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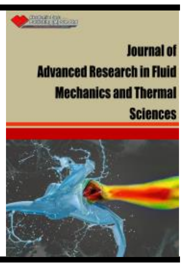

The Effects of Magnetic Casson Blood Flow in an Inclined Multi-stenosed Artery by using Caputo-Fabrizio Fractional Derivatives

Dzuliana Fatin Jamil[1], Salah Uddin[2], Muhamad Ghazali Kamardan[1], Rozaini Roslan[1,*]

[1] Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia, Pagoh, 84600 Muar, Johor, Malaysia
[2] Department of Physical and Numerical Sciences, Qurtuba University of Science and Information Technology D.I. Khan, Peshawar, 25000 Khyber Pakhtunkhwa, Pakistan

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ABSTRACT

This paper investigates the magnetic blood flow in an inclined multi-stenosed artery under the influence of a uniformly distributed magnetic field and an oscillating pressure gradient. The blood is modelled using the non-Newtonian Casson fluid model. The governing fractional differential equations are expressed by using the fractional Caputo-Fabrizio derivative without singular kernel. Exact analytical solutions are obtained by using the Laplace and finite Hankel transforms for both velocities. The velocities of blood flow and magnetic particles are graphically presented. It shows that the velocity increases with respect to the Reynolds number and the Casson parameter. Meanwhile, the velocity decreases as the Hartmann number increases. These results are useful for the diagnosis and treatment of certain medical problems.

1. Introduction

Hemodynamic is the knowledge of blood circulation, which is useful in the diagnosis of coronary illness. The reason behind the malfunction of cardiovascular system is the presence of fats, cholesterol and lipoproteins at the sites of atherosclerotic lesion in the artery [1]. In recent years, due to its great importance in the human cardiovascular system, the study of blood flow through constricted arteries has received a great deal of attention [2–4]. Prasad and Radhakrishnamacharya [5] considered the steady blood flow through an inclined non-uniform tube with multiple stenoses. Agarwal and Varshney [6] studied the flow of Herschel-Bulkley fluid through an inclined tube of non-uniform cross-section with multiple stenoses. Biswas and Paul [7] observed the steady blood flow through an inclined tapered vessel, where the blood was modelled as Newtonian fluid and the slip vessel wall condition was applied. Also, their analysis includes one-dimensional Poiseuille blood flow through tapered vessels with inclined geometries. Ismail and Jamali [8] explored the dynamic response of heat transfer in the steady laminar blood flow through the stenotic bifurcated artery.

[*] *Corresponding author.*
E-mail address: rozaini@uthm.edu.my

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Extensive research has been done since the last few decades on the dynamics of biological fluid in the presence of magnetic field with implications in bio-engineering and medical technology. Bég *et al.,* [9] investigated a mathematical model for blood flow through an inclined artery under the influence of an inclined magnetic field. Many researchers considered blood as viscous and non-viscous fluid in stenotic arteries with magnetic field effects but limited number of research works focused on the effect of induced magnetic field on blood flow through stenosis [10–11]. Mukhopadhyay and Layek [12] worked on a mathematical model to study blood flow through a variable shape stenosed artery under the influence of magnetic field and demonstrated the effect of stenosis shape and magnetic field on the flow resistance. Gudekote *et al.,* [13] investigated a mathematical model for the impact of slip velocity on the peristaltic flow of blood using the Herschel-Bulkley model in a flexible tube.

Plasma is classified as Newtonian fluid, but blood exhibits non-Newtonian behavior [14]. It is well known that blood being a suspension of cells behaves as a non-Newtonian fluid at low shear rate and while flowing through small blood vessels, especially in diseased states when clotting effects in small arteries are present [15]. As blood flows at low shear rate into narrow arteries, it behaves like a Casson fluid [16]. Many researchers have been working on the Casson fluid model for modelling blood flow through narrow arteries [17–19]. Nagarani and Sarojamma [20] studied the effect of body acceleration on pulsatile flow of Casson fluid through a mild stenosed artery. Gudekote and Choudhari [21] examined the combined effects of slip and inclination on peristaltic transport of Casson fluid in the porous tube.

