SEMARAK ILMU PUBLISHING

Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage:
https://semarakilmu.com.my/journals/index.php/fluid_mechanics_thermal_sciences/index
ISSN: 2811-3950

Accurate Assessment Model for Solving Resistance-Temperature Equations of Platinum Thermometers in Range from -40 °C up to 660 °C

Ahmed El-Matarawy[1,*], Doaa Abd-ElGaliel[1], Eman Mohy El-Dien[2]

[1] National Institute of Standards, Egypt
[2] Pyramids Higher Institute for Engineering and Technology, Egypt

ARTICLE INFO

Article history:
Received 18 November 2021
Received in revised form 5 February 2022
Accepted 7 February 2022
Available online 8 March 2022

Keywords:
ITS-90; IPRT; SFE; CVD; high order polynomials; Interpolating equations

ABSTRACT

The missions of the thermal metrology laboratory are to maintain, disseminate, develop and realize the International Temperature Scale 1990 (ITS-90). One of the services that the laboratory has introduced is the routine calibration to all industrial sectors that covers almost all fields inside and outside Egypt. This work compares different interpolating equations of set of industrial Platinum Resistance Thermometers (IPRTs) in the temperature range from -40 °C up to 660 °C that convert the resistance of thermometer sensing element to temperature and find the optimum method in which range applicable. The used analytical methods are Callendar–van Dusen (CVD) equation and the (4^{th} -5^{th}) order polynomials of resistance as a function of temperature. The study splits the ranges of investigation into lower subranges, test the significant standard error and residual for each case, and give advice to the lower level calibration laboratories that does not own primary standard about the best method used. From the results it is found that the standard fit error (SFE) for 4^{th} order polynomial on average = 30 % compared to the CVD equation, =50 % compared to the ITS-90 deviation function and = 73% compared to the 3^{rd} order polynomial. It is preferable to used 5^{th} order polynomial than CVD to decrease 1 point in the calibration and reduce the cost of the calibration with nearly the same accuracy if the mathematical model is applied. The study gives advice to the end user who wants to use CVD equation by selecting the best two-calibration points above 0 °C which are 100 °C and 400 °C.

1. Introduction

Thermal Metrology Laboratory (ThML) at National Institute of Standards (NIS) is one of the leading laboratory that satisfy the metrological traceability of temperature to all medical, industrial, agriculture and other sectors inside and outside Egypt. Our mission is to maintain, disseminate and develop the unit of thermodynamic temperature that defined by Boltzmann constant on the advanced research branch and the approximated thermodynamic temperature that realized by International Temperature Scale 1990 (ITS-90) [1, 2]. Furthermore, there are another mission to ThML is to introduce the industrial routine calibration of Industrial Platinum Resistance

[*] *Corresponding author.*
E-mail address: ahmed.matarawey@nis.sci.eg

https://doi.org/10.37934/arfmts.93.1.123135

Thermometers (IPRT) either by calibration directly according to the requirement of ITS-90 in fixed point or by comparison with one SPRT or Pt-100 that traceable to ITS-90. As the impurities increase in crystal seeds of platinum as the behaviour of resistance change. The comparison methods should be satisfying either by the deviation function that is described by the text of ITS-90 or Callendar Van-Dusen equation (CVD). In industrial field, ThML is working according to the basics of (ITS-90) to calibrate high precision Standard Platinum Resistance Thermometers SPRT in all defining fixed point from 13.8 K up to 962 ^{o}C [3, 4]. Precise measurements play a vital role in industrial work and by achieving the accurate measurements; we will introduce high quality services. Temperature Measurement is indirect operation because there is no straight method able to measures it directly achieving, the measurements procedures should be indirect that measure certain physical property change with respect to the temperature. For example, measuring the temperature by Liquid In Glass Thermometers (LIGTs) is basically depend on linear expansion coefficient of the mercury column against the temperature, thermocouple thermometers based on generation of electromotive force (EMF) with respect to the temperature and platinum resistance thermometers also based on change the resistance as temperature change. Platinum resistance thermometers are used for precise measurement of temperature by taking advantage of the linear change in resistance (R) with respect to temperature (t). In contrast, (IPRTs) are used in secondary precision measurements in the temperature range between −200 °C and 1000 °C or higher with little higher uncertainty and has a nominal value of 100 Ω and 1000 Ω. The resistance – temperature characteristics of IPRTs are usually expressed using CVD equation. In the CVD equation, the resistance ratio $R(t)/R(t = 0\,°\text{C})$ above 0 °C is expressed as

$$w(t_{CVD}) = \frac{R(t)}{R(t=0\ ^{o}\text{C})} = 1 + At + Bt^2,\ t \geq 0 \tag{1}$$

where t is the temperature expressed in °C, A and B are the polynomial coefficient and vary from thermometer to another one. The CVD equation is expressed by 2nd order polynomial as a function in t. In the CVD equation, the resistance ratio R(t)/R(t = 0 °C) below 0 °C is expressed as

$$\frac{R(t)}{R(t=0\ ^{o}\text{C})} = 1 + At + Bt^2 + Ct^3\,(t - 100),\ t < 0 \tag{2}$$

