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Numerical Simulation of Cascade Flow: Vortex Element Method for Inviscid Flow Analysis and Axial Turbine Blade Design

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ABSTRACT

This study aims to design an axial turbine rotor blade and predict the turbine performance at preliminary design stage. Quasi three dimensional method was applied to design including blade to blade flow analysis. The blade profile uses a NACA 0015 airfoil by varying the profile thickness from hub to tip. The profile is divided into eleven segments which has different parameters. The profile was analysed using blade to blade flow/cascade flow analysis called vortex panel method to obtain lift coefficient. The analysis of cascade flow was performed in potential flow and prediction of turbine perfomance is carried out involving common best practice to give drag effect on the blade. The design of the turbine was applied on three different rotors, which also have a different discharge, head, and design rotation. The outer diameter of turbine 1 is 0.65 m, while turbine 2 and turbine 3 have an outer diameter of 0,60 m. The calculation result show that the efficiency of turbines 1, 2, and 3 were 88,32%, 89,67%, and 89,04%, respectively.

1. Introduction

Research on the cascade blade is currently increasing quite rapidly. Cascade blades can be applied to blade design for gas turbine, steam turbine, hydro turbine, pump compressors, blowers, and many other applications. Khattak *et al.*, [1] described the range turbine efficiency for gas turbine, steam turbine and hydro turbine including the advantages and disadvantages. A turbine design tool with high accuracy and efficiency is a challenge in the middle of the era of sophisticated computer technology. Initially, the surface vorticity method was implemented as a tool for analyzing cascade blades. However, Lewis [2] modified it into a tool for designing cascade blades. He used Martensen method derived for the potential flow to calculate the velocity distribution on the airfoil and cascade [3]. Then, back diagonal correction was used to check the design calculation of cascade blades. By adopting the surface vorticity method, the researcher is allowed to design the shape of the airfoil

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and cascade according to desired shape or prescribed pressure distribution. The method of surface vorticity is then called the vortex element method or vortex method.

The vortex method could be implemented in a reverse sense by determining the point of stagnation in the leading edge. This approach can be used to get desired shape. Calculations on airfoils and cascades using the vortex method show promising results compared to the original profile [4]. During its development, the vortex method could be combined with viscous flow and is widely adopted for the design of single airfoil, ground airfoil, airfoil cascade or the impeller using the random walk technique with the Lagrangian scheme [5-8]. The panel method may be applied to increase performance of the sail on the sailboat autonomous part. Sun *et al.,* [8] used the same two airfoils (before and after sail) with various angle of attack. The thicker the airfoil, the farther the airfoil position from the sail and this is a sensitive matter similiar with cascade blade.

The vortex method is a panel method that uses panel discretization during the calculation process. There are several different calculations process for the vortex method, Lewis [2] in his study using vortex constant for the calculations, meanwhile Liu [10] and Boorsma *et al.,* [11] used linear vortex calculation. The difference between the vortex constant and the linear vortex is in the number of matrix coefficients when performing the calculation. Furthermore, vortex method could be optimized to obtain stagger and camber angles on the airfoil cascade to get shock-free conditions [12]. Optimization using the panel method is also widely used, both for potential flow and viscous flow. Hothazie and Mirica [13] and Huang *et al.,* [14] combined the metaheuristic technique with the panel method to obtain a profile shape with a higher lift coefficient than the base profile, while Akram and Kim [15] used another method called Parsec.

The vortex method could be implemented both on airfoil and cascade design. Ng *et al.,* [16] and Yazik *et al.,* [17] investigated the effect of the inlet or angle of attack on the airfoil towards the cascade performance using viscous analysis. However, a study to examine the effect of the inlet or attack angle on the airfoil toward inviscid cascade performance is limited. In general, increasing the inlet or angle of attack will increases the lift coefficient due to lack of skin friction. Susan-Resiga *et al.,* [18], in their study, designed an inviscid cascade at multiple inlet angles with more than 80% accuracy using the finite element method. However, Rose and Raguraman [19] devised the cascade using the vortex method and used CFD airfoil, but the CFD results show a barrier in the periodic section and make the flow like an isolated airfoil.

This study aims to develop a vortex panel computation program for designing axial turbine blades. The input data for the calculation can be obtained from the triangular velocity and meridional flow analysis. Meridional flow analysis was performed based on the mixed vortex criteria described in the studies of Ng *et al.,* [15] and Yazik *et al.,* [16]. The criteria for a mixed vortex are a combination of fee vortex and forced vortex. There are three turbine models to be studied, however this research is limited to modeling the rotor blades and predicting turbine performance. Finally, it is hoped that this program can be used as a design tool with good accuracy.