Due to the increasing concern in modelling by using fractional derivatives, several fractional derivative models have been formulated by inferring the existing fluid models [22]. Ali *et al.,* [23] has developed a fractional order model for blood flow (Casson fluid) with the help of Hankel and Laplace transform techniques to obtain the exact solutions. The so-called Caputo and Fabrizio fractional derivative is employed to solve different real problems [24–25]. Saqib *et al.,* [26] developed a mathematical model for MHD blood flow in a magnetite dusty particle tube by replacing the ordinary time derivative with a fractional time derivative of Caputo. Some other recent studies can be found in Alkahtani and Atangana [27], Shah *et al.,* [28], Shah *et al.,* [29] and the references therein.

A thorough search of the relevant literature has witnessed the fact that the existing literature did not present the exact solution of MHD blood flow model in the context of Caputo-Fabrizio fractional derivative for inclined multi-stenosed artery. In the present study, the model of the non-Newtonian Casson fluid has been applied subject to previous studies subject to multiple stenosed artery. The blood flow is due to the oscillating pressure gradient in the z-direction and the external magnetic field. The exact solutions are then calculated by means of significant transformations like Laplace and finite Hankel transforms. Bessel functions of the zero-order have been used for numerical computations to generate graphical results by using Mathcad for various values of fractional parameters and some important physical parameters.

2. Methodology

The physical domain consists of a multi-stenosed artery with the physical dimensions as shown in Figure 1. Consider the unsteady blood flow in an inclined multi-stenosed artery aligned in the axial direction (z-axis). r-axis is the radial direction. Blood is treated as an incompressible non-Newtonian Casson fluid subjected to an oscillating pressure gradient. The corresponding momentum equation is therefore a generalization of the preview study conducted by [29] with the adding factors of Casson fluid and in the inclined multi-stenosed artery. At t=0, the blood and the magnetic particles are treated as stationary.

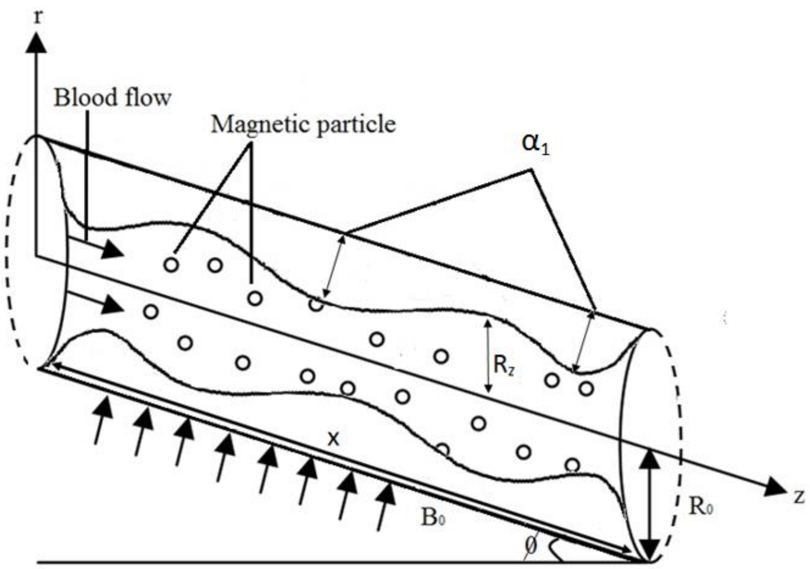


Fig. 1. Geometry of an inclined arterial segment with multi-stenosis

The unsteady blood flow in an axisymmetric cylindrical tube of radius R_0 under the influence of uniform transverse magnetic field and pressure gradient of the form Agarwal and Varshney[6]

$$-\frac{\partial p}{\partial z} = A_0 + A_1 \cos(\omega t), \qquad A_0 > 0. \tag{1}$$

is considered, where the constants A_0 and A_1 are the amplitudes of the pulsatile magnetic field and pressure gradient that give rise to systolic or diastolic pressure.

The geometry of the multi-stenosis in the arterial lumen may be described mathematically by Tashtoush and Magableh [10] as follows

$$R_z = 1 - \alpha_1(1.48x - 0.7398x^2 + 0.1485x^3 - 0.013955x^4 + 0.0006145x^5 - 0.000010243x^6) \tag{2}$$

where R_z is the radius of the artery in the constricted region, R_0 is the radius of the normal artery, x is the length of stenosis and α_1 is the degree of the stenosis.

