Dual Solutions in Mixed Convection Stagnation-Point Flow Over a Vertical Stretching Sheet with External Magnetic Field and Radiation Effect

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ARTICLE INFO	ABSTRACT
	The purpose of this research is to study the problem of mixed convection stagnation-point flow on a vertical stretching sheet with external magnetic field, with the effect of radiation is taken into account. The partial differential equations are reduced to ordinary differential equations using similarity transformation. The transformed boundary layer equations are then solved numerically via bvp4c in MATLAB software. The effects of different values of radiation parameter and Hartmann number on the skin friction coefficient and local Nusselt number, velocity and temperature profiles are presented and discussed. The effect of radiation parameter and Hartmann number are also considered for both assisting and opposing flows. Dual solutions are found to exist in the opposing flow only, while for assisting flow, unique solution exists. It is also found that the radiation parameter enhances the boundary layer separation, and the Hartmann number delays the boundary layer separation.

1. Introduction

The study of the fluid flow and heat transfer with thermal radiation effect has picked a high interest amongst researchers due to its importance applications in industrial and engineering, namely, in nuclear reactor cooling system, gas turbines and various propulsion devices or aircraft, missiles, satellites and space vehicles. The effect of thermal radiation on the heat transfer processes is very important in high operating temperature [1]. Das [2] investigated the radiation effect together with melting effect on MHD boundary layer flow over a moving surface. Ishak [3] studied MHD boundary layer flow due to an exponentially stretching sheet with radiation effect. Later, Hamid *et al.,* [4] investigated Marangoni convection over a flat surface with the effect of radiation. Recently,

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Bakar *et al.,* [5] solved the problem of mixed convection flow in a boundary layer saturated by a nanofluid in a porous medium with the effect of Soret and Dufour, suction and thermal radiation using Runge-Kutta-Fehlberg method. All the above-mentioned papers showed only unique solution exists in their results.

Many results regarding multiple solutions or specifically dual solutions obtained in boundary layer problems [6-16]. Dual solutions reported for a certain range values of suction and unsteadiness parameters by Ali *et al.,* [6] and they concluded that the variation in the temperature distribution is not significant when high radiation parameter is applied. Radiation also found to increase the heat transfer rate at the surface. Ingham [7] provided non-unique solutions (dual solutions) for the flow due to free convection near continuously moving vertical plate using Runge-Kutta Merson method. Later, Ramachandran *et al.,* [8] observed dual solutions in opposing flow case only for the problem of mixed convection in stagnation flow adjacent to vertical surface. On the other hand, dual solutions for both opposing and assisting flows reported by Ridha [9], Ishak *et al.,* [10, 11], Bachok *et al.,* [12], Rostami *et al.,* [13] and Ali *et al.,* [14].

In most cases of dual solutions, researchers verified which branch solution is stable and valid physically by performing a stability analysis. Thus, Bakar *et al.,* [15,16] and Adnan *et al.,* [17] performed a stability analysis to verify which solution is stable, and concluded that the upper branch solution is stable and physically realizable, while the lower branch solution is not stable. This is due to the upper branch solution initiated decaying disturbances, while the second solution initiated growing disturbances. Other dual solutions problems can be found in [18-24].

In this present paper, we study the effects of external magnetic field together with radiation effect on mixed convection stagnation-point flow over a vertical stretching sheet. The nonlinear partial differential equations are reduced to similarity or nonlinear ordinary differential equations, which are solved numerically using bvp4c function in MATLAB.

