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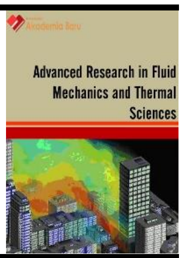

Boundary Layer Flow, Heat and Mass Transfer of Cu-Water Nanofluid over a Moving Plate with Soret and Dufour Effects: Stability Analysis

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ABSTRACT

Our main focus in this paper is to investigate the effects of Soret and Dufour known as thermodiffusion and diffusion-thermo on moving plate in copper water nanofluid. The set of partial differential equations are converted into set of ordinary differential equations using the appropriate similarity variables before being solved numerically using bvp4c code in Matlab software. The results of heat and mass transfer, temperature and concentration profiles on Soret as well as Dufour effects are presented graphically. Soret effect increases the heat transfer rate at the surface while Dufour effect decreases the mass transfer rate at the surface. Since the solutions exist in dual, we carry out the stability solutions to determine which solution is stable and hence the physical meaning is realized physically.

1. Introduction

The consideration of Soret effect (thermodiffusion) and Dufour effect (diffusion-thermo) in some literature has gain an attraction due to their significance of study when density difference existed in the flow regime [1]. Soret effect can be described as diffusion of particles from higher temperature towards lower temperature (temperature gradient) due to mass flux. Dufour effect is a reverse phenomenon of Soret effect where particles are diffused from higher concentration to the lower concentration due to energy flux. Both effects have been set up in crystal growth process where the behaviour of convective flows has a strong effect on the temperature and solute temporal variations which lead to nonuniform crystal growth and undesirable nonhomogeneous crystals [2]. Apart from that, many applications of thermodiffusion in industrial processes such in fabrication of semiconductor devices in molten metal and semiconductor mixtures, separation of polymers and DNA as well as in optimum oil recovery from hydrocarbon reservoirs [3]. Kafoussias and Williams [4] are among the first researchers who discovered the existence and development of Soret and Dufour effects on mixed convection with temperature dependent viscosity. Some literatures on moving plate under consideration of Soret and Dufour effects have been listed in [5-10].

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The purpose of this work is to extend the work by Bachok *et al.,* [11] in the presence of thermodiffusion (Soret effect) and diffusion-thermo (Dufour effect) in copper water nanofluid on a moving plate. The stability analysis is performed due to dual solutions obtained. Therefore, we implemented the pioneer research on stability solutions done by Merkin [12], Weidman *et al.,* [13], and Harris *et al.,* [14]. The consideration on stability solutions can be discovered in the studies by Roşca and Pop [15], Bachok *et al.,* [16], Ismail *et al.,* [17], Najib *et al.,* [18], and Najib *et al.,* [19].

2. Methodology

Consider a two-dimensional laminar boundary layer flow on a fixed or continuously moving flat surface in a water-based nanofluid containing copper (Cu) nanoparticles in the presence of Soret and Dufour effects. It is assumed that the plate moves in the same or opposite direction to the free stream, both with constant velocities. The nanoparticles are assumed to have a uniform spherical shape and size. The boundary layer equations are given by Bachok *et al.,* [11], and Balla and Naikoti [2]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}+\frac{D_m K_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

along with the initial and boundary conditions

$$\begin{aligned}
&t<0: u=v=0, \quad T=T_\infty, \quad C=C_\infty \ \text{ for any } x,y\\
&t\geq 0: u=U_w, \quad v=0, \quad T=T_f, \quad C=C_f \ \text{ at } y=0\\
&u\to U_\infty, \quad T\to T_\infty, \quad C\to C_\infty, \text{ as } y\to\infty.
\end{aligned} \tag{5}$$

where U_w and U_∞ are constants and correspond to the plate velocity and the free stream velocity, respectively. Further, u and v are the velocity components along the x and y directions, respectively. D_m is the diffusion coefficient, K_T is the thermal diffusion ratio, T_m is mean fluid temperature, c_p is the specific heat at constant pressure and c_s is concentration susceptibility. T is the temperature of the nanofluid, μ_{nf} is the viscosity of the nanofluid, α_{nf} is the thermal diffusivity of the nanofluid and ρ_{nf} is the density of the nanofluid, which are given by Oztop and Abu-Nada [20]

