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Numerical Solutions of Hybrid Nanofluids Flow Via Free Convection Over a Solid Sphere

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ABSTRACT

The purpose of the existing study is to examine how heat transfer enables consolidated by variations in the basic advantages of fluids in the existence of free convection with the assistance of suspended hybrid nanofluids. Iron-graphene oxide suspended in water as a hybrid nanofluid flow on a solid sphere is also considered in this work. The partial differential equations are gotten, for this problem, by transforming the mathematical governing equations using similarity equations (stream function). These partial differential equations are solved numerically by Keller-Box method and programmed by MATLAB program. The acquired numerical results are in excellent agreement with the preceding literature results. Graphical results of the influence of the hybrid nanofluid parameters on some physical quantities regarded to examine the behavior of hybrid nanofluid flow were attained, and they proved that hybrid nanofluid flow represents a more essential role in the operation of heat transfer than a regular nanofluid flow.

1. Introduction

At present time, the nano technology is continued to heat transfer enhancement, and it stays a matter of senior attention in the studies and sciences. When nanofluids expression was introduced by Choi and Eastman [1], considerable many searches have been widely achieved related to the charactaristcs of nanofluids on heat transfer and fluid flow. After that, two important kinds of simulations models permanently applied to discuss the demeanors of nanofluid, such as single-phase model and two-phase model, which are conducted by Tiwari and Das [2] and Buongiorno [3], respectively. Swalmeh *et al.,* [4-8] and Alwawi *et al.,* [9-12] used the one phase model to study the convection boundary layer flow in nanofluid over solid sphere and horizontal circular cylinder. And many papers investigated the two phase model like Sheikholeslami *et al.,* [13], Garoosi *et al.,* [14], Rea *et al.,* [15]. Furthermore, many researchers reported distinct articles concerning the in boundary layer flow in a nanofluid. Noor *et al.,* [16] stidied the problem of convection heat transfer on stagnation point in micropolar nanofluid. The convection boundary-layer flow over a horizontal

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circular cylinder with a porous medium in presence of nanofluid were investigated by Rashad *et al.,* [17]. Hussanan *et al.,* [18] looked in convection heat transfer in micropolar nanofluids with oxide nanoparticles. Also, Hussain *et al.,* [19] studied the analysis of micropolar nanofluid flow past a stretching surface [20-24].

A few years ago, experimental study of nanofluids called "hybrid nanofluids" are considered by Suresh *et al.,* [25]., which is suggested to present better heat trensfer advantages and rheological conduct along with improved thermo-physical features. Hybrid nanofluid is an stretching of nanofluid which collected of two various nanoparticles suspended in the base fluid. Hybrid nanofluids are vastly utilized in numerous fields of enhancement heat transfer, such as electronic cooling, acoustics, coolant in machining, supercomputers, transportation, military, pharmaceutical, biomedical, nuclear safety. Devi and Devi [26] numerically studied the problem of hydro-magnetic hybrid Cu-Al_2O_3/water nanofluid flow. On stagnation-point flow of an aqueous TiO_2 -Cu hybrid nanofluid on a wavy cylinder was considered by Yousefi *et al.,* [27]. Hayat *et al.,* [28] studied the rotating hybrid flow of Ag-CuO/H_2O nanofluid under radiation and partial slip boundary effects. Rehman *et al.,* [29] conducted the three-dimensional in existence of micropolar hybrid nanofluid flow past an exponentially stretched surface. And in the survey articles, such as Hussien *et al.,* [30], Tlili *et al.,* [31], Murray [32], Hussain *et al.,* [33], Babu *et al.,* [34], Ahmadi *et al.,* [35], Ali *et al.,* [36].

Depending on the aforementioned above publications for this special hybrid nanofluid flow, the efforts were gone to inspect heat transfer elaboration in free convection flow of iron-graphene oxide suspended in water as a hybrid nanofluid over a solid sphere, with two boundary conditions, namely constant wall temperature (CWT) and constant heat flux (CHF). In the engineering field, the amount of enhancement of composition of nanofluid as hybrid nanofluid brings a lot of wide prospects especially in the development of modern industries. Besides, this study problem can be extended to another studies, like influences of magneto-hydrodynamics, micropolar, or Casson fluid, in presence of convection boundary layer flow with hybrid nanofluid. Hence, the numerical results for physical quantities can be gained for these influences parameters. Moreover, this research is an expansion and stretching of some previous research, check Manjunatha *et al.,* [37], Waini *et al.,* [38], Nadeem *et al.,* [39], and Hamarsheh *et al.,* [40].

