined between two homocentric cylinders were numerically studied in the present work. The annulus was filled by ethylene glycol / water-based Titania nanofluid. Both internal and external cylinders are preserved at an isothermal hot and cold temperatures respectively, whereas, the upper and lower walls are adiabatic. The numerical solution is obtained by applying the finite volume method along with the SIMPLER, and TDMA algorithms. In the current study, the solid volumetric fraction is varied as ( $0\% \leq \phi \leq 3\%$ ), the volume ratios of EG to water are varied as (0:100%, 40:60%, 100:0%), while both the Rayleigh number and radii ratio are considered fixed at ( $Ra = 104$  and  $\lambda = 2$ ). The obtained results indicated that the average Nusselt number increases as the solid volumetric fraction and the volume ratio of ethylene glycol in the base fluid increase. Moreover, the velocity profiles reach their maximum value in the half region adjacent the internal hot wall when  $TiO_2$ -water nanofluid is used. Also, the temperature profiles decrease along the radial distance for all considered values of volume ratios of EG to water.

#### Keywords:

Natural convection; coaxial cylinders; annular region; numerical modelling; nanofluid; ethylene glycol / water mixture

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## 1. Introduction

The natural convection phenomena in annuli confined between two homocentric cylinders received a considerable attention in the last ten years among the scientists and researchers around the world. This increasable attention can be returned to their various significant practical applications in many industrial fields. Samples of these applications include thermal building design, nuclear reactors, chemical processing equipments, cooling of electronic components, X-ray tubes, drying technology, shell and tube heat exchangers, lubrication and solar collectors [1-10]. From another side, many published papers [11-21] insured that the addition of metallic or non-metallic nanoparticles to the well-known base fluids causing a dramatic improvement in their low thermal conductivity. The mixture consists from the suspension of these nanoparticle in the traditional fluid is called the nanofluid [22,23]. For further details about the nanofluid and its applications, one can see many extensive review articles like Hussein [24,25], Hussein *et al.*, [26,27] and more recent by Kamel *et al.*, [28]. It is useful to mention also that, one can add nanoparticles to a mixture consists from two conventional fluids such as ethylene glycol / water mixture. This mixture was used abundantly as a coolant in many industrial applications such as microelectronics, renewable energy, automotive technologies, transportations, car radiators and air conditioning systems [29-32]. Sundar *et al.*, [33] experimentally studied the viscosity deviations of Fe<sub>3</sub>O<sub>4</sub>-ethylene glycol/water nanofluid for different working temperatures and the solid volumetric fraction. According to their results, the value of fluid viscosity was considerably improved for bigger values of the solid volumetric fraction and lower temperatures. Also, they noted that the viscosity of the ethylene glycol / water mixture was increased by about 2.9 times for (1 %) nanoparticles volume fraction. The study of the effect of the working temperature on forced convection of TiO<sub>2</sub>-ethylene glycol / water nanofluid was undertaken by Azmi *et al.*, [34]. The authors noticed that, the nanofluid viscosity was increased by about (12 %) when the solid volumetric fraction increased from (0.5 %) to (1.5 %).

On the other side, the natural convection in an annulus fill with the nanofluid was studied by many researchers. The influence of nanofluid flow conditions, geometrical parameters and solid volumetric fraction on average Nusselt number was pointed out by Oztuna *et al.*, [35]. They performed mathematical modeling of natural convection of copper oxide – water nanofluid confined between two vertical homocentric cylinders. This study was conducted under a range Rayleigh number ( $10^5 \leq Ra \leq 10^7$ ) (flow conditions), diameter of the internal cylinder ( $0.25 \leq d \leq 0.75$ ) (geometrical parameters) and the solid volumetric fraction ( $0 \leq \varphi \leq 0.08$ ). The authors deduced that, the average Nusselt number was considerably improved for higher values of the Rayleigh number and the solid volumetric fraction. Abu-Nada *et al.*, [36] utilized various types of nanofluids to enhance the natural convection in the circular region between two homocentric cylinders horizontally aligned. They utilized water base fluid, while (Cu, Ag, Al<sub>2</sub>O<sub>3</sub> and TiO<sub>2</sub>) were utilized as nanoparticles. They found that, the addition of Al<sub>2</sub>O<sub>3</sub>-water nanofluid increased the Nusselt number in the vicinity of the internal cylinder especially for low values of Rayleigh number. Abouali and Falahatpisheh [37] undertook a numerical simulation of the free convection in vertical annuli fill of alumina-water nanofluid. The horizontal walls were assumed thermally isolated, whereas the vertical walls were preserved at uniform temperatures. Their results were illustrated for various Grashof number ( $10^3 \leq Gr \leq 10^5$ ), the aspect ratio ( $1 \leq H/L \leq 5$ ) and the solid volumetric fraction ( $0 \leq \varphi \leq 0.06$ ). The influence of the thermal conductivity and viscosity models on the Nusselt number was discussed also. Abu-Nada [38] examined numerically the influence of several models of the thermal conductivity and viscosity on natural convection in horizontally aligned circular gap subjected to differential heating and fill of alumina-water nanofluid. He deduced that for relatively low values of Rayleigh numbers ( $Ra = 10^3$ ), the average Nusselt number was found to be directly proportional to solid volumetric

