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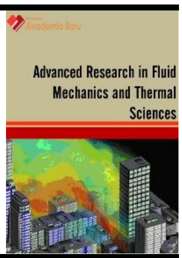

Heat Transfer of Thin Film Flow Over an Unsteady Stretching Sheet with Dynamic Viscosity

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ABSTRACT

This research paper explains the impact of dynamics viscosity of water base GO-EG/GO-W nanofluid over a stretching sheet. The impact of different parameter for velocity and temperature are displayed and discussed. The similarity transformation is used to convert the partial differential equation to nonlinear ordinary differential equation. The solution of the problem is obtained by using the optimal homotopy analysis method (OHAM). The BVPh 2.0 package function of Mathematica is used to obtain the numerical results. The result of important parameter such as magnetic parameter, Prandtl number, Eckert number, dynamic viscosity, nanoparticles volume fraction and unsteady parameter for both velocity and temperature profiles are plotted and discussed. The BVPh 2.0 package is used to obtain the convergences of the problem up to 25 iteration. The skin friction coefficient and Nusselt number is explained in table form.

1. Introduction

The flow in which one dimensional is lesser than the other/s is usually named the flow of the thin film. This implies and observed practically inside many industries. In recent years the requirement for emerging higher thermal management and a more effective heat transporting system has frequently increase. The progressing of strong energy storage technologies is the core issue of the engineer and scientist. The aforesaid difficulties produce due to high rising demands of heating/cooling and other needs in manufacturing processes. It is the focal challenge for a wide research community to boost the heat exchange performance of common pure fluids such as water, various oils, and ethylene glycol, etc., which has weak thermal performances. It is a well-known fact that nano liquids can reduce the difficulties associated with the low thermal performance of traditional fluids and also fulfill the growing demand for heating/cooling and other needs in engineering processes. The development of heat transport via nanofluid has involve several scientists

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due to a lot of uses in various sectors such as distillation and separation of bio-molecules, biosensors, atomic system cooling, and manufacture of glass fiber.

The homogeneous mixture of nanosized particle and base fluid is called nanofluid. The ordinary base fluids are water, oil and ethylene. It is known that the heat transfer rate of these nanofluids is very poor. To increase the thermal conductivity and heat transfer rate of the ordinary base fluid, we combine nanoparticles in the ordinary base fluid. Due to this combination of the ordinary base fluid and nanoparticles, the thermal conductivity and heat transfer of the ordinary base fluid increasing, but this method is very expensive and time vast. Remember that, because of these factors, many methods are introduced to increase the thermal conductivity of the ordinary fluid. One of these methods is the suspension of nanoparticle with diameter less than 50nm in the ordinary fluid.

The word of nanofluid was defined for the first time by Choi. Many researchers presented measurement for the thermal conductivity with different nanoparticle volume fraction material and dimension, and from all of these results, they observe the thermal conductivity of the nanofluid is more than that of the ordinary fluid. Haq *et al.,* [1] used finite element method, they investigate the temperature will be enhance if we heated the domain partially. Prasannakumara *et al.,* [2] do the comparison of linear and nonlinear stretching sheet and show that the Nusselt and Sherwood number is greater for the nonlinear. Rudraswamy *et al.,* [3] investigated three dimensional nanofluid by using moving plate in the presence of magnetic field. Soomro *et al.,* [4] used moving sheet to discuss the movement of nanoparticle, they use the passive method. Usman *et al.,* [5] do the compression of skin fraction for shrinking channels and stretching channel, and show that the skin fraction for shrinking channel is greater than that of stretching for this compression by using analytical method. Takhar *et al.,* [6] used stretching surface to discuss transient magnetohydrodynomic flow. Kumar *et al.,* [7] study Marangoni convection effect of nanofluid treatment in the existence of heat source sink. Sheikholeslami and Rokni [8] discussed the effect of thermal radiation on nanofluid by using porous channel. Sheikholeslami and Shehzad [9] discussed the convective heat transfer of nanofluid in the presence of magnetic field. They investigate the electric field influence on nanofluid by using numerical method.

Laminar flow is very important area of research in the field of viscous fluid. There are mainly two types of laminar flows problems such as Blasius and Wedge flow problems. When the fluid velocity is constant and the free stream appear parallel to the surface, then the situation is called Blasius problem. Similarly, the situation in which surface form an angle with the free stream and velocity of the fluid is not constant, then such situation is known as Wedge problem. In current period, most of the researcher take interest in Wedge flow problem because due to its wide use in engineering and industries. Falkner and Skan [10] are the first to work in this area. They investigate wedge problem by using the 2D flow of incompressible and viscous fluid. Yih [11] investigated the problem of boundary film flow of forced convection through a wedge by using the condition of suction/injection. Watanabe [12] and Rajagopal *et al.,* [13] examined different flows of wedge problems by using second grade fluid. Na [14] introduced a group of suitable transformation to transform the famous Falkner equation into two initial value problem and they solved the transformed problem by applying forward integration scheme. Also, Asaithambi [15] applied the finite difference scheme to explain the solution of Falkner and Skan equation.