2. Mathematical Formulation

The problem of steady laminar free convection boundary layer flow in presence of an incompressible hybrid nanofluid, on a solid sphere, is investigated. The *x*-axis measured in the circumference of the solid sphere surface motion from the lower stagnation point ($x \approx 0$), and the *y*-axis is measured perpendicular to it. Also, the constant wall temperature (CWT) (T_w) and constant heat flux (CHF) (q_w) boundary conditions, are studied in this problem, as shown in Figure 1. T_w is the wall temperature, q_w is the heat flux constant, T_∞ the ambient temperature of the fluid which does not change, *g* is the gravity vector which affects in the opposite direction.

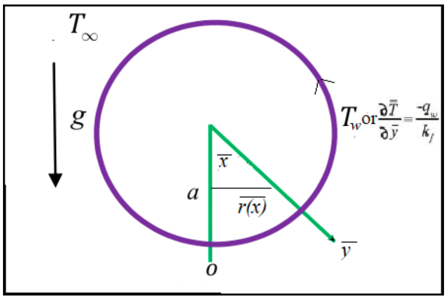

Fig. 1. Schematic physical model

Subject to the above suppositions, the continuity, momentum, and thermal equations, of the boundary layer flow of the hybrid nanofluids, on a solid sphere, are obtained by Salleh *et al.,* [48], Manjunatha *et al.,* [37]

$$\frac{\partial(\bar{r}\bar{u})}{\partial\bar{x}}+\frac{\partial(\bar{r}\bar{v})}{\partial\bar{y}}=0, \tag{1}$$

$$\rho_{hnf}\left(\bar{u}\,\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right)=\mu_{hnf}\left(\frac{\partial^2\bar{u}}{\partial\bar{x}^2}\right)-(\beta)_{hnf}\,\mathrm{g}\,(T-T_\infty)\sin\frac{\bar{x}}{a}, \tag{2}$$

$$\bar{u}\,\frac{\partial T}{\partial\bar{x}}+\bar{v}\,\frac{\partial T}{\partial\bar{y}}=\alpha_{hnf}\left(\frac{\partial^2\bar{T}}{\partial\bar{y}^2}\right), \tag{3}$$

subject the two boundary conditions defined by Swalmeh *et al.,* [6] and Nazar *et al.,* [41] as

$$\begin{aligned}&\bar{u}=0,\bar{v}=0,T=T_w\,(\mathrm{CWT})\mathrm{or}\,\frac{\partial T}{\partial\bar{y}}=\left(-\frac{\bar{q}_w}{k_f}\right)(\mathrm{CHF}),\mathrm{as}\,\bar{y}=0,\\&\bar{u}\to0,T\to T_\infty\,,\ \mathrm{as}\,\bar{y}\to\infty.\end{aligned} \tag{4}$$

Here, $(\bar{u},\bar{v})$-velocity components along (x,y) coordinate respectively, $\bar{r}(\bar{x})$ is the radial distance from the symmetrical axis to the surface of the sphere, *a* is constant. ρ_{hnf}, μ_{hnf}, g, $(\beta)_{hnf}$, $(\alpha)_{hnf}=\frac{k_{hnf}}{(\rho c_p)_{hnf}}$ are the density of hybrid nanofluid, viscosity of hybrid nanofluid, gravity acceleration, coefficient of thermal expansion of hybrid nanofluid, and Thermal diffusivity coefficient of the hybrid nanofluid, respectively. T, T_∞ are the temperature of fluid and ambient temperature, k_f and k_{hnf} are the thermal conductivity of based fluid and hybrid nanofluid, $(\rho c_p)_{hnf}$ is the heat capacity of hybrid nanofluid, $\bar{q}_w$ is constant heat flux. The thermo-physical properties for used nanoparticles and based fluid are presented in Table 1. Also, the hybrid nanofluid properties are displayed by Table 2.