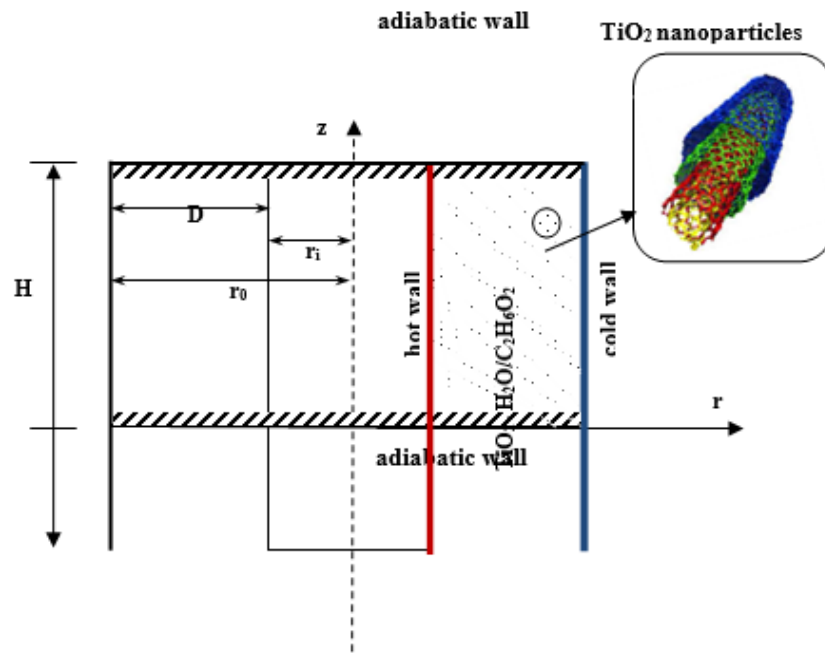
fraction. Theoretical analysis of the natural convection in circular gap between two homocentric cylinders aligned horizontally was conducted by Cianfrini *et al.*, [39]. The space was occupied with various kinds of nanofluids and the cylinder's surface were preserved at different temperatures. Two different fluids were used as base fluids, namely water and ethylene glycol, while (Cu, Al<sub>2</sub>O<sub>3</sub> and TiO<sub>2</sub>) were utilized as nanoparticles. It was found that, the optimum volume fraction was increased by decreasing the nanoparticle size and increasing the nanofluid average temperature. Parvin *et al.*, [40] undertook a numerical investigation on the natural convection in an annulus fill of Al<sub>2</sub>O<sub>3</sub>-water nanofluid. Two thermal conductivity models of the nanofluid were used in their analysis. The annulus internal wall was exposed to a constant heat flux, whereas the external one was assumed cold. The influence of the solid volumetric fraction, Grashof and Prandtl numbers on the flow and thermal fields were reported. They deduced that for both models, heat transfer rate was directly proportional to solid volumetric fraction and the Prandtl number, when the Grashof number was medium or large. The numerical study of the transient natural convection of CuO-water nanofluid contained in horizontally aligned circular gap confined between a two coaxial cylinders aligned horizontally was reported by Yu *et al.*, [41]. The authors pointed out that for fixed Rayleigh number, the time-averaged Nusselt number was decreased by increasing the solid volumetric fraction. The numerical analysis of the natural convection in horizontal annuli subjected to differential heating and fill of Al<sub>2</sub>O<sub>3</sub>-water nanofluid was undertaken by Corcione *et al.*, [42]. Their simulation were done for a considerable number of solid volumetric fractions, radius ratio, the temperatures of the internal and external cylinders, nanoparticle diameter and the internal cylinder radius. The authors pointed out that, heat transfer process was significantly influenced by all parameters considered in the study. Ashorynejad *et al.*, [43] examined numerically the radial external magnetic field impact on the natural convection of silver – water nanofluid enclosed in an annuli formed from a pair of homocentric cylinders aligned horizontally. Both internal and external cylindrical walls were preserved at different constant temperatures. The impact of the solid volumetric fraction, Hartmann and Rayleigh numbers on the fluid flow and thermal fields were presented and discussed. The authors pointed out that, the average Nusselt number was diminished when the Hartmann number was increased. Whereas, it was significantly improved for higher Rayleigh numbers and the solid volumetric fractions. Sheremet and Pop [44] undertook a numerical investigation on the natural convection in a homocentric annulus aligned horizontally and fill of copper – water nanofluid saturated with a porous medium. Their results were presented for several values of the annulus radius ratio, solid volumetric fraction, porosity of porous medium and Rayleigh number. The natural convection in a horizontally aligned homocentric cylindrical annulus contained discrete source-sink pairs was performed by Mirzaie and Lakzian [45]. The annulus was fill of copper-water nanofluid. The external annulus wall was preserved at an isothermal cold temperature, whereas the internal one was preserved at isothermal hot temperature at three different arrangements. The remaining regions of them were assumed adiabatic. The results showed that, the average Nusselt number was directly proportional to the Rayleigh number. Tayebi and Chamkha [46] performed numerical study on natural convection of Cu–Al<sub>2</sub>O<sub>3</sub> - water hybrid nanofluid contained in an eccentric annulus between two cylinders aligned horizontally. Both external and internal cylinder walls were preserved at isothermal cold and hot temperatures respectively. They used also Al<sub>2</sub>O<sub>3</sub> - water nanofluid for the purpose of comparison. Effects of both hybrid and normal nanofluids were investigated as a function of significant parameters such as solid volumetric fraction ( $0 \leq \varphi \leq 0.12$ ) and Rayleigh number ( $10^3 \leq Ra \leq 10^6$ ). As a conclusion, the authors stated that the usage of hybrid nanofluid results in enhancing heat transfer characteristics compared with the water and the normal nanofluid. Smusz [47] numerically studied the free convection in eccentric annuli fill of Cu-water nanofluid. According to the obtained results, the authors pointed out that, the solid volumetric fraction and the eccentricity had a major influence