Rehman *et al.,* [16] used stretching cylinder to discuss the Marangoni convection of water-based CNT nanofluid. Rehman *et al.,* [17] used stretching sheet to discuss viscous dissipation of thin film unsteady nanofluid. Due to the high oxidize structure, graphene is used as graphene oxide (GO). Graphene is a single layer and graphene oxide is two-dimension materials. Sir second Baronent was the first one introduced graphene oxide in 1859. Now graphene oxide is produced by modified hummeros method. The thermal conductivity of the metallic and nonmetallic nanofluid is very higher than the base fluid. Due to these properties, ethylene glycol (EG) can be used as a cooling fluid and anti-freezing agent. The rotating disk model of nanofluid is the active research area of GO. Balandin *et al.,* [18] discussed the improvement of thermal efficiency of the base fluid. For this, they used single layer graphene as a solvent. Wei *et al.,* [19] discussed the stable diffusion of graphene oxide in ethylene glycol for the improvement of heat transfer. Gul and Firdous [20] used rotating disk model to discuss stable dispersion of graphene nanoparticle and graphene oxide water GO-W nanofluid. Recently, Gul *et al.,* [21] discussed Marangoni convection by using water and ethylene glycol-based graphene oxide as a nanofluid.

Maleque [24] discussed the effects of binary on MHD Boundary layer heat and mass transfer flow with viscous dissipation and heat generation. Gireesha *et al.,* [25] studied the exploration of activation energy and binary chemical reaction effects on nano Casson fluid flow with thermal and exponential space-based heat source. Prasnnakumara *et al.,* [26] discussed the effects of chemical reaction and nonlinear thermal radiation on Williamson nanofluid slip flow over a stretching sheet embedded in a porous medium. Khodabandeh *et al.,* [27] studied the energy saving with using of elliptic pillows in turbulent flow of two-phase water-silver nanofluid in a spiral heat exchanger. Rehman *et al.,* [28] discussed the effect of dynamic viscosity of GO-W and GO-EG nanofluid by using stretching cylinder. Rehman *et al.,* [29] discussed the effect of dynamic viscosity of CNTs nanofluid by using stretching sheet. Rehman *et al.,* [30] used stretching sheet to discuss the effects of Marangoni convection, viscous dissipation and magnetic field of thin film unsteady nanofluid. Rehman *et al.,* [31] used stretching sheet to discuss the effects of viscous dissipation and magnetic field of thin film unsteady nanofluid. Rehman *et al.,* [32] used stretching sheet and discuss analytical study of unsteady squeezed flow of water base CNTs nanofluid with magnetic field and variable thermal conductivity. Raza [33] discussed thermal radiation and slip effects on magnetohydrodynamic (MHD) stagnation point flow of Casson fluid over a convective stretching sheet. Raza *et al.,* [34] discussed heat transfer study of convective fin with temperature-dependent internal heat generation by hybrid block method. Alkasassbeh *et al.,* [35] discussed the effects of Stefan blowing and slip conditions on unsteady MHD Casson nanofluid flow over an unsteady shrinking sheet. Lund *et al.,* [36] discussed the magnetohydrodynamic flow of $Cu\text{-}Fe_3O_4/H_2O$ hybrid nanofluid with effect of viscous dissipation. Lund *et al.,* [37] discussed the triple solutions of micropolar nanofluid in the presence of radiation over an exponentially preamble shrinking surface.

Inspired by the above analysis, the objective of this research paper is to provide an analytical method to explain the problem of water base GO-W and GO-EG nanofluid over an unsteady stretching sheet with dynamic viscosity. The model system of PDEs is converted to nonlinear ODEs by using similarity transformation. The analytical method OHAM is used to find analytical result of the model of nonlinear problem. Liao [23] used this method for the first time to solve nonlinear differential equation. The BVPh 2.0 package is used to obtain the convergences of the problem up to 25 iterations. The impact of different physical factors on the flow and heat transport features are discussed and presented in the graphical form. For the first time, this model is solved for dynamics viscosity analytically.

2. Mathematical Formulation

The unsteady flow of graphene oxide water and ethylene glycol (GO-W/GO-EG) based nanofluids with dynamic viscosity over a flexible sheet are considered. The unsteady flexible of the sheet in x direction is $U_{\varpi} = \frac{bx}{1-\gamma t}$ where t is time and b, γ is constant. The magnetic field is defined as $B(t) =$

$B_0(1-\gamma t)^{-\frac{1}{2}}$ where $T_{\varpi}(x,t) = T_0 - T_r\left(\frac{bx^2}{2\nu_f}\right)(1-\gamma t)^{-\frac{3}{2}}$ is the surface temperature vary with the distance x from the slit. The geometry of the proposed problem is sketched as follows (Figure 1).

