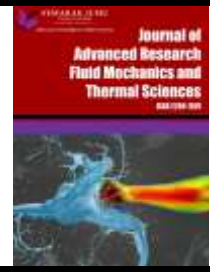




Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage:
https://semarakilmu.com.my/journals/index.php/fluid_mechanics_thermal_sciences/index
ISSN: 2289-7879



Improvement of Modeling of Laminar Flows in Pressure Collector-Pipelines

Volodymyr Cherniuk¹, Oleksandr Kravchuk², Vadym Fasuliak^{1,*}, Matvii Cherniuk³

¹ Department of Hydraulic and Water Engineering, Lviv Polytechnic National University, Stepana Bandery str., 12, Lviv, 79013, Ukraine

² Department of Water Supply and Drainage Kyiv National University of Construction and Architecture, Povitroflotskyi ave., 31, Kyiv, 03037, Ukraine

³ Department of Electronic Devices, Institute of Telecommunications, Radioelectronics and Electronic Engineering, Lviv Polytechnic National University, Stepana Bandery str., 12, Lviv, 79013, Ukraine

ARTICLE INFO

Article history:

Received 10 March 2024

Received in revised form 31 July 2024

Accepted 15 August 2024

Available online 30 August 2024

Keywords:

Pressure collector-pipeline; drainage perforated pipes; laminar flows; non-uniformity of inflow along drain

ABSTRACT

In the now-a-day's methods of hydraulic calculation of pressure collector-pipelines, the value of the coefficient of hydraulic friction (Darcy coefficient) is assumed to be constant. However, the fluid flow rate along the collector pipeline is variable. It increases along the pipeline at the expense of fluid inflow from jets through inlets in its wall. In the initial segments of long CP the fluid flow is laminar. Farther, with the increase of fluid flow rate, a turbulent stream is formed. In this paper, laminar streams are considered. The variability of Darcy coefficient is taken into account. Based on the differential equation of enforced variable flow rate fluid flow along the stream, a mathematical relationship for calculating the fluid inflow into the collector-pipeline has been derived. A formula for calculation of variable along the stream total working head in a collector-pipeline is obtained. The calculational relationships take into account geometric parameters of the CP, the angle of inflow of the jets to the stream, the angle of inclination of the pipeline to the horizon.

1. Introduction

Flows surrounded on all sides by solid walls are called pressure streams. Accordingly, pressureless are the flows that have a free surface, for example, rivers. The flow of liquid in the pressure collector-pipelines (CP) is non-uniform along their length. It increases in the direction of flow in the CP. There occurs a flow with a variable fluid flow rate along the CP. CPs which work in pressure mode are common in many branches of technology in the implementation of various production processes.

In agricultural hydromelioration, in order to lower the level of groundwater in waterlogged areas, drains are arranged. The main working elements of meliorative drainage systems are CP, which are perforated pipelines [1].

In water supply, for the purpose of cleaning from mechanical impurities, water taken from surface sources, infiltration water intakes are used. The water receivers of these water intakes are CP. To

* Corresponding author.

E-mail address: vadym.y.fasuliak@lpnu.ua

improve the inflow of water to the CP, reverse filters are placed around the outer surfaces of the latter. The grain sizes of the reverse filter layers increase as they approach the walls of the CP. The infiltration intake, which is shown in Figure 1, built on the coast of the Pacific Ocean in the city of Long Beach, near Los Angeles, in California [2].

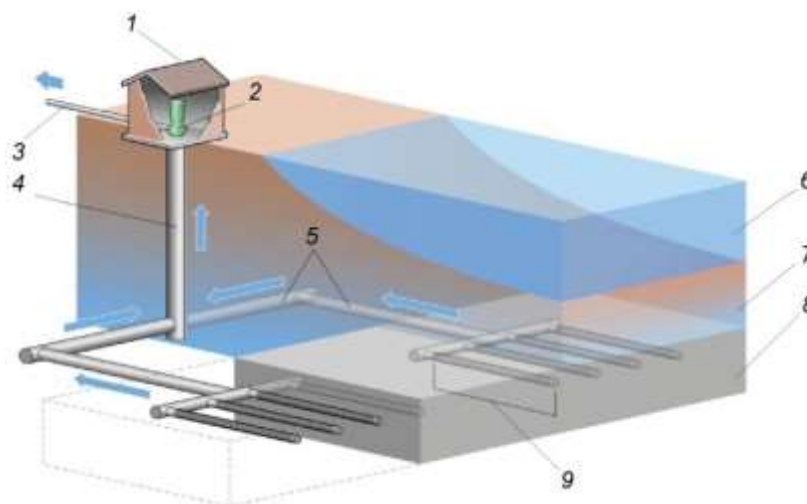
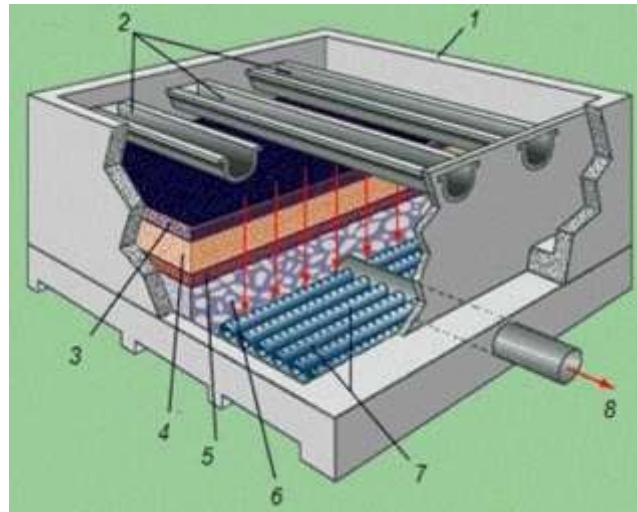


