



Boundary Layer Flow of Dusty Williamson Fluid with Variable Viscosity Effect Over a Stretching Sheet

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ABSTRACT

Numerical investigation of the boundary layer flow of Williamson fluid with the presence of dust particles over a stretching sheet is carried out by taking into account the variable viscosity effect and Newtonian heating boundary condition. The genuinely two-phase flow model which has been proved to be compatible to present the mutual relationship between non-Newtonian fluid and solid particles is considered in this present study. To be precise, the governing equations are initially transformed into ordinary differential equations through formulation process before proceeding further with the numerical computation by using Keller-box method. The resulting equations are then programmed in MATLAB software. The obtained numerical results are validated with existing study found in open literature and a good agreement is achieved. The influence of pertinent parameters on velocity and temperature profiles, skin friction coefficient together with Nusselt number is presented in graphical and tabular forms. Results revealed that the increasing Williamson parameter decreases the fluid velocity of both fluid and dust phases. It is expected that the present numerical results could conceivably help in predicting the boundary layer problem arising in two-phase flow in the future.

1. Introduction

Research activities on boundary layer flow associated with two-phase problem have been extensively published due to its many engineering applications such as purification of crude oil, blood flow, sedimentation, and etc. This problem involves mutual relationship between two-phase or components in a flow system and able to interact simultaneously with each other. Note that, the main focus of this present study is to delve into the flow of fluid (fluid phase) embedded with solid (solid phase) in the form of dust particles. One of the important aspects of this problem for fluid

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phase is that it can be treated as Newtonian and non-Newtonian fluids which stimulates continuing interest within this area of research. Theoretical works covering on suspension of dust particles in Newtonian fluid with dissimilar circumstances have been reported by several researchers, as documented in [1-5]. A recent study reported by Kumar *et al.*, [6] caters the problem on dusty fluid flow under the effect of mixed convection over a stretching cylinder by taking into account the nanoparticles. Meanwhile, Gajjela and Nandkeolyar [7] concentrated on the viscous dissipation effect on dusty fluid and Koriko *et al.*, [8] unravel the behaviour of fluid flow displayed by dusty Carreau nanofluid. A few more interesting results on non-Newtonian based fluid incorporated with dust particles for diverse situations have been acquired by many authors [9-13]. With regard to Jeffrey fluid, Kasim *et al.*, [14] presented the solution of the aligned magnetic flow containing spherical particles over a vertical stretching sheet and in subsequent years, they took the initiative to measure the thermal radiation effect with similar condition [15]. On the other hand, recently, Dinesh *et al.*, [16] and Bilal *et al.*, [17] obtained the exact solution for magnetohydrodynamic (MHD) flow problem of dusty viscoelastic fluid in irregular channel and rotating frame, respectively. Apart from that, research activities on non-Newtonian fluid with miscellaneous circumstances have been widely investigated [18-28].

All the studies reviewed so far focused on the fixed physical properties which suggested that the viscosity of fluid is constant or changes with shear stress. Nevertheless, there are certain elements that are noteworthy to be considered which may influence the particular property such as temperature and in view of this, further investigation is required [29]. In the case of single phase flow of Newtonian fluid, Tshehla [30] examined the temperature dependent variable viscosity on boundary layer flow across inclined plane. They also point out the pronounced changes in the fluid flow structure on account of varying the viscosity with temperature. Under the category of non-Newtonian fluid, a number of studies concerning stretching sheet flow for Williamson fluid have comprehensively described in [31-33] and for Eyring Powell fluid, can be found in [34-35]. Some more problems on the impact of viscosity dependency have also been published, however, the main focus is on two-phase flow [36-38].

In view of all that have been mentioned so far, the growing literature in investigating viscosity dependency for various conditions can be noticed which proved that this effect is gaining prominence and there has been little discussion involving two-phase flow. Besides, the present study aims to evaluate the two-phase flow of dusty Williamson fluid over a vertical stretching sheet by highlighting the effect of variable viscosity. Numerical method of Keller-box is applied as a tool for obtaining the solutions with the help of MATLAB software.