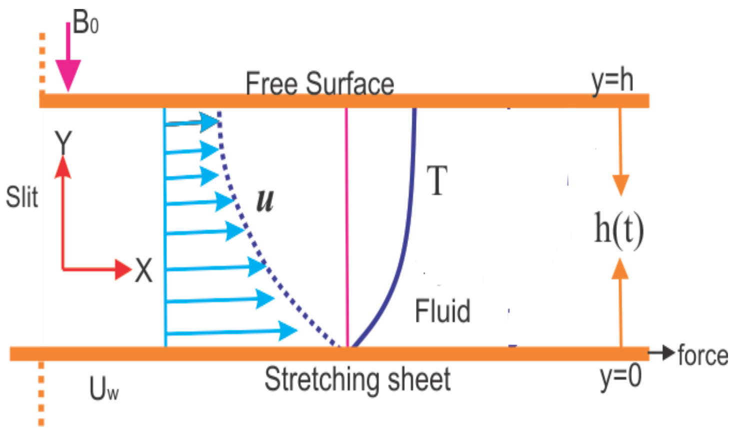

Fig. 1. Geometry of the problem

The continuity, momentum and thermal boundary layer equations are stated, see [22]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{nf}B^2(t)}{\rho_{nf}}u \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{\mu_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

where u and v are the velocity component along x and y direction, respectively. The thermal diffusivity of the base fluid is $\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}$.

The associated boundary condition is

$$u = U_{\varpi}, v = 0, T = T_{\varpi} \text{ at } y = 0 \tag{4}$$

$$\frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = 0, v = \frac{\partial h}{\partial t} \text{ at } y = h(t), \tag{5}$$

where $h(t)$ is the film thickness. The similarity transformation is defined as

$$\eta = \left(\frac{b}{v(1-\gamma t)}\right)^{\frac{1}{2}} y, \psi(x,y,t) = \left(\frac{vb}{1-\gamma t}\right)^{\frac{1}{2}} xf(\eta),$$

$$T(x,y,t) = T_0 - T_r\left(\frac{bx^2}{2v}\right)(1-\gamma t)^{-\frac{3}{2}}\theta(\eta). \tag{6}$$

The stream function $\psi(x,y)$ is defined as $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$ where β is the dimensionless film thickness and is defined as

$$\beta = \left(\frac{b}{\nu(1-\gamma t)}\right)^{\frac{1}{2}} h(t). \tag{7}$$

From Eq. (7),

$$\frac{dh}{dt} = -\frac{\gamma\beta}{2}\left(\frac{\nu}{b}\right)^{\frac{1}{2}}(1-\gamma t)^{-\frac{1}{2}}. \tag{8}$$

Putting Eq. (8) into Eq. (2)-(5), we obtain the following coupled system of ordinary differential equation.

$$\frac{d^3 f}{d\eta^3} + \frac{1}{(1+\alpha\theta)(1-\phi)^{2.5}}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\left[f\frac{d^2 f}{d\eta^2} - \left(\frac{df}{d\eta}\right)^2 - \frac{S}{2}\left(\frac{df}{d\eta} + \frac{\eta}{2}\frac{d^2 f}{d\eta^2}\right)\right] - M(1-\phi)^{2.5}\frac{df}{d\eta} = 0 \tag{9}$$

$$\frac{k_{nf}}{k_f}(1+\alpha\theta)\frac{d^2\theta}{d\eta^2} + Pr\left[(1+\alpha\theta)(1-\phi) + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right]\left[f\frac{d\theta}{d\eta} - 2\theta\frac{df}{d\eta} - \frac{S}{2}\left(3\theta + \eta\frac{d\theta}{d\eta}\right)\right] + Ec\left(\frac{d^2 f}{d\eta^2}\right)^2 = 0. \tag{10}$$

The dimensionless parameters, S, M, Pr, and Ec, and are defined as

$$S = \frac{\gamma}{b}, M = \frac{\sigma_f B_0^2}{\rho B}, Pr = \frac{\upsilon_{nf}}{\alpha_f}, Ec = \frac{U_\varpi^2}{C_p(T_\varpi - T_0)} \tag{11}$$

The transform boundary condition is

$$f(0) = 0, \frac{df}{d\eta}(0) = 1, f(\beta) = \frac{S\beta}{2}, \frac{d^2 f}{d\eta^2}(\beta) = 0, \theta(0) = 1, \frac{d\theta}{d\eta}(\beta) = 0. \tag{12}$$

The physical quantities of interest are the skin friction coefficient C_{nf}, the local Nusselt number Nu_x and are defined as

$$C_{nf} = \frac{\tau_\varpi}{\frac{1}{2}\rho U_\varpi^2}, Nu = \frac{q_\varpi}{k(T_\varpi - T_0)}x. \tag{13}$$