2. Problem Formulation

This study takes into consideration of two-dimensional boundary layer flow of viscous fluid over a plate surface where it is assumed that the velocity of the far flow on the stretching surface with the velocity of $u_e(x) = ax$ and the surface is stretched with the velocity of $u_w(x) = cx$ where *a* and *c* are positive constants. It is also assumed that the temperature of the plate is $T_w(x) = T_\infty + bx$ where T_∞ is the temperature of the ambient fluid, T_w is the wall temperature, and *b* is a constant. It is good to mention that for a heated surface $T_w(x) > T_\infty$ (when *b* > 0), there exist assisting flow, while, when *b* < 0, the plate is cooled $(T_w(x) < T_\infty)$, therefore, opposing flow occurs. Another assumption for this study is the magnetic Reynolds number is small so that the induced magnetic field is negligible. The schematic diagram of the present problem is displayed in Figure 1.
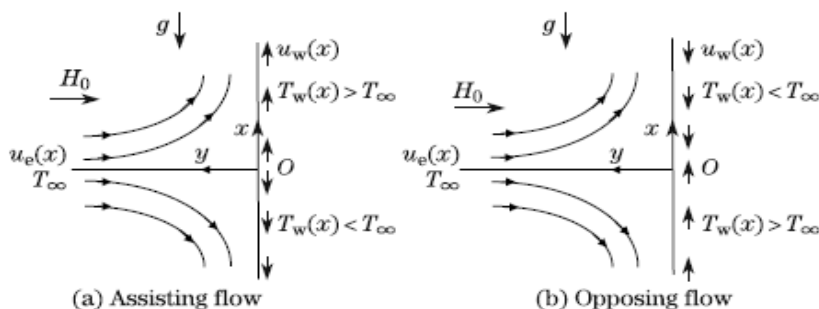

Fig. 1. Physical model and coordinate system

The effect of an external magnetic field of constant strength H_0 is included, which is also applied normal to the stretching surface. With all these assumptions, together with the Boussinesq approximations and Rosseland approximation for radiation, neglecting the viscous dissipation effect, the steady two-dimensional flow of viscous and electrically conducting fluid with radiation effect can be described by the following governing equations.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_e \mu_e^2 H_0^2}{\rho}u + g\beta(T - T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

subject to boundary conditions

$$v = 0, \, u = u_w(x) = cx, \, T = T_w(x) = T_\infty + bx \quad \text{at } y = 0$$
$$u = u_e(x) = ax, \, T = T_\infty \quad \text{as } y \to \infty, \tag{4}$$

where u and v are the velocity components along the *x* and *y* axes, respectively. The fluid pressure is *p*, *T* is the fluid temperature, *g* is the acceleration due to gravity, and $k, c_p, \nu, \rho, \beta, \sigma_e, \mu_e$ and q_r are the thermal conductivity, the specific heat of the fluid at a constant pressure, the kinematic viscosity, the fluid density, the thermal expansion coefficient, the electrical conductivity, the magnetic permeability, and the Rosseland approximation for radiation, respectively. Following Sutton and Sherman [25], we take

$$-\frac{1}{\rho}\frac{dp}{dx} = u_e\frac{du_e}{dx} + \frac{\sigma\mu_e^2 H_0^2}{\rho}u_e \tag{5}$$

where the forces due to the hydrostatic and magnetic pressure gradient would be in equilibrium. Therefore, Eq. (2) becomes

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_e \mu_e^2 H_0^2}{\rho}(u - u_e) + g\beta(T - T_\infty). \tag{6}$$

Using the Rosseland approximation for radiation [26], the radiative heat flux q_r is simplified as

$$q_r = \frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{7}$$

where $\sigma*$ and $k*$ are the Stefan–Boltzmann constant and the mean absorption coefficient, respectively. Following Bataller [27], it is assumed that the temperature differences within the flow are sufficiently small such that the term T^4 can be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, we obtain

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \tag{8}$$

In view of Eq. (7) and Eq. (8), Eq. (3) can be written as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\frac{k}{\rho c_p} - \frac{16\sigma*T_\infty^3}{3\rho c_p k*} \right)\frac{\partial^2 T}{\partial y^2}. \tag{9}$$

Applying the following similarity transformations

$$\varphi = x\sqrt{av}\, f(\eta),\ \eta = \sqrt{\frac{a}{v}}\, y,\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{10}$$

to Eq. (1), Eq. (6) and Eq. (9), where φ is the stream function, which is defined as $u = \dfrac{\partial \varphi}{\partial y}$ and

$$v = -\frac{\partial \varphi}{\partial x}.$$

Therefore, the continuity equation, Eq. (1) is satisfied, and from Eq. (6) and Eq. (9), we obtain the following ordinary differential equations.