The momentum equation for fluid flow in an inclined stenosed artery can be written as [23], [29], [30]

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \upsilon\left(1+\frac{1}{\beta}\right)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + \frac{KN}{\rho}(v-u) - \frac{\sigma B_0^2 \sin\theta}{\rho}u + g\sin\phi, \tag{3}$$

The motion of magnetic particles is governed by the Newton's second law

$$m\frac{\partial v}{\partial t} = K(u-v). \tag{4}$$

Here, ρ, υ, p, N, K, u and v is the fluid density, the kinematic viscosity, the pressure, the number of magnetic particles per unit volume, the Stokes constant, the fluid velocity, and the particle velocity, respectively. $\beta = \frac{\mu_B \sqrt{2\pi_c}}{\tau_r}$ is the material parameter of Casson fluid where μ_B is the plastic dynamic viscosity, τ_r is the yield stress of fluid, $2\pi_c$ is the critical value of this product based on the non-Newtonian model, and *m* is the average mass of the magnetic particles. $\frac{KN}{\rho}(v-u)$ is the force due to the relative motion between fluid and magnetic particles. It is assumed that the Reynolds number of the relative velocity is small. As such, the force between the magnetic particles and the blood is proportional to the relative velocity.

The initial boundary conditions of the fluid inside the cylindrical domain of radius R_0 are

$$\begin{aligned} &u(r,0)=0, \quad v(r,0)=0, \quad at\ r\in[0,R_z] \\ &u(r,t)=0, \quad v(r,t)=0, \quad at\ r=R_z \end{aligned} \tag{5}$$

The non-dimensional parameters can be introduced as

$$\begin{aligned} &r^* = \frac{r}{R_0}, \quad t^* = \frac{t}{\lambda}, \quad u^* = \frac{u}{u_0}, \quad v^* = \frac{v}{u_0}, \\ &A_0^* = \frac{\lambda A_0}{\rho u_0}, \quad A_1^* = \frac{\lambda A_1}{\rho u_0}, \quad \omega^* = \lambda\omega, \quad g^* = \frac{g}{u_0^2 / R_0} \end{aligned} \tag{6}$$

where u_0 is the characteristics velocity.

The non-dimensional forms of governing equations subject to Eq. (6) after dropping the * notation are given as follows

$$D_t^\alpha u = A_0 + A_1\cos(\omega t) + \beta_1\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r^*}\frac{\partial u^*}{\partial r^*}\right] + R(v-u) - Ha^2 u + \frac{\sin\phi}{F} \tag{7}$$

$$G\cdot D_t^\alpha v = u - v \tag{8}$$

Here, $\beta_1 = \frac{1}{\mathrm{Re}}\left[1+\frac{1}{\beta}\right]$ where $\mathrm{Re} = \frac{R_z^2}{\lambda v}$ is the Reynolds number, $R = \frac{kN\lambda}{\rho}$ is the particles concentration parameter, $Ha = B_0\sqrt{\lambda}\sqrt{\frac{\sigma}{\rho}\sin\theta}$ is the Hartmann number, and $F = \frac{R_0}{\lambda u_0 g}$ is the inclination angle parameter.

The non-dimensional boundary conditions are

$$\begin{aligned} &u\left(\frac{r}{R_z},0\right)=0, \quad v\left(\frac{r}{R_z},0\right)=0, \quad at\ \frac{r}{R_z}\in[0,1] \\ &u\left(\frac{r}{R_z},t\right)=0, \quad v\left(\frac{r}{R_z},t\right)=0, \quad at\ \frac{r}{R_z}=1 \end{aligned} \tag{9}$$

Applying finite Hankel transform of order zero, then the following equation can be obtained

$$\left[\frac{s}{s+\alpha(1-s)}-R\left(\frac{s+\alpha(1-s)}{s+sG+\alpha(1-s)}\right)+R+Ha^2\right]\bar{u}_H(r_n,s)= \left[\frac{A_0}{s}+\frac{A_1 s}{s^2+\omega^2}+\frac{\sin\phi}{sF}\right]\frac{J_1(r_n)}{r_n}-\beta_1 r_n \bar{u}_H(r_n,s) \tag{10}$$