on the heat transfer characteristics inside the annuli. Mihoubi *et al.*, [48] undertook numerical study on natural convection of sliver-water nanofluid confined in an annular region formed by two horizontal coaxial cylinders. Both internal and external cylinder walls were preserved at different isothermal temperatures. Their analysis were done for a range of Rayleigh number ( $10^3 \leq Ra \leq 10^5$ ) and solid volumetric fraction ( $0 \leq \varphi \leq 0.12$ ). The authors deduced that, heat transfer rate was directly proportional to Rayleigh number as well as solid volumetric fraction. Mebarek-Oudina and Bessaïh [49] reported a numerical study on free convection in a vertical cylindrical annulus fill of copper – water nanofluid. The internal cylinder wall contained two discrete heat sources of various lengths, while the portions of this wall, upper and lower walls were adiabatic. An isothermal cold temperature boundary condition was assigned for the external wall of the cylinder. The influence of several important parameters such as Rayleigh number ( $10^3 \leq Ra \leq 10^6$ ) and the volume fraction of nanoparticles ( $0 \leq \varphi \leq 0.1$ ) on flow and thermal fields were reported and discussed. The authors reported that, both heat transfer characteristics and heaters temperature were significantly influenced by these parameters together with the length of heat sources. The natural convection in a circular region between two coaxial cylinders vertically aligned and filled with Titania nanoparticles immersed with three different types of base fluids was carried out numerically by Mebarek-Oudina [50]. A discrete heat source was assigned for the internal wall of the cylinder, while the rest of the domain was maintained thermally isolated. The remaining conditions were similar to Mebarek-Oudina and Bessaïh [49]. His findings were consistent with previous works, the average Nusselt number was directly proportional to both the solid volumetric fraction and the Rayleigh number. Belabid *et al.*, [51] undertook a numerical study on the free convection in a homocentric cylindrical annulus horizontally aligned fill of copper – water nanofluid saturated with a porous medium. They observed a favorable influence of the increasing of the solid volumetric fraction on the average Nusselt number when the Rayleigh number was relatively low. Very recently, Ashraf *et al.*, [52] employed the MRT lattice Boltzmann method to simulate the natural convection in a symmetrical annulus vertically aligned filled with Titania – water nanofluid. Both internal and external cylinder walls were persevered at different isothermal temperatures, whereas the upper and lower walls were assumed thermally isolated. The study was performed under a range of Rayleigh numbers ( $10^3 \leq Ra \leq 10^6$ ), aspect ratio ( $0.2 \leq \gamma \leq 0.8$ ) and the solid volumetric fraction varied as ( $0 \leq \varphi \leq 0.5$ ). They pointed out that the average Nusselt number was significantly improved with increasing the Rayleigh number and the solid volumetric fraction. For further references regarding the natural convection in an annulus full of nanofluid, the reader can refer to [53–70].

The main objective of this work is to study the effect of the variation of volume fraction of  $TiO_2$  nanoparticles and the volume ratios of EG to water on the heat transfer characteristics. From the above detailed literature review it is clear that natural convection in the annuli between homocentric cylinders fill of Titania-ethylene glycol / water nanofluid was not investigated previously in any paper up to date. So, the present work can be considered as a positive addition to help the researchers to deep their knowledge in this field.

## 2. Mathematical Formulation

Figure 1 presents the geometry considered in the present work, it consists from a cylindrical annulus that is made by a pair homocentric cylinders of internal radius ( $r_i$ ) and external radius ( $r_o$ ). The internal and external cylinders are maintained at an isothermal different temperature ( $T_H$  and  $T_C$ ) respectively [1,4]. While, the upper and lower walls are adiabatic. The radii ratio and the aspect ratio were considered fixed at ( $\lambda = r_o/r_i = 2$  and  $\gamma = 1$ ) respectively.



**Fig. 1.** Configuration of problem

The base fluid is a mixture between ethylene glycol and water, while ( $TiO_2$ ) was considered as a nanoparticle. The natural convection phenomena are generated by the temperature gradient between the internal and external cylinders. Based on the considered Rayleigh number ( $10^4$ ) the flow is considered as laminar flow, the assumption of thermal equilibrium state of the base fluids and the nanoparticles is adopted, the nanofluid is considered as an incompressible Newtonian fluid. The Boussinesq approximation is also adopted in the present analysis. The thermo-physical properties of the base fluids and nanoparticles are presented in Table 1. While, the relations of the nanofluids properties are listed in Table 2. The dimensionless governing equations of the considered problem are given by [2,70-72]

$$\frac{1}{r} \frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} = -\frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial r} + Pr \frac{\nu_{nf}}{\nu_f} \nabla^2 u, \quad (2)$$

$$u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = -\frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial z} + Pr \frac{\nu_{nf}}{\nu_f} \nabla^2 w + Ra Pr \frac{(\rho\beta)_{nf}}{\rho_{nf} \beta_f} T, \quad (3)$$

$$u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \frac{\alpha_{nf}}{\alpha} \nabla^2 T. \quad (4)$$

where

$$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{\partial^2}{\partial z^2}.$$

The following dimensionless variables are listed below

$$r = \frac{r^* - r_i}{D}, z = \frac{z^*}{D}, u = u^* / \left(\frac{\alpha_f}{D}\right), w = w^* / \left(\frac{\alpha_f}{D}\right), P = P^* / \rho_{nf} \left(\frac{\alpha_f}{D}\right)^2, T = \frac{T^* - T_C}{T_H^* - T_C^*}.$$

$$Pr = \frac{\nu}{\alpha} \text{ and } Ra = \frac{g\beta(T_H - T_C)D^3}{\nu \alpha}.$$