Fig. 1. Infiltration water intake: 1 – above-ground pavilion of the pumping station; 2 – pump motor; 3 – pressure pipeline; 4 – vertical shaft; 5 – water pipelines; 6 – water layer; 7 – sand layer; 8 – gravel layer; 9 - is one of the collector-pipelines [3]

A type of infiltration water intake is a radial water intake. It is a combination of a vertical shaft well and horizontal collector-pipelines, which are laid in the layer of water-bearing soil. CPs converge radially to the vertical shaft. It is advisable to use such water intakes on shallow rivers which in the period of floods, and floods carry a lot of silt, sand and larger pieces of solid soil, which are rolled by the flow of water along the bottom of the river. Radial water intakes are used for water supply in the following cities: Warsaw, Krakow, Przemyśl (Poland), Penza, Orenburg, Ufa (Russia), Rustava (Georgia), Turin (Italy), Eilenburg (Germany), Prince George (Canada), and others [4].

After preliminary treatment of water by means a reagent, to clean it from suspended flakes, fast (Figure 2) or slow filters are used at water treatment facilities [5]. The water purified by filtering is captured by collecting pipelines (Figure 2(b) and Figure 2(c)) located in the lower part of the filter housing (Figure 2(a)) [6]. In the process of periodic washing of the filter backfill, water is supplied to the filter through pipe 8 in the reverse direction. Then the system of collector-pipelines es switches to the mode of water distribution, that is, CP temporarily become pressure distribution-pipelines.



(a)



(b)



(c)

Fig. 2. Fast filter at water treatment plants: a – schematic diagram of the filter; b – system of collecting pipelines; c – collector-pipeline: 1 – filtration tank; 2 – washing trays; 3 – backfilling of a layer of coal (anthracite); 4 – ditto for sand, 5 – ditto for mineral sands which consist of hard granules of high density; 6 - layer of large-grained gravel; 7 – perforated pipes; 8 – draining of filtered water [5,7]

Drainage systems are used to distribute liquid in the case of discharge of domestic wastewater from primary settling tanks into the soil. At the same time, groundwater is being polluted. It is recommended to stop this practice [8]. Measures are being developed to prevent the spread of contaminated groundwater to neighboring territories [9].

Pressure collector-pipelines in combination with pressure distribution-pipelines make up the main part of solar collectors (Figure 3), which have recently been increasingly used for domestic and industrial water heating [10,11].



(a)

(b)

Fig. 3. The combination of the operation of the distribution-pipeline and the collector-pipeline in the solar collector: a – general view; b – longitudinal section of the heating element [12]

Pressure equipment is also used in exhaust ventilation, energy industry (circulating water cooling systems at nuclear and thermal power plants), aviation industry (removal of spent toxic gases from chambers for factory testing of jet engines), etc. [13].

The widespread use of pressure equipment requires the availability of reliable methods of their hydraulic calculation. Many efforts are being made to create adequate mathematical models to describe the inflow of liquid into CP.

2. Analysis of Literary Data

The flow of liquid into pressure CPs depends on the characteristics of the environment in which they are installed. In circulating water cooling systems at nuclear and thermal power plants, water enters the water intake windows of the heads of water intake structures from the cooling pond [13]. The water intake heads of these water intake structures are CPs with internal diameters that reach 4 m. The non-uniformity of water inflow into pressure CPs laid in the body of water is reduced by the following: a) increase the distance between water-receiving windows with the same areas of water-receiving windows; b) reduction of area of water-receiving windows along the stream in the CP, provided that the distances between the windows remain the same; c) increase the diameter of the CP along the flow of liquid in it [14,15].

In solar collectors (Figure 3) and in chemical reactors the liquid is brought to the holes in the wall of the CP through parallel tubes [11,16,17]. Inclined, vertical and horizontal water pipes are used. With vertical microtubes, the non-uniformity of the inflow to the CP is less than with horizontal ones [17]. Tong *et al.*, [2] set the goal of achieving uniformity of fluid flow in parallel tubes that connect the distributive pipeline and CP. By the method of numerical modeling, they selected the geometric parameters of the shutters, installed in the places of connection of parallel tubes with the CP, which would ensure the given condition.

The coefficient of flow rate of inlet nozzles in CP (like for outlet nozzles in distributive pipelines) [18,19] significantly depends on the angle of joining of the inflowing jets to the flow that is formed inside the CP. Therefore, the non-uniformity of the inflow of liquid into the pressure CP is regulated by changing the angle of joining of the inflowing jets to the flow in the CP [1,19]. The inflow of liquid into the CP increases with the increase in the value of the Reynolds criterion calculated for the inflowing jets [1].

During experimental studies of the operation of CP laid in a mass of porous soils, various aspects of CP operation were studied, depending on their intended purpose. Theoretically, laboratory and physical experiments investigated the work of drainage during the construction of tunnels for subway, railway and road transport [20]. Given the urgency of the problem, Zhou *et al.*, [21] analyzed 27 publications, which present the results of research into methods of reducing groundwater pressure on tunnel walls during their construction in sandy dolomite rocks. It was found that the arrangement of a large number of drainage holes in the walls of the tunnel has a negative effect on the construction effectiveness and stability of the tunnel. Lowering of the groundwater level with the help of horizontal drainage collectors arranged along the outer surface of the tunnel walls disturbs the balance of the surrounding soil and can cause natural disasters, which are common phenomena during the construction of tunnels. Studies have shown that it is more appropriate to reduce the pressure of the ground water which acts on the walls of the tunnel, than to lower the water level. Drains that radially converge in the vertical plane to the tunnel walls reduce the groundwater pressure most effectively [21].