2. Problem Formulation

This study is devoted to examine the behaviour of two-phase flow problem on dusty Williamson fluid under the influence of variable viscosity over a vertical stretching sheet by considering Newtonian heating (NH) as a thermal boundary condition. Note that, the analysis is conducted by means of a steady, two-dimensional and incompressible Williamson fluid defined here. Additionally, dust particles are assumed to be uniform in size, spherical shape and non-interacting. The physical configuration of the sheet can be clearly seen in Figure 1, where x -axis is positioned vertically upward along the sheet with y -axis being perpendicular to it.

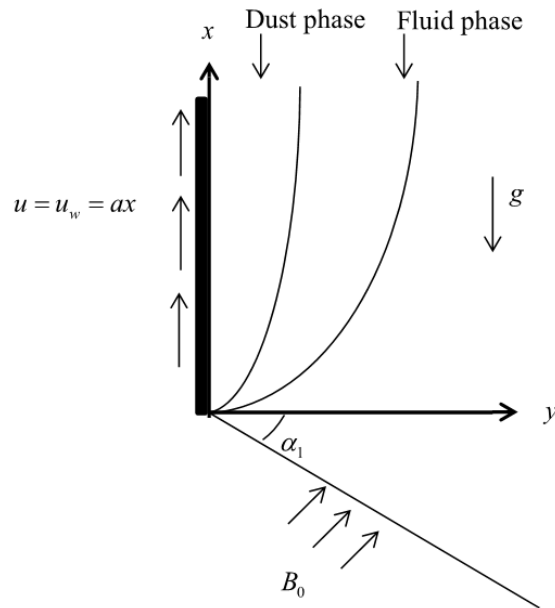


Fig. 1. Flow configuration of dusty Williamson fluid

To solve the problem, the following governing boundary layer equations of Williamson fluid and dust particles are taken into account.

Fluid phase:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left(\mu(T) \frac{\partial u}{\partial y} \right) + \frac{\sqrt{2}\Gamma}{\rho} \left[\frac{\partial}{\partial y} \left(\mu(T) \frac{\partial u}{\partial y} \right) \frac{\partial u}{\partial y} \right] + \frac{\rho_p}{\rho \tau_v} (u_p - u), \tag{2}$$

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial y^2} \right) + \frac{\rho_p c_s}{\gamma_T} (T_p - T), \tag{3}$$

Dust phase:

$$\frac{\partial}{\partial x} (u_p) + \frac{\partial}{\partial y} (v_p) = 0, \tag{4}$$

$$\rho_p \left(u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} \right) = \frac{\rho_p}{\tau_v} (u - u_p), \tag{5}$$

$$\rho_p c_s \left(u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} \right) = - \frac{\rho_p c_s}{\gamma_T} (T_p - T) \tag{6}$$

The boundary conditions of NH are in the form

$$u = u_w(x) = ax, \quad v = 0, \quad \frac{\partial T}{\partial y} = -h_s T \quad \text{at } y = 0$$

$$u \rightarrow 0, \quad u_p \rightarrow 0, \quad v_p \rightarrow v, \quad T \rightarrow T_\infty, \quad T_p \rightarrow T_\infty \quad \text{at } y \rightarrow \infty \quad (7)$$

where (u, v) and (u_p, v_p) are the velocities components of the fluid and particle phases along x and y axes, respectively. $\mu(T)$ is the variable viscosity, ρ and ρ_p are the density of fluid and dust phase, $\tau_v = 1/k$ is the relaxation time of particles phase, k is the Stoke's resistance (drag force), c_p and c_s are specific heat of fluid and dust particle, T and T_p are the temperature of fluid and particle phases, γ_T is the thermal relaxation time, a is positive constant and h_s is heat transfer parameter. It is important to note that, the viscosity, μ considered herein follows the robust Reynold exponential model which can be defined as [39]