$$f''' + f f'' - f'^2 + 1 + Ha^2\left(1 - f'\right) + \lambda\theta = 0 \tag{11}$$

$$\frac{1}{Pr}\left(1 + \frac{4}{3}Rd\right)\theta'' + f\theta' - f'\theta = 0 \tag{12}$$

subject to the new boundary conditions

$$\begin{aligned}
&f(0) = 0,\quad f'(0) = A,\quad \theta(0) = 1,\\
&f'(\infty) \to 1,\quad \theta(\infty) \to 0
\end{aligned} \tag{13}$$

where primes denote differentiation with respect to η, and

$$A = \frac{c}{a},\ Ha = \mu_e H_0\sqrt{\sigma_e/\rho a},\ Pr = \frac{\mu c_p}{k},\ Rd = \frac{4\sigma*T_\infty^3}{kk*},\ \lambda = \frac{Gr_x}{Re_x^2} \tag{14}$$

are the wall ratio velocity parameter, the Hartmann number, the Prandtl number, the radiation parameter, and the mixed convection or buoyancy parameter, respectively, and μ is the dynamic viscosity.

Here, $Gr_x = g\beta(T_w - T_\infty)x^3/\nu^2$ is the local Grashof number, and $Re_x^2 = u_e(x)x/\nu$ is the local Reynolds number. In this study, $\lambda > 0$ refers to the assisting flow, $\lambda < 0$ refers to the opposing flow, and $\lambda = 0$ refers to the forced convection flow. For a non-static surface ($A \neq 0$) and when a magnetic field and radiation are absent ($Ha = 0$ and $Rd = 0$), the present problem reduces to the problem studied by Ishak *et al.,* [28] and Ali *et al.,* [29]. The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho u_e^2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)} \tag{15}$$

with τ_w is the surface shear stress and q_w is the surface heat flux. Following Bakar *et al.,* [16],

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -\left(k + \frac{16\sigma * T_\infty^3}{3k*}\right)\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{16}$$

and using variables in Eq. (10), Eq. (15) and Eq. (16), we obtain

$$Re_x^{1/2}C_f = f''(0), \quad Re_x^{-1/2}Nu_x = -\left(1 + \frac{4}{3}Rd\right)\theta'(0). \tag{17}$$

3. Results and Discussion

The set of the ordinary differential Eq. (11) and Eq. (12) with the boundary conditions Eq. (13) have been solved numerically by applying the bvp4c method. To verify the numerical results obtained, a comparison study is done with those of Ishak *et al.,* [28] and Ali *et al.,* [29] as shown in Table 1 and 2, for the cases of assisting flow and opposing flow. It is found that a good agreement between these results. For both assisting and opposing flows, it is also found that the skin friction coefficient, $|f''(0)|$ decrease when Prandtl number, Pr increases, however, the heat transfer rate at the surface, $-\theta'(0)$ increase with Pr.

Table 1
The skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ for different values of Pr when $A = 1$, $Ha = 0$, $Rd = 0$ and $\lambda = 1$ (assisting flow)

	Ishak *et al.,* [28]		Ali *et al.,* [29]		Present	
Pr	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0.72	0.3645	1.0931	0.3645	1.0931	0.3645	1.0931
6.8	0.1804	3.2902	0.1804	3.2897	0.1804	3.2896
20	0.1175	5.6230	0.1175	5.6208	0.1175	5.6201
40	0.0873	7.9463	0.0873	7.9403	0.0873	7.9383

60	0.0729	9.7327	0.0729	9.7217	0.0728	9.7180
80	0.0640	11.2414	0.0640	11.2244	0.0639	11.2187
100	0.0578	12.5726	0.0578	12.5490	0.0577	12.5411

Table 2

The skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ for different values of Pr when $A = 1$, $Ha = 0$, $Rd = 0$ and $\lambda = -1$ (opposing flow)

| | Ishak *et al.,* [28] | | Ali *et al.,* [29] | | Present | |
Pr	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0.72	−0.3852	1.0293	−0.3852	1.0293	−0.3852	1.0293
6.8	−0.1832	3.2466	−0.1832	3.2463	−0.1832	3.2461
20	−0.1183	5.5923	−0.1183	5.5903	−0.1183	5.5896
40	−0.0876	7.9227	−0.0876	7.9169	−0.0876	7.9145
60	−0.0731	9.7126	−0.0731	9.7018	−0.0730	9.6982
80	−0.0642	11.2235	−0.0641	11.2068	−0.0641	11.2012
100	−0.0579	12.5564	−0.0579	12.5329	−0.0578	12.5252