Using the initial guessed values and auxiliary linear operators from Eq. (8) and Eq. (9), we obtain

$$f_0(\eta) = \frac{3(2-S)}{2\beta^2}\left[\frac{x^3}{6} - \frac{\beta x^2}{2}\right] + x, \; \theta_0(\eta) = 1, \tag{14}$$

$$L_f = \frac{d^3 f}{d\eta^3}, \; L_\theta = \frac{d^2\theta}{d\eta^2}. \tag{15}$$

3. Method of Solution

The analytical method namely OHAM is used to find the approximate analytical solution of the nonlinear ordinary differential equation. The BVPh 2.0 package function of Mathematica is used to obtain the numerical results of important parameters. The given Eq. (9) and Eq. (10) are solved analytically by analytical method, i.e., optimal homotopy asymptotic method (OHAM) which is described below

$$L\big(u(x)\big) + N\big(u(x)\big) + g(x) = 0, B\big(u(x)\big) \tag{16}$$

where L is linear operator, x is independent variable, $g(x)$ is the unknown function, N is the nonlinear operator and $B(u)$ is a boundary operator. By using this method, we first find a family of equations

$$H(\phi(x), p) = (1 - p)\big[L\big(\phi(x, p)\big) + g(x)\big] - H(p)\big[L\big(\phi(x, p)\big) + g(x) + N\big(\phi(x, p)\big)\big] = 0$$

$$B\big(\phi(x, p)\big) = 0 \tag{17}$$

where p is an embedding parameter and lies in $[0,1]$, $H(p)$ is nonzero auxiliary function for $p \neq 0$ and $H(0) = 0$ and $\phi(x, p)$ is an unknown function.

When $p = 0$, Eq. (17) become as

$$L\big(\phi(x, p)\big) + g(x) = 0. \tag{18}$$

Let the solution of Eq. (18) be

$$v_0(x). \tag{19}$$

When $p = 1$, Eq. (17) become as

$$L\big(\phi(x, p)\big) + g(x) + N\big(\phi(x, p)\big) = 0, \tag{20}$$

which have a solution, let the solution is

$$v(x). \tag{21}$$

Thus, by increasing p from 0 to 1, the solution change from $v_0(x)$ to $v(x)$. Choose the supplementary function as

$$H(P) = PC_1 + P^2C_2 + P^3C_3 + \cdots \tag{22}$$

where $C_1, C_2, C_3, \ldots$ are constants to be determined. Expanding $\phi(x, p)$ in a series with respect to p, we have

$$\phi(x, p, C_i) = v_0(x) + \textstyle\sum_{k=1}^{\infty} v_k(x, C_i)p^k \text{ for } i = 1,2,3, \ldots. \tag{23}$$

Put Eq. (23) into Eq. (22) and comparing the coefficient of different power of p, we get differential equations with boundary condition. By solving this equation, we find a series solution in the form of functions $v_0(x), v_1(x, C_1), v_2(x, C_2)$ and the solution of Eq. (22) can be written as

$$v(x, C_i) = v_0(x) + \textstyle\sum_{k=1}^{\infty} v_k(x, C_i). \tag{24}$$

The residual is as follow

$$R(x, C_i) = L\big(v(x, C_i)\big) + N\big(v(x, C_i)\big) + g(x). \tag{25}$$

In Eq. (25), if $R(x, C_i) = 0$, then Eq. (24) have exact solution but for nonlinear differential equation, it is not possible. So, we try to minimize the $R(x, C_i)$ to get best approximate analytical solution of Eq. (17).

4. Result and Discussion

The main objective of this section is to study the effect of various model factors like $\phi, M, \alpha, Pr, Ec, S, \beta$ (nanoparticle volume fraction, magnetic field, dynamic viscosity, Prandtl number, Eckert number, unsteady parameter and dimensionless film thickness parameter on velocity and temperature distribution. In Table 1 and Table 2, the numeric results illustrate the impacts of different model factors on the skin friction coefficient and Nusselt number of both GO-W and GO-EG. The effect of different parameters on local skin friction coefficient is shown in Table 1. Table 1 depicts that skin friction coefficient enhances in the case of GO-W and reduce in the case of GO-EG for the increasing values of both unsteady parameter S and magnetic field M. Whereas, Table 3 shows the Nusselt number coefficient decline in the case of both GO-W and GO-EG for rising magnitude of Prandtl number and Eckert number. The convergence for the both GO-W and GO-EG nanofluids has been obtained up to the 25th iteration in Table 4 and Table 5 one by one. Table 4 and Table 5 show that by increasing the number of iterations, reduces the order of residual error and strong convergence attained. Figure 1 is the physical geometry of the problem. Figure 2 shows the effect of the dynamic viscosity against the velocity field. The relation between $\frac{df}{d\eta}(\eta)$ velocity field and dynamic viscosity α is inverse relation. The increasing values of the dynamic viscosity α decreases the $\frac{df}{d\eta}(\eta)$ velocity field as displayed in Figure 2. Physically, by increasing dynamic viscosity, it will produce resistant forces due to which fluid particles does not move easily, so velocity field is decreasing by increasing the dynamic viscosity α. Relatively, this effect is strong in the GO-W. Figure 3 shows the impact of the nanoparticles volume fraction versus the velocity field. The relation between $\frac{df}{d\eta}(\eta)$ and ϕ is inverse relation. For the growing magnitude of ϕ, decreases the $\frac{df}{d\eta}(\eta)$ as displayed in Figure 3, because the large value of ϕ produce viscous forces due to which velocity field is decreasing. Relatively this effect is strong in the GO-W as equated to GO-EG.

