



Numerical Solution for Unsteady Acceleration MHD Third-Grade Fluid Flow in a Rotating Frame Through Porous Medium Over Semi-Infinite Boundary Condition with a Presence of Heat Transfer

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ABSTRACT

The aim of this work is to present a suitable numerical solution for unsteady non-Newtonian third-grade fluid which rotates at z -axis and pass through a porous medium. The fluid flows in magnetic field with constant acceleration and the semi-infinite boundary condition are highlighted. The fluid problem is also deal with heat transfer. The nonlinear partial differential equation is discretised using the finite difference method (FDM). The linear system obtained for three different domains (lengths). Consequently, the asymptotic interpolation method is merged to solve problems of large sizes. This hybrid method yielded results that satisfied the boundary condition that reaches zero as length grows to infinite length. For velocity profile and temperature distribution, a comparison of FDM and hybrid method is shown. It is discovered that the hybrid method produces better results than FDM for this infinitely large problem. Several analyses have been carried out to investigate the effect of various fluid parameter values. The findings reveal that as the porosity parameter increases, the velocity decreases. The Grashof and Prandtl numbers demonstrate the relationship to the temperature distributions. The effects of the magnetic field and the non-Newtonian parameters were also illustrated, as these parameters influence the velocity distribution of the fluid flow.

1. Introduction

Recently, application of non-Newtonian fluid problem in a various of disciplines, covering meteorology, geophysics, engineering, and mathematics have been notified [1-3]. The non-Newtonian equations offer difficulties for researchers because they are more complex than those of Newtonian fluid.

The finite difference method (FDM) is the most well numerical method that most researchers use to find the numerical solution especially in fluid flow problems. The FDM substitutes difference

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quotients for derivatives in the governing field equations, which include values of the solution at discrete mesh points in the domain under study where this is called discretisation. This representation is used repeatedly to create algebraic systems of equations in terms of unknown mesh point values. The main problem with this system is the inaccuracy in dealing with regions of complicated domain, even though it can be solved using coordinate transformation techniques [4,5].

Tome [6] applied the FDM to address two-dimensional modified Newtonian flow with several free surfaces within an arbitrary domain. After eight years, the problem has been extended to three-dimensional modified Newtonian free surface flows with additional features, and it has been resolved using the same mathematical approach [7]. The numerical method of implicit FDM is used to convert partial differential equations to a system of non-linear equations by using the damped-Newton method to solve non-Newtonian third-grade fluid which pass a porous medium [8]. The MHD rotating flow equations of fourth-grade fluid bounded between two non-conducting rigid plates in a rotating frame with uniform transverse magnetic field is modelled. The second-order Central difference formula and successive under relaxation scheme have been used to solve the equation. The finding reveals that the flow behaviour is parameter dependent [9].

Parida and Padhy [10] used the FDM in 2018 to discretise the differential equation of MHD third-grade in a non-Darcian porous medium. Garg [11] and Anitha [12] have explored the chemical reaction with radiation in greater details. Nayak [13] performed an investigation of heat transfer exists in fluid flow with the issue of focusing on differential form of MHD third grade with viscous dissipation. FDM is used to solve the fluid problem. According to the results, FDM is still relevant today as it can provide an approximate system for solving nonlinear differential equations of fluids.

The implicit scheme finite difference Keller-box approach was used to address the problem of MHD incompressible viscous fluid between two parallel porous plates and velocity slip boundary conditions at the permeable boundaries [14]. The Keller-box approach also been used to evaluate heat transfer and the effects of an external magnetic field on the non-Newtonian micropolar fluid flow between a porous and a non-porous disc [15].

MHD also called magnetic field fluid dynamics, is the analysis of the dynamics of electrically conducting fluids, like salt water and electrolytes [16]. Interactions exist between the motion in fluid flow where electric current is generated when fluid flows across magnetic lines and the transverse magnetic lines of forces which lead to other factors on fluid. This present work also focuses on other two parameters which are rotating frame and a porous medium. A rotating frame is one that rotates with respect to an inertial reference frame. Fluid flow in a porous media has recently emerged as a research subject of interest. Porous medium is a parameter in the momentum equation that applies to porous materials such as rocks or soil. Several works have been published, including the development of new exact solutions for MHD transient Rotating flow of a second-grade fluid in a porous medium [2,17], third-grade fluid in a porous medium and a rotating frame [18], MHD rotating flow of a fourth-grade fluid [8] and MHD flow of a third-order fluid in a porous medium [19].