Collector-pipelines should also be used to collect rainwater and meltwater in cities [22]. Excess supply of irrigation water to agricultural lands in arid regions often leads to excessive rise of saline

groundwater and salinization of the root layer of the soil. Saline soils become unsuitable for growing plants. They lower the level of groundwater in irrigation systems with the help of CP. Laboratory experiments have established that 76% of excess soil water is removed from the irrigated area through drainage pipes, and 24% through perforated collectors to which these drainage pipes are connected [11].

It was experimentally established that when a siphon outlet is installed at the end of the pressure CP of the reclamation drainage, the flow rate and water consumption in the CP increases by 27.1-89.8%, depending on the value of the vacuum pressure on the siphon [23].

The analysis of the data of the physical experiment and the analytical equation of the energy balance of the flow in the pressure CP showed the following. The energy losses of the jets on their inflow from the loose soil into the CP through the holes in its wall significantly exceed the energy losses to overcome the frictional forces of the flow in the CP [24].

Clemo [25] used a numerical experiment to find pressure losses in a pressure pipeline-collector for collecting oil from porous soils. The proposed method takes into account the increase in oil consumption along the CP due to its inflow through the holes in the CP walls. The results of calculations based on this model were compared with the data of three physical experiments: one for air flow and two for water flows. The proposed model is not fully consistent with the data of physical experiments. The author of the method believes that the reason for this is the unreliable results of individual physical experiments.

The carrying capacity of pressure cylindrical pipeline can be increased by means of introduction of polymer admixtures into the fluid flow, by optimization of geometric characteristics of the pipeline [6,26,27]. New methods of reducing the hydraulic resistance of pipelines are being developed on the bases of Navier-Stokes equation [28].

The results of the experiments provide objective information about the inflow of liquid into the collector-pipeline with specific operating conditions. However, the design of the CP requires mathematical models that take into account all the geometric parameters of the CP and the hydraulic characteristics of the jets that flow into the CP through the holes in its walls, and the flow of liquid that is formed in the CP. Many efforts are being made to create adequate mathematical models to describe the inflow of fluid into the CP. However, it is known that high accuracy of design of CP can be achieved based on the methods of differential calculation [29].

The differential equation of fluid movement with variable flow rate along the path for pressure fluid flow was first obtained by Prof. Nenko Ya. T. in 1937 [30]. During its integration, various simplifications were introduced. Considering pressure cylindrical CP with discrete fluid inflow. Konstantinov and Smyslov [31], Kravchuk [32], and Smyslov [33] used the assumption that the coefficient λ of hydraulic friction is constant along the CP. In effect, the coefficient λ is variable, since the fluid flow along the flow in the CP increases.

Nowadays, there has been no proposal to solve the DEFMVFR (the Differential Equation of Fluid Movement with Variable Flow Rate) taking into account the variability of the coefficient λ of hydraulic friction for the inflow of liquid into CP. Experience shows that incomplete consideration of the geometric parameters of the pressure CP and the hydrodynamic characteristics of the flow in it causes serious miscalculations in the design of the CP. For this reason, the reliability and efficiency of the CP is weakened [26].

The purpose of the work is to develop a method of hydraulic calculation of pressure collector-pipelines for the laminar mode of fluid flow, the calculation being based on the differential equation of fluid movement with a variable flow rate along the path, taking into account the geometric parameters of the CP and the hydraulic characteristics of the flow.

3. Differential Equation

The differential equation of fluid movement with variable flow rate (DEFMVFR) for the enforced flow in a cylindrical collector-pipeline has the following form [34]

$$\frac{\alpha_o(2V - v \cos \beta)dV}{g} + d\left(\frac{p}{\rho g}\right) + \sin \psi \cdot dx + dh_x = 0, \quad (1)$$

where,

α_o is the coefficient of the fluid flow momentum, $\alpha_o = 1.03 \dots 1.05$, dimensionless;

V is the average speed inside the CP, m/s;

v is the average speed of the jet flowing into the CP, m/s;

β is the angle between the vectors of the average velocity \vec{V} in CP and the average velocity \vec{v} of the jet that flows into the CP, degrees;

p is the fluid pressure inside the CP, N/m²;

ρ is the density of the liquid flowing in the CP, kg/m³;

ψ is the angle of inclination of the longitudinal axis of the CP to the horizon, degrees;

dh_x is the losses of head for friction along the length of the segment dx of the collector-pipeline, m.

All the terms in Eq. (1) have dimension m. This equation contains six variable quantities: V , dV , v , dp , dh_x , dx . The variable length dx is an independent quantity. In order to solve the Eq. (1), five additional equations must be added.

4. Formation of the Flow in CP

The flow in the collector-pipeline is formed from discrete jets that flow into the CP through holes in its wall (Figure 4). In the segments of the CP located between the adjacent inlets, the flow rate Q of the liquid is constant. However, at the center of each inlet opening the flow rate Q of the liquid flow in the CP increases by the amount dQ of the inflow through this opening. Since the flow of liquid along the CP increases, the pressure losses p increases due to the following: a) overcoming the frictional forces along the CP length, b) creation of a velocity head $V^2/2g$ of the flow, the loss of which increases along the length of the CP, c) overcoming the local hydraulic resistance of the inlets under the influence of liquid jets in the CP. As a result of these pressure losses, the piezometric line 3 and the full head line 4 decreases along the flow (Figure 4). Accordingly, under the condition of a constant level 5 of the liquid outside the CP, the working pressure increases, under the influence of which the jets flow into the CP.