$$\mu(T) = e^{-\alpha\theta(\eta)} = 1 - \alpha\theta(\eta) \quad (8)$$

where α is the viscosity parameter. Eq. (1)-(7) are required to be transformed into a convenient form to facilitate the numerical computation of this study. Therefore, the similarity transformation as expressed in Eq. (9) is implemented, given by

$$u = axf'(\eta), \quad v = -(av)^{1/2} f(\eta), \quad \eta = \left(\frac{a}{v}\right)^{1/2} y, \quad \theta(\eta) = \frac{T - T_\infty}{T_\infty}$$

$$u_p = axF'(\eta), \quad v_p = -(av)^{1/2} F(\eta), \quad \theta_p(\eta) = \frac{T_p - T_\infty}{T_\infty}, \quad (9)$$

where ψ is the stream function defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. The equations can now be expressed as follows

$$(1 - \alpha\theta(\eta))f'''(\eta)[1 + f''(\eta)] - \alpha\theta'(\eta)[\lambda_3(f''(\eta))^2 + f''(\eta)] - (f'(\eta))^2 + f(\eta)f''(\eta) + \beta N(F'(\eta) - f'(\eta)) = 0, \quad (10)$$

$$\theta''(\eta) + \text{Pr} f(\eta)\theta'(\eta) + \frac{2}{3}\beta N(\theta_p(\eta) - \theta(\eta)) = 0, \quad (11)$$

$$(F'(\eta))^2 - F(\eta)F''(\eta) + \beta(F'(\eta) - f'(\eta)) = 0, \quad (12)$$

$$\theta_p'(\eta)F(\eta) + \frac{2}{3}\frac{\beta}{\text{Pr}\gamma}(\theta(\eta) - \theta_p(\eta)) = 0 \quad (13)$$

and the boundary conditions (7) are reduced to

$$f(0) = 0, \quad f'(0) = 1, \quad \theta'(0) = -b(1 + \theta(0)) \quad \text{at } \eta = 0$$

$$f'(\eta) \rightarrow 0, F(\eta) \rightarrow 0, F(\eta) \rightarrow f(\eta), \theta(\eta) \rightarrow 0, \theta_p(\eta) \rightarrow 0 \text{ at } \eta \rightarrow \infty \quad (14)$$

where a prime (') denotes differentiation with respect to η . $N = \rho_p / \rho$ is the mass concentration of particle phase, $\beta = 1 / \alpha \tau_v$ is the fluid-particle interaction parameter, $\text{Pr} = \mu c_p / k$ is the Prandtl number, $\gamma = c_s / c_p$ is the specific heat ratio of mixture, $b = -h_s (v / a)^{1/2}$ is the conjugate parameter for NH and $\lambda_3 = \sqrt{2a^3 / \nu \Gamma x}$ is the Williamson parameter. As referred to Crane [40], the exact solution for Eq. (10) by ignoring the effects of viscosity dependency and dust particles can be expressed as

$$\theta(\eta) = C_1 \int_{\eta}^{\infty} e^{-\text{Pr} \int_{\eta}^{\infty} f \, d\eta} \, d\eta, \quad C_1 = \frac{-b(1 + \theta(0))}{e^{-\text{Pr} \int_{\eta}^{\infty} f \, d\eta}}. \quad (15)$$

Furthermore, the physical quantities of skin friction coefficient and Nusselt number are given as

$$C_f \text{Re}_x^{1/2} = [1 - \alpha \theta(\eta)] \left(f''(0) + \frac{\lambda_3}{2} (f''(0))^2 \right), \quad Nu_x \text{Re}_x^{-1/2} = b \left(\frac{1}{\theta(0)} + 1 \right). \quad (16)$$

3. Numerical Procedure

The Keller box method is employed for the purpose of obtaining numerical results of the present problem. In view of this, Eq. (10)-(14) are translated into programming languages and further computed in MATLAB software. A direct comparison between the results generated here and previous published work is therefore needed in order to prove the authentic of applied numerical method. Once accomplished, the computational process continues by selecting the boundary layer thickness, $\eta_{\infty} = 8$ and step size, $\Delta\eta = 0.02$ to which the results are revealed in the forthcoming figures and tables.