The CVD equation is expressed by 4th order polynomial as a function in t. The Alpha (α) parameter is a detector factor that indicates the characteristics and the sensitivity of the PRT that is

$$\alpha = \frac{R(100\ ^{o}\text{C}) - R(0\ ^{o}\text{C})}{100R(0\ ^{o}\text{C})} \tag{3}$$

The α value for PRT is usually » ≈ 0.00385. Pure platinum sensing elements has α ≥ 0.003925 Ω/(Ω·°C) that ranged from 0 ^{o}C up to 100 ^{o}C is used in the construction of the required classes and grades of PRT. IEC 60751 standard specify the equations, coefficient constant and the α value. These different α values for platinum are coming from the impurities that become embedded in the lattice structure of the platinum and result in a different R vs. T curve.

2. Development and Results Refinements

Day after day, the technology development has enabled high quality IPRTs with better repeatability, reproducibility and well comparison comparative thermostat mediums (bathes, dry well, furnaces and ovens) with better stability, homogeneity, consistency, and uniformity with less hysteresis. Those developments in equipment hardware and software have allowed minimize the uncertainty of some IPRT calibration to be limited by the accuracy of the CVD equation. Furthermore, the mathematical software is developed rapidly that gives the advantage to handle massive data to get fine results. On the other hand, a polynomial of higher order has additional terms to define the resistance characteristic behaviour better than CVD equation of less uncertainty but results in higher degree of freedom.

The problem is that many temperature indicators devices may not contains the appropriate functions to convert the temperature from resistance using such interpolating functions, they may provide improper calibration results in describing the R(t) behaviour of low-α PRTs. In order to provide a methodology on the best interpolation equation might be used for IPRTs for both National Metrology Institute (NMI) and lower level calibration laboratories to thermometer readout equipment, higher-level polynomials and CVD inverse equation has been experimentally compared in order to quantify the best subrange zone should be used CVD or polynomials. In the previous articles, papers and literatures, several studies and scientists compare the behaviour of ITS-90 model and polynomials model from second to ninth order as well as the CVD model to describe the characteristic T-R behaviour of IPRT. The conclusions, proposals and recommendations of the most accurate results that included but not limited to

- Inseok Yang *et al.,* [5] have worked in temperature range from 0 °C to 500 °C, they compare several interpolating functions and found that the fitting residual of the 3rd, 4th order polynomial and the deviation function of ITS-90 was around 33%, 70% and 50% compared to the CVD equation respectively.
- Fernicola *et al.,* [6] have worked in temperature range from -196 °C to 420 °C, they recommended the deviation function of the ITS-90 to calculate the coefficients a and b at three points near but not exactly TPW, SnFP and ZnFP with deviation error between - 0.020 °C and 0.035 °C.
- Hasheiman *et al.,* [7] have worked in temperature range from 0 °C to 300 °C, they determined that polynomials of 6th to 8th order provide a good approximation of ITS-90 than the CVD model.
- Zhang *et al.,* [8] have worked in temperature range from 0 °C to 800 °C, they used polynomials of 2nd to 9th order, they conclude that 4th and 5th orders are not proper functions for interpolating.
- Kaiser [9] has worked in temperature range from -50 °C to 420 °C, he concludes that the reference function of ITS-90 and polynomials of higher orders than 4th one is suitable for working on IPRT than CVD equations.
- Mèndez-Lango. *et al.,* [10] have worked in temperature range from 0 °C to 420 °C, they suggested the ITS-90 equations for IPRTs, where the coefficients a and b are calculated by least-squares method [11].
- Moiseeva [12, 13] has worked in temperature range from 0 °C up to 230 °C, she investigated the relationship between B and A for different PRTs and nominal values W (100) and proposed a proper correction to the coefficients by using ratio B/A of the coefficients of the quadratic approximation of ITS-90.
- Hahtela *et al.,* [14] have worked in temperature range from 200 °C to 700 °C, they conclude that the deviations from the 2nd order polynomial fit follow more diligently to the resultant deviation reference function of ITS-90.

- Jiang Yingying *et al.,* [15] have worked in temperature range from -80 °C to 300 °C using two comparative functions, they concluded that the measurement error using the ITS-90 deviation function is within ±23mK and ±24mK using CVD function in the temperature range from -80 °C up to 0 °C. Furthermore, no significant different between two functions in temperature range from 0 °C up to 100 °C was found. They were suggested the deviation function for wide temperature range and CVD for narrow temperature ranges between 100 °C up to 300 °C [16, 17].

The aim of this work is to investigate the performance of different interpolating equations that were tested by a quantitative assessment of residual, which is the major factor that provide the best model for qualitative range of temperature. The advantage here is a combination of two overlapped ranges with extension to 660 °C that are not studied before in one issue. Moreover, this novel work solves the equation of CVD in low temperature range (absolute T as a function in R) which is consider an essential feature and no study discussed it directly without approximations as discussed in previous work.