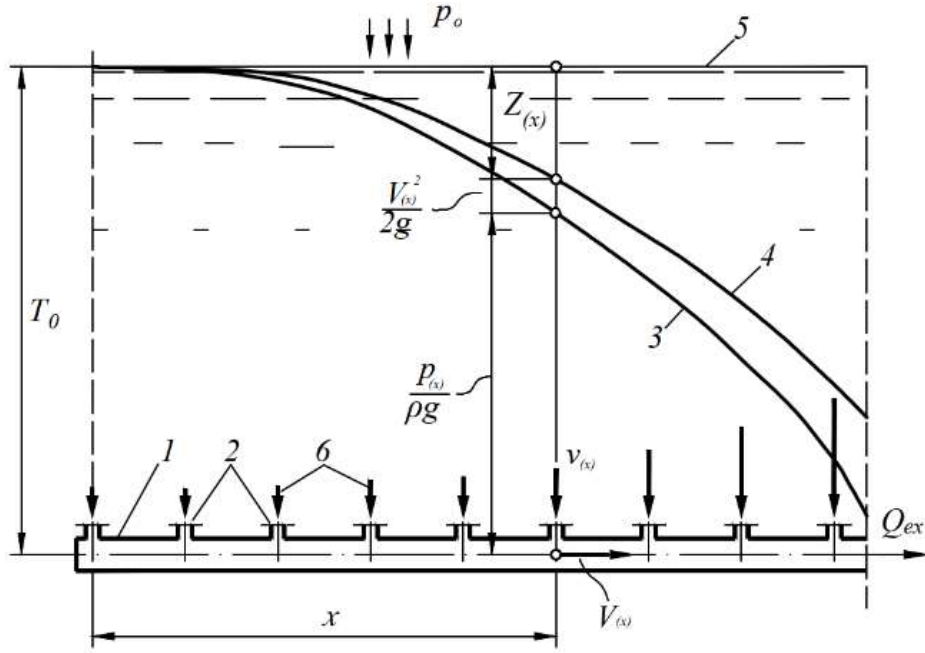


Fig. 4. Schematic diagram of formation of liquid flow in the pressure pipeline-collector: 1 – CP; 2 – inflow holes; 3 – piezometric line for the flow flowing in the CP; 4 – ditto for, full pressure line; 5 – liquid level outside the CP; 6 – inflow of jets into the CP

5. Solving the Differential Eq. (1)

To solve the differential Eq. (1), we express all its variables in terms of the operating pressure $Z_{(x)}$ (Figure 4), under the influence of which the jets flow into the CP through the holes in its wall, and in terms of the independent variable distance dx measured along the CP axis in the direction of flow.

The flow rate of the liquid stream that flows to the CP through a hole located in the cross-section x :

$$q_{(x)} = \mu\omega\sqrt{2gZ_{(x)}} = aZ_{(x)}^{1/2}, \quad (2)$$

where μ is the inlet flow rate coefficient, dimensionless;

ω is the area of one inlet hole, m^2 ;

a is the constant multiplier,

$$a = \mu\omega\sqrt{2g} = \text{const}, \quad m^{2.5}/s. \quad (3)$$

The average velocity v_x of the jet flowing to the collector-pipeline is obtained from the formula (2):

$$v_{(x)} = \frac{q_{(x)}}{\omega} = \frac{aZ_{(x)}^{1/2}}{\omega}. \quad (4)$$

The differential of flow rate, in m^3/s , of the liquid that flows into the CP through a series of holes made in its wall in a segment with the length of dx is the following [34]

$$dQ_{(x)} = nq_{(x)}dx = n\mu\omega\sqrt{2gZ_{(x)}}dx = bZ_{(x)}^{1/2}dx, \quad (5)$$

where b is the constant multiplier,

$$b = n\mu\omega\sqrt{2g} = \text{const}, m^{1.5}/s; \quad (6)$$

n is the number of inlets arranged per unit length CP, m^{-1} .

The flow rate of liquid transported in the section x of CP, in m^3/s , we obtain by means of integrating the mathematical expression (5)

$$Q_{(x)} = Q_{tr} + b\int_0^x Z_{(x)}^{1/2} dx, \quad (7)$$

where Q_{tr} is the transit flow rate of liquid at the inlet to the considered calculated segment of the CP whose length is x , m^3/s .

The average velocity of the liquid flow in the section x of CP is obtained from the formula (7)

$$V_{(x)} = \frac{Q_{(x)}}{\Omega} = \frac{Q_{tr} + b\int_0^x Z_{(x)}^{1/2} dx}{\Omega}, \quad (8)$$

where Ω is the cross-sectional area of CP, m^2 .

The differential of the average velocity of the liquid flow in the section x of CP is obtained by differentiating the mathematical expression (8)

$$dV_{(x)} = d\left(\frac{Q_{(x)}}{\Omega}\right) = \frac{bZ_{(x)}^{1/2}dx}{\Omega}. \quad (9)$$

The pressure of the fluid flow inside the CP laid horizontally (Figure 4)

$$\frac{p_{(x)}}{\rho g} = T_{(0)} - Z_{(x)} - \frac{\alpha V_{(x)}^2}{2g}, \quad (10)$$

where $T_{(0)}$ is the depth of immersion of the inflow section below the liquid level of the calculated segment of the CP.

The pressure differential for the flow of liquid in a horizontal CP was obtained from formula (10) by substituting into it the dependence (8) for the speed $V_{(x)}$

$$d\left(\frac{P_{(x)}}{\rho g}\right) = d\left(T_{(0)} - Z_{(x)} - \frac{\alpha V_{(x)}^2}{2g}\right) = -dZ_{(x)} - \frac{\alpha\left(Q_{tr} + b\int_0^x Z_{(x)}^{1/2} dx\right)}{g\Omega^2} bZ_{(x)}^{1/2} dx. \quad (11)$$

The term $\sin \psi \cdot x$ in the Eq. (1) takes into account the inclination of the longitudinal axis of the CP to the horizon (Figure 5).