The stream function is given by [73]

$$u = \frac{1}{r} \frac{\partial \psi}{\partial z}, w = -\frac{1}{r} \frac{\partial \psi}{\partial r} \tag{5}$$

The initial conditions read

$$\text{For } t = 0, u = w = T = 0, \tag{6}$$

For  $t > 0$ ,

$$\text{when } r = r_i : u = w = 0, T = 1, \tag{7}$$

$$\text{when } r = r_o : u = w = 0, T = 0, \tag{8}$$

$$\text{when } z = 0 : u = w = 0, \frac{\partial T}{\partial z} = 0, \tag{9}$$

$$\text{when } z = \frac{H}{D} : u = w = 0, \frac{\partial T}{\partial z} = 0. \tag{10}$$

The average Nusselt number of the hot cylinder wall reads

$$Nu_{avg}|_{r=1} = \frac{1}{A_r} \int_0^{A_r} Nu \cdot dz, \tag{11}$$

where,  $Nu = -\frac{\kappa_{nf}}{\kappa_f} \frac{\partial T}{\partial r} \Big|_{r=1}$  is the local Nusselt number.

**Table 1**  
 Thermophysical properties [30]

Property	0:100% EG/Water	40:60% EG/Water	100:0% EG/Water	TiO <sub>2</sub>
$\rho$ [Kg/m <sup>3</sup> ]	997.1	1057.60	1132	4250
$C_p$ [J/Kg.K]	4179	3485	2349	686.2
$\kappa$ [W/m.K]	0.613	0.408	0.258	8.9538
$\beta$ [K <sup>-1</sup> ]	$21 \times 10^{-5}$	$35 \times 10^{-5}$	$57 \times 10^{-5}$	$0.9 \times 10^{-5}$
Pr	6.2	21.95	137.48	-

**Table 2**  
 Relations of the nanofluid properties [71]

Nanofluid properties	Model
Thermal conductivity	$\kappa_{nf} = \kappa_f \left[ \frac{(\kappa_s - 2\kappa_f) - 2\phi(\kappa_f - \kappa_s)}{(\kappa_s - 2\kappa_f) + \phi(\kappa_f - \kappa_s)} \right]$
Heat capacity	$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s$
Thermal expansion coefficient	$(\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s$
Density	$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s$
Thermal diffusivity	$\alpha_{nf} = \kappa_{nf} / (\rho C_p)_{nf}$
Dynamic viscosity	$\mu_{nf} = -\frac{\mu_f}{(1 - \phi)^{2.5}}$

### 3. Numerical Solution

The numerical solution of the set of equations system that governs the laminar natural convection along with the associated boundary conditions is obtained by applying the finite volume method. The coupling of flow pressure field and velocity is established by using the SIMPLER algorithm [50]. The iterative resolution is obtained by Thomas (TDMA) algorithm [32]. The second-order central difference scheme is used to discretize the terms of diffusion and convection in the equations. Table 3 shows a good agreement between our numerical results with those in the reference [74]. In order to be able to obtain numerical results that are independent of grid resolution, a grid with (82×82) nodes is used for the calculations in order to give the best comparison between the calculation and precision (see Table 4) [1,2]. The same results can be found using commercial software like Fluent ANSYS (Finite Volume Method) or COMSOL Multiphysics (Finite Element Method).

**Table 3**  
 Validation of our result with those of ref. [74]

$\phi$	Oztop and Abu-Nada [74] results	Current study results
0.00	2.0125	2.487033
0.05	2.0875	2.651727
0.10	2.14375	3.114118