3. Accurate Solving of CVD

It is easy to find the inverse of Eq. (1) to obtain the temperature as a function in resistance but it is very difficult to find it in Eq. (2). As well, we used strong mathematical and statistical environment language also Monte Carlo simulation to obtain the temperature from the resistance with limited residual. The inverse of Eq. (1) can be expressed as

$$t_h = \begin{pmatrix} -\frac{\sqrt{Ro\,(Ro\,A^2-4\,B\,Ro+4\,B\,Rt)}+A\,Ro}{2\,B\,Ro} \\ \frac{\sqrt{Ro\,(Ro\,A^2-4\,B\,Ro+4\,B\,Rt)}-A\,Ro}{2\,B\,Ro} \end{pmatrix} \tag{4}$$

The first root is real and taken into consideration; the code used to get the Eq. (4) given the following math script.

```
==** Math-Script Code**==
syms Ah Bh
th1= to be inserted from measurements (numerical value);
th2= to be inserted from measurements (numerical value);
Rth1= to be inserted from measurements (numerical value);
Rth2= to be inserted from measurements (numerical value);
eqnh1=Ro*(1+Ah*th1+Bh*th1^2)-Rth1==0;
eqnh2=Ro*(1+Ah*th2+Bh*th2^2)-Rth2==0;
thsol=solve(Rt== Ro*(1+Ah*t+Bh*t^2),th)
thsol=solve(eqnh1,eqnh2,Ah,Bh);
```

The inverse of Eq. (3) is the solution of CVD in low temperature range

$$t_l = \begin{pmatrix} 25 - \sigma_2 - \sigma_3 \\ \sigma_2 - \sigma_3 + 25 \\ \sigma_3 - \sigma_1 + 25 \\ \sigma_3 + \sigma_1 + 25 \end{pmatrix} \tag{5}$$

where

$$\sigma_1 = \frac{\sqrt{\begin{array}{c} -9\,\sigma_6^{\frac{2}{3}}\,\sigma_5 - 12\,\sigma_8\,\sigma_5 - \left(\frac{B}{C} - 3750\right)^2 \sigma_5 - 3\sqrt{6}\,\sigma_9 \\ \sqrt{2\left(\frac{B}{C} - 3750\right)^3 - 72\left(\frac{B}{C} - 3750\right)\sigma_8 + 27\,\sigma_9^2 + 3\sqrt{3}\,\sigma_7} \\ -12\left(\frac{B}{C} - 3750\right)\sigma_6^{\frac{1}{3}}\,\sigma_5 \end{array}}}{\sigma_4} \tag{6}$$

$$\sigma_2 = \frac{\sqrt{\begin{array}{c} 3\sqrt{6}\,\sigma_9\,\sqrt{\begin{array}{c} 2\left(\frac{B}{C} - 3750\right)^3 - 72\left(\frac{B}{C} - 3750\right)\sigma_8 \\ +27\,\sigma_9^2 + 3\sqrt{3}\,\sigma_7 \end{array}} \\ -12\,\sigma_8\,\sigma_5 - \left(\frac{B}{C} - 3750\right)^2 \sigma_5 - 9\,\sigma_6^{2/3}\,\sigma_5 - 12\left(\frac{B}{C} - 3750\right)\sigma_6^{1/3}\,\sigma_5 \end{array}}}{\sigma_4} \tag{7}$$

$$\sigma_3 = \frac{\sigma_5}{6\,\sigma_6^{1/6}} \tag{8}$$

$$\sigma_4 = 6\,\sigma_6^{1/6} \left(\begin{array}{c} \left(\frac{B}{C} - 3750\right)^2 + \frac{300\,A}{C} + \frac{7500\,B}{C} + 9\,\sigma_6^{\frac{2}{3}} \\ -6\left(\frac{B}{C} - 3750\right)\sigma_6^{1/3} + \frac{12\,(Ro - Rt)}{C\,Ro} - 14062500 \end{array} \right)^{1/4} \tag{9}$$

$$\sigma_5 = \sqrt{\begin{array}{c} \left(\frac{B}{C} - 3750\right)^2 + \frac{300\,A}{C} + \frac{7500\,B}{C} + 9\,\sigma_6^{\frac{2}{3}} \\ -6\left(\frac{B}{C} - 3750\right)\sigma_6^{1/3} + \frac{12\,(Ro - Rt)}{C\,Ro} - 14062500 \end{array}} \tag{10}$$

$$\sigma_6 = \frac{\left(\frac{B}{C} - 3750\right)^3}{27} - \frac{4\left(\frac{B}{C} - 3750\right)\sigma_8}{3} + \frac{\sigma_9^2}{2} + \frac{\sqrt{3}\,\sigma_7}{18} \tag{11}$$