$$\begin{aligned}
&\rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s, \qquad \alpha_{nf}=\frac{k_{nf}}{(\rho C_p)_{nf}}, \qquad \mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}},\\
&(\rho C_p)_{nf}=(1-\varphi)(\rho C_p)_f+\varphi(\rho C_p)_s, \quad \frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\varphi(k_f-k_s)}{(k_s+2k_f)+\varphi(k_f-k_s)}
\end{aligned} \tag{6}$$

where φ is the nanoparticle volume fraction, $(\rho C_p)_{nf}$ is the heat capacity of the nanofluid, k_{nf} is the effective thermal conductivity of the nanofluid and C_p is the specific heat at constant pressure, k_f

and k_s are the thermal conductivities of the fluid and of the solid fractions, respectively, ρ_f and ρ_s are the densities of the fluid and of the solid fractions, respectively.

2.1 Steady-State Equation $\left(\frac{\partial}{\partial t}=0\right)$

Introducing the following similarity transformation

$$\eta=\left(\frac{U}{\nu_f x}\right)^{1/2}y,\quad \psi=\left(\nu_f xU\right)^{1/2}f(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty},\quad \phi(\eta)=\frac{C-C_\infty}{C_f-C_\infty},\tag{7}$$

where η is the similarity variable, where U is the composite velocity defined as $U=U_w+U_\infty$. This definition of U was first introduced by Afzal *et al.,* [21] and ψ is the stream function defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$, which automatically satisfied Eq. (1). Substituting the similarity variables (7) into Eq. (2) to Eq. (4) we obtain the following ordinary (similarity) differential equations

$$\frac{1}{(1-\varphi)^{2.5}\left(1-\varphi+\varphi\rho_s/\rho_f\right)}f'''+\frac{1}{2}ff''=0\tag{8}$$

$$\frac{k_{nf}}{k_f}\theta''+Pr\left(1-\varphi+\varphi\left(\rho c_p\right)_s/\left(\rho c_p\right)_f\right)\left(\frac{1}{2}f\theta'+Du\phi''\right)=0\tag{9}$$

$$\phi''+Sc\left(\frac{1}{2}f\phi'+Sr\theta''\right)=0\tag{10}$$

subject to the boundary conditions (5) which become

$$\begin{aligned}&f(0)=0,\quad f'(0)=\lambda,\quad \theta(0)=1,\quad \phi(0)=1\\&f'(\eta)\to 1-\lambda,\quad \theta(\eta)\to 0,\quad \phi(\eta)\to 0\ \text{ as }\ \eta\to\infty\end{aligned}\tag{11}$$

In the above equations, primes denote the differentiation with respect to η. Here Pr is the Prandt number, Sc is the Schmidt number, Sr is the Soret number, Du is the Dufour number and λ is the velocity ratio parameter which are defined as

$$Pr=\frac{\nu_f}{\alpha_f},\quad Sc=\frac{\nu_f}{D_m},\quad Sr=\frac{D_mK_T}{\nu_fT_m}\frac{T_f-T_\infty}{C_f-C_\infty},\quad Du=\frac{D_mK_T}{\nu_fc_sc_p}\frac{C_f-C_\infty}{T_f-T_\infty},\quad \lambda=\frac{U_w}{U}\tag{12}$$

where $\lambda>0$ corresponds to assisting flow and $\lambda<0$ corresponds to reverse flow.

