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Unsteady Boundary Layer Flow Over a Permeable Stretching/Shrinking Cylinder Immersed in Nanofluid

Nor Fadhilah Dzulkifli[1], Norfifah Bachok[2], Nor Azizah Yacob[1,*], Norihan Arifin[2], Haliza Rosali[2], Ioan Pop[3]

[1] Faculty of Computer and Mathematics Sciences, Universiti Teknologi MARA Pahang, Jengka Campus, Pahang, Malaysia
[2] Department of Mathematics and Institute for Mathematical Research, Universiti Putra Malaysia, UPM, Serdang Selangor, Malaysia
[3] Department of Mathematics, Babes-Bolyai University, Cluj-Napoca, 400084, Romania

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ABSTRACT

In this study, the unsteady boundary layer flow over a stretching/shrinking cylinder immersed in copper (Cu)-water nanofluid with the presence of suction effect is analyzed. The governing partial differential equations are converted to ordinary differential equations using similarity transformation. The bvp4c solver in Matlab software is applied to solve the system of ordinary differential equations where the numerical solutions are obtained and presented graphically. The study aims to investigate the effects of nanoparticle volume fraction, the unsteadiness parameter, the stretching/shrinking parameter on the velocity and temperature gradients. It is found that the dual solutions are obtained in a specific range of these parameters for both stretching and shrinking cylinders. Besides, a high volume of the nanoparticles in the base fluid increases the velocity gradient and decreases the temperature gradient at the surface. Also, increasing nanoparticle volume fraction in the base fluid expands the range of solutions, which denotes the boundary layer separation from the surface has been delayed. The stability analysis is performed by introducing a new dimensionless variable to determine the stability of the solutions. In this phase, the smallest eigenvalue obtained shows that the first solution is stable and physically realizable while the second solution is not stable.

1. Introduction

The unsteady boundary layer problems have recently attracted the attention of many researchers where the considered parameters were depending on time; for example, Fang *et al.,* [1] studied the expanding stretching cylinder in a viscous fluid where they identified that the Reynold number and the unsteadiness parameter controlled the fluid flow. The shrinking cylinder of the unsteady problem in viscous flow was proposed by Zaimi *et al.,* [2] where the impact of mass suction and unsteadiness parameters on the velocity profile and velocity gradient has been analysed numerically. Marinca and Ene [3] solved a similar problem with Zaimi *et al.,* [2] using an analytical method, namely the optimal

* *Corresponding author.*
E-mail address: norazizah872@uitm.edu.my

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homotopy asymptotic method (OHAM) to obtain the solutions. Their results showed a decent concurrence with the results generated numerically.

After that, Zaimi *et al.,* [4] expanded their study by considering heat and mass transfer in nanofluid using Buongiorno's model, where the numerical solutions were obtained by the shooting method. They found that the unsteadiness parameter decreases the skin friction coefficient, local Nusselt number, and Sherwood number. Besides, the Brownian motion parameter also decreases both local Nusselt and Sherwood numbers, respectively. A similar nanofluid model had been used by Mahdy and Chamkha [5] in their study and besides, they considered non-Newtonian nanofluid. As a result, both the increasing unsteadiness parameter and Brownian motion decreased the heat and mass transfer rates at the surface.

Abbas *et al.,* [6] extended the work by Zaimi *et al.,* [4] by considering the partial slip condition of the flow over a porous stretching/shrinking cylinder. They presented the numerical solutions graphically and found that dual solutions exist for a shrinking surface. It was found that increasing the unsteadiness parameter decreases the magnitude of the skin friction coefficient but increases the temperature profile. A similar study had been considered by Azam *et al.,* [7] in MHD Carreau nanofluid where the effect of thermal radiation is taken into account. They reported unique solutions exist and found that increasing the value of radiation parameters decreases the thermal boundary layer for dilatant and Pseudoplastic fluids. Further, Khan *et al.,* [8] studied the effects of partial slip and suction parameters on the shrinking/stretching surface immersed in Williamson nanofluid. It was observed that the magnitude of skin friction coefficient decreases with increasing partial slip parameter while the heat and mass transfer rates at the surface increase. Further, by increasing suction parameters, all the magnitude of skin friction coefficient, heat and mass transfer rate are increased for the first solution.