2. Cascade Blade Design

2.1 Construction of Blade Profile Geometry

Blade profile needs special requirements to provide a stable diffusion flow, especially for the upper surface (suction side). The pressure gradient is always found on the suction side and separation may occur in the viscous flow. On a single airfoil, this could be minimized by injecting the flow into the suction side to reduce aerodynamic losses. Liang *et al.,* [20] showed that this technique could increase C_l and reduce C_d in NACA 0012 with various angles of attack.

There are two methods for designing profile shapes, the direct method and the inverse method. In the direct method, the profiles are designed based on experiment or theoretical analysis to identify the most efficient profile shape with the best aerodynamic performance. On the other hand, the inverse method allows the researcher to determine the velocity or pressure distribution on the blade surface.

In this study, the inverse method was adopted to design the profile shape. Construction of blde profile as shown in Figure 1. Profiles are formulated based on half the thickness of the profile base y_t perpendicular to the camber line. Then, we construct a circle with radius *1/2* and divide it into segments "M" to obtain the the segment φ. Thus, the coordinates of the camber line x_c and airfoil coordinate can be calculated as follows:

$$\frac{x_c}{l} = \frac{1}{2}(1 - cos\phi) \quad (1)$$

$$x_a = x_c - y_t sin\theta_c \quad \text{and} \quad y_a = x_c - y_t cos\theta_c \quad \text{for upper surface} \quad (2)$$

$$x_b = x_c - y_t sin\theta_c \quad \text{and} \quad y_b = x_c - y_t cos\theta_c \quad \text{for lower surface} \quad (3)$$

Let x_a and y_a are coordinate of upper surface of airfoil, x_b and y_b are cordinate at lower surface and ϑ_c is camber angle. If the arifoil is simetry, then $x_a = x_b$ and $y_a = y_b$.

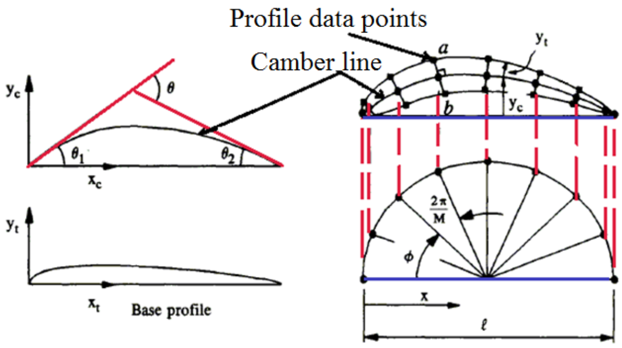

Fig. 1. Construction of blade based on camber line

2.2 Cascade Blade Parameter

Cascade blade is consist of infinite number of airfoils with unique parameters. Some of these parameter are pitch chord ratio *t/l*, stagger angle λ and camber angle ϑ, inlet angle β_1 and outlet angle β_2. These paramater describe in the Figure 2 below. The analysis of the cascade blades can be carried out using the volume control method as shown in Figure 2. Fluid flow through the blade cascade with velocity W_1 and inlet angle β_1 will be deflected and produce velocity vector W_2 and outlet angle β_2. The flow produces aerodynamic forces on the blade surface as in the cascade velocity triangle in Figure 2.