The physical quantities of practical interest are the local skin friction coefficients C_f, local Nusselt number Nu_x and local Sherwood number Sh_x which are defined as

$$C_f=\frac{\tau_w}{\rho_f{U_e}^2},\quad Nu_x=\frac{xq_w}{k_f\left(T_f-T_\infty\right)},\quad Sh_x=\frac{xq_m}{D_m\left(C_f-C_\infty\right)}\tag{13}$$

where τ_w is the skin friction or the shear stresses on the stretching/shrinking sheet, q_w is the heat flux from the surface of the plate and q_m is the mass flux from the surface of the plate, which are given by

$$\tau_w=\mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0},\quad q_w=-k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0},\quad q_m=-D_m\left(\frac{\partial C}{\partial y}\right)_{y=0},\tag{14}$$

Using Eq. (7) in Eq. (13) and Eq. (14), we obtain

$$(Re_x)^{1/2}C_f = \frac{1}{(1-\varphi)^{2.5}}f''(0), \quad (Re_x)^{-1/2}Nu_x = -\frac{k_{nf}}{k_f}\theta'(0), \quad (Re_x)^{-1/2}Sh_x = -\phi'(0) \tag{15}$$

where $Re_x = Ux/\nu_f$ is the local Reynolds number.

2.2 Stability Analysis

Weidman *et al.,* [13] and Roşca and Pop [15] have shown that the lower branch solutions are unstable (not realizable physically), while the upper branch solutions are stable (physically realizable). We test these features by considering the unsteady Eq. (2)-(4). Thus, we introduce the new dimensionless time variable τ. The use of τ is associated with an initial value problem and is consistent with the question of which solution will be obtained in practice (physically realizable). Using the variables τ and (7), we have

$$\eta = \left(\frac{U}{\nu_f x}\right)^{1/2} y, \ \psi = \left(\nu_f x U\right)^{1/2} f(\eta, \tau), \ \theta(\eta,\tau) = \frac{T-T_\infty}{T_f-T_\infty}, \quad \phi(\eta,\tau) = \frac{C-C_\infty}{C_f-C_\infty}, \ \tau = \frac{Ut}{x} \tag{16}$$

so that Eq. (2)-(4) can be written as

$$\frac{1}{(1-\varphi)^{2.5}(1-\varphi+\varphi\rho_s/\rho_f)}\frac{\partial^3 f}{\partial\eta^3} + \left(\frac{1}{2}f - \tau\frac{\partial f}{\partial\tau}\right)\frac{\partial^2 f}{\partial\eta^2} - \left(1-\tau\frac{\partial f}{\partial\eta}\right)\frac{\partial^2 f}{\partial\eta\partial\tau} = 0 \tag{17}$$

$$\frac{k_{nf}}{k_f}\frac{1}{Pr\left(1-\varphi+\varphi(\rho c_p)_s/(\rho c_p)_f\right)}\frac{\partial^2\theta}{\partial\eta^2} + \frac{1}{2}f\frac{\partial\theta}{\partial\eta} + Du\frac{\partial^2\phi}{\partial\eta^2} - \left(1-\tau\frac{\partial f}{\partial\eta}\right)\frac{\partial\theta}{\partial\tau} = 0 \tag{18}$$

$$\frac{1}{Sc}\frac{\partial^2\phi}{\partial\eta^2} + \frac{1}{2}f\frac{\partial\phi}{\partial\eta} + Sr\frac{\partial^2\theta}{\partial\eta^2} - \left(1-\tau\frac{\partial f}{\partial\eta}\right)\frac{\partial\phi}{\partial\tau} = 0 \tag{19}$$

subject to the boundary conditions

$$\begin{aligned} &f(0,\tau) = 0, \quad \frac{\partial f}{\partial\eta}(0,\tau) = \lambda, \quad \theta(0,\tau) = 1, \quad \phi(0,\tau) = 1 \\ &\frac{\partial f}{\partial\eta}(\eta,\tau) \to 1-\lambda, \quad \theta(\eta,\tau) \to 0, \quad \phi(\eta,\tau) \to 0 \end{aligned} \tag{20}$$