$$\sigma_7 = \sqrt{\begin{array}{c} 27\,\sigma_9^4 - 256\,\sigma_8^3 - 16\left(\frac{B}{C} - 3750\right)^4 \sigma_8 + 4\left(\frac{B}{C} - 3750\right)^3 \sigma_9^2 \\ +128\left(\frac{B}{C} - 3750\right)^2 \sigma_8^2 - 144\left(\frac{B}{C} - 3750\right)\sigma_9^2\,\sigma_8 \end{array}} \tag{12}$$

$$\sigma_8 = \frac{25\,A}{C} + \frac{625\,B}{C} + \frac{Ro - Rt}{C\,Ro} - 1171875 \tag{13}$$

$$\sigma_9 = \frac{A}{C} + \frac{50\,B}{C} - 125000 \tag{14}$$

4. Experimental Setup and Data Preparation

Six platinum resistance thermometer (PRT) class A Fluke model 5626 with stability ± 0.003 °C and accuracy ranged as ± 0.006 °C at –40 °C to 0 °C and from ± 0.015 °C to ± 0.022 °C at 420 °C to 661 °C respectively. The thermometers are connected with F18 ASL thermometry resistance bridge that conjugate with a 100 Ω Tinsley standard resistor box (calibrated value Rs = 99.99979530 Ω). This technique is experimentally assembled to measure the RTD resistance in the fixed point. A multimeter (Fluke Model 8864A) was used to measure the resistance of six RTD class A and four lower class RTD (α ≥ 0.003985 $°C^{-1}$) by comparison with SPRT in different well-controlled equipment. All measurements are automatically acquired using LabVIEW software environment (Figure 1).

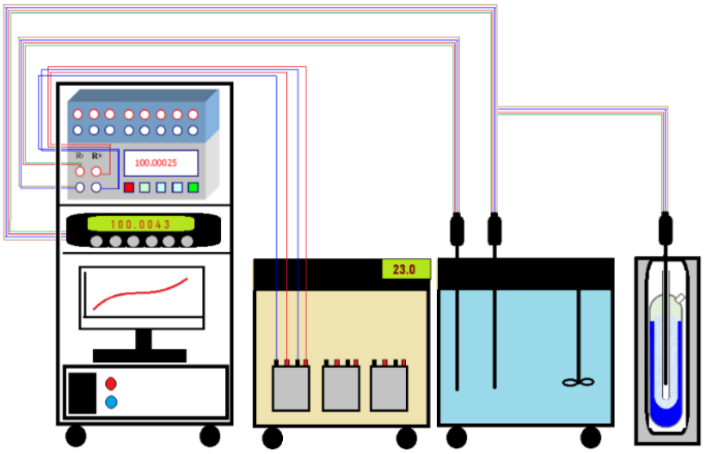

Fig. 1. Schematic diagram of the experimental setup

At the beginning of measurements, Ten IPRT were annealed for 10 hours at 450 °C then their nominal values (Ro) were checked before and after another exposure. If the Ro reading changed more than 0.01 °C, the procedure was repeated. The ten thermometers are split into two sets, the first set is composed of six long stem IPRT (with high value of α ≈ 0.003925 $°C^{-1}$). The thermometers are calibrated in well-controlled maintenance bath and dry well by comparison with SPRT that calibrated according to ITS-90, the calibration points are -40 °C, -25 °C, -10 °C, 0 °C, 50 °C, 100 °C, 200 °C, 300 °C, 400 °C, 500°C and 600°C (Table 1). Another set of thermometers are consists of four IPRT (with low value of α ≈ 0.003985 $°C^{-1}$) are calibrated by comparison at -70 °C, -40 °C, -20 °C, -10 °C, 0 °C, 50 °C, 100 °C, 150 °C, 250 °C, 350 °C and 450°C, the results are summarized in Table 2.

After stability, the data was treated after acquired, collected then corrected from self-heating effect, hydrostatic head pressure and high-low gain triggering as shown in Figure 2.

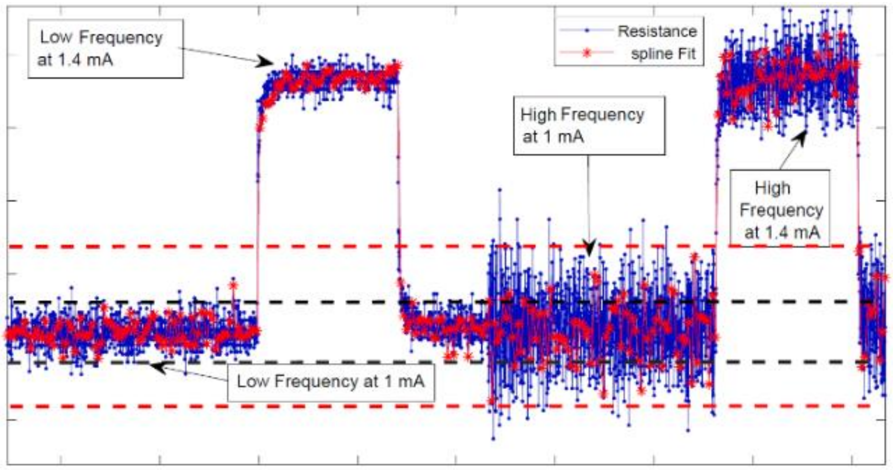

Fig. 2. Data correction from self-heating effect and high frequency

The corrected data were statistically treated to calculate the arithmetic median from normal fitting of the portability distribution function (PDF) at confidence level 95% as shown in Figure 3, the results are summarized in Table 1.