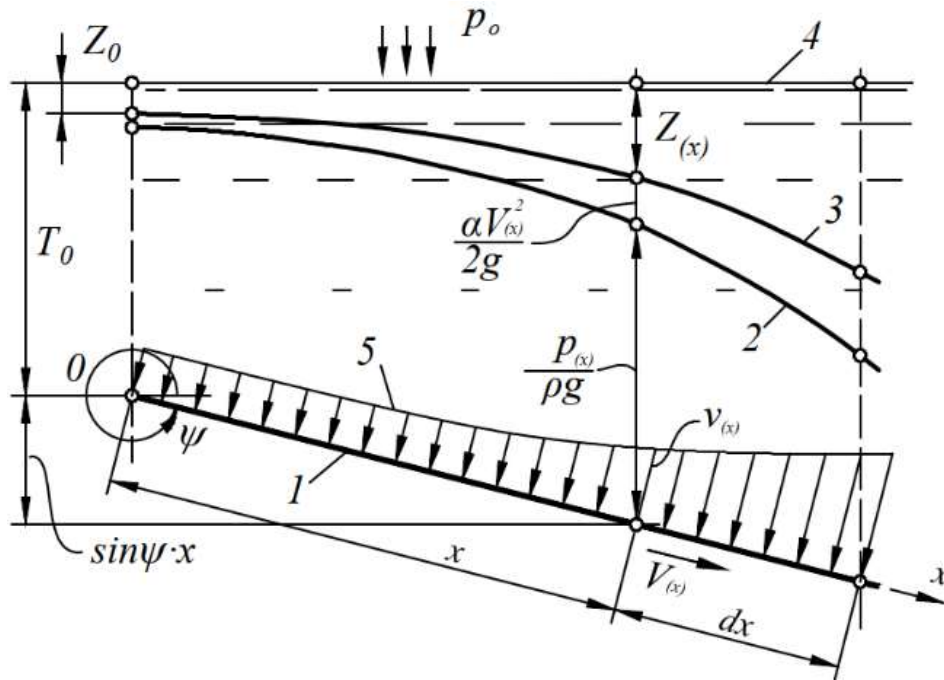


Fig. 5. Pressures acting on the collector-pipeline: 1 – CP; 2 - piezometric line for the flow of liquid inside the CP; 3 – ditto for, full pressure; 4 – liquid level outside the CP; 5 – graph of jet velocities into the CP; V - average velocity of the liquid flow in the CP; v - ditto for the jet acting into the CP

The last term of Eq. (1) is the differential of frictional head losses dh_x along the length of the CP. It is calculated using the Darcy-Weisbach formula [35]

$$dh_{(x)} = d\left(\lambda_{(x)} \frac{x}{D} \frac{V_{(x)}^2}{2g}\right) = \frac{l}{2g\Omega^2 D} d\left[\lambda_{(x)} \left(Q_{tr} + b\int_0^x Z_{(x)}^{1/2} dx\right)^2 x\right], \quad (12)$$

where D is the inner diameter CP, m . For cylindrical CP $D = const$. Therefore, it is placed in front of the differential sign.

For the laminar mode of enforced flow of liquid, the dimensionless coefficient of hydraulic friction (Darcy coefficient) λ_x is calculated according to the Poiseuille's formula

$$\lambda_{(x)} = \frac{64}{Re_{(x)}} = \frac{64\Omega \cdot \nu}{\left(Q_{tr} + b\int_0^x Z_{(x)}^{1/2} dx\right) D}, \quad (13)$$

where $Re_{(x)}$ is the Reynolds criterion, dimensionless

$$Re_{(x)} = \frac{V_{(x)}D}{\nu} = \frac{(Q_{tr} + b \int_0^x Z_{(x)}^{1/2} dx)D}{\Omega \nu}, \quad (14)$$

where ν is the kinematic viscosity, m^2/s .

After substituting the mathematical expressions (13) and (14) into the formula (12), we obtained the dependence for the differential of pressure loss for laminar flow in the CP [35]

$$dh_{(x)} = \frac{32 \cdot \nu}{g \Omega \cdot D^2} \left(Q_{tr} + b \int_0^x Z_{(x)}^{1/2} dx + b Z_{(x)}^{1/2} x \right) dx. \quad (15)$$

Substituting mathematical dependencies for $V_{(x)}$ (8); $\nu_{(x)}$ (4); $dV_{(x)}$ (9); $d(p_{(x)}/\rho g)$ (11); $dh_{(x)}$ (15) into the differential Eq. (1), we obtained a new differential equation of fluid movement of variable flow rate in one unknown variable $Z_{(x)}$, which is the operating pressure under the influence of which the jets flow into the CP through the holes in its wall

$$\begin{aligned} & \frac{(2\alpha_0 - \alpha)b}{g\Omega^2} \left(Q_{tr} + b \int_0^x Z_{(x)}^{1/2} dx \right) Z_{(x)}^{1/2} dx + \frac{32\nu}{g\Omega D^2} \left(Q_{tr} + b \int_0^x Z_{(x)}^{1/2} dx \right) dx - \frac{\alpha_0 abc \cos \beta}{g\Omega \omega} Z_{(x)} dx + \\ & + \frac{32b\nu}{g\Omega D^2} Z_{(x)}^{1/2} x dx - dZ_{(x)} + \sin \psi \cdot dx = 0. \end{aligned} \quad (16)$$

To simplify the writing of Eq. (16), we denote its constant quantities by the following coefficients

$$a_1 = \frac{(2\alpha_0 - \alpha) \cdot b}{g\Omega^2}; a_2 = \frac{32\nu}{g\Omega D^2}; a_3 = \frac{\alpha_0 abc \cos \beta}{g\Omega \omega}; a_4 = \frac{32b\nu}{g\Omega D^2}. \quad (17)$$