Figure 2 and 3 show the velocity and temperature profiles for different values of radiation parameter, *Rd* for both assisting and opposing flows, when *A* < 1, respectively. For assisting flow, the velocity profiles increase with *Rd*, however, opposite effect found in opposing flow. On the other hand, higher value of *Rd*, results in increasing the temperature profiles for both assisting and opposing flows. The effect of buoyancy parameter (λ < 0) on the velocity profiles, when *A* < 1, can be seen from Figure 4. The velocity profiles of the first solution reduce with λ, however, for the second solution, the profiles increase with λ. Figure 4 also displays dual velocity profiles, for the case of opposing flow, thus, there exist second solution of the problem.

The skin friction coefficient and the local Nusselt number for various values of *Rd* and λ, when *A* < 1 are displayed in Figure 5 and 6, respectively. Dual solutions exist for opposing flow, while for assisting flow, only unique solution exists. For upper solution, the skin friction coefficient and the local Nusselt number reduce as *Rd* increases. It is found that *Rd* enhances the boundary layer separation. This can be observed in Figure 5 and 6, by looking at the critical values, λ_c, where $\lambda_c = -8.46, -8.20, -8.02$ when *Rd* = 0.6, 1.0, 1.4, where beyond these critical values, the boundary layer separation occurs.

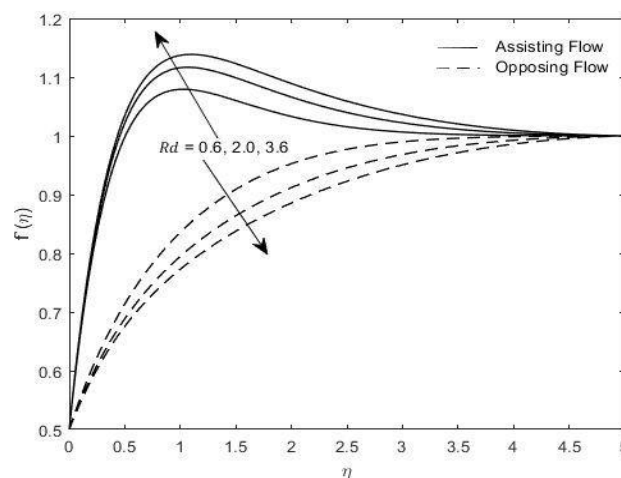**Fig. 2.** Velocity profiles for different values of *Rd* when *A* = 0.5 when λ = 2.0 (assisting flow) and λ = −2.0 (opposing flow)
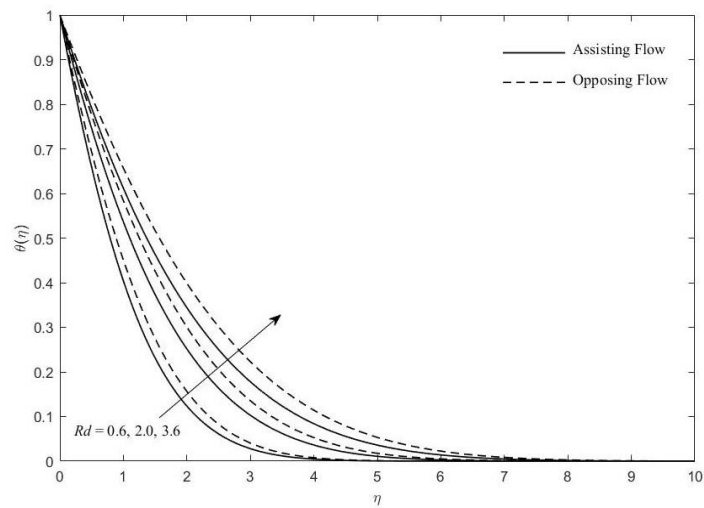
Fig. 3. Temperature profiles for different values of Rd when A = 0.5 when λ = 2.0 (assisting flow) and λ = −2.0 (opposing flow)
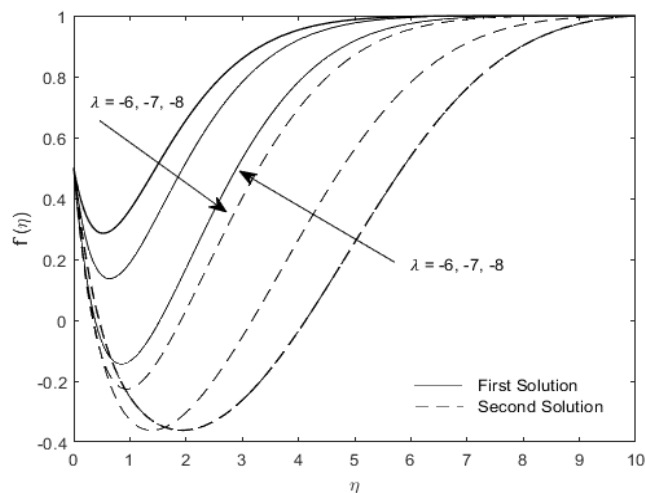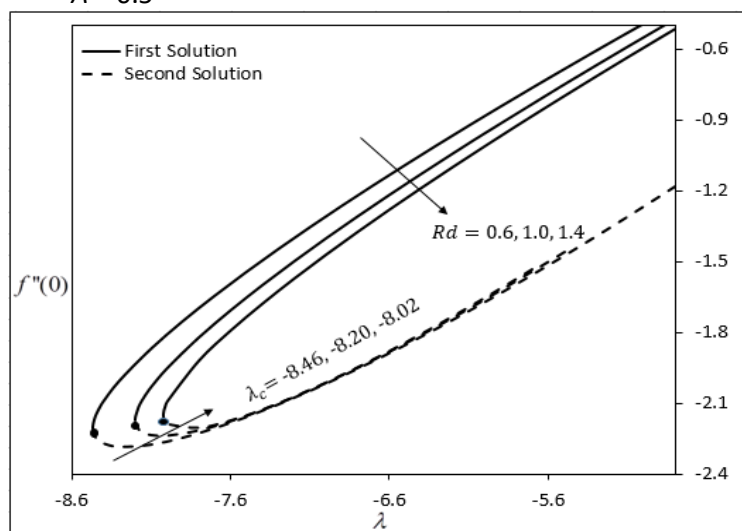