Table 1
Different values of thermo-physical properties of nanoparticles of one base fluid [42]

Physical properties	Water	GO	Fe
k (W/mK)	0.613	5000	9.7
ρ (kg/m^3)	997.1	1800	5180
ρc_p (J/kgK)	4179	717	670
Pr	6.2	…	…

Table 2
Thermo-physical model [37]

	Properties of nanofluid	Properties of hybrid nanofluid
1.	$(\beta)_{nf}=(\gamma_2\,(\beta)_s+(1-\gamma_2)(\beta)_f\,),$	$(\beta)_{hnf}=(1-\gamma_2)[\,(1-\gamma_1)\,(\beta)_f+\gamma_1(\beta)_{s1}]+\gamma_2(\beta)_{s2},$
2.	$(\mu)_{nf}=\frac{\mu_f}{(1-\gamma_2)^{2.5}},$	$(\mu)_{hnf}=\frac{\mu_f}{(1-\gamma_1)^{2.5}(1-\gamma_2)^{2.5}},$
3.	$(\rho c_p)_{nf}=(\gamma_2\,(\rho c_p)_s+(1-\gamma_2)(\rho c_p)_f\,),$	$(\rho c_p)_{hnf}=(1-\gamma_2)[\,(1-\gamma_1)(\rho c_p)_f+\gamma_1(\rho c_p)_{s1}]+\gamma_2(\rho c_p)_{s2},$
4.	$(\alpha)_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}},$	$(\alpha)_{hnf}=\frac{k_{hnf}}{(\rho c_p)_{hnf}},$
5.	$\frac{k_{nf}}{k_f}=\frac{(k_s+2\,k_f\,)-2\,\gamma_2\,(\,k_f-\,k_s\,)}{(\,k_s+2\,k_f\,)\,+\,\gamma_2\,(\,k_f-\,k_s\,)}.$	$\frac{k_{hnf}}{k_{bf}}=\frac{(k_{s2}+2\,k_f)-2\gamma_2\,(\,k_{bf}-k_{s2}\,)}{(k_{s2}+2\,k_{bf}\,)+\gamma_2\,(\,k_{bf}-k_{s2}\,)},$ $\frac{k_{bf}}{k_f}=\frac{(\,k_{s1}+2\,k_f\,)-2\,\gamma_1\,(\,k_f-\,k_{s1}\,)}{(\,k_{s1}+2\,k_f\,)\,+\,\gamma_1\,(\,k_f-\,k_{s1}\,)}.$

where γ_2 is the volume fraction for the Fe, γ_1 is the volume fraction for GO. ($\gamma_2 = \gamma_1 = 0$) are represent to a regular Newtonian fluid. Besides, to simplify the above problem equations, we get the following non-dimensional variables, as follows [9,10]

$$r = \left(\frac{\bar{r}}{a}\right), x = \left(\frac{\bar{x}}{a}\right),$$
$$\left\{y = (Gr)^{(1/4)}\left(\frac{\bar{y}}{a}\right), \theta = \frac{T - T_\infty}{T_w - T_\infty}, u = (Gr)^{(-1/2)}\left(\frac{a\bar{u}}{v_f}\right), v = (Gr)^{(-1/4)}\left(\frac{a\bar{v}}{v_f}\right)\right\}(\mathrm{CWT}),$$
$$\left\{y = (Gr)^{(1/5)}\left(\frac{\bar{y}}{a}\right), \theta = (Gr)^{(1/5)}\left(\frac{T - T_\infty}{\frac{a\bar{q}_w}{k_f}}\right), u = (Gr)^{(-2/5)}\left(\frac{a\bar{u}}{v_f}\right), v = (Gr)^{(-1/5)}\left(\frac{a\bar{v}}{v_f}\right)\right\}(\mathrm{CHF}), \tag{5}$$
$$Gr = g(\beta)_f (T_w - T_\infty)\, a^3/v_f^2 (CWT), Gr = g(\beta)_f \left(\frac{a\bar{q}_w}{k_f}\right) a^3/v_f^2(\mathrm{CHF}),$$

such that *Gr* is the Grashof number. v_f is Kinematic viscosity of the fluid. The above non-dimensional variables along (5) have utilized in Eq. (1) to Eq. (4) to obtain the next non-dimensional equations, subject to two boundary conditions, namely constant wall temperature and constant heat flux.

$$u\,\frac{\partial ru}{\partial x} + v\,\frac{\partial ru}{\partial y} = 0, \tag{6}$$

