



MHD Glauert Flow of a Hybrid Nanofluid with Heat Transfer

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ABSTRACT

This paper examines the wall jet flow and heat transfer of the Glauert problem with the effect of the hybrid nanoparticles. Also, the influence of the magnetic field and the variable surface temperature are taken into consideration. Here, we consider copper (Cu) and alumina (Al_2O_3) as the hybrid nanoparticles while water is the base fluid. The governing equations are reduced to the similarity equations using similarity transformations. Then, the numerical solutions are obtained by using the `bvp4c` function in MATLAB software. The findings reveal that hybrid nanofluid provides a higher heat transfer rate compared to regular nanofluid. Besides, the heat transfer rate and the skin friction coefficient increase in the presence of nanoparticles. Moreover, the rise of the temperature index parameter contributes to the enhancement of the heat transfer rate, but it does not affect the skin friction coefficient. The stronger magnetic strength led to the reduction of the heat transfer rate and the skin friction coefficient.

1. Introduction

These days, the process of fluid flow with heat transfer is crucial in designing and optimizing an efficient system [1]. Therefore, scientists and engineers have worked to intensify the thermal properties of fluid by adding nano-sized solid particles in the heat transfer fluid, and this mixture is called nanofluid [2].

Although nanofluid can improve thermal efficiency, better fluids in those aspects are still sought after to this day. By the innovations in science and technology, hybrid nanofluid has been developed which consists of two different nanoparticles in the base fluid and is believed to be able to provide better thermal properties. Furthermore, hybrid nanofluid is used in several applications, for example, in the heat exchanger, transformer, solar water heating, vehicle brake fluid, and domestic refrigerator [3].

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The studies of hybrid nanofluid flows were examined by Devi and Devi [4]. The new thermophysical model for hybrid nanofluid is developed in their studies, and being compared with the experimental results of Suresh *et al.*, [5]. Moreover, the magnetic field effects on hybrid nanofluid flow have been studied by several authors [6–12]. Besides, many researchers have studied the hybrid nanofluid flow with different aspects [13–19]. For further reading, the reader is encouraged to refer to the review papers by Sarkar *et al.*, [20], Babu *et al.*, [21], Huminic and Huminic [22], Yang *et al.*, [23], and Sidik *et al.*, [24].

The wall jet flow on a rigid surface bounded by the fluid at rest was pioneered by Glauert [25]. Basically, a wall jet is defined as the flow that spreads out over a surface by striking it at the right angle. The spray-paint process is one of the examples that used the concept of the wall jet flow. Inspired by the work of Glauert [25], similar problems have been considered by Bansal and Tak [26,27] with the heat transfer analysis. Later, Merkin and Needham [28] reported the effect of suction or injection on the wall jet flow by considering the moving wall. Since that, just to name a few, there are several studies on the wall jet flow has been reported by the researchers, for examples, Cohen *et al.*, [29], Magyari and Keller [30], Raees *et al.*, [31], Turkyilmazoglu [32], Zaidi *et al.*, [33], Jafarimoghaddam [34,35], Jafarimoghaddam and Pop [36], and Selimefendigil and Öztop [37].

Therefore, the hybrid nanofluid flow of the Glauert problem with the magnetic field and variable surface temperature effects are studied in this paper.

2. Mathematical Formulation

Consider the wall jet flow of hybrid nanofluid blown from a thin slit on the upper of a static flat surface, as displayed in Figure 1. The surrounding fluid is assumed in the rest condition.

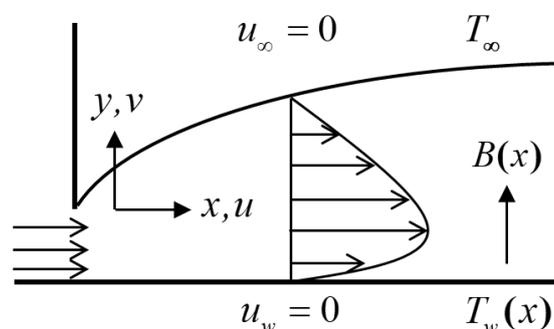


Fig. 1. Physical configuration

Besides, to obtain similarity equations, the variable surface temperature should be within the form of $T_w(x) = T_\infty + T_0 x^{m/4}$ with $m \geq 0$, while the ambient temperature T_∞ is constant. Also, the magnetic field is taken as $B(x) = B_0 x^{-3/4}$ with the magnetic field strength B_0 [34,35]. Thus, the governing equations are (see Glauert [25], Raees *et al.*, [31], Zaidi *et al.*, [33])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{hmf}}{\rho_{hmf}} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{hmf}}{\rho_{hmf}} B^2 u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho C_p)_{hnf}} \frac{\partial^2 T}{\partial y^2} \tag{3}$$