Recently, Awaludin *et al.,* [9] investigated the flow stability past a shrinking cylinder by considering prescribed surface heat flux as the boundary condition. They discovered that increasing the curvature parameter delays the boundary separation and dual solutions only obtained for a shrinking surface. A new class of unsteady flow was introduced by Fang [10], which flow over a long thin cylinder was considered and the end of the cylinder is either accretion or ablation. It was noted that for accretion, a unique solution was obtained, whereas dual solutions exist for ablation. Further, Yashkun *et al.,* [11] studied the stagnation flow over a stretching/shrinking surface in Casson fluid with heat source and injection effect. They found dual solutions exist only for shrinking surface and a stability analysis was performed to determine the stability of solutions. Parvin *et al.,* [12] analysed the stability of the solutions of MHD unsteady mixed convection flow towards an exponentially stretching/shrinking surface in Casson fluid and found that the first solution is stable whilst the second solution is not stable. Moreover, the most recent studies on the boundary layer flow over a cylinder can be found in [13-16].

In the present study, we extend the work of Abbas *et al*., [6] by investigating the flow and heat transfer characteristics of unsteady boundary layer flow towards a stretching/shrinking cylinder immersed in Cu-water nanofluid using a model introduced by Tiwari and Das [17]. The governing partial differential equations are transformed into a system of ordinary differential equations by applying similarity transformation using the appropriate similarity variables. The numerical solutions are obtained using a built-in bvp4c solver in Matlab and presented graphically. A stability analysis is performed since dual solutions are obtained to identify the stability of solutions.

2. Problem Formulation

Consider an unsteady two-dimensional laminar flow past a permeable stretching/shrinking surface immersed in nanofluid as illustrated in Figure 1. The fluid is assumed incompressible and the radius of the cylinder is a function of time with unsteady radius, $r = a(t) = a_0\sqrt{1-\beta t}$ where a_0 is a constant, β is an expansion/contraction strength constant, and t is a time. Let u and w represent the velocity component in r and z directions, respectively.

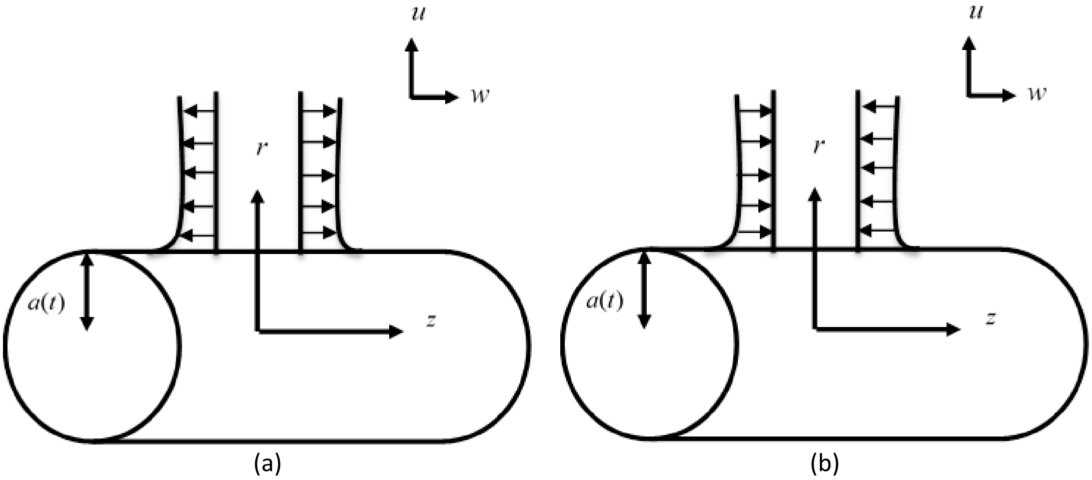

Fig. 1. Physical models and coordinate system; (a) stretching cylinder, (b) shrinking cylinder