**Table 4**  
 Grid independence test for 3 % of 40 : 60 % EG / water and Ra = 10<sup>4</sup>

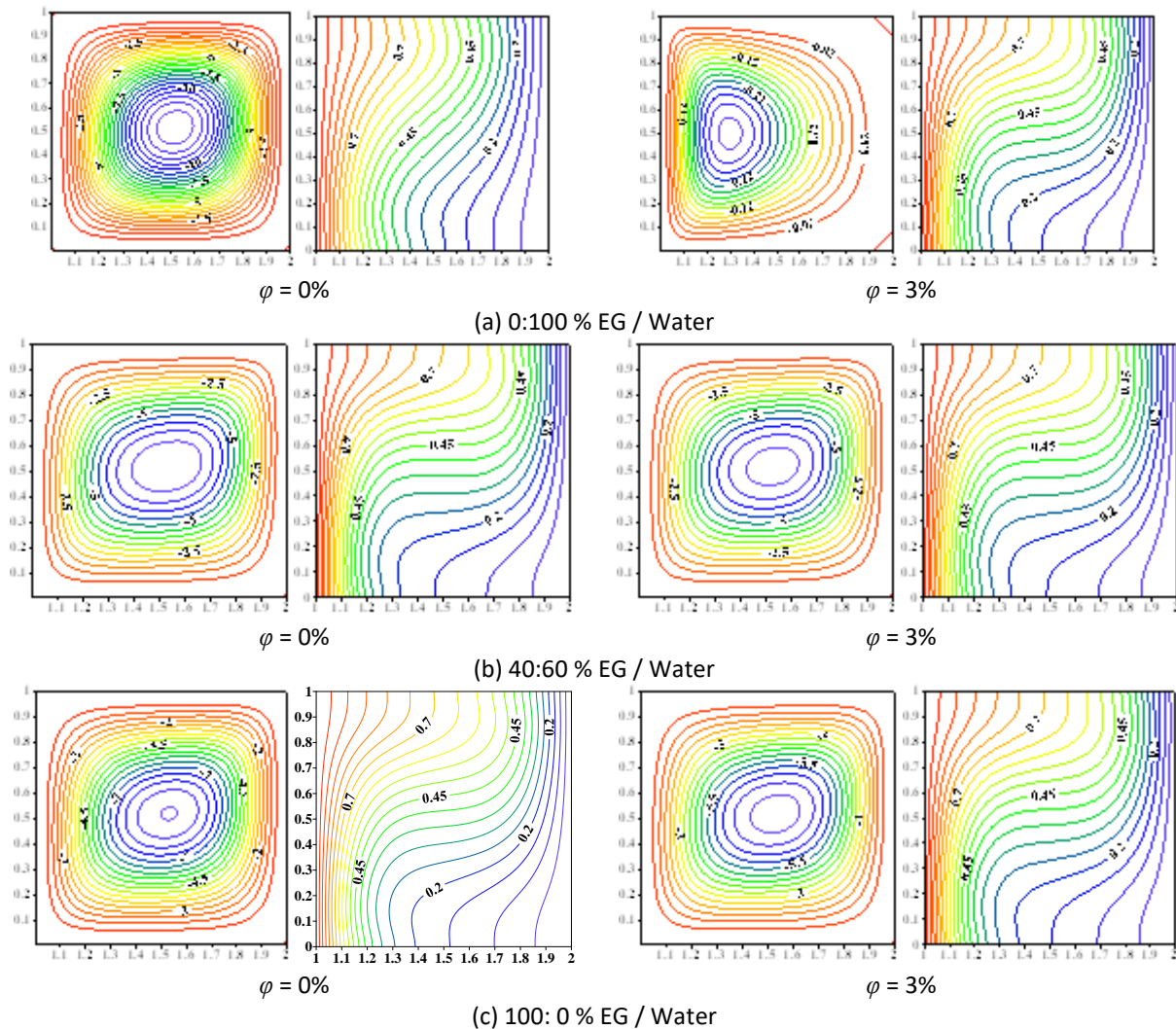
	24×24	48×48	64×64	82×82	102×102
$Nu_{avg}$	2.046463	3.331142	3.485302	3.782261	3.790160
$w_{max}$	23.3632	23.295660	23.640490	26.821750	26.82977

### 4. Results and Discussion

In the current study, the solid volumetric fraction ranged between (0 % ≤ φ ≤ 3 %), the volume ratios of EG to water are varied as (0:100 %, 40: 60 %, 100:0 %). While, both the Rayleigh number and radii ratio are considered fixed at (Ra = 10<sup>4</sup> and λ = 2).

#### 4.1 Effects of the Solid Volumetric Fraction and Volume Ratios of EG to Water on Streamlines and Isotherms

The influence of the solid volumetric fraction and volume ratios of EG to water on streamlines (left) and isotherms (right) were illustrated in Figure 2.



**Fig. 2.** Streamlines (left) and isotherms (right) for different values of the solid volume fraction and volume ratios of EG to water

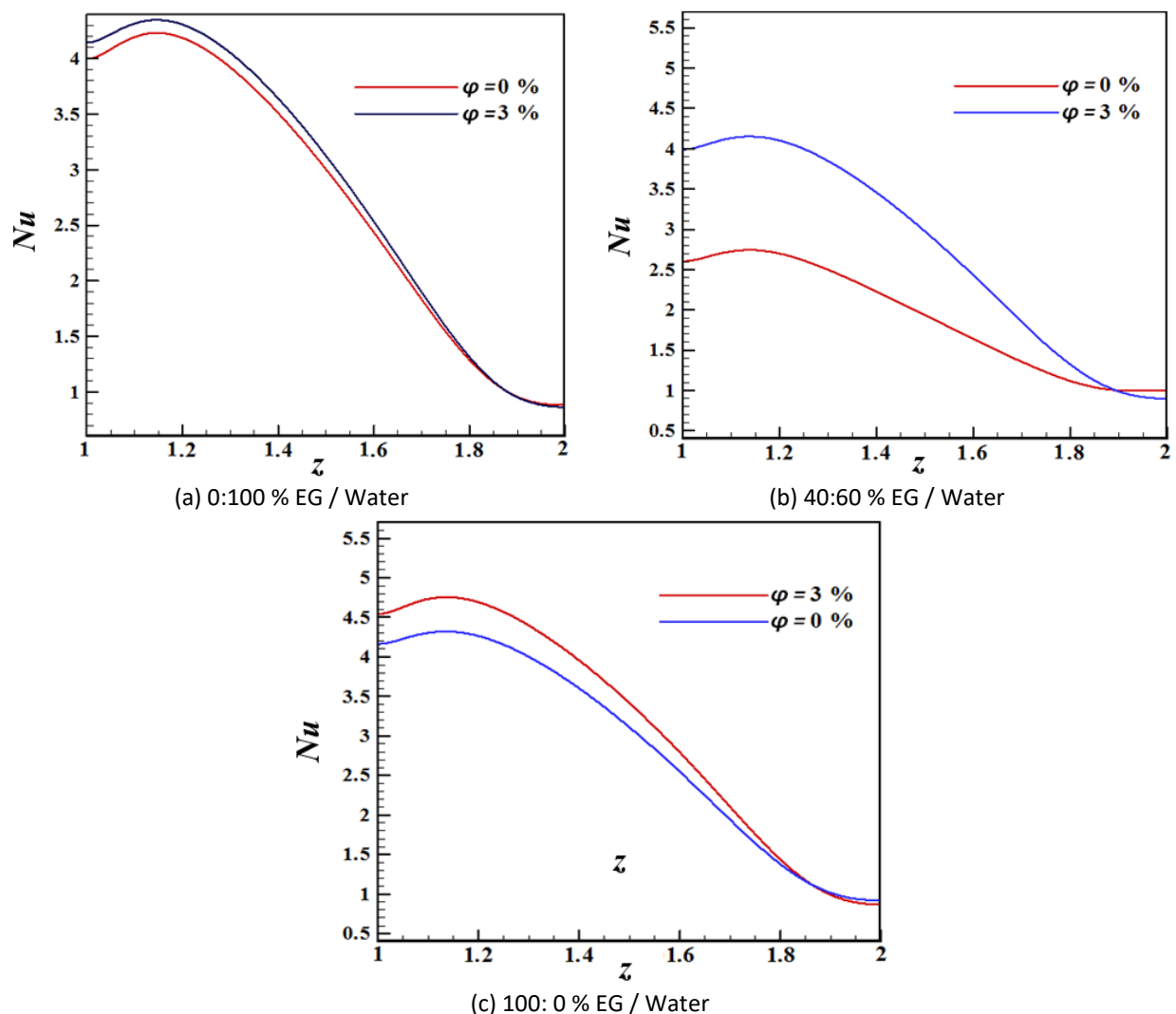
The results indicated that when working fluid is only water (0:100 % EG / water), the flow inside the annulus can be represented by a rotating vortex that cover all the region of it. It is clear that, stream function values are high inside the core of the annulus and decrease gradually towards the walls of it. This satisfies the problem boundary conditions. Now, when the pure water ( $\phi = 0\%$ ) is utilized inside the annulus a uniform pattern of the flow vortices and a high values of the stream function can be seen. But, when  $\text{TiO}_2$ -water nanofluid is used ( $\phi = 3\%$ ) inside the annulus a clear change can be seen. The flow vortices begin to elongate towards the external cold wall. Whereas, the stream function values begin to decrease sharply. Therefore, it can be concluded that for volume ratio (0:100 % EG / water), the using of the nanofluid decreases the flow circulation strength inside the annulus. For isotherms, it is also clear that increasing solid volumetric fraction from 0 % to 3 % results in changing the pattern of isotherm contours from uniform, semi-parallel lines to the curved