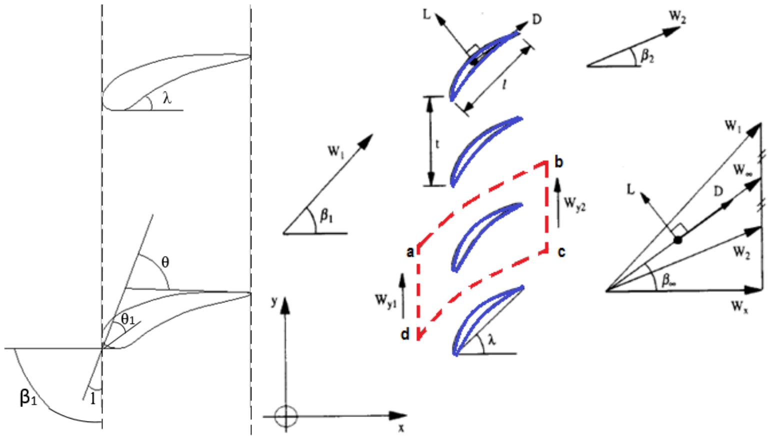

Fig. 2. Geometry and triangle velocity of cascade

3. Analysis of Cascade Blade

3.1 Surface Vorticity Model and Martensen Method

To apply the panel vortex method on the body surface, the uniform flow with prescribed velocity induced by vortex strength $\gamma(s)$ along with x direction, while s is measured clockwise through body surface. The velocity of the body surface (dq_{mn}) on point s_m induced by the vortex element $\gamma(s)ds_n$ at surface s_n will be perpendicular to the radial vector r_{mn} (see Figure 3). Following Bio Savart's law

$$dq_{mn} = \frac{\gamma(s)ds_n}{2\pi r_{nm}} \quad (4)$$

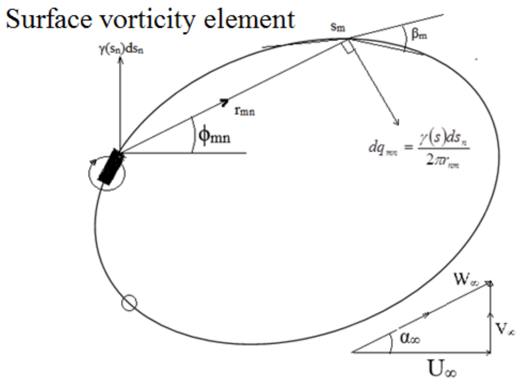

Fig. 3. Surface vorticity model for a two-dimensional body

To simplify the calculation, the velocity dq_{mn} is divided into two components in the x and y direction by solving the Eq. (4). The velocity dq_{mn} is assumed to be parallel with the body surface at m, where the profile slope is defined as βm at s_m. Hence the velocities of the x and y directions are obtained as follows

$$dU_{smn} = \frac{\gamma(s_n)ds_n}{2\pi r_{nm}} \sin\varphi_{nm} = \left(\frac{y_m - y_n}{2\pi r_{mn}^2}\right)\gamma(s_n)ds_n \quad (5)$$

$$dV_{smn} = \frac{\gamma(s_n)ds_n}{2\pi r_{nm}} \cos\varphi_{nm} = \left(\frac{x_m - x_n}{2\pi r_{mn}^2}\right)\gamma(s_n)ds_n \quad (6)$$

U and *V* are the components of velocity parallel to the *x*-axis and *y*-axis. Suppose a parallel flow passes through the body surface s_m. In this case, the calculation is carried out through a simple method in the form of a straight line or a panel with several pivotal points M located in the center of the panel representing the body surface. The vital point M is represented as several straight line elements of length Δs_n at the vital point (x_n, y_n). Furthermore, the Martensen's integral could transformed into a linear equation by replacing the vortex sheet with some vortex elements $\gamma(s_n)\, ds_n$ on the surface of the object.

$$\sum_{n=1}^{M\sum_m} K(s_m, s_n)\gamma(s_n) = -U_\infty \cos\beta_m - V\sin \tag{7}$$

where *K* is matrix coupling cooefficient. *K* (s_m, s_n) could modify by linking points s_m and s_n, then,

$$K(s_m, s_n) = \frac{\Delta s_n}{2t}\left\{\frac{(y_m - y_n)\cos\beta_m - (x_m - x_n)\sin\beta_m}{(x_m - x_n) - (y_m - y_n)}\right\} \tag{8}$$

If *m* = *n* then the coupling coefficient become:

$$K(s_m, s_m) = -\frac{1}{2} - \frac{\Delta\beta_m}{4\pi} \tag{9}$$

$$K(s_m, s_m) \approx -\frac{1}{2} - \frac{\beta_{m+1} - \beta_{m-1}}{8\pi} \tag{10}$$

3.2 Back Diagonal Correction

The application of the coupling coefficient to airfoil faces two problems, the inaccuracy and the need of Kutta conditions on the trailing edge. In this case, the back diagonal correction could be used to increase the accuracy of the coupling coefficient. Back diagonal correction indicates the mutual impact of vortex elements on the coupling matrix. Figure 4(a) shows the procedure of the back diagonal correction in the coupling matrix.

$$K(s_{M+1-m}, s_n) = \frac{1}{\Delta s_{M+1-m}} \sum_{\substack{n=1 \\ n \neq M+1-m}}^{M} K(s_n, s_m)\Delta s_n \tag{11}$$

3.3 Wilkinson Kutta Condition

The flow condition strongly influences the total vortex on the airfoil surface at the trailing edge (TE). Therefore, the vorticity could not be determined at the beginning of the calculation. The flow on the upper and lower surface would achieve a trailing edge with velocity $v_s = \gamma(s)$. In order to increase the accuracy of the calculation, the fluid flow passing through the body surface must be treated at the same conditions, on velocity and vorticity distribution. It could be achieved by giving the opposite sign for the velocity and vorticity distribution on the upper and lower surface shown in Eq. (12). This condition is known as the Kutta condition, as illustrated in Figure 4(b).