subject to

$$\begin{aligned} u = 0, v = 0, T = T_w(x) \quad \text{at} \quad y = 0 \\ u \rightarrow 0, T \rightarrow T_\infty \quad \text{as} \quad y \rightarrow \infty \end{aligned} \tag{4}$$

where (u, v) are the corresponded velocity components in the x - and y - directions, and T is the temperature. The thermophysical properties of nanoparticles, water, and hybrid nanofluid are provided in Table 1 and Table 2.

Table 1
 Thermophysical properties of nanoparticles and water [9,38]

Properties	Nanoparticles		Base fluid
	Cu	Al ₂ O ₃	water
ρ (kg / m ³)	8933	3970	997.1
C_p (J / kgK)	385	765	4179
k (W / mK)	400	40	0.613
σ (S / m)	5.96×10 ⁷	3.69×10 ⁷	0.05
Prandtl number, Pr			6.2

Table 2
 Thermophysical properties of nanofluid and hybrid nanofluid [4]

Properties	Nanofluid	Hybrid nanofluid
Dynamic viscosity	$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$	$\mu_{hnf} = \frac{\mu_f}{(1-\phi_1)^{2.5} (1-\phi_2)^{2.5}}$
Density	$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_n$	$\rho_{hnf} = (1-\phi_2)[(1-\phi_1)\rho_f + \phi_1\rho_{n1}] + \phi_2\rho_{n2}$
Heat capacity	$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_n$	$(\rho C_p)_{hnf} = (1-\phi_2)[(1-\phi_1)(\rho C_p)_f + \phi_1(\rho C_p)_{n1}] + \phi_2(\rho C_p)_{n2}$
Thermal conductivity	$\frac{k_{nf}}{k_f} = \frac{k_n + 2k_f - 2\phi(k_f - k_n)}{k_n + 2k_f + \phi(k_f - k_n)}$	$\frac{k_{hnf}}{k_f} = \frac{k_{n2} + 2k_{nf} - 2\phi_2(k_{nf} - k_{n2})}{k_{n2} + 2k_{nf} + \phi_2(k_{nf} - k_{n2})}$
Electrical conductivity	$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3\left(\frac{\sigma_n}{\sigma_f} - 1\right)\phi}{2 + \frac{\sigma_n}{\sigma_f} - \left(\frac{\sigma_n}{\sigma_f} - 1\right)\phi}$	$\frac{\sigma_{hnf}}{\sigma_f} = \frac{\sigma_{n2} + 2\sigma_{nf} - 2\phi_2(\sigma_{nf} - \sigma_{n2})}{\sigma_{n2} + 2\sigma_{nf} + \phi_2(\sigma_{nf} - \sigma_{n2})}$

where

$$\frac{k_{nf}}{k_f} = \frac{k_{n1} + 2k_f - 2\phi_1(k_f - k_{n1})}{k_{n1} + 2k_f + \phi_1(k_f - k_{n1})}$$

where

$$\frac{\sigma_{nf}}{\sigma_f} = \frac{\sigma_{n1} + 2\sigma_f - 2\phi_1(\sigma_f - \sigma_{n1})}{\sigma_{n1} + 2\sigma_f + \phi_1(\sigma_f - \sigma_{n1})}$$

The similarity transformation is considered as follows

$$\psi = 4(v_f^2 x)^{1/4} f(\eta), \quad \eta = (v_f^2 x^3)^{-1/4} y, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (5)$$

with the stream function ψ . Besides, the velocity components are defined by $u = \partial\psi / \partial y$ and $v = -\partial\psi / \partial x$. Then, we have

$$u = 4x^{-1/2} f'(\eta), \quad v = -\sqrt{v_f} x^{-3/4} (f(\eta) - 3\eta f'(\eta)) \quad (6)$$