Table 1
The thermo physical properties

Physical properties		Thermal conduct $k\ (W/mk)$	Specific heat $c_p\ (Jkg^{-1}K^{-1})$	Density $\rho\ (kgm^{-3})$
Base fluid		0.492	3594	1053
Solid particles	GO-W	2400	1600	3600
	GO-EG	3000	2600	1500

Table 2
Effect of unsteady parameter S, magnetic field M on skin friction coefficient

M	S	$-\frac{d^2f}{d\eta^2}(0)$ $\varphi = 0.01$ GO-W	$-\frac{d^2f}{d\eta^2}(0)$ $\varphi = 0.02$ GO-W	$-\frac{d^2f}{d\eta^2}(0)$ $\varphi = 0.01$ GO-EG	$-\frac{d^2f}{d\eta^2}(0)$ $\varphi = 0.2$ GO-EG
0.1		30.3162	30.3335	28.7696	30.3535
0.2		30.5140	30.3412	28.3040	303414
0.3		30.5991	30.3635	28.2918	30.3293
	0.1	30.6162	30.3735	27.2629	30.3135
	0.2	30.6883	30.5861	25.2830	30.264
	0.3	307430	30.7777	25.1609	28.7778

Table 3
Effect of Prandtl number Pr and Eckert number Ec on Nusselt number coefficient

r	Ec	$-\frac{d^2f}{d\eta^2}(0)$ $\varphi = 0.01$ GO-W	$-\frac{d^2f}{d\eta^2}(0)$ $\varphi = 0.02$ GO-W	$-\frac{d^2f}{d\eta^2}(0)$ $\varphi = 0.01$ GO-EG	$-\frac{d^2f}{d\eta^2}(0)$ $\varphi = 0.02$ GO-EG
	0.1	1.1433	0.4409	1.437	0.441
	0.2	1.1245	0.4371	1.224	0.257
0.5		1.1143	0.4370	1.131	0.231
0.6		1.1011	0.4369	1.121	0.229
	0.3	1.0976	0.4236	0.8700	0.2223
0.7		0.5870	04211	0.587	0.21235

Table 4
Individual averaged squared residual errors for GO-W when $Pr = 6.5, Ec = 0.5, M = 0.1, S = 0.4, = 0.1, \beta = 0.3, \phi = 0.1$

m	ε_m^f GO-W	ε_m^θ GO-W
5	2.1321×10^{-2}	1.9911×10^{-1}
10	2.5721×10^{-4}	1.2211×10^{-2}
15	3.15721×10^{-5}	2.2211×10^{-4}
20	3.22721×10^{-6}	3.2211×10^{-5}
25	3.9911×10^{-7}	4.2211×10^{-6}

Table 5
Individual averaged squared residual errors for GO-EG when $Pr = 6.5, Ec = 0.5, M = 0.1, S = 0.4, = 0.1, \beta = 0.3, \phi = 0.1$

m	ε_m^f GO-EG	ε_m^θ GO-EG
5	1.2211×10^{-1}	1.4711×10^{-1}
10	3.2211×10^{-3}	3.4711×10^{-3}
15	4.2211×10^{-5}	5.4711×10^{-5}
20	5.2211×10^{-6}	7.1111×10^{-6}
25	6.4711×10^{-7}	5.5232×10^{-7}