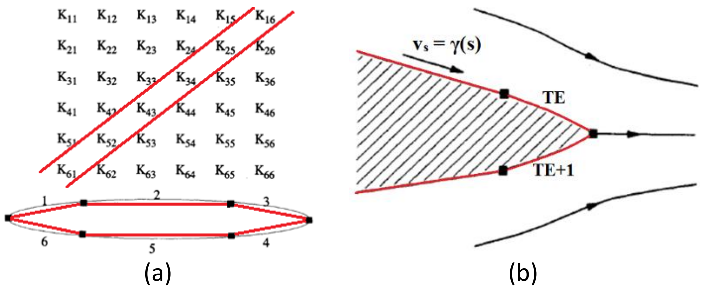

Fig. 4. (a) Back diagonal correction of matrix coefficient, (b) Kutta condition

$$\gamma(s_{te}) = -\gamma(s_{te+1}) \tag{12}$$

The final result of this calculation is the prediction of the lift coefficient C_l. In this study, the lift coefficient is an important parameter for predicting turbine performance. The pitch chord ratio *t/l* influences the calculation of the lift coefficient on the cascade. If *t/l* > 30, the calculation would be considered a single airfoil. The calculation of the lift coefficient on the cascade is calculated as follows

$$C_l = 2\frac{t}{l}(tan\,\beta_1 - tan\,\beta_2)\,cos\,\beta_\infty \tag{13}$$

3.3 Calculation Test of Cascade

The cascade flows were analyzed using a computation program (author's program) based on the panel vortex method. The authors used the C4 profile to compare the outlet angle (see Table 1) and pressure coefficient distribution Cp (see Figure 5) with Goestelow's exact solution, surface vortices theory, and Schlichting linearization theory. According to Lewis [21], Goestelow's exact solution could predict the best shape of blade profile through experiments with conformal transformations. However, the author tried to develop a program to approach Goestelow's with more accurate results. Computations were carried out at two different inlet angles, namely -35° and 35°, with a camber angle of 70°, a stagger angle of 0°, and a chord pitch ratio of *t/l* 0.900364. The results are presented in Table 1 and Figure 5 with slight differences but are satisfactory. Hence, further studies could use this program to design cascade blades for turbines and compressors.

Table 1
Comparison of cascade analysis calculation test

Method	β_1 = 35°	β_1 = -35°
Exact solution (Goestelow, 1984)	23.80°	24.84°
Surface vorticity theory	23.85°	25.04°
Schlichting linearised theory	20.28°	22.57°
Author program	23.13°	24.38°