Using Eq. (5) and Eq. (6), Eq. (1) is identically fulfilled, and Eq. (2) and Eq. (3) become

$$\frac{\mu_{hmf} / \mu_f}{\rho_{hmf} / \rho_f} f''' + ff'' + 2f'^2 - \frac{\sigma_{hmf} / \sigma_f}{\rho_{hmf} / \rho_f} M^2 f' = 0 \quad (7)$$

$$\frac{1}{Pr} \frac{k_{hmf} / k_f}{(\rho C_p)_{hmf} / (\rho C_p)_f} \theta'' + f\theta' - mf'\theta = 0 \quad (8)$$

subject to

$$f(0) = 0, \quad f'(0) = 0, \quad \theta(0) = 1 \\ f'(\eta) \rightarrow 0, \quad \theta(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (9)$$

where (') represents the differentiation with respect to η . Besides, m and M denote the temperature index and the magnetic parameters, while Pr is the Prandtl number and these parameters are expressed as

$$M = B_0 \sqrt{\frac{\sigma_f}{\rho_f}}, \quad Pr = \frac{(\mu C_p)_f}{k_f} \quad (10)$$

The skin friction coefficient C_f and the local Nusselt number Nu_x are defined as

$$C_f = \frac{\mu_{hmf}}{\rho_f u_r^2} \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad Nu_x = -\frac{x k_{hmf}}{k_f (T_w - T_\infty)} \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (11)$$

By substituting Eq. (5) into Eq. (11), we get

$$2Re_x^{1/2} C_f = \frac{\mu_{hmf}}{\mu_f} f''(0), \quad 2Re_x^{-1/2} Nu_x = -\frac{k_{hmf}}{k_f} \theta'(0) \quad (12)$$

where $Re_x = u_r x / v_f$ represents the local Reynolds number with $u_r = 4x^{-1/2}$ denotes the reference velocity as in Raees *et al.*, [31]. It should be noticed that for the regular fluid case ($\varphi_1 = \varphi_2 = 0$), Eq. (7) reduces to the classical Glauert [25] problem by replacing $f'(\eta) \rightarrow 0$ with $f(\eta) \rightarrow 1$ as $\eta \rightarrow \infty$.

3. Results and Discussion

Eq. (7) to Eq. (9) are solved numerically by the bvp4c solver in MATLAB software [39]. In this study, various volume fractions of Cu and Al₂O₃ are considered ($0 \leq \varphi_1, \varphi_2 \leq 0.04$). Meanwhile, water is used as the base fluid. The magnetic parameter M is taken from 0 to 0.05, meanwhile the temperature index parameter m is considered from 0 to 2.

Table 3 provides the numerical values of $f''(0)$ and $-\theta'(0)$ when $\varphi_1 = \varphi_2 = 0$ (regular fluid), $M = 0$ and $Pr = 6.2$ for different values of m . The increase of $-\theta'(0)$ is observed as m increase, whereas it does not affect the values of $f''(0)$. Also, in the present study, we obtained $f''(0) = 0.2222$ which consistent with the result of Glauert [25]. Furthermore, Table 4 shows the impact of M , m , φ_1 , and φ_2 on $2Re_x^{1/2}C_f$ and $2Re_x^{-1/2}Nu_x$ when $Pr = 6.2$. We observed that the values of $2Re_x^{1/2}C_f$ and $2Re_x^{-1/2}Nu_x$ are accelerated with the increase of φ_1 and φ_2 . Besides, the values of $2Re_x^{-1/2}Nu_x$ enhanced, whereas the values of $2Re_x^{1/2}C_f$ are not affected by the rising of m . Besides, the rising of M led to the reduction of $2Re_x^{1/2}C_f$ and $2Re_x^{-1/2}Nu_x$.