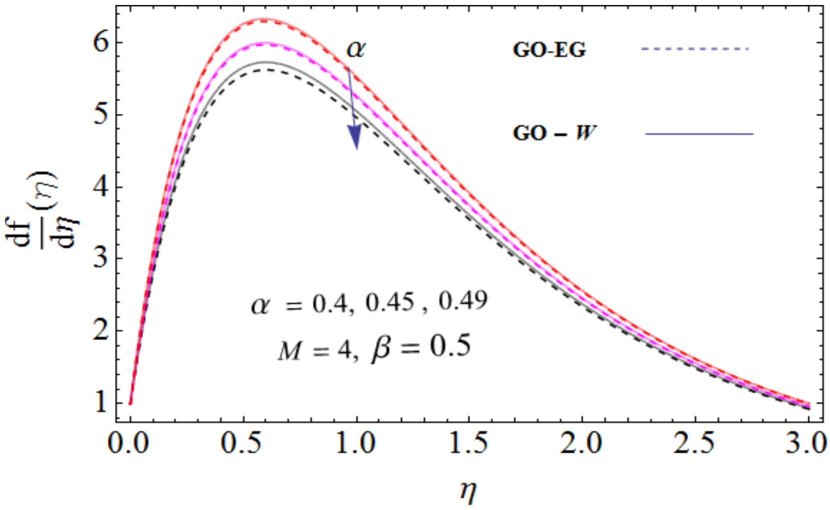

Fig. 2. The impression of the dynamic viscosity α against the velocity field

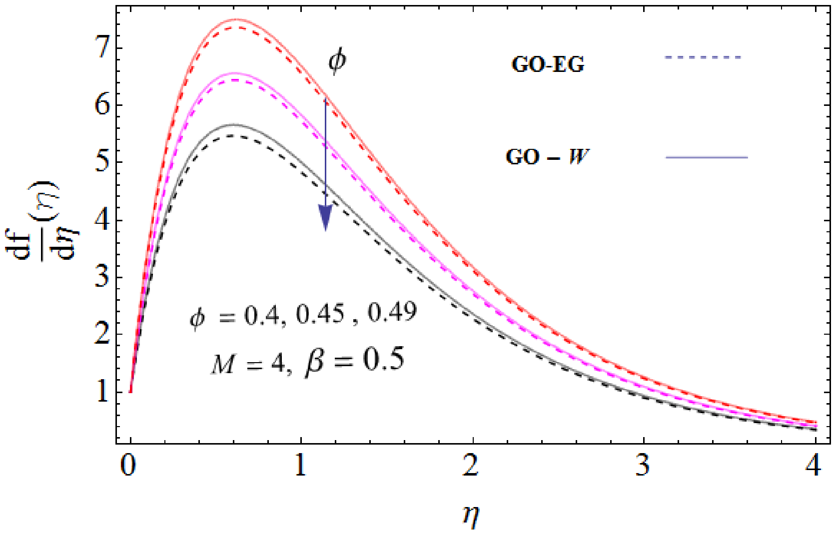

Fig. 3. The impression of the nanoparticles volume fraction ϕ against the velocity field

Figure 4 shows the impact of the magnetic field versus the velocity field. The relation between M and $\frac{df}{d\eta}(\eta)$ is inverse relation. For the growing magnitude of M, decreases the $\frac{df}{d\eta}(\eta)$ as displayed in Figure 4. Physically, by increasing magnetic field M, resistance forces are produced so that decreases the velocity profile. Relatively this effect is strong in the GO-W as equated to GO-EG. Figure 5 shows the impact of the unsteady parameter S versus the velocity field. The relation between S and $\frac{df}{d\eta}(\eta)$ is inverse relation. For the growing magnitude of S, decreases the $\frac{df}{d\eta}(\eta)$ because surface is greater for higher estimation of S. Therefore, the higher value of S augmenting the opposing force cause decline the fluid velocity as displayed in Figure 5. Relatively this effect is strong in the GO-W as equated to GO-EG.