To determine the stability of the solution $f = f_0(\eta)$, $\theta = \theta_0(\eta)$ and $\phi = \phi_0(\eta)$ satisfying the boundary-value problem (17)-(19), we write [13,15]

$$f(\eta,\tau) = f_0(\eta) + e^{-\gamma t}F(\eta), \ \theta(\eta,\tau) = \theta_0(\eta) + e^{-\gamma t}G(\eta), \ \phi(\eta,\tau) = \phi_0(\eta) + e^{-\gamma t}H(\eta), \tag{21}$$

where γ is an unknown eigenvalue parameter, and $F(\eta)$, G(η) and $H(\eta)$ are small relative to $f = f_0(\eta), \theta = \theta_0(\eta)$ and $\phi = \phi_0(\eta)$. Solutions of the eigenvalue problem (17)-(20) give an infinite set of eigenvalues $\gamma_1 < \gamma_2 < \gamma_3 \ldots$; if γ_1 is negative, there is an initial growth of disturbances and the flow is unstable but when γ_1 is positive, there is an initial decay and the flow is stable. Introducing Eq. (21) into Eq. (17)-(20), we get the following linearized problem

$$\frac{1}{(1-\varphi)^{2.5}(1-\varphi+\varphi\rho_s/\rho_f)}F_0{}''' + \frac{1}{2}f_0F_0{}'' + \frac{1}{2}f_0{}''F_0 + \gamma F_0{}' = 0 \tag{22}$$

$$\frac{k_{nf}}{k_f}G_0{}'' + Pr\left(1-\varphi+\varphi(\rho c_p)_s/(\rho c_p)_f\right)\left(\frac{1}{2}f_0G_0{}' + \frac{1}{2}F_0\theta_0{}' + DuH_0{}'' + \gamma G_0\right) = 0 \tag{23}$$

$$H_0{}'' + Sc\left(\frac{1}{2}f_0H_0{}' + \frac{1}{2}F_0\phi_0{}' + SrG_0'' + \gamma H_0\right) = 0 \tag{24}$$

subject to the boundary conditions

$$\begin{gathered} F_0(0) = 0, \quad F_0{}'(0) = 0, \quad G_0(0) = 0, \quad H_0(0) = 0, \\ F_0{}'(\eta) \to 0, \quad G_0(\eta) \to 0, \quad H_0(\eta) \to 0 \ \text{ as } \ \eta \to \infty. \end{gathered} \tag{25}$$

It should be mentioned that for particular values of λ, Pr, Sr, Du, Sc and φ the stability of the corresponding steady flow solution$f_0(\eta)$, $\theta_0(\eta)$and $\phi_0(\eta)$, is determined by the smallest eigenvalue γ. According to Harris *et al.,* [14], the range of possible eigenvalues can be determined by relaxing a boundary condition either on $F_0(\eta), G_0(\eta)$ or $H_0(\eta)$. For the present problem, we relax the condition that $F_0{}'(\eta) \to 0$ as $\eta \to \infty$ and for a fixed value of γ we solve the system (22)-(24) subject to Eq. (25) along with the new boundary condition$F_0{}''(0) = 1$.

3. Results

The system of nonlinear ordinary differential Eq. (8)-(10) along with subjected boundary condition (11) have been solved numerically using bvp4c code in Matlab software. Table 1 depicts the comparison of numerical results for $f''(0)$ values which clearly in a good agreement with Bachok *et al.,* [11]. Thus, give us confidence that our numerical results and also plotted figures are correct.

Table 1
Values of $f''(0)$ for some values of λ when $\varphi = 0.1$

λ	Bachok *et al.,* [11]		Present results	
	First Solution	Second Solution	First Solution	Second Solution
-0.5	0.4674	0.2009	0.46737	0.20091
-0.4	0.5117	0.0979	0.51171	0.09792
-0.3	0.5097	0.0431	0.50968	0.04314
-0.2	0.4844	0.0134	0.48442	0.01343
-0.1	0.4433	0.0012	0.44333	
0	0.3901		0.39008	
0.5	0		0	
1	-0.5218		-0.52133	