where $\bar{u}_H(r_n,s)=\int_0^1 r\bar{u}(r,s)J_0(r_n r)dr$ represents the finite Hankel transform of the velocity function $\bar{u}(r,s)=LT[u(r,t)]$ and $r_n, n=1,2,...$ are the positive roots of the equation $J_0(x)=0,$ here J_0 is the Bessel function of order zero and it belongs to the first kind. By simplifying the coefficient of $\bar{u}_H(r_n,s)$ in Eq. (10), the following equations can be formulated

$$\bar{u}_H(r_n,s)=\frac{s^2 y_{5n}+s y_{6n}+\alpha^2}{s^2 y_{2n}+s y_{3n}+y_{4n}}\left[\frac{1}{s}\left(A_0+\frac{\sin\phi}{F}\right)+\frac{A_1 s}{s^2+\omega^2}\right]\frac{J_1(r_n)}{r_n} \tag{11}$$

$$\bar{u}_H(r_n,s)=\left(A_0+\frac{\sin\phi}{F}\right)\left[\frac{1}{s}\frac{y_{5n}}{y_{2n}}\frac{s^{-1}}{s-x_{7n}}y_{9n}+\frac{s^{-1}}{s-x_{8n}}y_{10n}\right]\frac{J_1(r_n)}{r_n} +A_1\frac{s}{s^2+\omega^2}\left[\frac{y_{5n}}{y_{2n}}+y_{9n}\frac{1}{s-x_{7n}}+y_{10n}\frac{1}{s-x_{8n}}\right]\frac{J_1(r_n)}{r_n} \tag{12}$$

where

$$\begin{aligned}
&y_{1n}=Ha^2+R+\beta_1 r_n^2,\\
&y_{2n}=1+G-\alpha-R-R\alpha^2+2R\alpha+y_{1n}+\alpha^2 y_{1n}-2\alpha y_{1n}+Gy_{1n}-G\alpha y_{1n},\\
&y_{3n}=\alpha+2R\alpha^2-2R\alpha-2x_{1n}\alpha^2+2\alpha x_{1n}+G\alpha x_{1n},\quad y_{4n}=\alpha^2 y_{1n}-R\alpha^2,\\
&y_{5n}=1+\alpha^2-2\alpha+G-G\alpha,\quad y_{6n}=-2\alpha^2+2\alpha+G\alpha,\\
&y_{7n}=\frac{-y_{3n}+\sqrt{y_{3n}^2-4y_{2n}y_{4n}}}{2y_{2n}},\quad y_{8n}=\frac{-y_{3n}-\sqrt{y_{3n}^2-4y_{2n}y_{4n}}}{2y_{2n}},\\
&y_{9n}=\frac{y_{7n}\left(y_{6n}-\frac{y_{3n}y_{5n}}{y_{2n}}\right)+\frac{y_{4n}y_{5n}}{y_{2n}}+\alpha^2}{y_{7n}-y_{8n}},\quad y_{10n}=\frac{y_{8n}\left(y_{6n}-\frac{y_{3n}y_{5n}}{y_{2n}}\right)+\frac{y_{4n}y_{5n}}{y_{2n}}+\alpha^2}{y_{8n}-y_{7n}},
\end{aligned} \tag{13}$$

The Laplace transform of the image function $\bar{u}_H(r_n,s)$ in Eq. (12) is given as follows

$$\bar{u}_H(r_n,t)=\frac{J_1(r_n)}{r_n}\Bigg[\left(\frac{y_{5n}}{y_{2n}}\delta(t)\right)\left(A_0+\frac{\sin\phi}{F}\right)+\left(e^{y_{7n}t}-1\right)\left(\frac{A_0y_{9n}}{y_{7n}}+\frac{y_{9n}\sin\phi}{y_{7n}F}\right)+\left(e^{y_{8n}t}-1\right)\left(\frac{A_0y_{10n}}{y_{8n}}+\frac{y_{10n}\sin\phi}{y_{8n}F}\right)+A_1\cos(\omega t)\left(\frac{y_{5n}}{y_{2n}}\right)+A_1y_{9n}e^{y_{7n}t}*\cos(\omega t)+A_1y_{10n}e^{y_{8n}t}*\cos(\omega t)\Bigg] \tag{14}$$

with the aid of the Robotnov and Hartley's functions as follows

$$LT^{-1}\left[\frac{1}{s^w+y}\right]=F_w(-y,t)=\sum_{n=0}^{\infty}\frac{(-y)^n t^{(n+1)w-1}}{\Gamma((n+1)w)},w>0 \tag{15}$$

$$LT^{-1}\left[\frac{s^z}{s^w+y}\right]=R_{w,z}(-y,t)=\sum_{n=0}^{\infty}\frac{(-y)^n t^{(n+1)w-1-z}}{\Gamma((n+1)w-z)},\mathrm{Re}(w-z)>0 \tag{16}$$