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \frac{\rho_f}{\rho_{hnf}}\left(\frac{\mu_f}{(1-\gamma_1)^{2.5}(1-\gamma_2)^{2.5}}\right)\left(\frac{\partial^2 u}{\partial y^2}\right) + \frac{1}{\rho_{hnf}}\left((1-\gamma_2)\left[(1-\gamma_1)\,\rho_f + \gamma_1\frac{\rho_{s1}\beta_{s1}}{\beta_f}\right] + \gamma_2\frac{\rho_{s2}\beta_{s2}}{\beta_f}\right)\theta\sin x, \tag{7}$$

$$u\,\frac{\partial\theta}{\partial x} + v\,\frac{\partial\theta}{\partial y} = \frac{1}{\Pr}\left(\frac{k_{hnf}/k_f}{(1-\gamma_2)\left[(1-\gamma_1)\,\rho_f + \gamma_1\frac{\gamma_1(\rho c_p)_{s1}}{(\rho c_p)_f}\right] + \gamma_2\frac{\gamma_2(\rho c_p)_{s2}}{(\rho c_p)_f}}\right)\left(\frac{\partial^2\theta}{\partial\bar{y}^2}\right), \tag{8}$$

where, $\Pr = \frac{v_f}{\alpha_f}$ is the Prandtl number.

The boundary conditions (4) become

$$u = 0, v = 0, \theta = 1\ (\mathrm{CWT})\text{or } \frac{\partial\theta}{\partial y} = -1\ (\mathrm{CHF}), \text{as } y = 0,$$
$$u \to 0\,, \theta \to 0,, \text{as } y \to \infty. \tag{9}$$

To solve the Eq. (6) to Eq. (8), subject to the boundary conditions (9), we assume the following variables [9]

$$\psi = xr(x)\, f(x, y), \theta = \theta(x, y) \tag{10}$$

such that ψ is the stream function defined as

$$u = \frac{1}{r}\frac{\partial\psi}{\partial y}, \text{ and } v = \frac{1}{r}\frac{\partial\psi}{\partial x}, \tag{11}$$

which satisfies the continuity Eq. (6), then the Eq. (6) to Eq. (8) become

$$\frac{\rho_f}{\rho_{hnf}}\left(\frac{\mu_f}{(1-\gamma_1)^{2.5}(1-\gamma_2)^{2.5}}\right)\frac{\partial^3 f}{\partial x^3} + (1 + x\cot x) f\frac{\partial^2 f}{\partial x^2} - \left(\frac{\partial f}{\partial y}\right)^2 + \frac{1}{\rho_{nf}}\left((1-\gamma_2)\left[(1-\gamma_1)\rho_f + \gamma_1\frac{\rho_{s1}\beta_{s1}}{\beta_f}\right] + \gamma_2\frac{\rho_{s2}\beta_{s2}}{\beta_f}\right)\theta\frac{\sin x}{x} = x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right), \tag{12}$$

$$\frac{1}{\Pr}\left(\frac{k_{hnf}/k_f}{(1-\gamma_2)\left[(1-\gamma_1)\rho_f+\gamma_1\frac{\gamma_1(\rho c_p)_{s1}}{(\rho c_p)_f}\right]+\gamma_2\frac{\gamma_2(\rho c_p)_{s2}}{(\rho c_p)_f}}\right)\left(\frac{\partial^2\theta}{\partial y^2}\right) + f\frac{\partial\theta}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial\theta}{\partial y}\right), \tag{13}$$

subject to the boundary conditions

$$f = \frac{\partial f}{\partial y} = 0, \theta = 1 \text{ (CWT) or } \frac{\partial\theta}{\partial y} = -1 \text{ (CHF), as } y = 0,$$
$$\frac{\partial f}{\partial y} \to 0, \theta \to 0, \text{ as } y \to \infty. \tag{14}$$