The effects of Soret and Dufour are presented in Figure 1 and Figure 2. In Figure 1, we have set the value of Dufour effect Du is equal to 0.15 $(Du = 0.15)$ where we only focusing on different values of Soret effects Sr. The heat transfer is increasing when we increased the values of Sr but the different observation has been seen where mass transfer is decreasing as Sr increased. Soret effect also known as thermal diffusion (thermodiffusion) where the nanoparticles are diffused from higher temperature to the lower temperature due to the mass flux. The different effects on Dufour Du can be seen in Figure 2 where Soret effect is taken to be 0.15 $(Sr = 0.15)$. However, increasing Dufour effect Du cause to decrease heat and mass transfer. This is because Dufour effect is the reverse phenomenon of Soret effect called as diffusion-thermo. The nanoparticles diffused from higher concentration to the lower concentration due to energy (heat) flux. The dual temperature as well as concentration profiles have been illustrated graphically in Figure 3 and Figure 4 to support our numerical results and the Figure 1 and Figure 2. All profiles satisfy the far field boundary conditions (11) by fulfill the behavior of the flow asymptotically. The boundary layer thickness for the second solution is always thicker than the first solution.

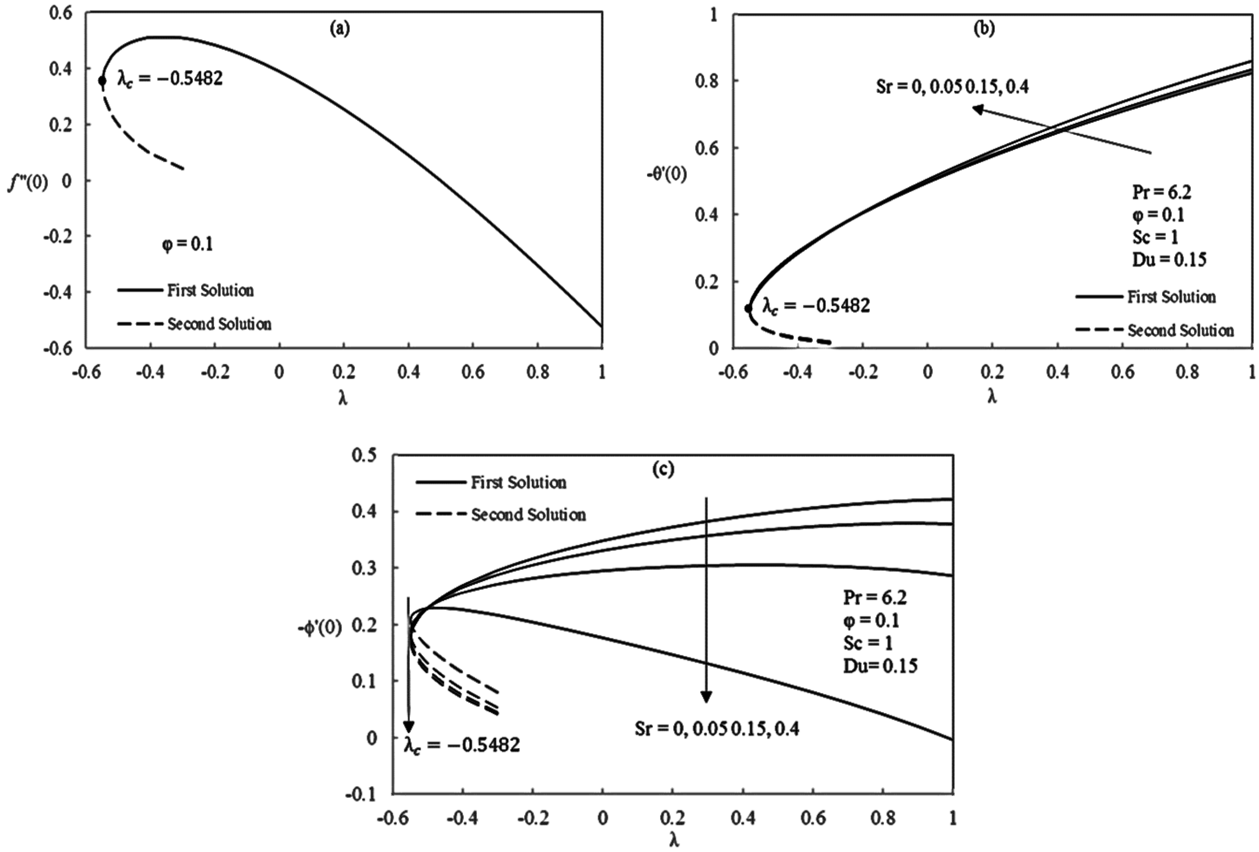

Fig. 1. (a) Skin friction coefficient $f''(0)$, (b) temperature gradient $-\theta'(0)$ and (c) concentration gradient $-\phi'(0)$ vs. λ for several values of Sr