Based on the assumptions above, Abbas *et al.,* [6] and Tiwari and Das [17], the governing equations of the system can be written as follows

$$\frac{1}{r}\frac{\partial(ru)}{dr}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial r}+\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}\right) \tag{2}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial r}+\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right) \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\frac{k_{nf}}{(\rho c_p)_{nf}}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right) \tag{4}$$

subject to boundary conditions

$$\begin{aligned}
&t<0: u=0,\ w=0,\ T=T_\infty,\ \text{ for all } r \text{ and } z,\\
&t\geq 0: u=w_w,\ w=U_w,\ T=T_w \text{ at } r=a(t),\\
&w\to 0,\ T\to T_w \text{ as } r\to\infty,
\end{aligned} \tag{5}$$

where z and r are cylindrical polar coordinates measured in axial and radial directions, p is the pressure of the nanofluid, ρ_{nf} is the density of nanofluid, μ_{nf} is the dynamic viscosity of nanofluid,

T is the temperature of the nanofluid, k_{nf} is the thermal conductivity of nanofluid, $\left(\rho c_p\right)_{nf}$ is the heat capacity of nanofluid and $U<0$ is the constant mass transfer (suction) and $U>0$ is the constant mass transfer (injection) velocities. The velocities in r and z directions and surface temperature T_w are given by

$$w_w = \frac{U}{\sqrt{1-\beta t}}, U_w = \frac{\varepsilon}{a_0}\frac{4\nu_f z}{1-\beta t}, T_w = T_\infty + \frac{bz}{a_0 \nu_f (1-\beta t)}, \text{where } b(>0) \text{ is a constant.} \tag{6}$$

In order to transform the partial differential Eq. (1)-(4) to ordinary differential equations, we apply the related parameters as mentioned by Tiwari and Das [17] (for Eq. (5)-(8)) and introduce the following similarity transformation:

$$u = -\frac{1}{a_0}\frac{2\nu}{\sqrt{1-\beta t}}\frac{f(\eta)}{\sqrt{\mu}}, \quad w = \frac{1}{a_0^{\ 2}}\frac{4\nu z}{1-\beta t} f'(\eta), \ \eta = \left(\frac{r}{a_0}\right)^2 \frac{1}{1-\beta t}, \quad \theta(\eta) = \frac{T-T_\infty}{T_w - T_\infty}. \tag{7}$$

Substituting Eq. (6) and (7) into Eq. (1)-(5), then we obtain the following ordinary differential equations

$$\chi(\eta f''' + f'') + f''f - f'^2 - A(\eta f'' + f') = 0 \ \text{ where } \chi = 1/(1-\varphi)^{2.5}\left[1-\varphi+\varphi\left(\rho_s/\rho_f\right)\right], \tag{8}$$

$$\frac{\zeta}{\Pr}(\eta\theta'' + \theta') - \left[\theta f' - f\theta' + A(\eta\theta' + \theta)\right] = 0 \ \text{ where } \zeta = k_{nf}/k_f\left[1-\varphi+\varphi\left(\left(\rho c_p\right)_s/\left(\rho c_p\right)_f\right)\right], \tag{9}$$

subject to the boundary conditions

$$\begin{aligned} &f(\eta) = s, \ f'(\eta) = \varepsilon, \ \theta(\eta) = 1 \text{ at } \eta = 1, \\ &f'(\eta) \to 0, \ \theta(\eta) \to 0 \text{ as } \eta \to \infty, \end{aligned} \tag{10}$$

where primes denote the differentiation with respect to η, $s = -ua_0/2\nu > 0$ is the mass suction parameter whereas $s = -ua_0/2\nu < 0$ is the mass injection parameter. Further, $A = a_0^{\ 2}\beta/4\nu$ is the unsteadiness parameter and ε is the stretching/shrinking parameter where $\varepsilon<0$ for shrinking sheet $\varepsilon>0$ for stretching sheet, φ is the Cu nanoparticle volume fraction, ρ_s is the density of solid fraction, ρ_f is the density of fluid, $\left(\rho c_p\right)_s$ is the heat capacity of solid fraction and $\left(\rho c_p\right)_f$ is the heat capacity of fluid.