By inverting the finite Hankel transforms, the fluid velocity $u(r,t)=2\sum_{n=1}^{\infty}\frac{J_0\left(\frac{r}{R_z}r_n\right)}{r_nJ_1^2(r_n)}\times u_H(r_n,t)$ is given as follows

$$u(r,t)=2\sum_{n=1}^{\infty}\frac{J_0\left(\frac{r}{R_z}r_n\right)}{r_nJ_1(r_n)}\Bigg[\left(\frac{y_{5n}}{y_{2n}}\delta(t)\right)\left(A_0+\frac{\sin\phi}{F}\right)+\left(e^{y_{7n}t}-1\right)\left(\frac{A_0y_{9n}}{y_{7n}}+\frac{y_{9n}\sin\phi}{y_{7n}F}\right)+\left(e^{y_{8n}t}-1\right)\left(\frac{A_0y_{10n}}{y_{8n}}+\frac{y_{10n}\sin\phi}{y_{8n}F}\right)+A_1\cos(\omega t)\left(\frac{y_{5n}}{y_{2n}}\right)+A_1y_{9n}e^{y_{7n}t}*\cos(\omega t)+A_1y_{10n}e^{y_{8n}t}*\cos(\omega t)\Bigg] \tag{17}$$

where $*$represents the convolution product. Note that the convolution product of f and g, $f*g$ can be calculated as $(f*g)(t)=\int_0^t f(\tau)g(t-\tau)d\tau$.

While, the magnetic particle velocity $\bar{v}(r,s)=\frac{s+\alpha-\alpha s}{s+Gs+\alpha-\alpha s}\bar{u}(r,s)$ becomes

$$v(r,t)=\frac{G\alpha}{(1+G-\alpha)^2+\alpha}u(r,t),0<\alpha<1 \tag{18}$$

3. Results and Discussions

Mathcad code has been implemented to obtain the numerical results extracted from the analytical solutions Eq. (17) and Eq. (18). For numerical computations, the following values are set as $A_0=0.5, A_1=0.6, G=0.8, R=0.5, Re=3, \omega=\frac{\pi}{4}, Ha=2$ and $\beta=0.4$ [23], [29]. The numerical results have been compared with Shah *et al.,* [29] for having the same blood flow with magnetic particles as shown in Figure 2. In the current work, the Casson fluid is considered with magnetic particles that flow through an inclined multiple stenosed artery. Meanwhile, Shah *et al.,* [29]studied the blood flow with magnetic particles travelling through a cylindrical tube under the influence of a magnetic field and an oscillating pressure gradient. In the calculation the following values $A_0 = 0.5$, $A_1 = 0.1, G = 0.8, R = 0.5, Re = 5, \omega = \frac{\pi}{4}, Ha = 2, z = 1$ and $\beta = 0.25$ are used for comparison purposes, so that both problems become similar.