4. Results and Discussion

The response of pertinent parameters on velocity and temperature profiles, skin friction coefficient as well as Nusselt number is primary concerned in this section. For this, the variations in considered parameters are evaluated by fixing $\text{Pr} = 7, \alpha = b = 0.1, \gamma = 0.3, \lambda = B = N = 0.5$ unless for the measured one. The computation is initially started by transforming the present model into the existing model which arising Newtonian fluid flow problem for the verification purpose. Table 1 shows the comparative data between exact solution and numerical solution reported by Salleh *et al.*, [41] as well as current study. From the table, the obtained results are observed to experience exactness up to 3 decimal digits with exact solution. Thus, the numerical algorithm developed here is considered to be accurate and computation process for the involved parameters can be carried out accordingly. Next, Table 2 presents the skin friction coefficient, $C_f \text{Re}_x^{1/2}$ and Nusselt number, $Nu_x \text{Re}_x^{-1/2}$ for assorted values of α, β and λ_3 . Both physical quantities share similar trend correspond to increasing values of β and λ_3 , respectively, for which intensifying performance is observed for the first parameter while reducing in latter parameter. The opposite trend in two quantities is noticed as α increases.

The distribution of velocity and temperature profiles for both fluid and dust phases are portrayed in Figure 2 to 7, where insignificant changes can be discovered in almost figures. Figures 2 and 3 show

the influence of α on velocity and temperature profiles. It can be seen that, the velocity profile of dust phase decreases near to surface and then conversely found to increase as α escalates. Meanwhile, the fluid remains increased across the surface when α is enhanced. For the temperature profile, it is noticed to decrease in all phases as α grows.

Figure 4 and 5 present the behaviour of Williamson fluid and dust particles in response to the variations in λ_3 . In both figures, no marked reduction is found in velocity profile, whilst temperature profile elevates with any increase in values of λ_3 . This result may be explained by the fact that, the viscosity of the fluid is physically ascended which then leads to additional resistance to the fluid flow and ultimately decelerates the velocity distribution.

On the other hand, Figure 6 and 7 display the effect of dust particles in Williamson fluid by measuring the parameters β . As a result of enlarging values of β , the velocity of dust particles is noticed to escalate, with a corresponding decrease in Williamson velocity. This result can be reasoned that an enhancement in β deteriorates the velocity relaxation time of dust particles in which the particles alter itself to reach the fluid velocity. Thus, there exists the propensity for particles to accelerate and consequently decelerates the fluid motion. In the case of temperature profile, the two phases experience a decreasing trend as the effect of β becomes larger.

Table 1
 Comparative values of surface temperature $\theta(0)$ for various values of Pr when $\beta = N = \lambda_3 = \alpha = 0$, $\gamma \rightarrow \infty$ and $b = 1$

Pr	Exact Eq. (15)	Salleh <i>et al.</i> , [41]	Present
3	6.05159	6.02577	6.05176
5	1.760395	1.76594	1.76583
7	1.116815	1.13511	1.11996
10	0.764524	0.76531	0.76531

Table 2
 Variation of $-C_f Re_x^{1/2}$ and $Nu_x Re_x^{-1/2}$ for various values of α , λ_3 and β

α	λ_3	β	$-C_f Re_x^{1/2}$	$Nu_x Re_x^{-1/2}$
0.01	0.5	0.5	0.93825	2.26737
0.05			0.93561	2.26851
0.1			0.93234	2.26872
0.2			0.92586	2.27093
0.1	0.3	0.5	0.99703	2.30030
	0.4		0.96795	2.28734
	0.5		0.93234	2.26872
	0.6		0.82409	2.19961
0.1	0.5	0.5	0.93234	2.26872
		1.5	0.96953	2.30332
		2.5	0.98271	2.33303
		3.5	0.98869	2.35366