The physical quantities of interest in this problem are the skin friction coefficients C_f and Nusselt number Nu, which can be defined as

$$C_f = \frac{\tau_w}{\rho w_{\ w}^2}, \quad Nu = \frac{a(t)q_w}{k_f(T_w - T_\infty)} \text{ where } \tau_w = \mu_{nf}\left(\frac{\partial w}{\partial r}\right)_{r=a(t)} \text{ and } q_w = k_{nf}\left(\frac{\partial T}{\partial r}\right)_{z=a(t)} \tag{11}$$

Substituting (7) into (11), we obtain

$$C_f(z/r) = \frac{1}{(1-\varphi)^{2.5}} f''(1) \ , \ Nu = -2\frac{k_{nf}}{k_f}\theta'(1) \,. \tag{12}$$

3. Stability Analysis

A stability analysis has been applied to determine which solution is more stable since dual solutions are obtained for this case. Based on Weidman *et al.,* [18], a dimensionless time variable τ needs to be introduced then we have

$$\tau = \frac{4r^2}{a_0{}^4\beta(1-\beta t)},\ u = -\frac{2v_f}{a_0\sqrt{1-\beta t}}\frac{f(\eta,\tau)}{\sqrt{\eta}},\ w = \frac{4v_f z}{a_0{}^2(1-\beta t)}\frac{\partial f}{\partial\eta}(\eta,\tau),\ \eta = \left(\frac{r}{a_0}\right)^2\frac{1}{1-\beta t},$$
$$\theta(\eta,\tau) = \frac{T-T_\infty}{T_w-T_\infty}. \tag{13}$$

Further, by doing a similar derivation as reported by Parvin *et al.,* [12] and Ismail *et al.,* [19], the linearized eigenvalue problem is obtained as follows:

$$\chi(\eta F_0''' + F_0'') + f_0''F_0 + F_0''f_0 - 2f_0'F_0' - A\eta F_0'' - AF_0' + \eta\gamma F_0' = 0 \tag{14}$$

$$\frac{\zeta}{\Pr}(\eta G_0'' + G_0') + F_0\theta_0' + f_0G_0' - G_0f_0' - F_0'\theta_0 - A\eta G_0' - AG_0 + \eta\gamma G_0 = 0 \tag{15}$$

subject to boundary conditions,

$$F_0(\eta) = 0,\ F_0'(\eta) = 0, G_0(\eta) = 0 \text{ at } \eta = 1,$$
$$F_0'(\eta) \to 0,\ G_0(\eta) \to 0 \text{ as } \eta \to \infty \tag{16}$$

4. Results and Discussion

The effects of Cu nanoparticle volume fraction, φ, unsteadiness parameter, A and stretching/shrinking parameter, ε when Pr=6.2 on the velocity and temperature gradients were considered in this paper. By solving the ordinary differential Eq. (8) and Eq. (9) subject to boundary conditions (10) using bvp4c solver in Matlab, the numerical results obtained have been compared with Abbas *et al.,* [6] for certain values as shown in Table 1, which shows a good agreement. The thermophysical properties of water and Cu nanoparticles considered in this study are as reported by Oztop and Abu-Nada [20]. Figure 2 and Figure 3 depict the velocity and temperature gradients for a few values of nanoparticle volume fraction, φ and unsteadiness parameters, A where it shows that dual solutions are obtained. Based on Figure 2 and Figure 3, dual solutions can be categorized as the first solution and the second solution where the dual solutions exist for negative values of *A* in the range of $A < A_c$. Besides that, a unique solution can be found at $A = A_c$ and no solution for $A > A_c$. It is seen that the velocity gradient of the first solution is increasing due to the increase of φ but the second solution shows the opposite results, as shown in Figure 2. However, increasing φ decreases the temperature gradient for both solutions.