We write the Eq. (16) taking into account the denotation (17)

$$\begin{aligned} & 2a_1 \left(Q_{tr} + b \int_0^x Z_{(x)}^{1/2} dx \right) Z_{(x)}^{1/2} dx + a_2 \left(Q_{tr} + b \int_0^x Z_{(x)}^{1/2} dx \right) dx - a_3 Z_{(x)} dx + a_4 Z_{(x)}^{1/2} x dx - dZ_{(x)} \\ & + \sin \psi \cdot dx = 0. \end{aligned} \quad (18)$$

We make the following substitution in Eq. (18)

$$y = \left(Q_{tr} + b \int_0^x Z_{(x)}^{1/2} dx \right). \quad (19)$$

The first derivative of the mathematical dependence (19) is the following

$$y' = b Z_{(x)}^{1/2}. \quad (20)$$

From the mathematical expression (20), by squaring its right and left sides, we find the first derivative of the total pressure $Z_{(x)}$

$$Z_{(x)} = \left(\frac{y'}{b}\right)^2; \frac{dZ_{(x)}}{dx} = \frac{2y'y''}{b^2}, \quad (21)$$

where y'' is the second derivative of the function y with respect to independent variable distance x .

We write Eq. (18) taking into account the expressions (19)-(21), dividing all its terms by dx

$$\frac{a_1}{b} yy' + a_2 y - \frac{a_3}{b^2} (y')^2 + \frac{a_4}{b} y'x - \frac{2y'y''}{b^2} + \sin \psi = 0. \quad (22)$$

Eq. (18) and Eq. (22) are nonlinear quadratic ones. They are represented by a quadratic parabola. Therefore, in Eq. (22) the previously accepted notation $y = \left(Q_{tr} + b \int_0^x Z_{(x)}^{1/2} dx\right)$ is replaced by the following

$$y = Ax + Bx^2. \quad (23)$$

Then,

$$y' = A + 2Bx. \quad (24)$$

$$y'' = 2B. \quad (25)$$

We write Eq. (22) taking into account substitution (23) and mathematical dependencies (24) and (25)

$$\begin{aligned} & \frac{a_1}{b} (Ax + Bx^2) \cdot (A + 2Bx) + a_2 (Ax + Bx^2) - \frac{a_3}{b^2} (A + 2Bx)^2 + \\ & 4 + \frac{a_4}{b} (A + 2Bx)x - \frac{4}{b^2} (A + 2Bx)B + \sin \psi = 0. \end{aligned} \quad (26)$$

After performing algebraic operations in Eq. (26), it was given this form

$$\begin{aligned} & \left(\frac{a_1}{b}x - \frac{a_3}{b^2}\right)A^2 + \left(\frac{2a_1}{b}x^2 - \frac{4a_3}{b^2}x - \frac{8}{b^2}\right)B^2x + \left(\frac{3a_1}{b}x^2 - \frac{4a_3}{b^2}x - \frac{4}{b^2}\right)AB + \\ & + \left(a_2 + \frac{a_4}{b}\right)Ax + \left(a_2 + \frac{2a_4}{b}\right)Bx^2 + \sin \psi = 0. \end{aligned} \quad (27)$$

In Eq. (27), the variable coefficients A and B at the independent variable distance x are unknown. To find the values of A and B, we apply the boundary condition

$$x = 0. \quad (28)$$

By substituting condition (28) into Eq. (27), we obtain

$$\frac{a_3}{b^2} A^2 + \frac{4}{b^2} AB + \sin \psi = 0. \quad (29)$$

We equate the right sides of the Eq. (20) and Eq. (24)

$$bZ_{(x)}^{1/2} = A + 2Bx. \quad (30)$$

From Eq. (30) we obtain

$$A = bZ_{(x)}^{1/2} - 2Bx. \quad (31)$$

When $x=0$, from Eq. (31) we have

$$A = bZ_{(x)}^{1/2}. \quad (32)$$

Provided that $x = 0$, taking into account $A = bZ_{(x)}^{1/2}$ from Eq. (32), Eq. (29) takes the following form

$$a_3 Z_{(0)} + \frac{4}{b} Z_{(0)}^{1/2} B + \sin \psi = 0. \quad (33)$$

From Eq. (33) we obtain

$$B = \frac{b(a_3 Z_{(0)} + \sin \psi)}{4Z_{(0)}^{1/2}}. \quad (34)$$

We enter the coefficients A from (32) and B from (34) into Eq. (23), which is identical to Eq. (19)

$$y = \left(Q_{tr} + b \int_0^x Z_{(x)}^{1/2} dx \right) = Ax + Bx^2 = bZ_{(0)}^{1/2} x + \left(\frac{\alpha_0 ab Z_{(0)} b \cos \beta}{g \Omega \omega 4 Z_{(0)}^{1/2}} + \frac{b \sin \psi}{4 Z_{(0)}^{1/2}} \right)^2. \quad (35)$$

After performing algebraic operations in mathematical dependence (35), we obtain the equation for calculating the flow rate of liquid which, under the influence of external pressure, inflows through the holes into the collector-pipeline on the segment of length x and is transported to the CP through its section x

$$Q_{tr} + b \int_0^x Z_{(x)}^{1/2} dx = \left(Z_{(0)}^{1/2} + \frac{\alpha_0 ab Z_{(0)}^{1/2} \cos \beta}{4 g \omega \Omega} x + \frac{\sin \psi}{4 Z_{(0)}^{1/2}} x \right) bx. \quad (36)$$

The formula for calculating the full operating pressure of the liquid flow in the section x of the collector-pipeline was obtained by differentiating the Eq. (36)

$$Z_{(x)} = \left(Z_{(0)}^{1/2} + \frac{\alpha_0 ab Z_{(0)}^{1/2} \cos \beta}{2g\omega\Omega} x + \frac{x \sin \psi}{2Z_{(0)}^{1/2}} x \right)^2. \quad (37)$$

The mathematical model represented by Eq. (36) and Eq. (37) describes laminar flows in pressure collector-pipelines.