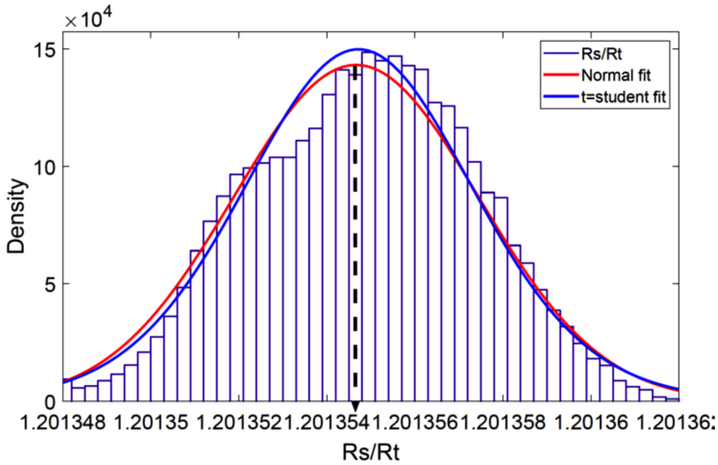

Fig. 3. Normal fitting of PDF to calculate the median

5. Data Analysis Protocol and Results

For all covered domain, the residuals from a fitted model (r_{ij}) are equivalent to the differences between the observed results and the fit to that value over number of samples defined in iteration i at specific appearance j.

$$r_{ij} = \left|R_i - \tilde{R}_j\right| \tag{15}$$

The fitting standard error of the regression estimate the absolute measured distance of the scattered data points around the regression curve, FSE is a numerically given by

$$SFE = \sqrt{\frac{\sum_i^m (R-\widetilde{R})^2}{\nu}} \tag{16}$$

where, ν (ν = n-1) is the degree of freedom and n is the number of observed samples.

Table 1
The Calibrated value of Class A IPRT by comparison

IPRT SN	T / °C -40.000 R / Ω	T / °C -25.000 R / Ω	T / °C -10.000 R / Ω	T / °C 0.00 R / Ω	T / °C 50.000 R / Ω	T / °C 100.000 R / Ω	T / °C 200.000 R / Ω	T / °C 300.000 R / Ω	T / °C 400.000 R / Ω	T / °C 500.000 R / Ω	T / °C 600.000 R / Ω
3489	83.4383	89.4431	95.4197	99.3890	119.0538	138.4199	176.2663	212.9484	248.4792	282.8512	316.036
3493	83.3233	89.3195	95.288	99.2518	118.8889	138.2281	176.0224	212.6537	248.1345	282.4568	315.5917
3487	83.7028	89.7268	95.7223	99.7047	119.4314	138.8591	176.8271	213.6266	249.2688	283.7450	317.0255
3375	83.5909	89.6063	95.5936	99.5699	119.2688	138.6686	176.5813	213.3274	248.9194	283.3498	316.5897
3504	83.367	89.3628	95.3339	99.2995	118.9460	138.2941	176.1059	212.5430	248.252	282.5914	315.7438
3523	83.2955	89.2898	95.2519	99.2186	118.8502	138.1832	175.9647	212.5833	248.0523	282.3646	315.4918

Table 2
The Calibrated value of α ≥ 0.003985 IPRT by comparison

SN	T / °C -69.883 R / Ω	T / °C -39.892 R / Ω	T / °C -20.366 R / Ω	T / °C -10.349 R / Ω	T / °C 0.000 R / Ω	T / °C 49.978 R / Ω	T / °C 100.441 R / Ω	T / °C ≈150 T / °C	R / Ω	T / °C 250.093 R / Ω	T / °C ≈345 T / °C	R / Ω	T / °C ≈443 T / °C	R / Ω
012	72.321	84.271	91.982	95.921	99.975	119.376	138.603	150.075	157.290	194.135	345.860	228.221	443.494	261.717
013	72.376	84.305	92.016	95.950	99.998	119.387	138.640	150.109	157.232	194.067	345.736	227.963	443.507	261.629
015	72.350	84.300	92.020	95.957	100.017	119.430	138.657	150.025	157.311	194.207	348.527	229.211	----------	---------
022	72.254	84.273	92.021	95.980	100.037	119.465	138.698	150.006	157.340	194.177	347.634	228.742	----------	---------

5.1 Set (1) Reference Model-Class A

In this model, we compare the temperature difference between the generated temperature at each possible resistance of CVD and ITS-90.