It can be seen that at the lower stagnation point of the sphere, $x \approx 0$, the above equations reduce to the following ordinary differential equations:

$$\frac{\rho_f}{\rho_{hnf}}\left(\frac{\mu_f}{(1-\gamma_1)^{2.5}(1-\gamma_2)^{2.5}}\right)\frac{\partial^3 f}{\partial x^3} + 2\frac{\partial^2 f}{\partial x^2} - \left(\frac{\partial f}{\partial y}\right)^2 + \frac{1}{\rho_{nf}}\left((1-\gamma_2)\left[(1-\gamma_1)\rho_f + \gamma_1\frac{\rho_{s1}\beta_{s1}}{\beta_f}\right] + \gamma_2\frac{\rho_{s2}\beta_{s2}}{\beta_f}\right)\theta\frac{\sin x}{x} = 0, \tag{15}$$

$$\frac{1}{\Pr}\left(\frac{k_{hnf}/k_f}{(1-\gamma_2)\left[(1-\gamma_1)\rho_f+\gamma_1\frac{\gamma_1(\rho c_p)_{s1}}{(\rho c_p)_f}\right]+\gamma_2\frac{\gamma_2(\rho c_p)_{s2}}{(\rho c_p)_f}}\right)\left(\frac{\partial^2\theta}{\partial y^2}\right) + f\frac{\partial\theta}{\partial y} = 0. \tag{16}$$

The boundary conditions become

$$f(0) = f'(0) = 0, \theta = 1 \text{ (CWT) or } \theta' = -1 \text{ (CHF), as } y = 0,$$
$$f' \to 0, \theta \to 0, \text{ as } y \to \infty, \tag{17}$$

where primes denote differentiation with respect to *y*.

The physical quantities of interest in this problem are the local skin friction coefficient C_f, the Nusselt number *Nu* and local wall temperature θ_w, and they can be written as

$$\left(C_f = \frac{Gr^{-3/4}a^2}{\mu_f v_f}\tau_w, Nu = \frac{a}{k_f(T_f - T_f)}q_w\right) \text{(CWT)}, \left(\theta_w = \theta(x,0), C_f = \frac{Gr^{-2/5}a^2}{\mu_f v_f}\tau_w\right) \text{(CHF)}, \tag{18}$$

where τ_w is Surface shear stress, and defined as

$$\tau_w = \mu_{hnf}\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)_{\bar{y}}, q_w = -k_{hnf}\left(\frac{\partial T}{\partial\bar{y}}\right)_{\bar{y}=0}, \tag{19}$$

By using the non-dimensional variables (5) and boundary conditions (9) the local skin friction coefficient C_f and Nusselt number Nu become

$$C_f = Gr^{-1/4}\left(\frac{\mu_f}{(1-\gamma_1)^{2.5}(1-\gamma_2)^{2.5}}\right) x \frac{\partial^2 f}{\partial y^2}(x,0),\ Gr^{-1/4}Nu = -Gr^{1/4}\frac{k_f}{k_{nf}}\left(\frac{\partial\theta}{\partial y}\right)(x,0),\ \text{(CWT)} \qquad (20)$$

3. Results and Discussions

Numerical solutions of the nonlinear partial differential equations, Eq. (12) and Eq. (11), under the two boundary conditions, such that (CWT) and (CHF), Eq. (14), are solved employing the Keller-box method available in the Matlab software. For a long time, This method well-known solver that has been vastly utilized by many researchers to solve the convection boundary layer flow problems, see Keller [43], Nazar [44], Tham *et al.,* [45], and Hamarsheh*, et al.,* [40]. Thermo-physical properties of used based fluids and nanoparticles in this study are shown in Table 1. The numerical solutions are gained using an initial estimation provided at an initial profile, at the lower stagnation point of the sphere x = 0, and follow up around the sphere up to x = 100°. In the special case, viscous Newtonian fluid, we get a suitable step size (x,y), and then comparing theses solutions with formerly published numerical results reported by Alwawi *et al.,* [10] and Cheng [46]. We checked that the present results are in good agreement accuracy, as displayed in Table 3 and Table 4.