Table 1
Various values of $f''(1)$ for different values of *A* when $\varepsilon = -1$ and $s = 0.1$

A	Abbas *et al.*, [6]		Present results	
	First solution	Second solution	First solution	Second solution
-4.0	3.84077	-24.88224	3.84077	-24.88224
-3.5	3.29909	-17.47316	3.29909	-17.47316
-3.0	2.73978	-11.45527	2.73978	-11.45527
-2.5	2.14665	-6.71496	2.14665	-6.71496
-2	1.46820	-3.10294	1.46820	-3.10294

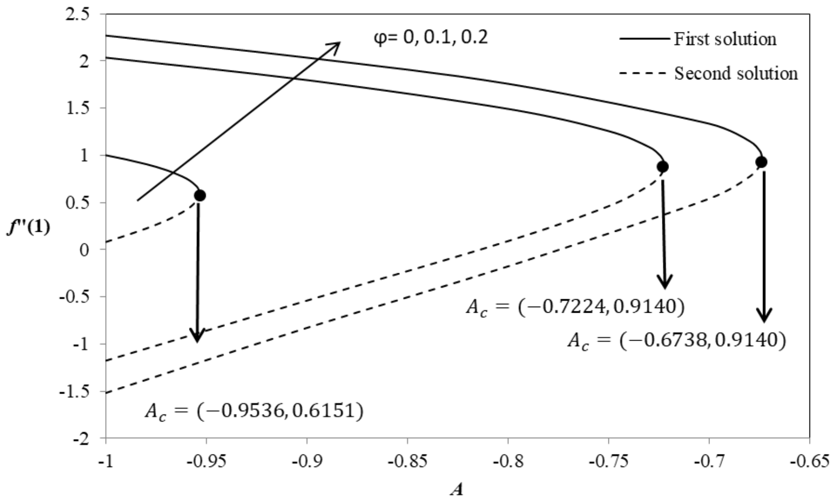

Fig. 2. Variations of $f''(1)$ with A for different values of φ when $s = 1$ and $\varepsilon = -1$

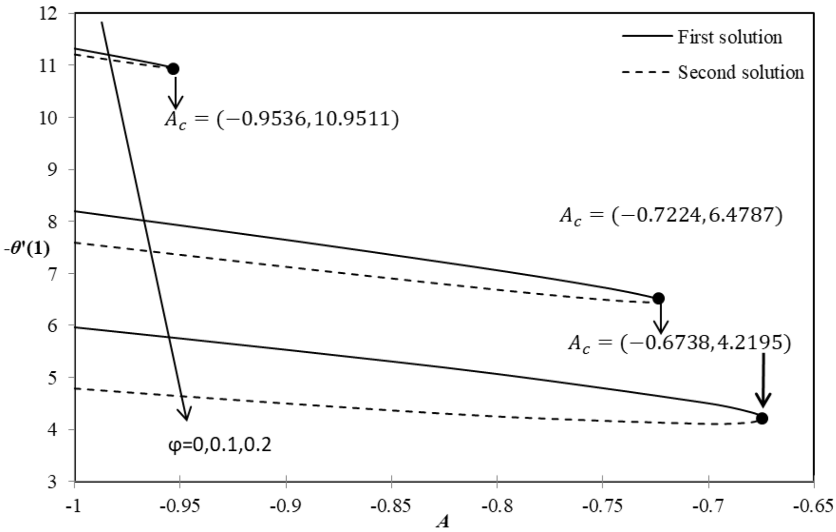

Fig. 3. Variations of $-\theta'(1)$ with A for different values of φ when $s = 1$ and $\varepsilon = -1$

The influence of Cu nanoparticles in the base fluid on the velocity gradient and temperature gradient with the effect of stretching/shrinking parameters are displayed in Figure 4 and Figure 5. Figure 4 proves that the existence of Cu nanoparticles in water increases the velocity gradient compared to the absence of Cu nanoparticles in the base fluid. Meanwhile, increasing φ is seen to decrease the temperature gradient for both solutions as illustrated in Figure 5.