Fig. 4. Velocity profiles for different values of λ when A = 0.5

Fig. 5. Skin friction coefficient as function of λ for various values of Rd when $A = 0.5$
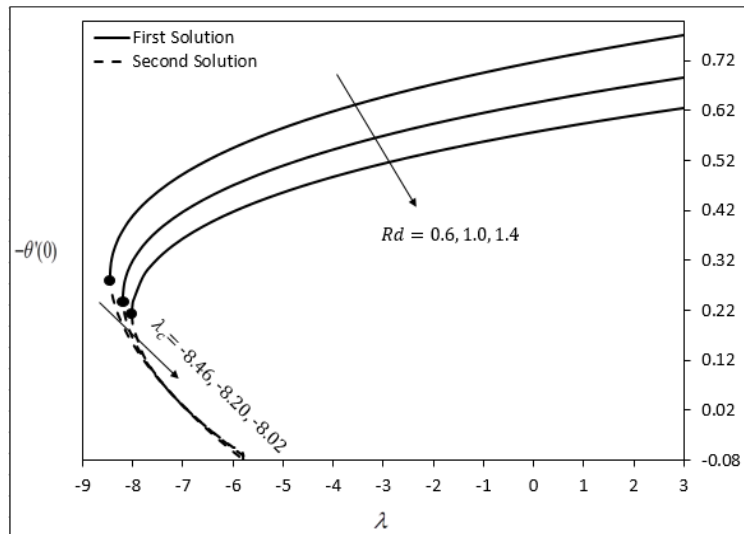

Fig. 6. Local Nusselt number as function of λ for various values of Rd when $A = 0.5$