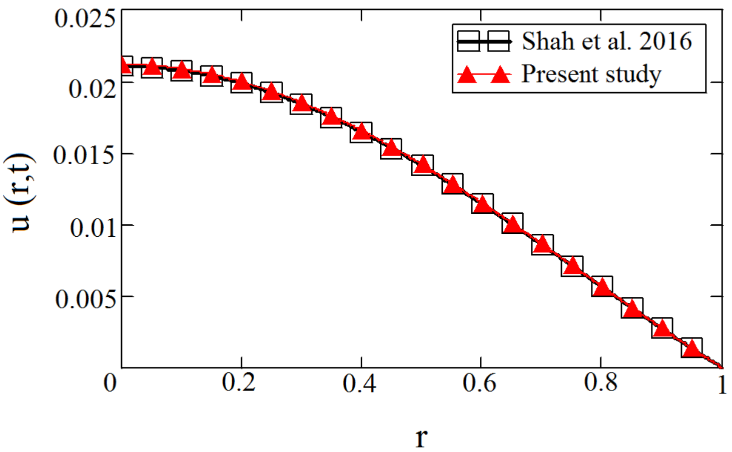
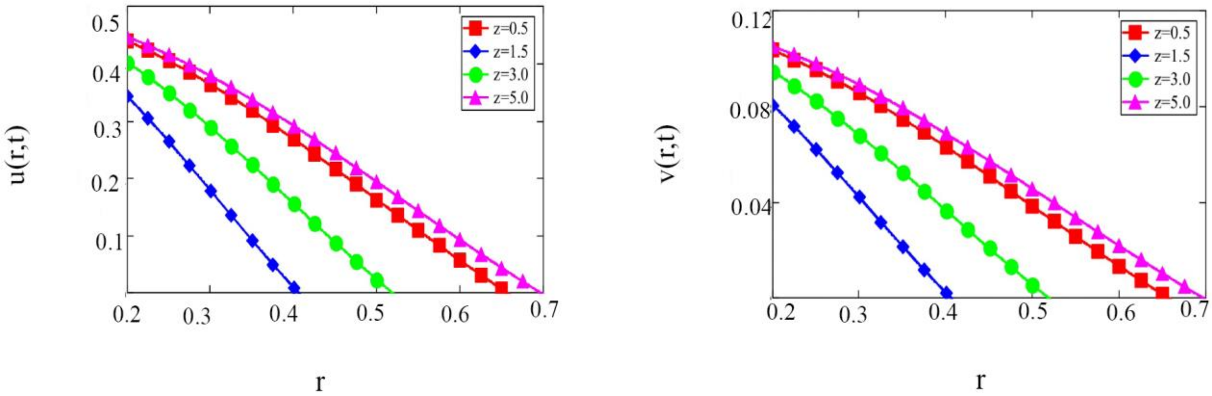


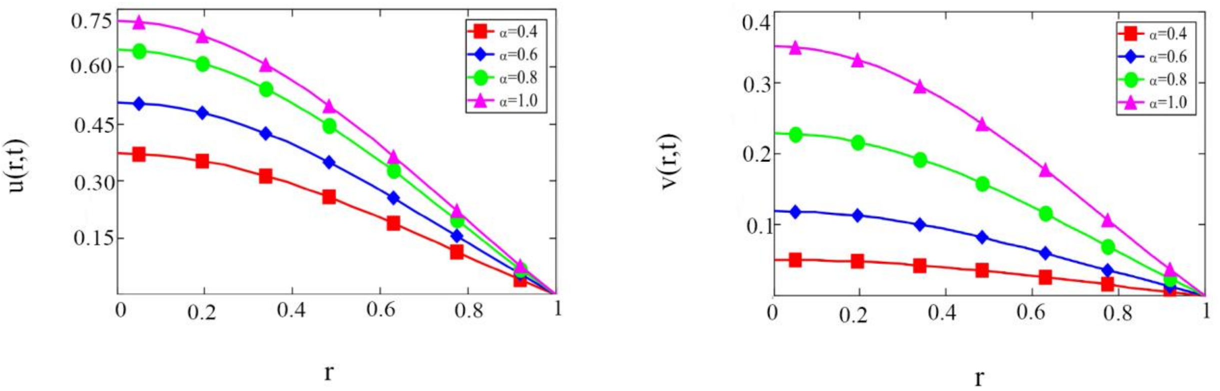
Fig. 2. Comparison of velocity distribution with previous study

For both profiles in Figure 3, it is found that the blood flow movement is slower as blood passes through the narrow stenotic region as compared to that in the wider part. It can be found that the patterns of flow resistance in both regions are similar for different stenotic sizes. Figure 4 is plotted to analyze the impacts of the fractional parameter on the blood flow and magnetic particle velocities. It indicates that the blood modeled using the ordinary model flows faster than that using the Casson fluid model with fractional derivatives.