Table 3
Comparison of local skin friction coefficient Nu with Newtonian fluid ($\gamma_1 = \gamma_2 = 0$) at Pr = 0.7

x°	Cheng [46]	Alwawi *et al.,* [10]	Present
0°	0.4576	0.4576	0.4576
10°	0.4565	0.4565	0.4565
20°	0.4534	0.4534	0.4533
30°	0.4481	0.4480	0.4480
40°	0.4407	0.4406	0.4405
50°	0.4310	0.4310	0.4311
60°	0.4191	0.4194	0.4192
70°	0.4049	0.4053	0.4050
80°	0.3881	0.3886	0.3882
90°	0.3686	0.3693	0.3685

Table 4
Comparison of local skin friction coefficient C_f with Newtonian fluid ($\gamma_1 = \gamma_2 = 0$) at Pr = 0.7

x°	Huang and Chen [47]	Alwawi *et al.,* [9]	Present
0°	0.0000	0.0000	0.0000
10°	0.2138	0.2123	0.2133
20°	0.4247	0.4157	0.4250
30°	0.6299	0.6252	0.6288
40°	0.8265	0.8201	0.8259
50°	1.0118	1.0033	1.0105
60°	1.1828	1.1676	1.1811
70°	1.3376	1.3198	1.3366
80°	1.4708	1.4519	1.4713
90°	1.5818	1.5609	1.5809

In this section, understanding the impact of hybrid nanofluid parameters γ_1 and $\gamma_2$on physical quantities for the problem of free convection boundary layer flow in a hybrid nanofluid is discussed. In this research, the nanoparticle of graphene oxide is added to the base fluid with nanoparticle volume fraction γ_1 equal to 0.1. Subsequently, Fe is added with a different value of nanoparticle volume fraction γ_2 to compose the hybrid nanofluid namely Fe-GO/water. The hybrid nanofluid parameter γ_2 values are studied from 0.007 to 0.06. Variations of the skin friction coefficient, local wall temperature, and the local Nusselt number, as well as the velocity and temperature profiles, are offered in plotted form through Figure 2 to Figure 8, with two boundary conditions (CWT) and (CHF). The influences of *Nu* and θ_w, are demonstrated in Figure 2 and Figure 3. It illustrates that an increase in γ_2 increases *Nu* (CWT) and θ_w (CHF), with an increase in the values in *x*. On the other hand, it is observed that there is a drop in the fluid moment in Nusselt number *Nu* along with the momentum angle *x*-direction. But the opposite case happens, there is a raise in the fluid moment in local wall temperature along with the momentum the angle *x*-direction. It is cleared that in the subsistence of a charismatic field, the change in the heat transfer *N*u and θ_w of hybrid nanofluid (Fe-GO/H_2O) is higher than that of nanofluid (Fe/H_2O). Therefore, we understand that the expected heat transfer rate can be got by a convenient complex of nanoparticle magnitude.

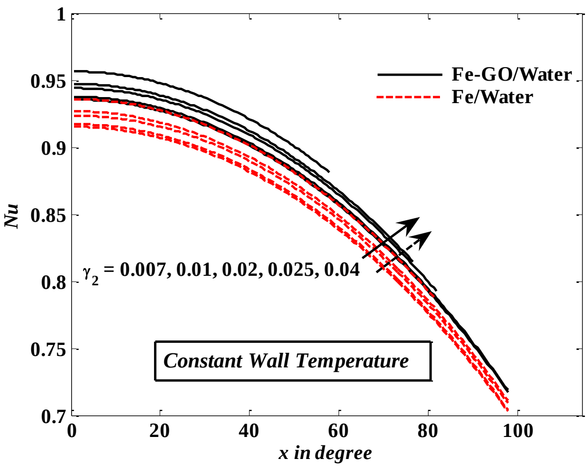

Fig. 2. The influence of$\gamma_2$on *Nu*

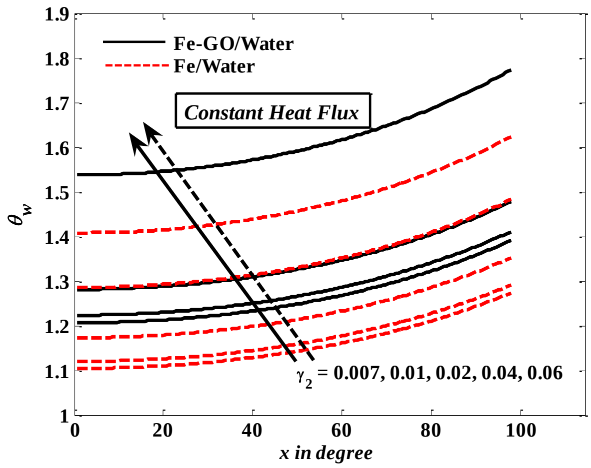

Fig. 3. The influence of γ_2 on θ_w

The characteristics of C_f are studied in Figure 4 and Figure 5. These figures signalize that with the increasing values of γ_2, the flow in local skin friction quantity is faster for both the nanofluid and the combination of nanofluids. Besides, the local skin friction values for hybrid nanofluid (Fe-GO/H_2O) are greater than nanofluid (Fe/H_2O), with two (CWT) and (CHF) boundary conditions. Also, It indicates that when the nanoparticle volume fraction values γ_2 are raising, the C_f quantity values are reducing, in presence of the (Fe-GO/H_2O) and (Fe/H_2O).