6. Conclusions

Main branches of application of pressure collector-pipelines are described. Problems of designing the pressure CP are considered. In the present-day mathematical models, in order to simplify the solution of differential equation, the coefficient of hydraulic friction (Darcy coefficient) is assumed to be constant along the collector pipeline. Based on the differential equation of enforced variable flow rate fluid flow along the stream, relationships for calculation of fluid inflow into a CP through orifices in its wall are derived. Laminar flows with variable Darcy coefficient are considered. A formula for calculation of total working head of fluid flow in a collector-pipeline has been obtained. In the obtained mathematical relationships, the geometric parameters of the CP, the angle of its inclination to horizon, the angle of jets inflow to the formed in the CP stream are taken into account.

Acknowledgement

This research was not funded by any grant.

References

- [1] Ivaniv, Vasyl, Volodymyr Cherniuk, and Vasyl Kochkodan. "Non-uniformity of water inflow into pressure collector-pipeline depending on the values of Reynolds criterion and of inflow jets angles." In *International Scientific Conference EcoComfort and Current Issues of Civil Engineering*, pp. 142-149. Cham: Springer International Publishing, 2020. https://doi.org/10.1007/978-3-030-57340-9_18
- [2] Tong, Jimmy CK, Ephraim M. Sparrow, and John P. Abraham. "Attainment of flowrate uniformity in the channels that link a distribution manifold to a collection manifold." *Journal of Fluids Engineering* 129, no. 9 (2007): 1186-1192. <https://doi.org/10.1115/1.2754319>
- [3] Voutchkov, Nikolay. "Overview of Desalination Plant Intake Alternatives." WaterReuse Association (2011).
- [4] Razumov, G. A. *Design and construction of horizontal water intakes and drainages*. Moscow: Stroyizdat, 1988.
- [5] Nathanson, Jerry A., and Richard A. Schneider. *Basic Environmental Technology: Water Supply, Waste Management and Pollution Control*, Pearson, 2014.
- [6] Gunawan, Gunawan, Yanuar Yanuar, A. S. A. Utomo, F. Fariz, and S. A. Lambang. "Reduction of Drag Resistance by Pressure Drop in Pipeline with 3-Dimensional Design Optimization." *Journal of Advanced Research in Experimental Fluid Mechanics and Heat Transfer* 1, no. 1 (2020): 1-10.
- [7] Federal Screen Products. "Under drains ion exchange internals." Federal Screen Products Inc. (2023).
- [8] Józwiakowski, Krzysztof, Michał Marzec, Agnieszka Listosz, Magdalena Gizińska-Górna, Agnieszka Micek, Aneta Pytka, Patrycja Pochwatka, and Kamila Rybczyńska-Tkaczyk. "The influence of household wastewater treatment plants with drainage system on the quality of groundwater in the Lublin Province, Poland." *Journal of Ecological Engineering* 22, no. 3 (2021). <https://doi.org/10.12911/22998993/132545>
- [9] Operacz, Agnieszka, Karolina Kurek, Dariusz Młyński, and Piotr Bugajski. "Untypical draining barriers efficiency as a method of pollutants limiting in the groundwater reservoir." *Journal of Ecological Engineering* 20, no. 3 (2019): 67-76. <https://doi.org/10.12911/22998993/99735>
- [10] Amirgaliyev, Y., M. Kunelbayev, A. Kalizhanova, O. Auelbekov, N. Kataev, A. Kozbakova, W. Wójcik, and B. Amirgaliyev. "Calculation and selection of flat-plate solar collector geometric parameters with thermosiphon circulation." *Journal of Ecological Engineering* 19, no. 6 (2018): 176-181. <https://doi.org/10.12911/22998993/91882>
- [11] Ghorbani, Karim, Aimrun Wayayok, Masumeh Abbaszadeh, and Ahmad Fikri. "The effect of pipe collectors in reducing the drainage coefficient rate." *Water Science and Technology: Water Supply* 15, no. 4 (2015): 675-682. <https://doi.org/10.2166/ws.2015.014>