Heat transfer in non-Newtonian fluids can be seen in many industrial processes, such as the plastic manufacturing process. Aiyesami [20] investigated the fluid motion with temperature for the MHD third-grade fluid using an semi-analytical solution called HAM. El-dabe [21] used homotopy perturbation to solve an MHD boundary-layer flow of an incompressible fourth-grade nanofluid through a porous medium down a vertical cylinder with heat transfer.

The aim of this paper is to solve a third-grade non-Newtonian fluid flows with constant acceleration in a rotating frame across a magnetic field and porous medium with thermal conductivity. The nonlinear differential equation is discretised using FDM. Since the fluid problem has a semi-infinite boundary domain which present an infinite length, a hybrid method of finite difference - asymptotic interpolation method is designed and applied to the problem system.

Asymptotic interpolation method is implemented to boost the result where this method has been applied in mechanical and chemical hydrodynamics [22,23] and investigating fluctuating fluid membranes to give accurate predictions at infinity [24]. The hybrid method has previously been used to solve MHD third grade fluid in a rotating frame [25,26]. The discussions are centred on the comparison between classical FDM and hybrid method, and the effect of velocity profile and temperature distribution with varying dimensionless parameter values.

2. Mathematical Formulation

Define the velocity field, $V = (u(z, t), v(z, t), w(z, t))$ that exists in fluid where u, v and w are the velocity elements that correspond to the x -axis, y -axis and z -axis. The equation of continuity is given by

$$\nabla \cdot V = 0. \tag{1}$$

The following equations govern the MHD fluid flow in a porous medium and a rotating frame

$$\left(\frac{\partial V}{\partial t} + (V \cdot \nabla)V + 2\Omega \times V + \Omega \times (\Omega \times r) \right) = -\nabla p + \text{div}T - \sigma\beta_0^2 V + R \tag{2}$$

where ρ is the density, p is the pressure, r is the radial coordinate and Ω is the rotating parameter which rotates about z -axis with $V = (u(z, t), v(z, t), 0)$. Lorentz force per unit volume is given by $J \times B = -\sigma\beta_0^2 V$ where J is current density and B is total magnetic field. According to Abelman [3], the flow resistance for third grade fluid is given by

$$R = -\frac{\varphi}{k} \left[\mu + \alpha_1 \frac{\partial}{\partial t} + 2\beta_3 \left(\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right) \right] V \tag{3}$$

$T = -pI + \sum_{j=1}^n S_j$ is the stress tensor, where p is force or pressure and I is the identity tensor [1-3]. The first three tensors are given by $S_1 = \mu A_1$, $S_2 = \alpha_1 A_2 + \alpha_2 A_1^2$ and $S_3 = \beta_1 A_3 + \beta_2 (A_2 A_1 + A_1 A_2) + \beta_3 (\text{tr} A_1^2) A_1$ where μ is the shear viscosity coefficient. The material constants are $\alpha_i (i = 1, 2), \beta_j (1, 2, 3), \gamma_k (k = 1, 2, \dots, 8)$ while A_n is the Rivlin-Ericson tensors $A_n = \frac{d}{dt} (A_{n-1}) + A_{n-1} (\text{grad}V) + (\text{grad}V)^T A_{n-1}, n > 1$ where $A_1 = (\text{grad}V) + (\text{grad}V)^T$ and $A_2 = \frac{dA_1}{dt} + A_1 (\text{grad}V) + (\text{grad}V)^T A_1$. The constitutive equation for third grade fluid ($n = 3$), is $T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2 (A_2 A_1 + A_1 A_2) + \beta_3 (\text{tr} A_1^2) A_1$ which satisfies $\mu \geq 0, \alpha_1 \geq 0, |\alpha_1 + \alpha_2| \leq \sqrt{24\mu\beta_3}, \beta_1 = \beta_2 = 0, \beta_3 \geq 0$ and $\gamma_k = 0$ [3,18-20,27,28].