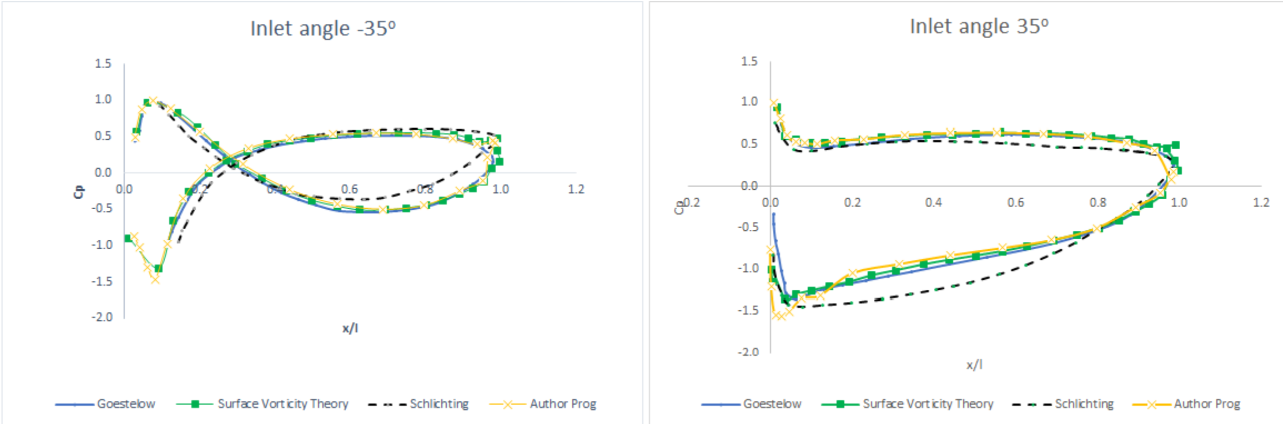

Fig. 5. Cp comparison of several cascade analysis methods

4. Result and Discussion

4.1 Global Parameter Design of Rotor

This study aims to design and predict the performance of three axial turbine rotors, which are Turbine 1, Turbine 2, and Turbine 3, with different design parameters. The global parameter design of the three axial turbine rotors is shown in Table 2.

Table 2
Design parameters of three axial turbine rotors

No.	Name of Turbine	Head (H)	Discharge flow (Q)	Inner Diameter	Outer Diameter
1	Turbine 1	4.5 m	1.500 l/sec	0.25 m	0.65 m
2	Turbine 2	3.5 m	2.500 l/sec	0.21 m	0.60 m
3	Turbine 3	5.0 m	1.200 l/sec	0.21 m	0.60 m

Meridional flow analysis and velocity triangle were performed based on Table 2. The details of its analysis were not discussed in this paper. Meridional flow analysis and velocity triangle provide β_1, β_2, camber angle ϑ, and stagger angle λ as input to the cascade analysis. Afterward, the input data is calculated using the panel vortex method, the results are shown in Table 3.

Table 3 shows the blade profiles on the hub, mean, and tip representing the eleven segments. Initially, each turbine was designed to use five blades, however there is a wide gap in the rotor due to chord length of the turbine blade. There are two ways to calculate the chord length. The first uses *t/l* as a function of the lift coefficient (see Eq. (13)), and the second uses *t/l* as a function of the diffusion factor (see Eq. (14)). In this study, the turbine chord length was analyzed by involving the diffusion effect on the blade surface. Therefore, the authors use Eq. (14) for calculated the chord length. The diffusion factor was 0.019 for all blade segments. The shape of the blade profiles on the hub, mean, and tip is shown in Figure 6.

$$DF = 1 - \frac{cos\beta_1}{cos\beta_2} + \frac{cos\beta_1}{2} + \frac{t}{l}(tan\,\beta_1 - tan\,\beta_2) \tag{14}$$

Figure 6 presents the comparison of the rotor blade profiles on the mean, hub, and tip. It shows that each rotor has different profiles. The profile was sketched into a 3-dimensional shape to create the actual rotor shape and become the basis for virtual and experimental testing. The 3-dimensional rotor blade model is shown in Figure 7.

Table 3
Design parameter of stator and rotor blade

Description	Turbine 1			Turbine 2			Turbine 3		
Section	Hub	Mean	Tip	Hub	Mean	Tip	Hub	Mean	Tip
Tangential vel., c_ϑ (*m/s*)	5.98	4.49	4.29	6.66	4.66	4.28	6.78	4.67	4.28
Axial vel. c_x (m/s)	5.31	5.31	5.31	10.52	10.52	10.52	4.84	4.84	4.84
Inlet Angle, β_1 (°)	-3.2	-56.79	-70.67	-16.98	-64	-78.01	-3.61	-58.42	-74.6
Outlet Angle, β_2 (°)	-46.28	-64.8	-75.98	-54.31	-71.45	-80.36	-50.51	-68.75	-78.09
Stagger Angle, λ (°)	-24.74	-57.8	-73.33	-35.65	-67.72	-79.18	-27.06	-63.58	-76.35
Camber Angle, ϑ (°)	64.62	21.02	7.97	56.00	11.17	3.53	70.34	15.49	5.23
Chord, l (mm)	173.25	308.59	479.48	116.53	131.91	176.44	141.57	135.73	178.26
Number of blades, z_r	5	5	5	5	5	5	5	5	5