Table 3

Numerical values of $f''(0)$ and $-\theta'(0)$ when $\varphi_1 = \varphi_2 = 0$ (regular fluid), $M = 0$, and $Pr = 6.2$ for different values of m

m	$f''(0)$		$-\theta'(0)$
	Glauert [25]	Present results	Present results
0	$2/9 \approx 0.2222$	0.2222	0.6735
0.5			0.8193
1			0.9304
1.5			1.0215
2			1.0993

Table 4

Numerical values of $2Re_x^{1/2}C_f$ and $2Re_x^{-1/2}Nu_x$ for several parameters when $Pr = 6.2$

M	m	φ_2	Cu/water ($\varphi_1 = 0$)		Al ₂ O ₃ -Cu/water ($\varphi_1 = 0.04$)	
			$2Re_x^{1/2}C_f$	$2Re_x^{-1/2}Nu_x$	$2Re_x^{1/2}C_f$	$2Re_x^{-1/2}Nu_x$
0	0	0	0.2222	0.6735	0.2514	0.7266
		0.02	0.2839	0.7442	0.3106	0.7941
		0.04	0.3488	0.8109	0.3720	0.8579
	1	0	0.2222	0.9304	0.2514	1.0052
		0.02	0.2839	1.0299	0.3106	1.1003
		0.04	0.3488	1.1239	0.3720	1.1906
0.05	0	0	0.2082	0.6591	0.2355	0.7110
		0.02	0.2673	0.7295	0.2921	0.7781
		0.04	0.3295	0.7958	0.3508	0.8414
	1	0	0.2082	0.9106	0.2355	0.9836
		0.02	0.2673	1.0094	0.2921	1.0780
		0.04	0.3295	1.1029	0.3508	1.1677

Moreover, Figure 2 displays the effect of φ_1 and φ_2 on $2\text{Re}_x^{1/2}C_f$. Note that, Pr and m does not affect the values of $2\text{Re}_x^{1/2}C_f$. It is observed that the values of $2\text{Re}_x^{1/2}C_f$ enhanced almost linearly with the rise of φ_1 and φ_2 . Besides, Figure 3 shows the variations of $2\text{Re}_x^{-1/2}Nu_x$ for φ_1 and φ_2 with m . Obviously, the values of $2\text{Re}_x^{-1/2}Nu_x$ enhanced with the rise of φ_1 and φ_2 , and also for larger m . According to Sarkar *et al.*, [20], the fluid that consists of nanoparticles can enhance the heat transfer rate due to the synergistic properties of the nanoparticles. In addition, Figure 4 and Figure 5 show the magnetic parameter M effects on $2\text{Re}_x^{1/2}C_f$ and $2\text{Re}_x^{-1/2}Nu_x$. It can be seen that the values of $2\text{Re}_x^{1/2}C_f$ and $2\text{Re}_x^{-1/2}Nu_x$ are reduced with the rise of M .