- For CVD, the six thermometers IPRT class A calibrated according to CVD equation by using three calibration point (t< 0 °C), the triple point of water or ice point (0 °C) and two point above 0 °C (t ≥0 °C). The selected points were -40 °C, -25.0 °C, -10.0 °C, 0 °C, 200°C and 600 °C, other point used to check and examine the quality of model assessment.
- For ITS-90, the six class A thermometers are calibrated according to the ITS-90 equations in two different subranges. Firstly, at -40 °C, 0 °C and 50 °C by comparison that is close to subrange Mercury triple point (HgTP), water triple point (WTP) and Gallium melting point (GaMP). The values of Wr (-40 °C) and Wr (50°C) are used to build a system of two equations in two variables to determine the magnitudes of coefficients (a) and (b). Secondly, for high temperature range from 0°C to 600 °C, the model build on Wr (100 °C), Wr (300 °C), Wr (600 °C), W (100 °C), W (300 °C) and W (600 °C) to create system of three equations in three variables to find the values of (a), (b) and (c) coefficients. As described previously, Wr for any temperature was to generate the temperature for 0.25 °C in low or high temperature sub-ranges respectively. The rest of calibrated points is used to verify the model and checks its validity. The temperature deviation between ITS-90 and CVD equation are ranged from -4 m°C ± 0.012 at 300 °C up to 13.8 m°C ± 0.012°C at 600 °C as shown in Figure 4, 5 and summarized in Table 3.

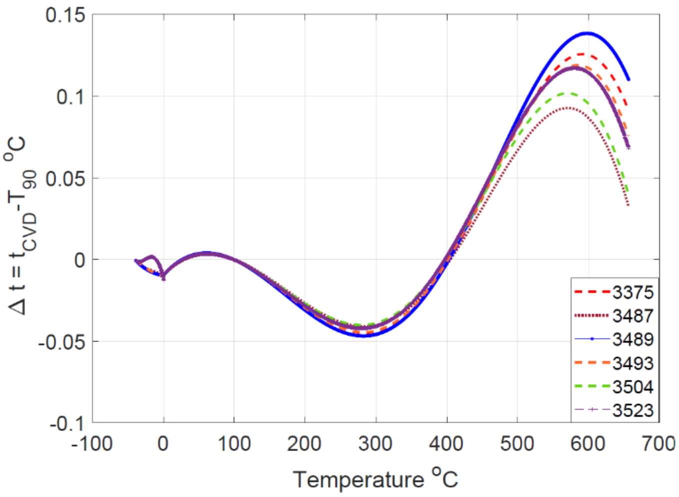

Fig. 4. Temperature difference (m°C) between CVD and W (ITS-90) for six primary thermometers

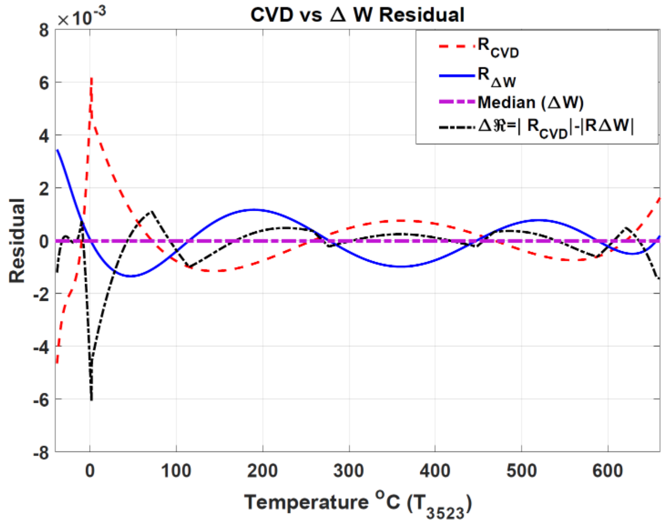

Fig. 5. Residual distribution for all covered temperature range for thermometer SN 3523 as an example of the calculation mechanism

Table 3
Temperature differences for reference set

SN	Min. Δt °C	Max. Δt °C	Residual CVD x 10^{-3} °C	Residual ΔW x 10^{-3} °C	SFE x 10^{-5} °C	Unc. ± °C
3375	0.026	0.04	1.32	0.69	0.62	0.012
3487	0.027	0.04	1.32	0.68	0.59	0.012
3489	0.026	0.04	1.30	0.66	0.59	0.012
3493	0.029	0.04	1.33	0.67	0.60	0.012
3504	0.028	0.037	1.26	0.62	0.56	0.012
3523	0.029	0.038	1.27	0.61	0.57	0.012

5.2 Set (2) Work Standard Model-Class B

In this model, we compare the temperature difference between the generated temperature at each possible resistance of CVD and higher order polynomials of orders from third up to fifth. All class B thermometers are calibrated at points summarized in Table 2. Using the equation of CVD all coefficients are determined and generated the rest of temperatures in between the calibrated points. The calibrated points in this model are -70 °C, -40 °C, -20 °C, -10 °C, 0 °C, 50 °C, 100 °C, 150 °C, 250 °C, 350 °C and 450 °C. The temperature differences between temperatures generated by CVD and others higher polynomials for thermometer ID-12 as an example has shown in the Figure 6, 7 and summarized in Table 4.