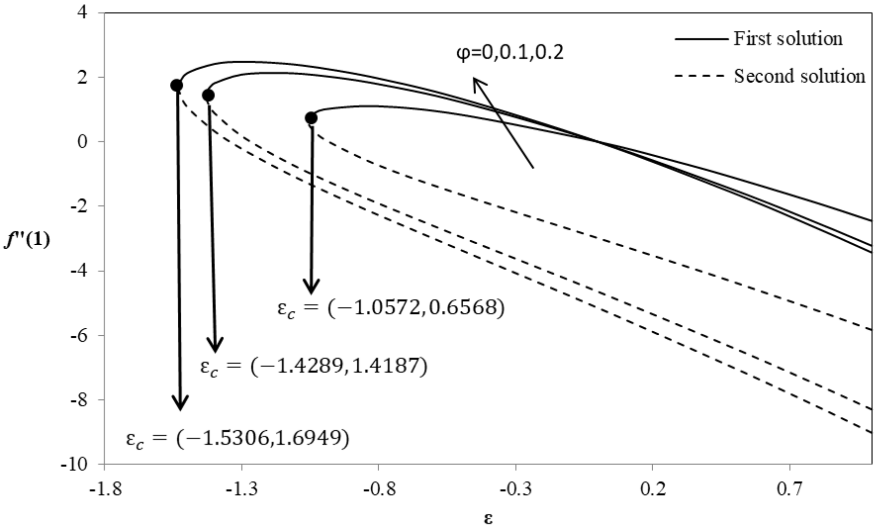

Fig. 4. Variations of $f''(1)$ with ε for different values of φ when $s=1$ and $A=-1$

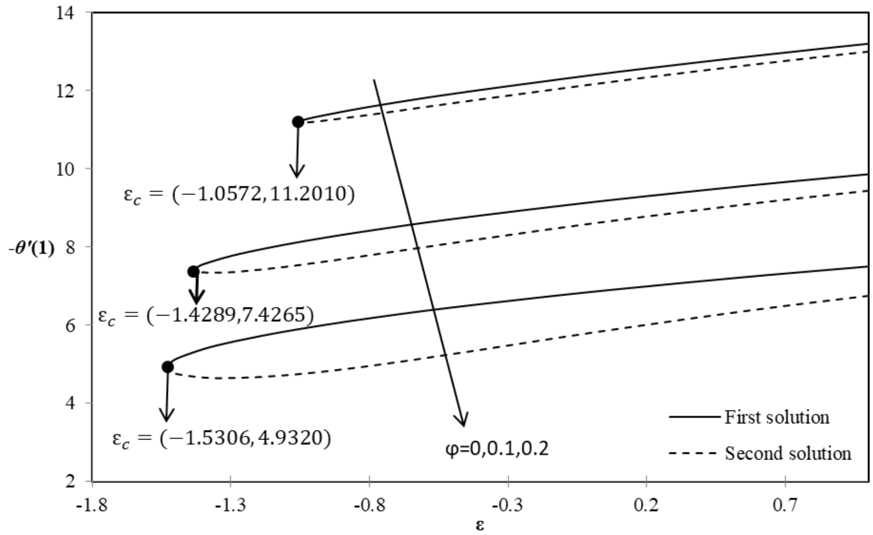

Fig. 5. Variations of $-\theta'(1)$ with ε for different values of φ when $s=1$ and $A=-1$

Table 2 is tabulated to present the smallest eigenvalues, which determine the stability of the solutions obtained for different φ. A solution is stable and physically realizable if the eigenvalues are positive and unstable if the values are negative. Based on Table 2, it is found that the smallest eigenvalues for the first solution are positive, which indicates that it is a stable and realizable solution but not for the second solution since it has negative eigenvalues.

Table 2
The smallest eigenvalue γ for different φ

φ	ε	γ (First solution)	γ (Second solution)
0	-1.5072	0.1903	-0.1718
	-1.5071	0.1908	-0.1738
	-1.5070	0.1912	-0.1752
0.1	-1.4288	0.2160	-0.2312
	-1.4287	0.2164	-0.2336
	-1.4280	0.2183	-0.2428
0.2	-1.5307	0.2226	-0.2394
	-1.5305	0.2292	-0.2470
	-1.5300	0.2308	-0.2552

5. Conclusion

The unsteady boundary layer flow over a stretching/shrinking cylinder in Cu-water nanofluid was investigated using Tiwari and Das model. It was found that dual solutions can be found for both stretching/shrinking cases for unsteady boundary layer flow problems passed through the cylinder when the value of the unsteadiness parameter is negative $(A<0)$. Besides that, increasing Cu nanoparticle volume fraction, φ in the water tended to delay the boundary layer separations, which lead to expanding the range of the solutions obtained. In addition, the velocity gradient at the surface increased when φ increased, however, the temperature gradient was decreased for the same effect. The stability analysis showed that the first solution was the stable solution and physically realizable.

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