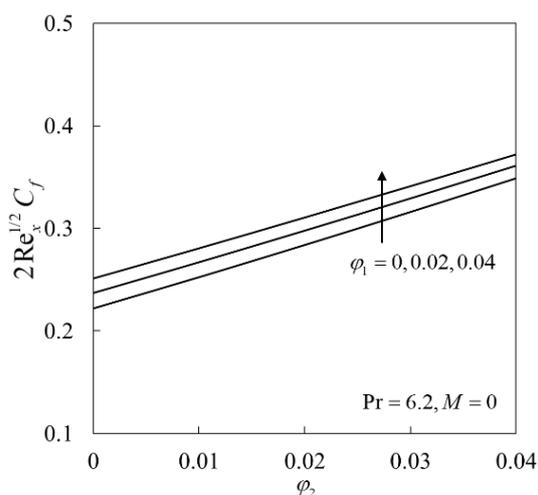


Fig. 2. $2\text{Re}_x^{1/2}C_f$ vs φ_1 and φ_2

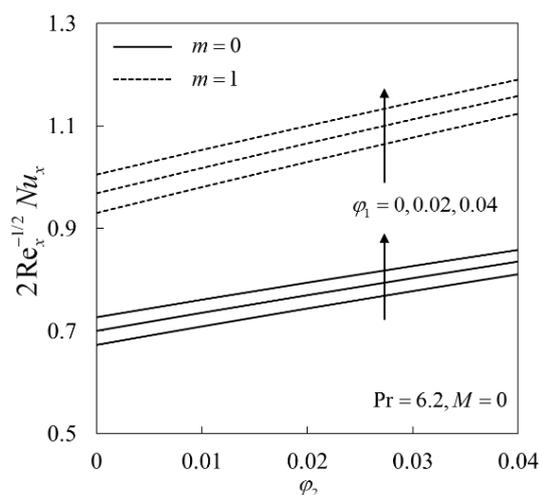


Fig. 3. $2\text{Re}_x^{-1/2}Nu_x$ vs φ_1 , φ_2 and m

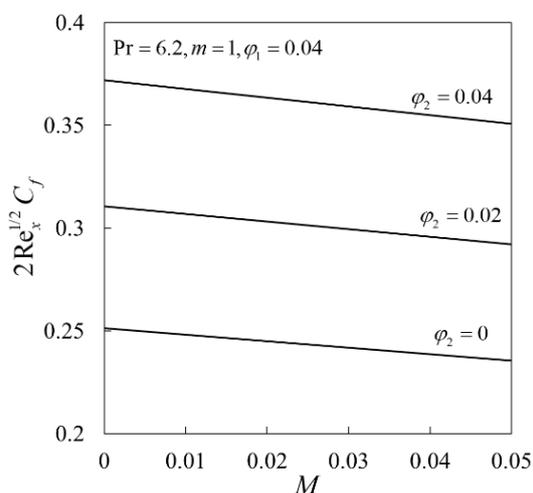


Fig. 4. $2\text{Re}_x^{1/2}C_f$ vs φ_2 and M

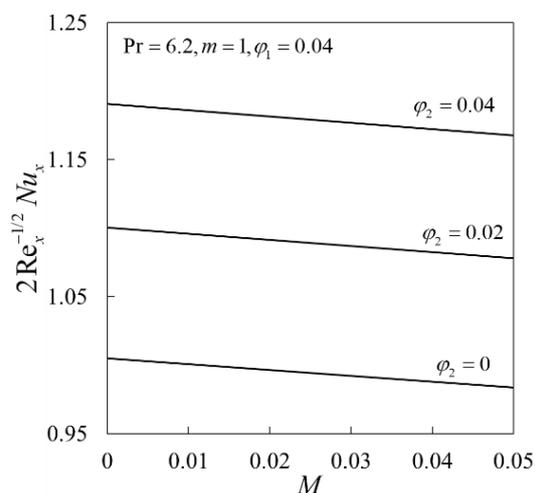


Fig. 5. $2\text{Re}_x^{-1/2}Nu_x$ vs φ_2 and M

Next, the profiles of the velocity $f'(\eta)$ and the temperature $\theta(\eta)$ for pertinent parameters are displayed in Figure 6 to Figure 9. It can be seen that these profiles asymptotically satisfy the infinity conditions (9), thus the precision of the current solution is achieved. The volumetric fraction of nanoparticles has a significant impact on $f'(\eta)$ and $\theta(\eta)$. The decreasing behaviour on the boundary layer thickness of $f'(\eta)$ and $\theta(\eta)$ are observed with the increase of φ_2 as shown in Figure 6 and

Figure 7. Also, we noted that the velocity $f'(\eta)$ increased near the surface and the maximum velocity rises as φ_2 increases. The effects of magnetic parameter M on $f'(\eta)$ and $\theta(\eta)$ are displayed in Figure 8 and Figure 9. It is noticed that the increasing of M led to rising the temperature profiles $\theta(\eta)$. Meanwhile, the maximum velocity occurs in the absence of M . The retardation on the velocity field is observed in the presence of the magnetic field due to the rising of the resistive force called Lorentz force. Besides, the profiles of temperature $\theta(\eta)$ for several values of m are displayed in Figure 10. It is noted that an upsurge in m led to the reduction of the temperature $\theta(\eta)$.