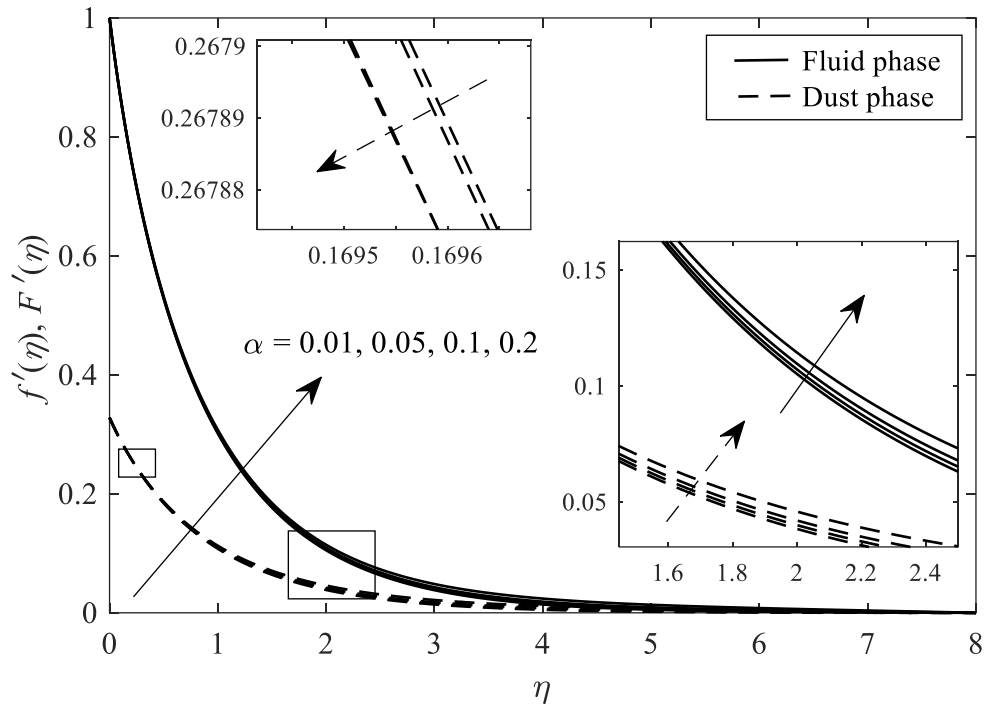


Fig. 2. Velocity profile for various values of α

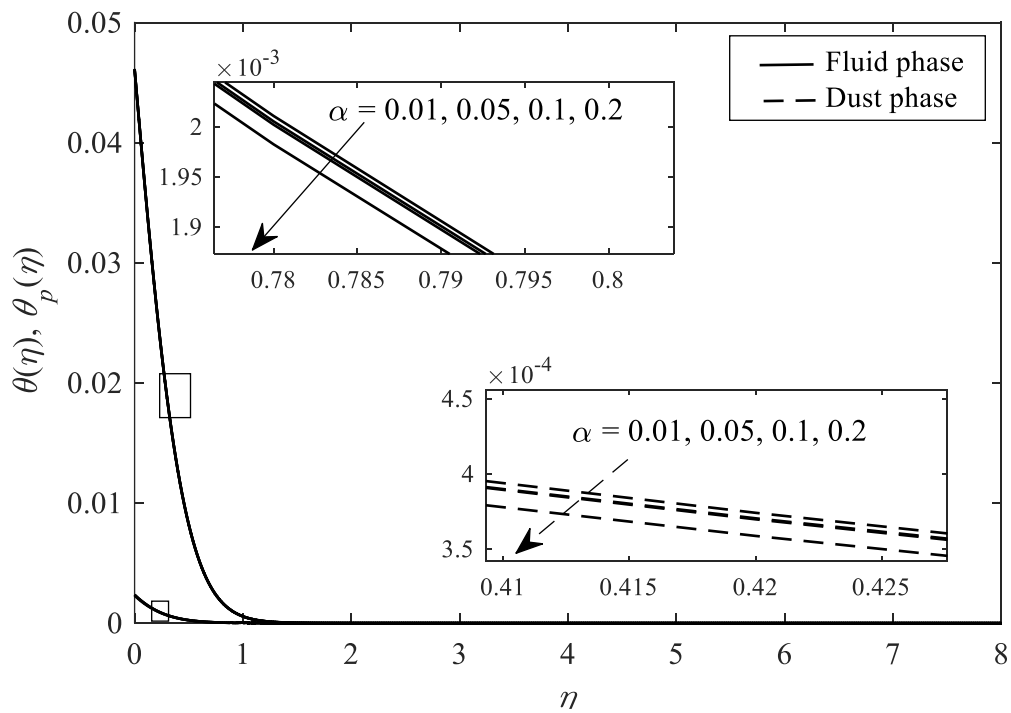


Fig. 3. Temperature profile for various values of α

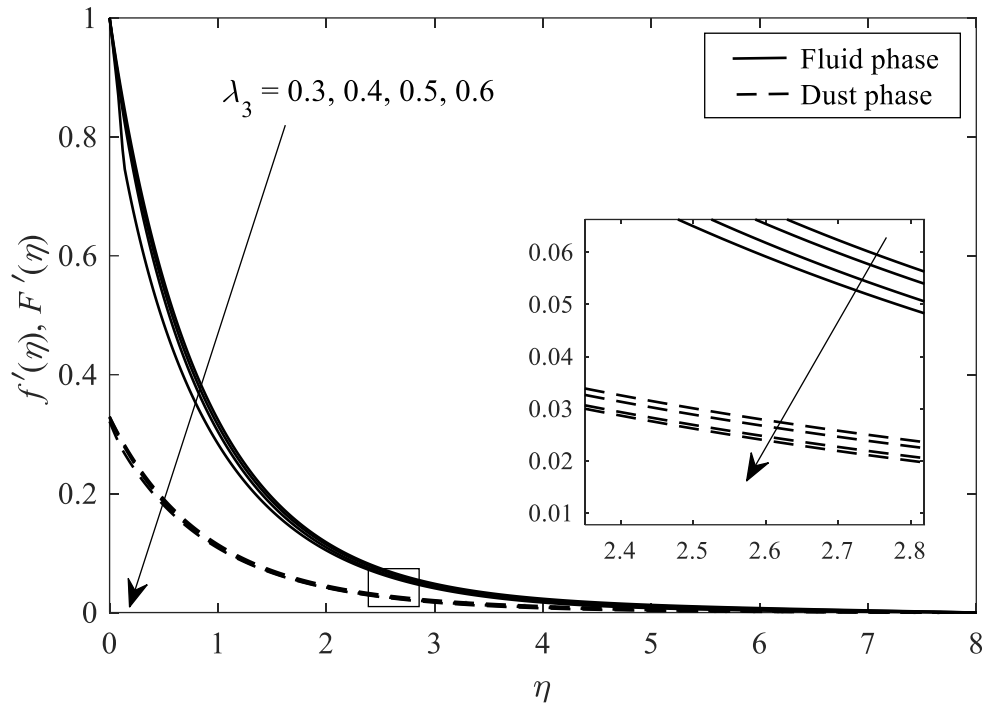


Fig. 4. Velocity profile for various values of λ_3

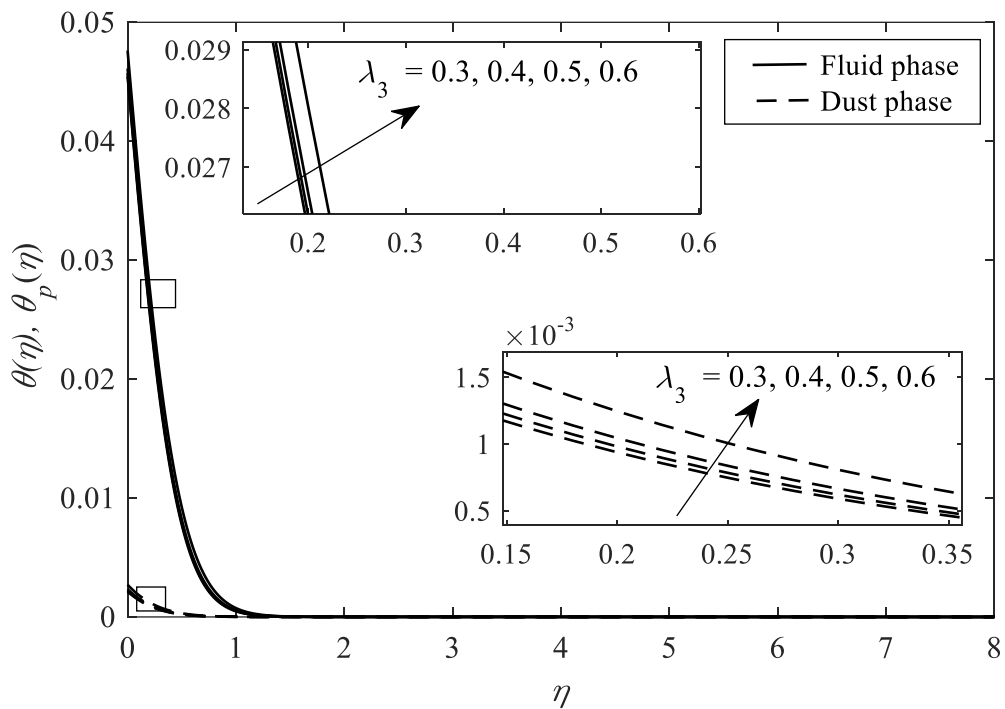


Fig. 5. Temperature profile for various values of λ_3