(a) (b)

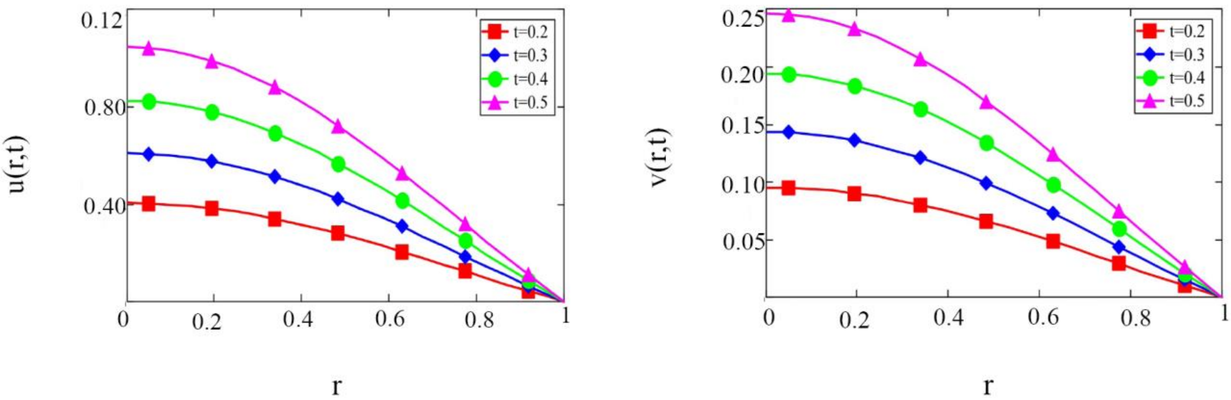
Fig. 3. Axial velocity profiles; (a) u(r,t) and (b) v(r,t) at different values of z



(a) (b)

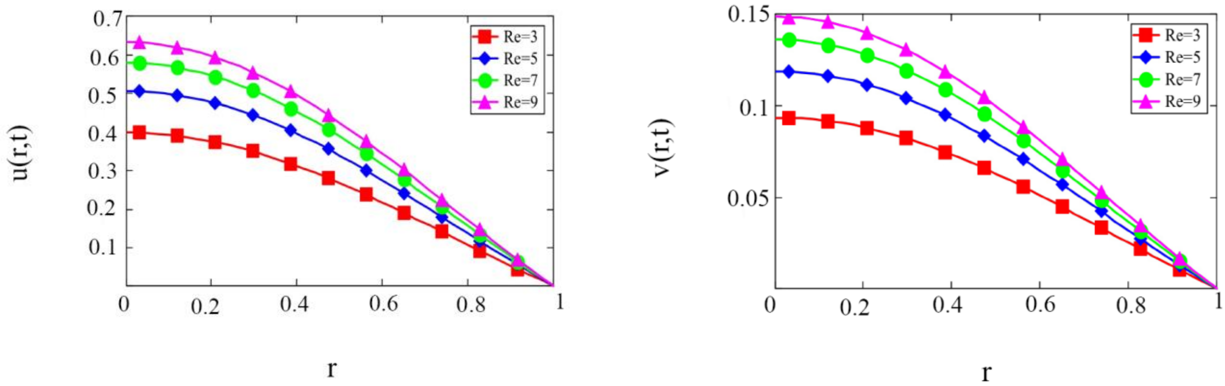
Fig. 4. Axial velocity profiles; (a) u(r,t) and (b) v(r,t) for different fractional parameters

Figure 5 represents the influence of fractional parameter has been shown for different time levels and it is observed that the blood velocity increases with respect to time. The fractional parameter is found to play a key role in regulating the blood distributions. Figure 6 shows the velocity profiles for different Reynolds numbers *Re*. It is clear that the Reynolds number is proportionally related to the velocity of the fluid. In general, increase in Reynolds number will increase the velocity of the fluid gradually.



(a) (b)