3. Governing Equation

The momentum equation of third grade fluid flow with heat transfer is derived as follows.

$$\rho \left(\frac{\partial u}{\partial t} - 2\Omega v \right) = \mu \frac{\partial^2 u}{\partial z^2} + \alpha_1 \frac{\partial^3 u}{\partial z^2 \partial t} + 2\beta_3 \frac{\partial}{\partial z} \left[\frac{\partial u}{\partial z} \left(\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right) \right] - \sigma\beta_0^2 u - \frac{\varphi}{k} \left[\mu + \alpha_1 \frac{\partial}{\partial t} + 2\beta_3 \left(\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right) \right] u + \frac{\beta g}{\rho} (T - T_\infty) u \tag{4}$$

$$\rho \left(\frac{\partial v}{\partial t} + 2\Omega u \right) = \mu \frac{\partial^2 v}{\partial z^2} + \alpha_1 \frac{\partial^3 v}{\partial z^2 \partial t} + 2\beta_3 \frac{\partial}{\partial z} \left[\frac{\partial v}{\partial z} \left(\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right) \right] - \sigma \beta_0^2 v - \frac{\varphi}{k} \left[\mu + \alpha_1 \frac{\partial}{\partial t} + 2\beta_3 \left(\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right) \right] v + \frac{\beta g}{\rho} (T - T_\infty) v \quad (5)$$

with a simple energy equation is

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} \quad (6)$$

where β is the coefficient of thermal expansion, g is gravity, T is fluid temperature, T_∞ is ambient temperature, C_p represents a fluid at constant pressure and k is thermal conductivity. Following Nazari [29] for constant acceleration, the boundary and initial condition are

$$\begin{aligned} u(0, t) &= At, v(0, t) = 0, t > 0, T = T_w \\ u(z, t) &\rightarrow 0, v(z, t) \rightarrow 0 \text{ as } z \rightarrow \infty \forall t, T \rightarrow T_\infty \\ u(z, 0) &= 0, v(z, 0) = 0, z > 0. \end{aligned} \quad (7)$$

Eq. (5) is multiplied by i and added to Eq. (4). The momentum equation becomes

$$\begin{aligned} \frac{\partial F}{\partial t} + 2i\Omega F \\ = v \frac{\partial^2 F}{\partial z^2} + \frac{\alpha_1}{\rho} \frac{\partial^3 F}{\partial z^2 \partial t} + \frac{2\beta_3}{\rho} \frac{\partial}{\partial z} \left[\left(\frac{\partial F}{\partial z} \right)^2 \left(\frac{\partial \bar{F}}{\partial z} \right) \right] - \frac{\sigma}{\rho} \beta_0^2 F - \frac{\varphi}{k\rho} \left[\mu + \alpha_1 \frac{\partial}{\partial t} + 2\beta_3 \left(\frac{\partial F}{\partial z} \right) \left(\frac{\partial \bar{F}}{\partial z} \right) \right] F + \frac{\beta g}{\rho} (T - T_\infty) F \end{aligned} \quad (8)$$

where $F = u + iv$ and $\bar{F} = u - iv$. The boundary and initial condition now are

$$\begin{aligned} F(0, t) &= At, T(z, 0) = 0 \text{ at } t = 0 \\ F(z, t) &\rightarrow 0, T(z, t) \rightarrow T_\infty \text{ as } z \rightarrow \infty \\ F(z, 0) &= 0, T(z, 0) = 0 \text{ at } t = 0 \end{aligned} \quad (9)$$