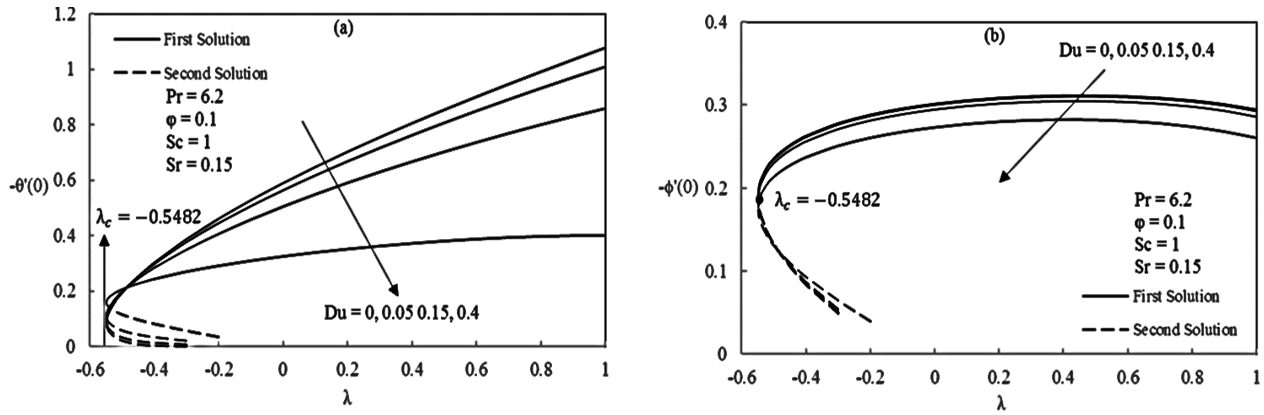

Fig. 2. (a) Temperature gradient $-\theta'(0)$ and (b) concentration gradient $-\phi'(0)$ vs. λ for some values of Du

The system of linearized problem (22)-(24) along with the new boundary condition (25) have been applied into code 3 of bvp4c to perform the stability solutions. The smallest eigenvalues γ for some values of λ are presented in Table 2. As shown, the eigenvalue γ will approaching zero $(\gamma \to 0)$ when the selected value λ is nearer to the critical point λ_c. Moreover, comparison of numerical result with previous research indicated that our smallest eigenvalues γ are in excellent agreement as stated by Bachok *et al.,* [16]. From our observation, γ is positive (stable solution) for the first solution and negative (unstable solution) for the second solution. The solution is said as a stable solution when there only slight disturbance on the flow system that does not affect the flow characteristics while the unstable solution is stated when there existed initial growth of disturbance that affect the flow system. Thus, the first solution is stable and hence they can be realized physically whereas the second solution is not.