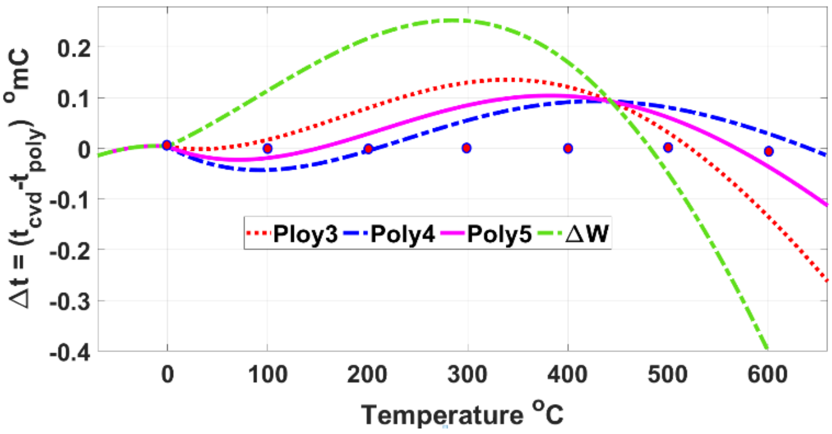

Fig. 6. Temperature difference between CVD and Polynomials

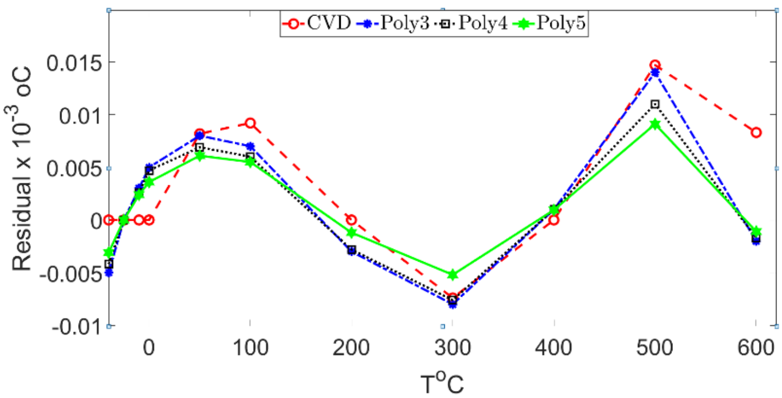

Fig. 7. Residual distribution for all covered temperature range for thermometer ID-12 as an example of the calculation mechanism

Table 4
Temperature differences for work standard set

SN	Min. Δt °C	Max. Δt °C	Residual CVD x 10^{-3} °C	Residual Poly. 4^{th} x 10^{-3} °C	Residual Poly. 5^{th} x 10^{-3} °C
012	0.034	0.051	0.015	0.011	0.009
013	0.033	0.053	0.016	0.014	0.011
015	0.035	0.057	0.018	0.019	0.010
022	0.037	0.058	0.016	0.012	0.009

For set (2), the results show that there is minimum deviation between the subrange 0 °C to up to 100 °C that does not exceed 0.008 °C (8 mC) in temperature. The deviation starts to increase and reach maximum value near 300 °C and return back to almost zero close to 400 °C. The deviation fluctuate in positive and negative direction until reach the maximum value near 600 °C to be near 0.038 °C, and decreases toward 0.008 °C.

6. Conclusion

Performance, behavior, thermal analysis and metrological characterization of different interpolation equations has been studied. The cost of calibration point is expensive so the study try to find suitable accurate method that gives more or at least equal accuracy for calibrating IPRT with less calibration points in most common temperature rage from -40 °C up to 600 °C. CVD equation required 6 calibartion points in this range, 3 points below 0 °C, 2 points above and 0 °C. We solve CVD with extremelyaccuracy for low temprature range and start to compare it with another function such as ΔW of ITS-90 and polynomials of higher orders from 3rd up to 5th.

- For set (1), from the results, it is found that The temperature deviation between ITS-90 and CVD equation are ranged from -4 m°C ± 0.012 at 300 °C up to 13.8 m°C ± 0.012°C at 600 °C. Those values are appeared as the maximum and minimum of the fluctuation peaks, the discerpancy between the two equations was well. We recomended to used ITS-90 equation insead of CVD because we can calibrate the thermometers at 6 point distributed well at the whole range and menimum residual. For set (2) from the results, it is found that the minimum deviation between the subrange 0 °C to up to 100 °C is around 8 m°C. The deviation fluctuates in positive and negative direction until reach the maximum value near 600 °C to be near 0.04 °C, and decreases toward 0.009 °C. The fourth-order polynomials showed distinctly better performance than the others did. Furthermore, the (SFE) for the fourth order polynomial in the temperature range between 0 °C and 660 °C was on average = 30 % compared to the CVD equation, =50 % compared to the ITS-90 deviation function and = 73% compared to the third-order polynomial. It is preferable to used 5th order polynomial than CVD to decrease 1 point in the calibration and reduce the cost of the calibration with nearly the same accuracy if the mathematical model is applied.
- If the end user wants to use CVD equation, we consider that the best two-calibration points above 0 °C are 100 °C and 400 °C which should be used in CVD model.