- [12] Eco Systems. "How to choose solar collectors? Principle of operation." Eco Systems (2018).
- [13] Ivaniv, Vasyl, and Iryna Bihun. "Application and methods of hydraulic calculation of pressure pipelines-collectors." *Bulletin of the National University of Water Management and Nature Management: Collection of Scientific Works, Rivne: The National University of Water and Environmental Engineering* (2019).
- [14] Kravchuk, A., G. Kochetov., and O. Kravchuk. "Pipelines designing for steady water collection along the path." *Problems of Water Supply, Drainage and Hydraulics. Scientific and Technical Collection* (2020): 34-40. <https://doi.org/10.32347/2524-0021.2020.33.34-40>
- [15] Kravchuk, Andriy, Volodymyr Cherniuk, Oleksandr Kravchuk, and Tamara Airapetian. "Assessing the value of the hydraulic friction factor in pipelines working with a flow connection along the path." *Eastern-European Journal of Enterprise Technologies* 5, no. 7 (2022): 61-67. <https://doi.org/10.15587/1729-4061.2022.265670>
- [16] Commenge, J. M., L. Falk, J. P. Corriou, and M. Matlosz. "Optimal design for flow uniformity in microchannel reactors." *AIChE Journal* 48, no. 2 (2002): 345-358. <https://doi.org/10.1002/aic.690480218>
- [17] Zoljalali, Mohammad Ali, and Elham Omidbakhsh Amiri. "Study of the flow distribution in Parallel micro-channels with a triangular manifold." *Journal of the Brazilian Society of Mechanical Sciences and Engineering* 42 (2020): 1-9. <https://doi.org/10.1007/s40430-019-2140-x>
- [18] Bosak, Nikolai, Volodymyr Cherniuk, Ivan Matlai, and Iryna Bihun. "Studying the mutual interaction of hydraulic characteristics of water-distributing pipelines and their spraying devices in the coolers at energy units." *Eastern-European Journal of Enterprise Technologies* 3, no. 8 (2019): 23-29. <https://doi.org/10.15587/1729-4061.2019.166309>
- [19] Cherniuk, Volodymyr, and Vasyl Ivaniv. "Influence of values of angle of jet-joining on non-uniformity of water inflow along the path in pressure collector-pipeline." In *Environmental Engineering. Proceedings of the International Conference on Environmental Engineering. ICEE*, vol. 10, pp. 1-7. Vilnius Gediminas Technical University, Department of Construction Economics & Property, 2017. <https://doi.org/10.3846/enviro.2017.073>
- [20] Wang, Xiuying, Mengshu Wang, Mi Zhang, and Huangfu Ming. "Theoretical and experimental study of external water pressure on tunnel lining in controlled drainage under high water level." *Tunnelling and Underground Space Technology* 23, no. 5 (2008): 552-560. <https://doi.org/10.1016/j.tust.2007.10.004>
- [21] Zhou, Feicong, Jinyi Li, Yifan Jiang, Yongli Qi, Huiling Zhang, Ping Zhou, Zhijie Wang, and Shougen Chen. "Optimization of radial drainage design for Water-Rich tunnel in sandy dolomite stratum considering pressure relief effect." In *Structures*, vol. 53, pp. 861-881. Elsevier, 2023. <https://doi.org/10.1016/j.istruc.2023.04.110>
- [22] Zhuk, Volodymyr, Ivan Matlai, Bohdan Zavoiko, Ihor Popadiuk, Vitaliy Pavlyshyn, Ihor Mysak, and Pavlo Mysak. "Experimental hydraulic parameters of drainage grate inlets with a horizontal outflow in the broad-crested weir mode." *Water Science & Technology* 88, no. 3 (2023): 738-750. <https://doi.org/10.2166/wst.2023.234>
- [23] Ren, Xiaolei, Shaoli Wang, Peiling Yang, and Yuan Tao. "Experimental and modeling evaluation of siphon-type subsurface drainage performance in flooding and waterlogging removal." *Agricultural Water Management* 275 (2023): 108031. <https://doi.org/10.1016/j.agwat.2022.108031>
- [24] Murphy, P., N. B. Kaye, and A. A. Khan. "Hydraulic performance of aggregate beds with perforated pipe underdrains flowing full." *Journal of Irrigation and Drainage Engineering* 140, no. 8 (2014): 04014023. [https://doi.org/10.1061/\(ASCE\)IR.1943-4774.0000740](https://doi.org/10.1061/(ASCE)IR.1943-4774.0000740)
- [25] Clemo, Tom. "Flow in perforated pipes: A comparison of models and experiments." *SPE Production & Operations* 21, no. 02 (2006): 302-311. <https://doi.org/10.2118/89036-PA>
- [26] Rashid, Farhan Lafta, Haider Nadhom Azziz, and Shaheed Mahdi Talib. "Experimental investigation of drag reduction by a polymeric additive in crude oil flow in horizontal pipe." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 60, no. 1 (2019): 15-23.
- [27] Okyere, Mavis Sika, Lucas Nana Wiredu Damoah, Emmanuel Nyankson, and David Sasu Konadu. "Flow Improvers and Pipeline Internal Coating Benefits and Limitations with Respect to Pipeline Capacity Enhancement." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 107, no. 1 (2023): 102-124. <https://doi.org/10.37934/arfmts.107.1.102124>
- [28] Mkhitarian, Suren, Arestak Sarukhanyan, and Eghine Kanetsyan. "On Exact Solutions to Problems of Steady Flow of Incompressible Viscous Fluid in a Channel and in a Circular Pipe." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 107, no. 2 (2023): 68-83. <https://doi.org/10.37934/arfmts.107.2.6883>
- [29] Navoyan, Kh. A. *Examples of hydraulic calculations of culvert structures*. Kyiv: Budivelnik, 1975.
- [30] Petrov, G. A. "Variable mass hydraulics (Movement of fluid with a change in flow rate along the path)." *Kharkov State University*, 1964.
- [31] Konstantinov, M. Yu, and V. V. Smyslov. "On the calculation of perforated pipes with the attachment of the flow along the path." *Hydraulics and Hydraulic Engineering: Scientific and Technical Collection* (1966): 118-127.
- [32] Kravchuk, A. M. "Determination of the coefficient of connecting mass in perforated prefabricated pipelines." *Hydraulics and Hydraulic Engineering: Scientific and Technical Collection* (1988): 7-9.

- [33] Smyslov, V. V. "To the calculation of collecting pipelines." *Hydraulics and Hydraulic Engineering: Scientific and Technical Collection* (1980): 60-65.
- [34] Cherniuk, V. V., V. Ye Fasuliak, I. V. Bihun, and M. V. Cherniuk. "Variability of Darcy Friction Factor in Differential Equation for Enforced Flow in Collector-Pipeline." In *International Conference Current Issues of Civil and Environmental Engineering Lviv-Košice-Rzeszów*, pp. 58-69. Cham: Springer Nature Switzerland, 2023. https://doi.org/10.1007/978-3-031-44955-0_7
- [35] Cherniuk, V. V., V. V. Ivaniv, I. V. Bihun, and Ja M. Wojtowicz. "Coefficient of flow rate of inlet cylindrical nozzles with lateral orthogonal inflow." In *International Conference Current Issues of Civil and Environmental Engineering Lviv-Košice-Rzeszów*, pp. 50-57. Cham: Springer International Publishing, 2019. https://doi.org/10.1007/978-3-030-27011-7_7