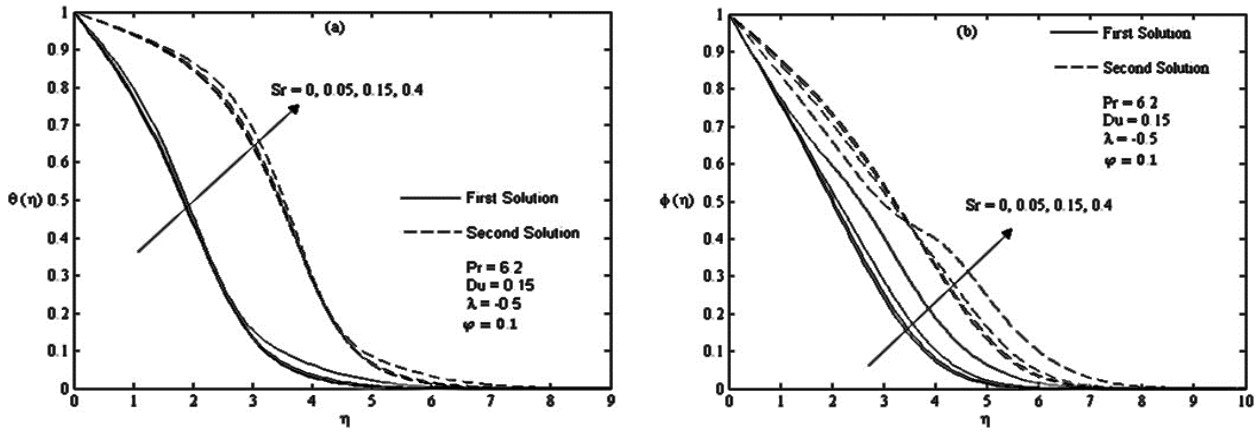

Fig. 3. (a) Dual temperature profile $\theta(\eta)$ and (b) concentration profile $\phi(\eta)$ for some values of Sr

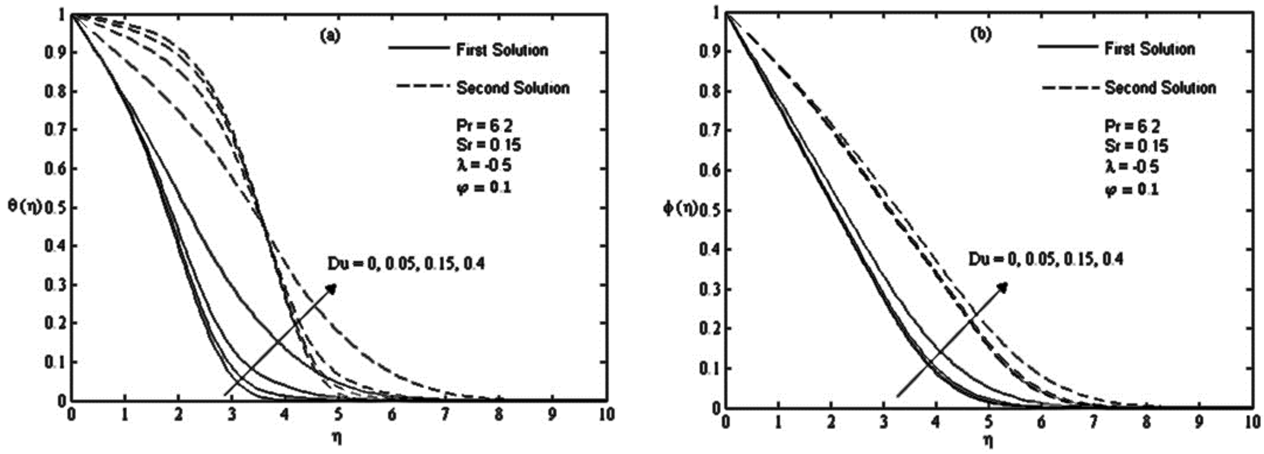

Fig. 4. (a) Dual temperature profile $\theta(\eta)$ and (b) concentration profile $\phi(\eta)$ for some values of Du

Table 2
Smallest eigenvalues γ for selected values of λ when $\varphi = 0.1$

λ	Bachok *et al.,* [16]		Present results	
	First Solution	Second Solution	First Solution	Second Solution
-0.5482			0.0029	-0.0028
-0.548			0.0066	-0.0065
-0.54	0.0406	-0.0350	0.0406	-0.0350
-0.52	0.0788	-0.0597	0.0788	-0.0597
-0.5	0.1059	-0.0733	0.1059	-0.0733
-0.4	0.1992	-0.0982	0.1992	-0.0982
-0.3	0.2662	-0.0949	0.2662	-0.0955

4. Conclusions

The effects of Soret and Dufour on boundary layer flow, heat and mass transfer over a moving surface in nanofluid is investigated numerically. The results revealed that

- dual solutions exist only for opposing flow (when the plate and free stream move in opposite direction to each other), $\lambda < 0$.
- the first solution is stable and physically realizable while the second solution is not.
- largest Sr is required to increase the heat transfer coefficient.
- smallest Du is sufficient to increase heat transfer coefficient.

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