References

[1] Preston-Thomas, Hugh. "The International Temperature Scale of 1990(ITS-90)." *metrologia* 27, no. 1 (1990): 3-10. https://doi.org/10.1088/0026-1394/27/1/002

[2] IEC, EN. "60 751. Industrial Platinum Resistance Thermometers and Platinum temperature sensors." (2008).

[3] El Matarawy, A. "Comparison of the realization of water triple point metallic cell through its preparation techniques in new modified adiabatic calorimeter at NIS-Egypt." *Journal of Thermal Analysis and Calorimetry* 136, no. 5 (2019): 2131-2138. https://doi.org/10.1007/s10973-018-7804-8

[4] El Matarawy, A. "New temperature controlling mechanism for thermal metrology regulation at NIS-Egypt." *Journal of Thermal Analysis and Calorimetry* 129, no. 3 (2017): 1799-1805. https://doi.org/10.1007/s10973-017-6340-2

[5] Yang, Inseok, Kee Sool Gam, and Yong-Gyoo Kim. "Interpolating equation of industrial platinum resistance thermometers in the temperature range between 0° C and 500° C." *Measurement Science and Technology* 26, no. 3 (2015): 035104. https://doi.org/10.1088/0957-0233/26/3/035104

[6] Fernicola, V. C., and L. Iacomini. "Approximating the ITS-90 temperature scale with industrial platinum resistance thermometers." *International Journal of Thermophysics* 29, no. 5 (2008): 1817-1827. https://doi.org/10.1007/s10765-008-0463-4

[7] H.M. Hasheiman, K.M. Petersen. "Temperature—Its Measurement and Control in Science and Industry." *Journal of the American Chemical Society* 78, no. 21 (1992): 427-432.

[8] J. Zhang, K. Fan, S. Wu, and Q. Yao. "Temperature—Its Measurement and Control in Science and Industry." *Journal of the American Chemical Society* 78, no. 21 (1992): 433-438.

[9] N.E. Kaiser. "Paper presented at TEMPMEKO'99, 7th International Symposium on Temperature and Thermal Measurements in Industry and Science." (1999):365-370.

[10] E. Mèndez-Lango, R. Ramìrez-Bazàn. "Paper presented at TEMPMEKO 2001, 8th International Symposium on Temperature and Thermal Measurements in Industry and Science." (1999):647-651.

[11] P. Marcarino, P.P.M. Steur, G. Bongiovanni, B. Cavigioli. "Paper presented at TEMPMEKO 2001, 8th International Symposium on Temperature and Thermal Measurements in Industry and Science." (1999):85-90.

[12] Moiseeva, N. P. "Investigation of Functions W (T90) for Low-a PRTs in the Sub-ranges above 0 C." *Temperature: Its Measurement and Control in Science and Industry* 7, no. 1 (2003): 377–382. https://doi.org/10.1063/1.1627147

[13] Moiseeva, N. P. "The choice of the interpolation equation for a platinum resistance thermometer." *Measurement Techniques* 53, no. 6 (2010): 657.Hahtela, Ossi, Martti Heinonen, Heikki Kajastie, Maija Ojanen, K. Riski, and R. Strnad. "Calibration of Industrial Platinum Resistance Thermometers up to 700° C." *International Journal of Thermophysics* 35, no. 3 (2014): 668-680. https://doi.org/10.1007/s10765-014-1588-2

[14] Hahtela, Ossi, Martti Heinonen, Heikki Kajastie, Maija Ojanen, K. Riski, and R. Strnad. "Calibration of Industrial Platinum Resistance Thermometers up to 700° C." *International Journal of Thermophysics* 35, no. 3 (2014): 668-680. https://doi.org/10.1007/s10765-014-1588-2
[15] Jiang, Yingying, Yichen Zhu, Bingyan Zhou, and Liyao Yao. "Interpolating Equation of Precision Platinum Resistance Thermometers in the Temperature Range between-80° C and 300° C." In *IOP Conference Series: Earth and Environmental Science*, vol. 526, no. 1, p. 012090. IOP Publishing, 2020. https://doi.org/10.1088/1755-1315/526/1/012090
[16] Irfan, Muhammad, and Faizir Ramlie. "Analysis of Parameters which Affects Prediction of Energy Consumption in Buildings using Partial Least Square (PLS) Approach." *Journal of Advanced Research in Applied Sciences and Engineering Technology* 25, no. 1 (2021): 61-68.
[17] Alawi, Omer A., and Haslinda Mohamed Kamar. "Performance of Solar Thermal Collector Using Multi-Walled Carbon Nanotubes: Simulation Study." *Journal of Advanced Research in Micro and Nano Engineering* 2, no. 1 (2020): 12-21.