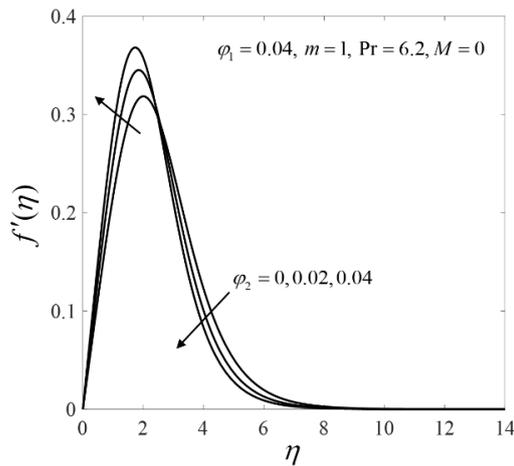


Fig. 6. $f'(\eta)$ vs φ_2

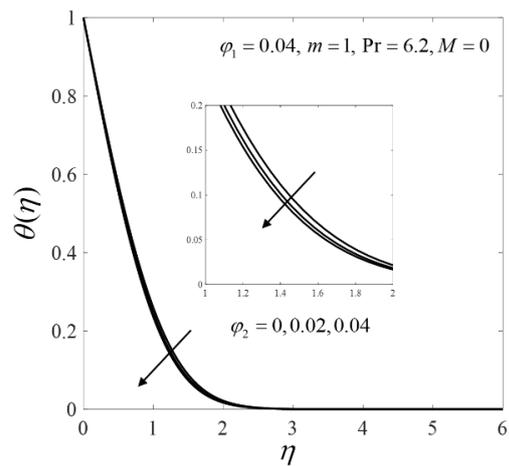


Fig. 7. $\theta(\eta)$ vs φ_2

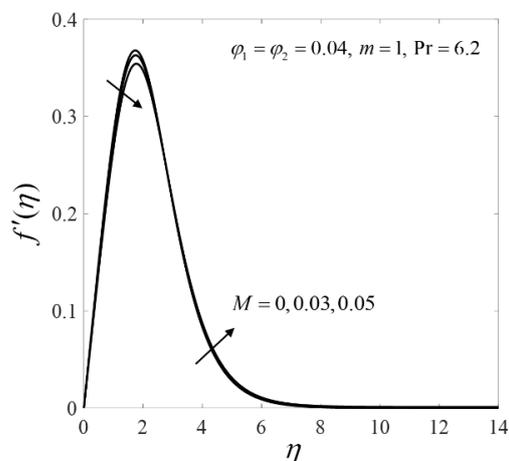


Fig. 8. $f'(\eta)$ vs M

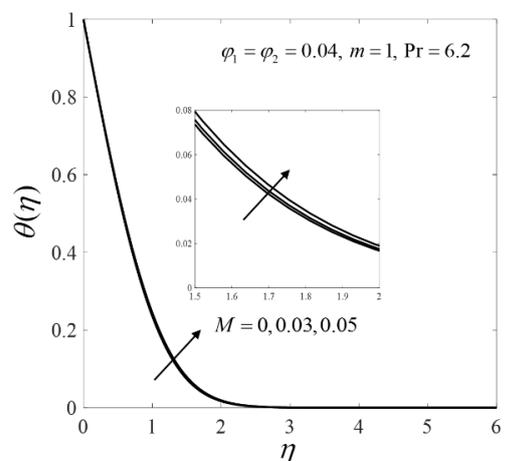


Fig. 9. $\theta(\eta)$ vs M

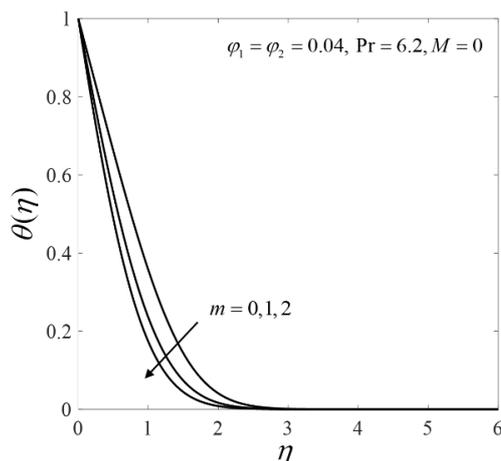


Fig. 10. $\theta(\eta)$ vs m

4. Conclusion

The Glauert flow of hybrid nanofluid with the magnetic field and variable surface temperature has been investigated. The present results for the special case are validated to the existing results and show a good comparison. The values of $2Re_x^{1/2}C_f$ accelerated with the rising values of φ_1 and φ_2 , but it is not affected by m . Meanwhile, the values of $2Re_x^{-1/2}Nu_x$ enlarged with the increase of φ_1 and φ_2 , and also for larger m . The values of $2Re_x^{1/2}C_f$ and $2Re_x^{-1/2}Nu_x$ are reduced with the increase of M . The value of $\theta(\eta)$ decreased with the increase of φ_2 and m . Moreover, we noticed that the boundary layer thickness of $f'(\eta)$ decreased, where the maximum velocity rises as φ_2 increases. The presence of the magnetic field produces the Lorentz force that retard the velocity field.

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