and confuse pattern. Since, increasing solid volumetric fraction results an increasing the base fluid thermal conductivity, this causes an enhancement of the buoyancy force and the natural convection inside the annulus. Now, when the volume ratio of ethylene glycol in the base fluid mixture increases respectively to (40: 60 % EG / water) and (100: 0 % EG / water), the flow pattern becomes more similar to each other. At ( $\phi = 0\%$ ), the stream function values and the size of vortices core begin to shrink when the volume ratio of EG in the base fluid increases. But, for ( $\phi=3\%$ ) it is also evident that the stream function values increase as the volume ratio of EG in the base fluid increases, while the size of vortices core remain invariant. With respect to isotherms, it is evident that there is a clear similarity between them at (40: 60 % EG / water) and (100: 0 % EG / water). Heat transfer by convection is dominating inside the annulus in this case.

#### 4.2 Effects of the Solid Volumetric Fraction and Volume Ratios of EG to Water on the Average Nusselt Number

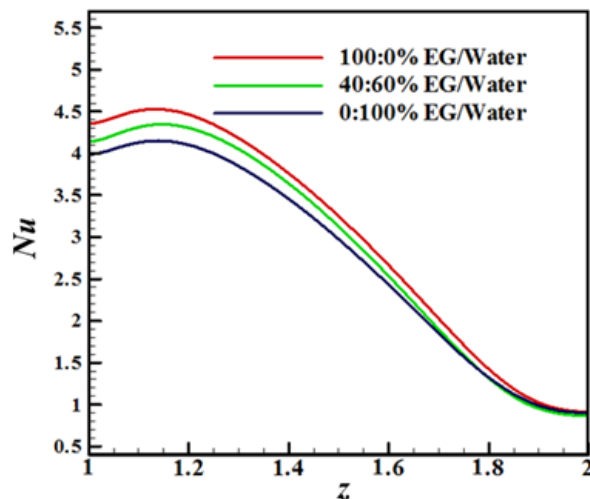
Figure 3 illustrates the average Nusselt number variation along the radial distance for different values of the solid volumetric fraction and volume ratios of EG to water.



**Fig. 3.** The local Nusselt number variation along the axial distance for different solid volume fractions and volume ratios of EG to water

The results show that, for all considered values of the volume ratios of EG to water, increasing solid volumetric fraction from 0 % to 3 % resulted in increasing the average Nusselt number. This means that, using of TiO<sub>2</sub>-ethylene glycol / water nanofluid enhances the natural convection heat transfer inside the annulus better than the pure ethylene glycol / water mixture. This is particularly due to the high increment in the thermal conductivity of ethylene glycol / water mixture by adding Titania nanoparticles to it. This addition helps also to activate the transport of the thermal energy inside the annulus as well as increasing the average Nusselt number.