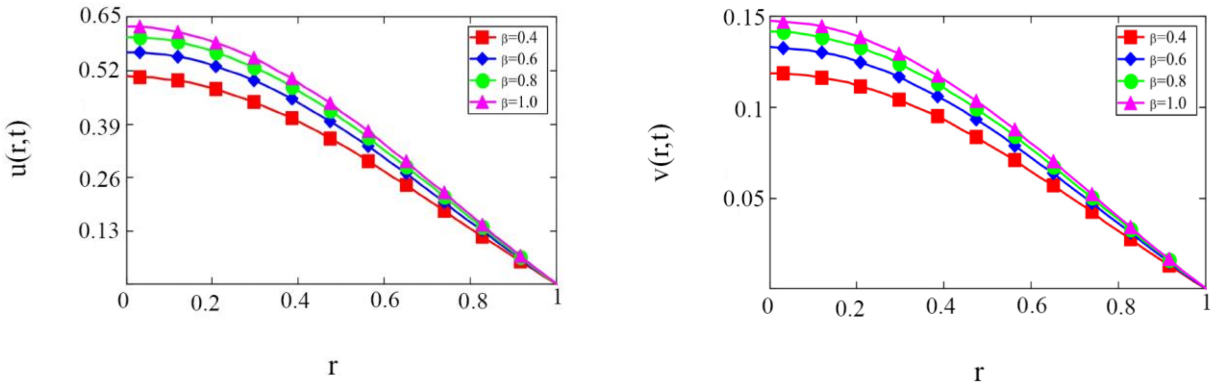
Fig. 5. Axial velocity profiles; (a) u(r,t) and (b) v(r,t) for different time levels *t*



(a) (b)

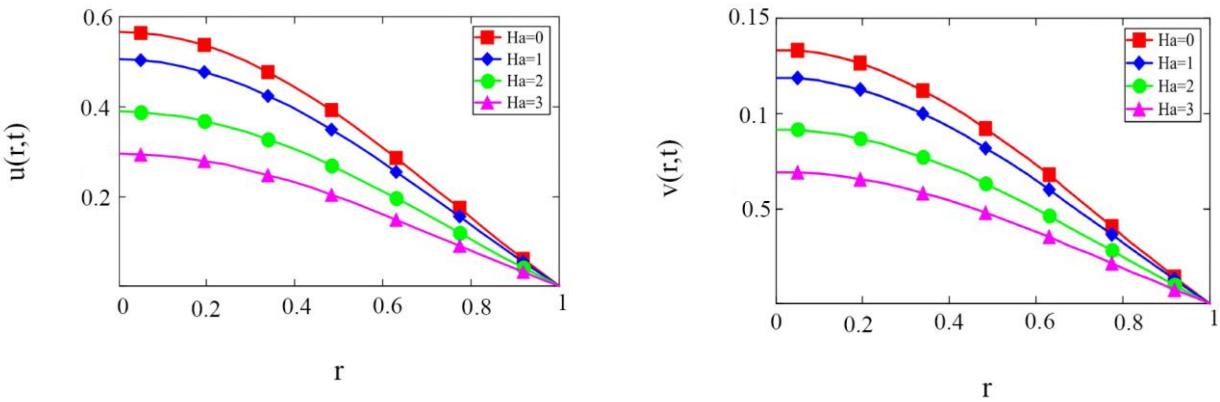
Fig. 6. Axial velocity profiles; (a) u(r,t) and (b) v(r,t) for different Reynolds numbers

The effects of Casson fluid parameter on the blood and magnetic particle motions are depicted in Figure 7. With an increase in the Casson fluid parameter, the fluid velocity increases. This statement is in perfect agreement with Ali *et al.,* [23] for a horizontal cylinder. It is hypothesized that the yield stress declines as β increases and the thickness of the boundary layer decreases. The effects of magnetic parameter on both fluid and magnetic particle velocities are shown in Figure 8. By increasing the Hartmann number *Ha*, the blood velocity decreases. It is noticeable that the magnetic field will reduce the axial velocities of blood and magnetic particles substantially. The usefulness of the magnetic field in the fluid flow model would increase the Lorentz force, thus restricting the blood flow in the system.



(a) (b)

Fig. 7. Axial velocity profiles; (a) u(r,t) and (b) v(r,t) for different Casson fluids



(a) (b)

Fig. 8. Axial velocity profiles; (a) u(r,t) and (b) v(r,t) for different Hartmann numbers

4. Conclusions

A mathematical analysis on the fractional order blood flow model under the influence of external magnetic field acting on the non-Newtonian Casson fluid that flows through a multi-stenosed artery has been performed. Generally, an extra-solution is required to extract the ordinary model; however, the velocity equation can be directly obtained using the current method since the equation is compatible. It should be noted that the particle has the same tendency as the blood; however, it moves slower. The blood velocity increases with respect to Reynolds number. In the meantime, the increase in Casson fluid parameters increases the velocities of blood and particles. On the other hand, the blood velocity decreases with respect to Hartmann number. These findings are technically important and interesting for drug distribution applications.

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