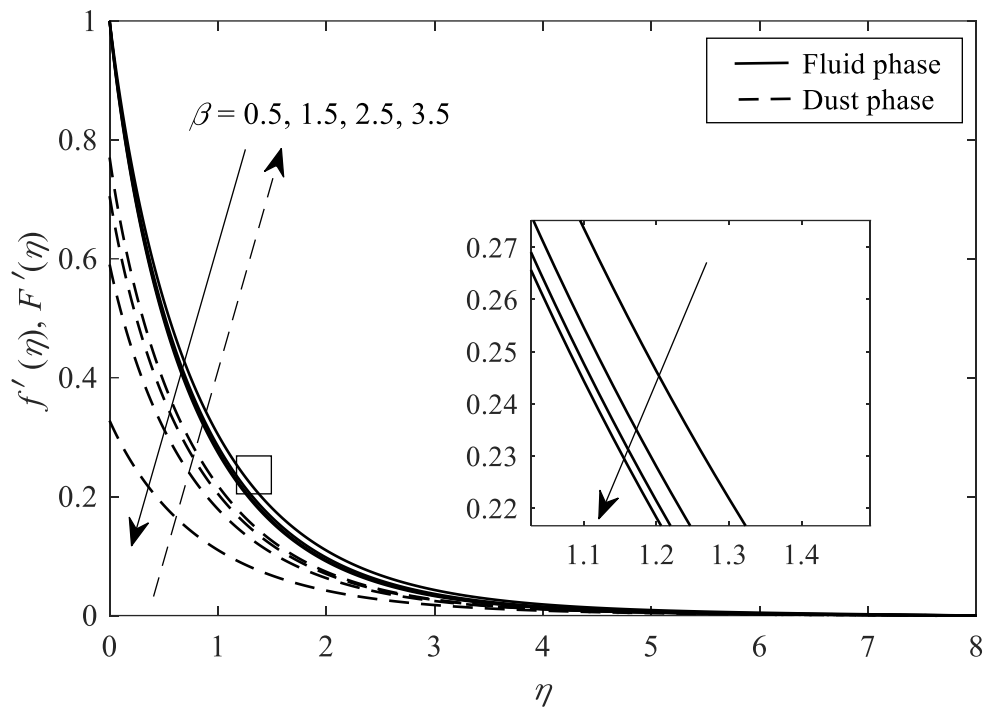


Fig. 6. Velocity profile for various values of β

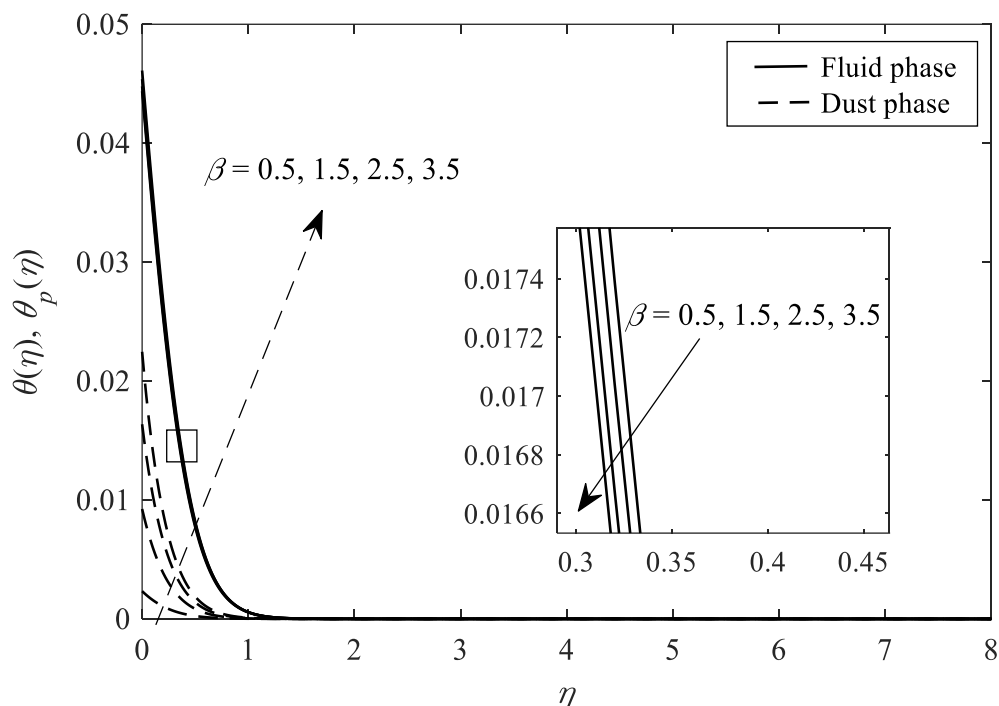


Fig. 7. Temperature profile for various values of β

5. Conclusion

The study undertaken here is to determine the influence of variable viscosity on the two-phase flow of both Williamson fluid and dust particles. Additionally, the surface of the sheet is exposed with boundary conditions of Newtonian heating (NH) and the Keller-box method is applied to aid the numerical computational process. Summing up from the obtained results that recorded on the aforementioned figures and tables, the findings to emerge from this study are

- i. the velocity profile of both phases shares the similar trend on which they increase as α and λ_3 increases, whereas reverse trend occurs for β ,
- ii. the temperature profile of both phases increases with the increase of λ_3 and β , whereas reverse trend occurs for λ_3 ,
- iii. the skin friction of fluid phase decreases with the increase of α and λ_3 , while opposite trend happens for β ,
- iv. the Nusselt number of fluid phase increases as α and β increases, while opposite trend happens for λ_3 .

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