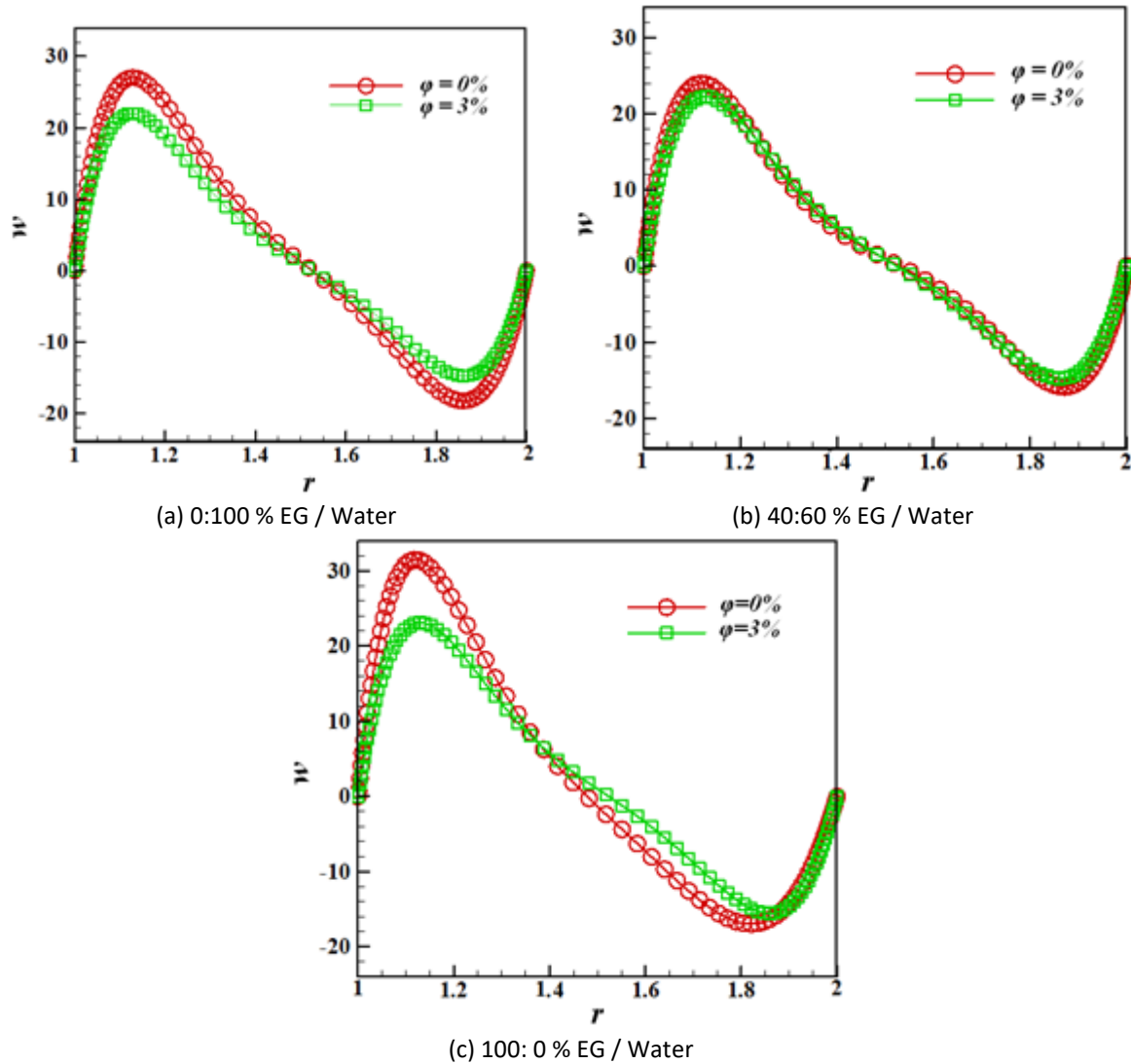
Also, it is clear that values of the average Nusselt number decrease along the radial distance. This was evident for all values of volume ratios considered in this study. With regards to the impact of volume ratios of EG to water on the average Nusselt number, Figure 3 and 4 show that the latter increases when EG to water volume ratio equals to 100:0 % EG / water. While, it starts to decline gradually as the volume ratio of ethylene glycol in the base fluid mixture decreases. This is a direct result of the high increase in Prandtl number of ethylene glycol compared with water as illustrated previously in Table 1. Since, the increase in it leads to increase the viscous forces and decrease the thermal forces and enhances the average Nusselt number. Thus, it is concluded that TiO<sub>2</sub>-ethylene glycol nanofluid is the best choice to improve the natural convection inside the annulus.



**Fig. 4.** The Local Nusselt number variation along the axial distance for different volume ratios of EG to water

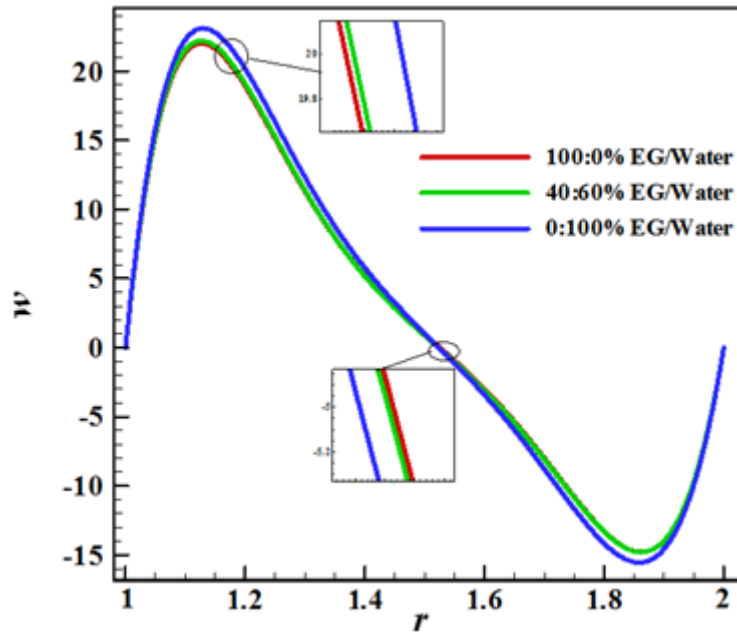
#### 4.3 Effects of the Solid Volumetric Fraction and Volume Ratios of EG to Water on the Velocity Profiles

The influence of solid volumetric fraction and volume ratios of EG to water on the axial velocity profiles at ( $Ra = 10^4$  and  $\lambda = 2$ ) were presented in Figure 5. In general, the velocity profiles along the radial distance exhibit a symmetrical behavior for the whole range of solid volumetric fractions and volume ratios of EG to water. It can be seen that; they increase at the beginning of the radial distance ( $r = 1$ ) until they reach its maximum value. After that, they begin to decrease to reach zero at the middle of the radial distance ( $r = 1.5$ ).