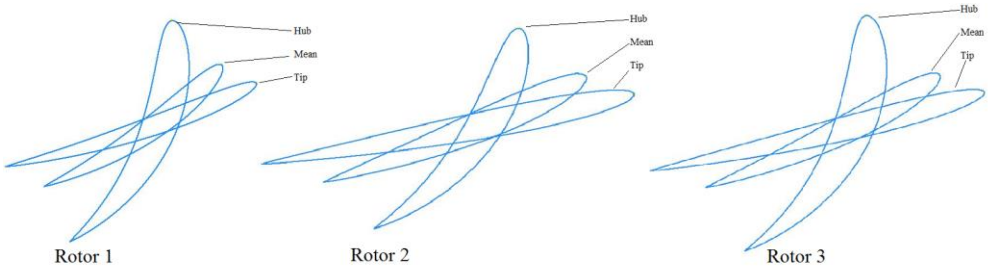

Fig. 6. Profile of Turbine 1, Turbin 2 and Turbine 3 at hub, mean and tip

The rotor model in Figure 7 shows a wide gap between blades. This condition occurred due to the diffusion factor at the design calculation. In this case, the number of blades would be multiplied by two when considering the diffusion factor [22]. Therefore, the rotor would have more blades, as presented in Figure 8.

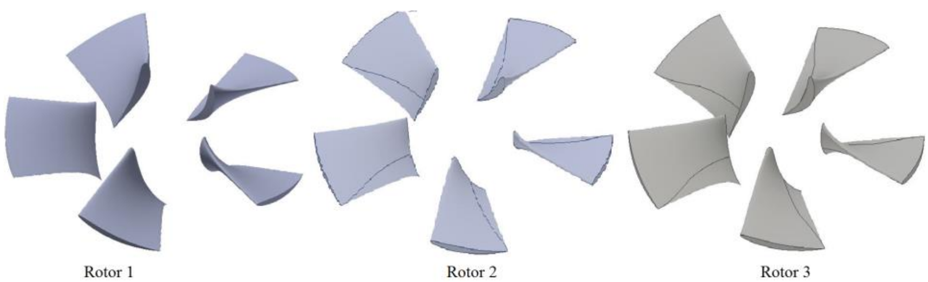

Fig. 7. Rotor model for Turbine 1, Turbin 2 and Turbine 3

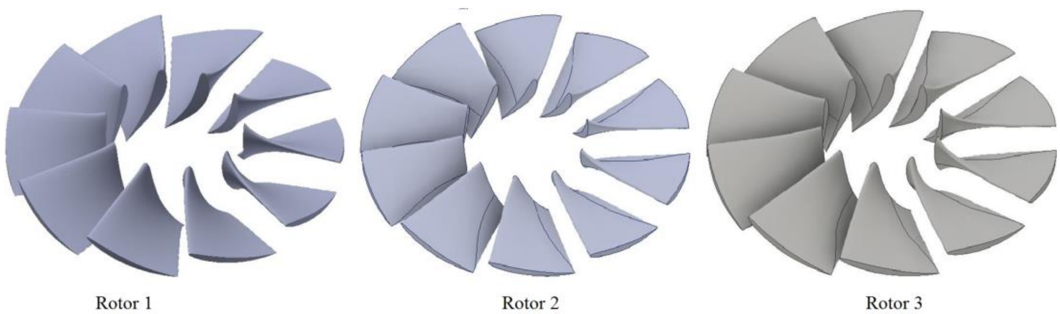

Fig. 8. Final model of Turbine 1, Turbin 2 and Turbine 3

The rotor, as shown in Figure 8, has more blades compared with Figure 7. It shows that the gap between the blades becomes narrow and would reduce losses. Furthermore, the prediction of rotor performance is carried out based on cascade analysis with several parameters such as C_l, drag effect

(5% of C_l), and chord length. The results of the rotor performance are quite satisfactory. It shows that each turbine's efficiency is higher than 88%, as presented in Table 4. However, Further studies require experimental or virtual testing to obtain actual results.

Table 4
Performance of turbine design at preliminary stage

No.	Rotor	Power of Fluid, P_f (Watt)	Power of Turbine, P_T (Watt)	Efficiency, η
1	Turbine 1	66,098.31	58,375.70	88.32%
2	Turbine 2	85,682.99	76,831.44	89.67%
3	Turbine 3	58,754.05	52,316.95	89.04%

4. Conclusions

This study has performed a cascade flow analysis using a computational program and compared it with a similar program. Overall, the results show that the development of the program has been accomplished and satisfying. This program could be used as a design tool for turbomachinery blades in compressors, turbines, and other applications in the preliminary design stage. However, to get more deep results regarding the cascade flow, a three-dimensional test should be carried out using virtual or experimental tests. One of the cheapest ways is through observation using CFD.

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