The effect of the Hartmann number on the skin friction coefficient and the local Nusselt number for $A < 1$ can be found in Figure 7 and 8, respectively. As the Hartmann number increases, the skin friction coefficient and the local Nusselt number also increase. The same phenomenon can be observed with λ. The Hartmann number are known related to the Lorentz force, whereby, when Ha increases, the Lorentz force also increases. Hence, this will slow down the momentum of the fluid flow and increase the drag at the surface, therefore, increases the skin friction coefficient and the local Nusselt number. The Hartmann number found to delay the boundary layer separation and broaden the range of λ for which the solutions exist, where the critical values $\lambda_c = -14.4, -25.5, -38.5$ when $Ha = 3.0, 4.0, 5.0$. In Figure 7 and 8, dual solutions also exist for certain region of λ (opposing flow).
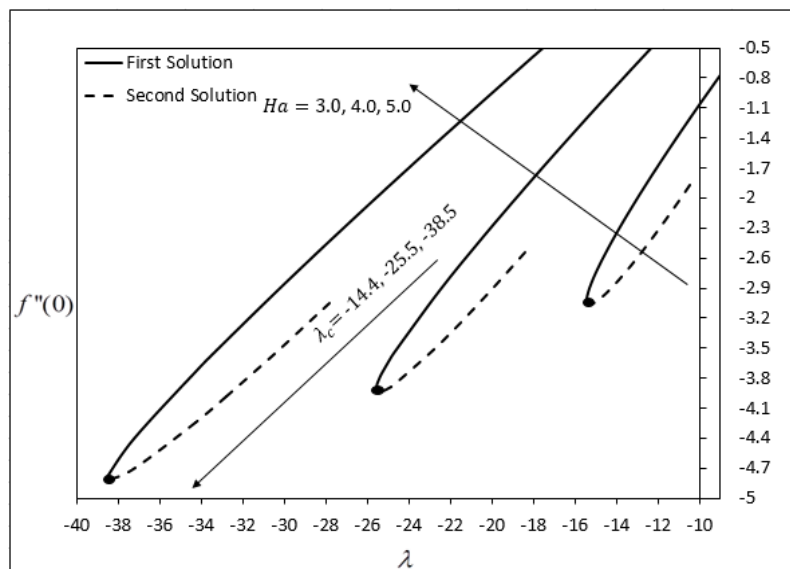

Fig. 7. Skin friction coefficient as function of λ for various values of Ha when $A = 0.5$
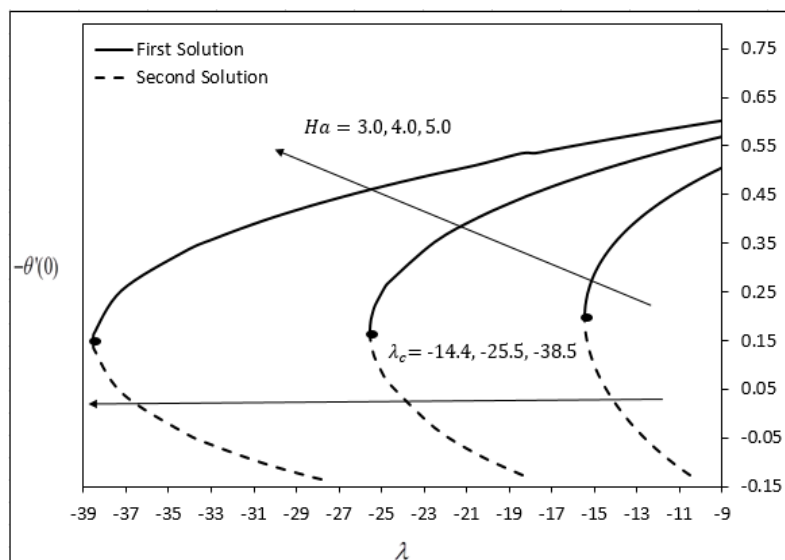
Fig. 8. Local Nusselt number as function of λ for various values of *Ha* when *A* = 0.5

4. Conclusions

The problem of mixed convection stagnation-point flow of an incompressible viscous fluid on a vertical stretching sheet with the presence of external magnetic field and radiation effect is studied. Dual solutions are found to exist for opposing flow only. However, for assisting flow, only unique solution exists. We also conclude that the radiation parameter enhances the boundary layer separation. While, the Hartmann number delays the boundary layer separation.

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