Figure 6 to Figure 9 depicted the behaviours of γ_1 and γ_2 on temperature and velocity profiles. It presents that an increase in γ_2 increases the temperature but the velocity decreases, with an associated thickness of the boundary layer increases. Physically, the boost in temperature and decrease in velocity between the surface and the ambient hybrid nanofluid is due to the increase in the concentration of nanoparticle density. It deduces that we get the optimizations in convection currents. In addition, GO-Fe/H_2O has high temperature and velocity profiles compared to Fe/Water with an increase in the values of γ_2.

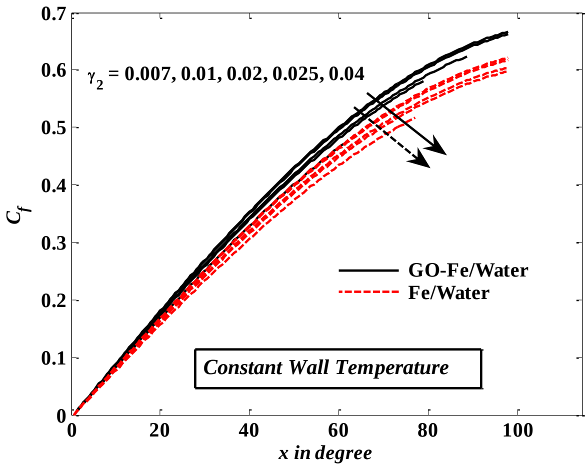

Fig. 4. The influence of γ_2 on C_f

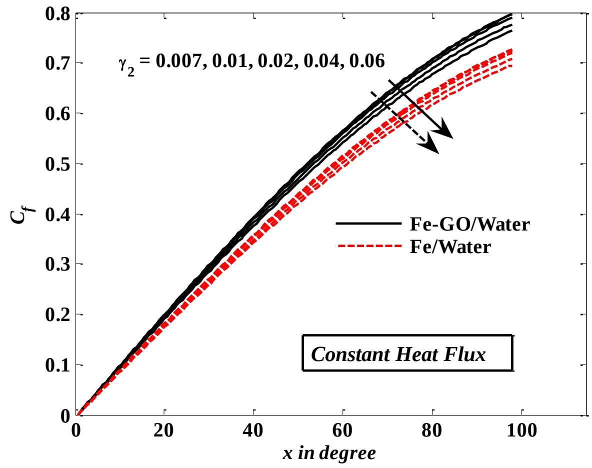

Fig. 5. The influence of γ_2 on C_f

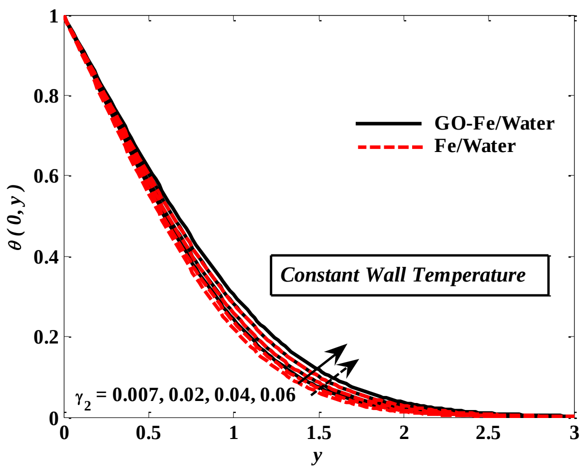

Fig. 6. The influence of γ_2 on $\theta(0,y)$

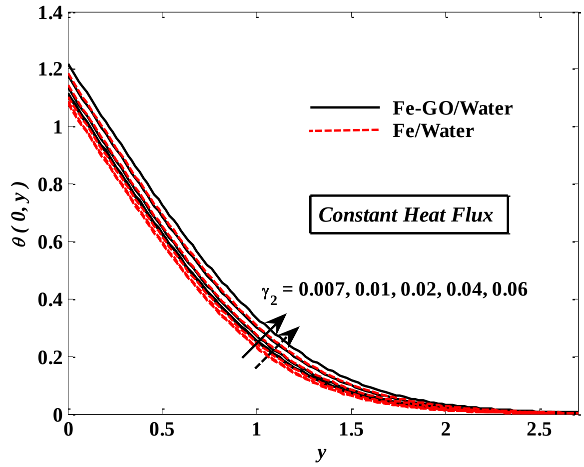

Fig. 7. The influence of γ_2 on $\theta(0,y)$

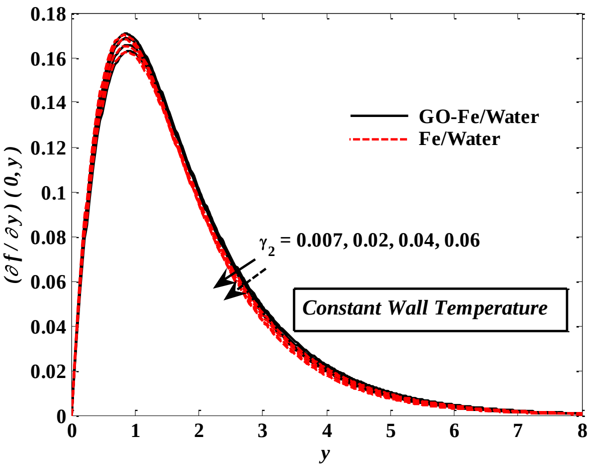

Fig. 8. The influence of γ_2 on $(\partial f/\partial y)(0,y)$

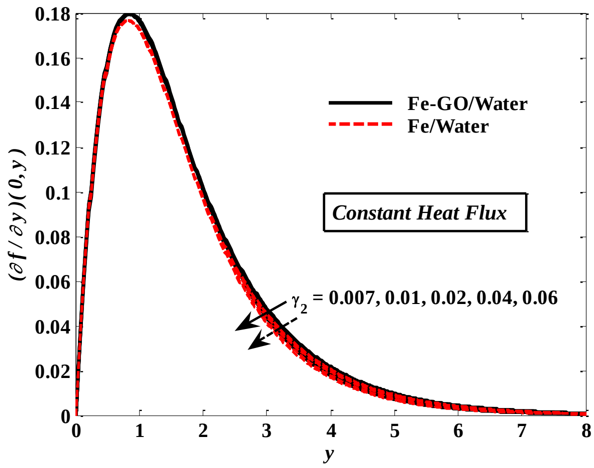

Fig. 9. The influence of γ_2 on $(\partial f/\partial y)(0,y)$

4. Conclusions

Research on the convection heat transfer boundary layer flow in presence of hybrid nanofluid are considered significant value in engineering sciences. To address the affair, this article presents mathematical model and numerical results for the heat transfer impacts for free convection in hybrid nanofluid flow over a solid sphere. The appropriate similarity transformation is utilized to convert the governing equations into partial differential equations. These partial differential equations are numerically solved by Keller box method and programmed with MATLAB program. The obtained numerical results for the effects of hybrid nanofluid parameters on the engineering interesting physical quantities are investigated in an attempt to discuss them through several figures and tables. Also, (CWT) and (CHF) boundary conditions have been considered in this investigation. The following significant observations are concluded of this study, as follows

i. When the nanoparticle volume fraction parameter γ_2 increases, the values of local Nusselt number, local wall temperature, and temperature profile are increased, of both regular nanofluid and hybrid nanofluid flows
ii. The local skin friction coefficient and the velocity profile are increased by an increment of nanoparticle volume fraction γ_2 of both regular nanofluid and hybrid nanofluids flows.
iii. Fe-GO/water hybrid nanofluid has a higher temperature and velocity profile compared with Fe/water nanofluid with an increase in the nanofluid parameter γ_2.
iv. Also, Fe/ Weter nanofluid has lower local skin friction, local wall temperature, and local Nusselt number than Fe-GO/ Water hybrid nanofluid, with (CWT) and (CHF) boundary conditions.
v. Hybrid nanofluid flow represents a more essential character in the operation of heat transfer than a regular nanofluid flow.

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