**Fig. 5.** Velocity profiles along the radial distance for different solid volume fractions and volume ratios of EG to water at ( $Ra = 10^4$  and  $\lambda = 2$ )

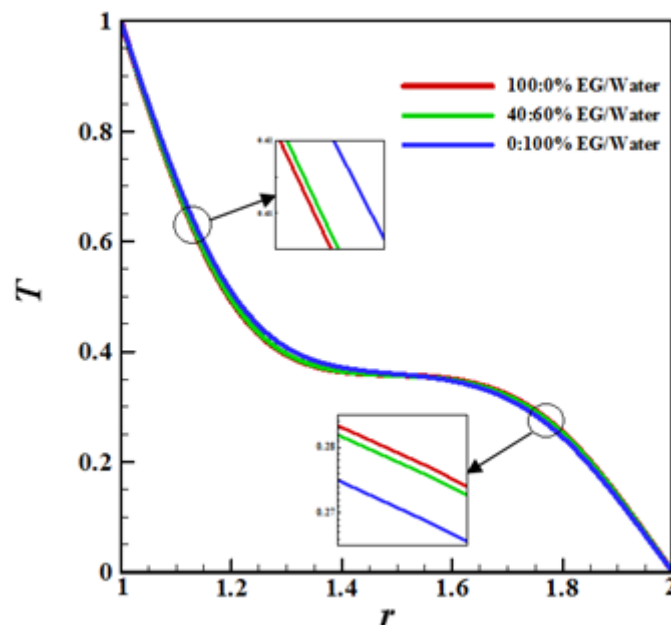
Now, with further increase of this distance, they decrease again until reach zero at the end of it ( $r = 2$ ). With regards to the impact of solid volumetric fraction on the velocity profiles, it is noticed that the velocity adjacent the internal hot wall ( $r = 1$ ) is varying inversely with the solid volumetric fraction. The nanofluid becomes denser adjacent this wall. Therefore, the water ( $\varphi = 0\%$ ) is better than nanofluid ( $\varphi = 3\%$ ) at the internal hot wall. While, for the external cold wall ( $r = 2$ ), it increases when the solid volumetric fraction is increased. Now, for volume ratios of EG to water impact on the velocity profiles, it can be deduced from results of Figure 5 and 6 that they attain their maximum value in the half region adjacent the internal hot wall for (0: 100 %) EG / water volume ratio.



**Fig. 6.** Velocity profiles along the radial distance for various values of the volume ratios of EG to water based  $\text{TiO}_2$  nanoparticle at ( $Ra = 10^4$ ,  $\phi = 3\%$  and  $\lambda = 2$ )

#### 4.4 Effects of the Solid Volumetric Fraction and Volume Ratios of EG to Water on the Temperature Profiles

Figure 7 illustrates the variation of the temperature profiles along the radial distance for various values of the volume ratios of EG to water based  $\text{TiO}_2$  nanoparticle at ( $Ra = 10^4$ ,  $\phi = 3\%$  and  $\lambda = 2$ ). Again, it can be observed that the temperature profiles decrease along the radial distance for all considered values of volume ratios of EG to water.



**Fig. 7.** Variation of the temperature profiles along the radial distance for various values of the volume ratios of EG to water based  $\text{TiO}_2$  nanoparticle at ( $Ra = 10^4$ ,  $\phi = 3\%$  and  $\lambda = 2$ )

Moreover, the results of Figure 7 show that, the temperature at the half region near the internal hot wall decreases as the volume ratio of ethylene glycol in the base fluid mixture increases. This is due to the high viscosity of ethylene glycol, and as a direct consequence of this high viscosity the temperature adjacent the hot wall is decreased. From the opposite side, the temperature at the half region close to the external cold wall increases as the ethylene glycol volume ratio increases.

## 5. Conclusions

The current work presents the numerical simulation of the natural convection enhancement in the annuli between two homocentric cylinders by using ethylene glycol / water-based Titania nanofluid. Based on the obtained results, the following points are concluded

- i. The average Nusselt number is increased as the solid volumetric fraction and the volume ratio of ethylene glycol in the base fluid are increased.
- ii. Utilization of TiO<sub>2</sub> - ethylene glycol / water nanofluid enhances the natural convection inside the annulus better than the pure ethylene glycol / water mixture.
- iii. The velocity profiles along the radial distance exhibit a symmetrical behavior for all selected values of the solid volumetric fraction and volume ratios of EG to water.
- iv. The velocity adjacent the internal hot wall is decreased when the solid volumetric fraction is increased, while an opposite behavior is observed at the external cold wall.
- v. The velocity profiles reach their maximum value in the half region adjacent the internal hot wall when TiO<sub>2</sub> – water nanofluid is used.
- vi. The temperature profiles decrease along the radial distance for all considered values of volume ratios of EG to water.
- vii. The temperature at the half region close to the hot wall decreases as the volume ratio of ethylene glycol in the base fluid mixture increases, while an opposite behavior can be seen near the cold one.
- viii. For nanofluid ( $\varphi=3\%$ ), the stream function values are increased when the volume ratio of EG in the base fluid is increased. While, an opposite behavior can be seen for ( $\phi = 0\%$ ).

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