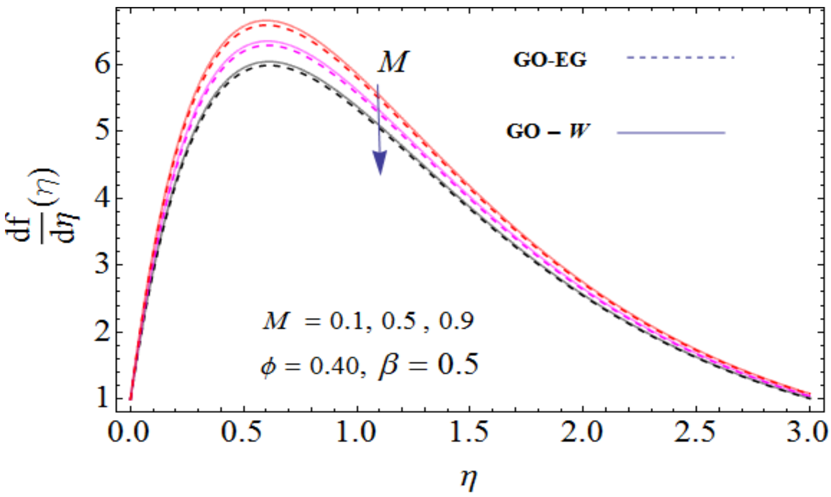

Fig. 4. The impression of the magnetic field M against the velocity field

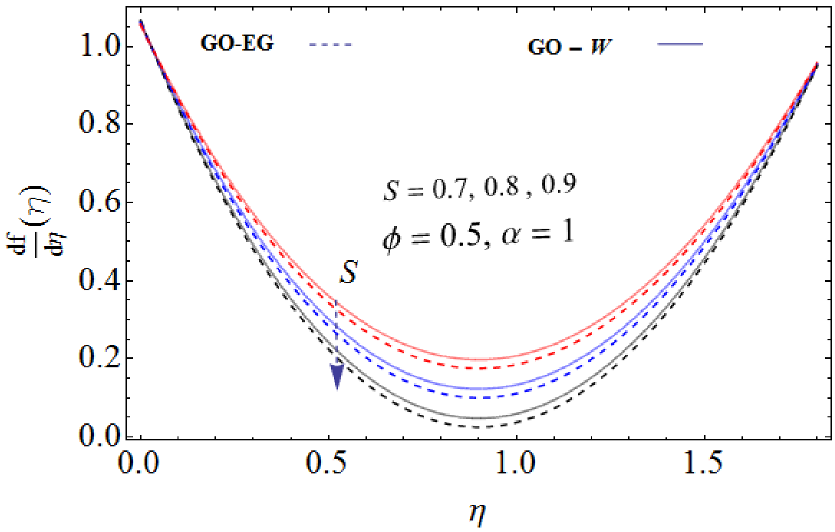

Fig. 5. The impression of the unsteady parameter S against the velocity field

Figure 6 shows the impact of the dimensionless film thickness parameter versus the velocity field. The dimensionless film thickness parameter shows dull effect on velocity profile, in the starting point velocity is decreasing near the sheet surface and after the point of convection the velocity profile is increasing as shown Figure 6. Relatively this effect is strong in the GO-W as equated to GO-EG. Similarly Figure 7 displays the effect of the ϕ against the $\theta(\eta)$ temperature field. From the figure, we observe that the relation between temperature field and ϕ is direct relation. So, by increasing values of the ϕ decreases the $\theta(\eta)$ temperature field as displayed in Figure 7. Relatively this effect is strong in the GO-W.

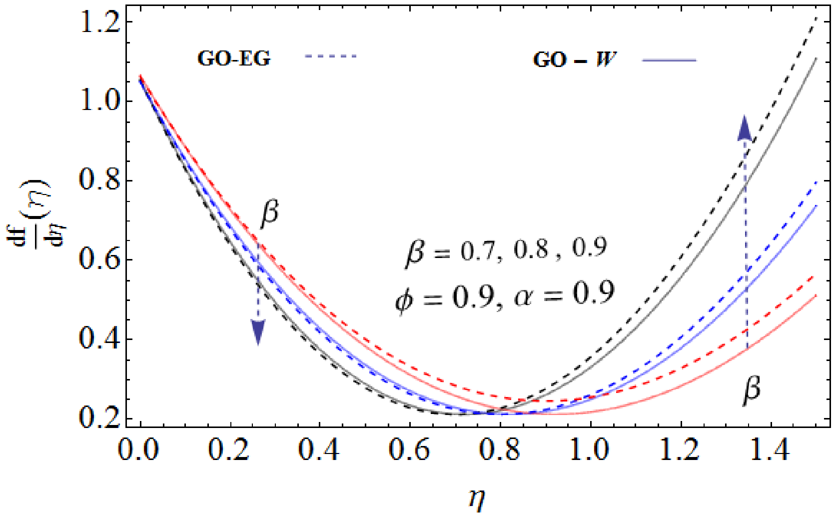

Fig. 6. The impression of the nanoparticles volume fraction ϕ against the velocity field

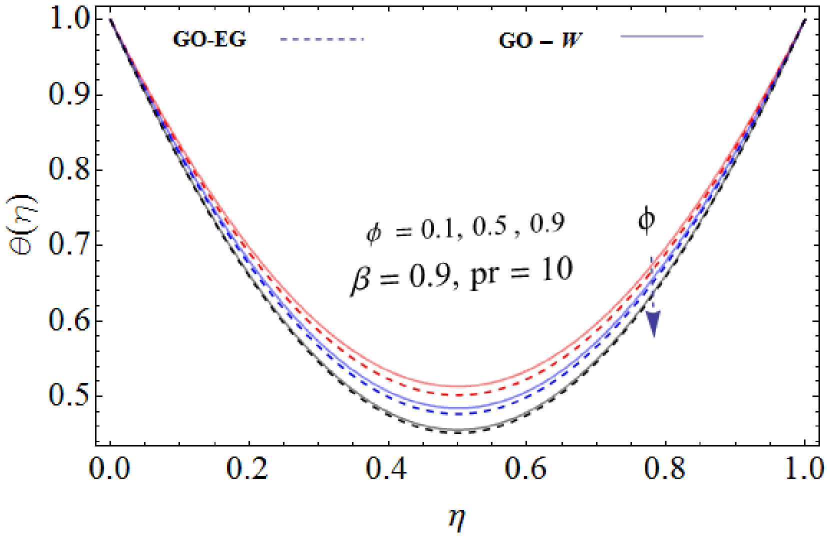

Fig. 7. The impression of the nanoparticles volume fraction ϕ against the temperature field

Figure 8 shows the effect of the Pr against the $\theta(\eta)$ temperature field. The relation between temperature field and Pr is inverse relation, so by enhance the Pr decreasing the temperature profile as displayed in Figure 8. In fact, the thickness of the momentum boundary layer is to be larger than that of the thermal boundary layer, or that the viscous diffusion is larger than the thermal diffusion and therefore, the larger amount of the Pr reduces the thermal boundary layer. Relatively this effect is strong in the GO-W. Figure 9 shows the impact of the Eckert number Ec versus the temperature field. From the figure, we notice that the relation between temperature field and Ec is direct relation, that is by increase the Ec, increasing the temperature profile as displayed in Figure 9. In fact, the thickness of the momentum boundary layer is to be small than that of the thermal boundary layer, or that the viscous diffusion is less than the thermal diffusion and therefore, the larger amount of the Ec increases the thermal boundary layer. Relatively this effect is strong in the GO-W.

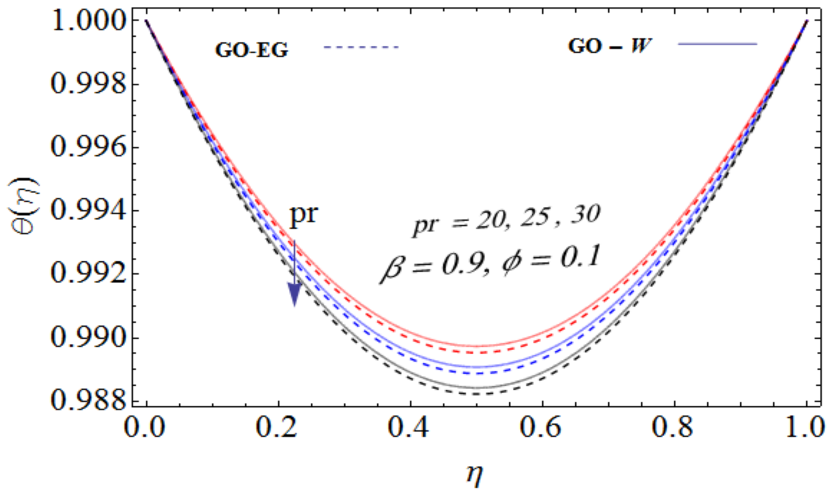

Fig. 8. Impression of the Prandtl number Pr against temperature field

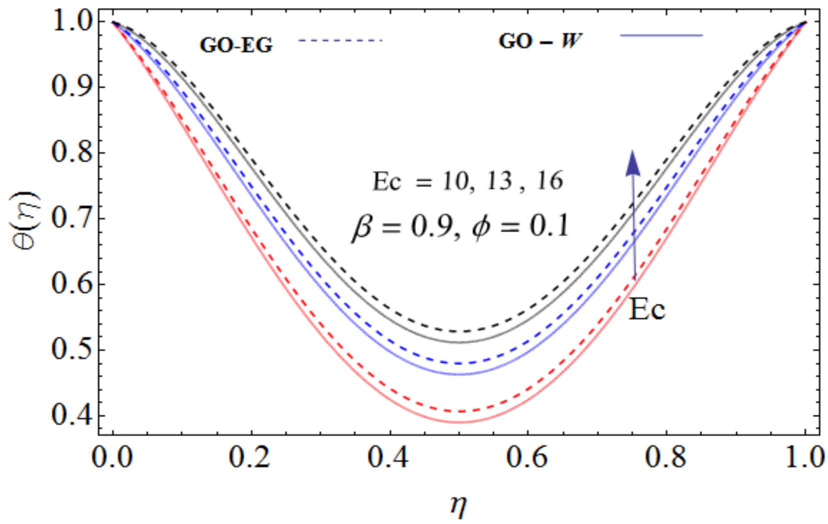

Fig. 9. Impression of the Eckert number Ec against temperature field

5. Conclusion

Laminar (wedge flow problem) thin film flow of water-based GO-EG and GO-W nanofluids over the stretching sheet of dynamic viscosity has been examined in this research. The two sorts GO-EG/GO-W water base nanofluids have been used for the impact of dynamic viscosity. The solution of the problem is obtained by using the optimal homotopy analysis method (OHAM). The BVPh 2.0 package function of Mathematica is used to obtain the numerical results. The result of important parameters such as magnetic parameter, Prandtl number, Eckert number, dynamic viscosity, nanoparticles volume fraction and unsteady parameter for both velocity and temperature profiles are plotted and discussed. The BVPh 2.0 package is used to obtain the convergences of the problem up to 25 iteration. The skin friction coefficient and Nusselt number is explained in table form. The obtained outputs are deliberated as follows

- It is observed that the increasing nanoparticles volume fraction ϕ decreases the velocity field.
- It is observed that by increasing dynamics viscosity α reduces the velocity field.
- It is observed that by increasing the values of the thin film thickness parameter β decreases the velocity in the starting and then increasing the velocity field.
- It is observed that by increase the Eckert number Ec increasing the temperature profile.

- It is observed that by increasing the Prandtl number Pr decreasing the temperature profile.

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