The dimensionless parameters in this problem are $\theta^* = \frac{T - T_\infty}{T_w - T_\infty}$, $Gr = \frac{g\beta(T_w - T_\infty)A^3}{\nu^2}$, $Pr = \frac{\mu C_p}{k}$ where θ^* is the temperature of the velocity profile, Gr is the Grashof number and Pr is the Prandtl number. The nonlinear equations with heat transfer are

$$\begin{aligned} (1 + \zeta) \frac{\partial f}{\partial \tau} + (2i\Omega + M + c - Gr^* \theta^*) f \\ = \frac{\partial^2 f}{\partial \eta^2} + a \frac{\partial^3 f}{\partial \eta^2 \partial \tau} + b \left[2 \left(\frac{\partial^2 f}{\partial \eta^2} \right) \left(\frac{\partial f}{\partial \eta} \right) \left(\frac{\partial \bar{f}}{\partial \eta} \right) + \left(\frac{\partial f}{\partial \eta} \right)^2 \left(\frac{\partial \bar{f}}{\partial \eta} \right) \right] - 2bcf \left[\left(\frac{\partial f}{\partial \eta} \right) \left(\frac{\partial \bar{f}}{\partial \eta} \right) \right] \end{aligned} \quad (10)$$

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial \eta^2} \quad (11)$$

where $Gr^* = \frac{Gr}{\rho} \left(\frac{\nu^7}{A^5} \right)^{\frac{1}{3}}$. The boundary and initial conditions are

$$\begin{aligned} f(0, \tau) = \tau, \theta(0, \tau) = 1 \text{ for } \tau > 0 \\ f(\eta, \tau) = 0, \theta(\eta, \tau) = 0 \text{ as } \eta \rightarrow \infty \\ f(\eta, 0) = 0, \theta(\eta, 0) = 0 \end{aligned} \quad (12)$$

4. Method of Solutions

4.1 Finite Difference Method (FDM)

The nonlinear equations of Eq. (10-11) are discretised using central and forward difference as follow

$$\begin{aligned} & \frac{(1 + \zeta)}{\Delta\tau} (f_i^{j+1} - f_i^j) + (2i\Omega + M + c - Gr^*\theta^*)f_i^j \\ & = \frac{f_{i+1}^j - 2f_i^j + f_{i-1}^j}{\Delta\eta^2} + \frac{a}{\Delta\eta^2\Delta\tau} [(f_{i+1}^{j+1} - 2f_i^{j+1} + f_{i-1}^{j+1}) - (f_{i+1}^j - 2f_i^j + f_{i-1}^j)] \\ & + \frac{b}{\Delta\eta^4} (f_{i+1}^j - 2f_i^j + f_{i-1}^j)(f_{i+1}^j - f_{i-1}^j) (\bar{f}_{i+1}^j - \bar{f}_{i-1}^j) + \frac{b}{2\Delta\eta^4} (f_{i+1}^j - f_{i-1}^j)^2 (\bar{f}_{i+1}^j - 2\bar{f}_i^j + \bar{f}_{i-1}^j) \\ & - \frac{bc}{2\Delta\eta} f_i^j (f_{i+1}^j - f_{i-1}^j) (\bar{f}_{i+1}^j - \bar{f}_{i-1}^j) \end{aligned} \quad (13)$$

and

$$\frac{\theta_i^{j+1} - \theta_i^j}{\Delta\tau} = \frac{1}{Pr} \left(\frac{\theta_{i+1}^j - 2\theta_i^j + \theta_{i-1}^j}{(\Delta\eta)^2} \right) \quad (14)$$

where i and j are indices of $f(\eta_i, \tau_j)$. Bring the time-step of $j + 1$ to the left side and hence the equations become

$$\begin{aligned} & \frac{(1 + \zeta)}{\Delta\tau} f_i^{j+1} - \frac{a(f_{i+1}^{j+1} - 2f_i^{j+1} + f_{i-1}^{j+1})}{\Delta\eta^2\Delta\tau} \\ & = \frac{f_{i+1}^j - 2f_i^j + f_{i-1}^j}{\Delta\eta^2} - \frac{a}{\Delta\eta^2\Delta\tau} (f_{i+1}^j - 2f_i^j + f_{i-1}^j) \\ & + \frac{b}{\Delta\eta^4} (f_{i+1}^j - 2f_i^j + f_{i-1}^j)(f_{i+1}^j - f_{i-1}^j) (\bar{f}_{i+1}^j - \bar{f}_{i-1}^j) \\ & + \frac{b}{2\Delta\eta^4} (f_{i+1}^j - f_{i-1}^j)^2 (\bar{f}_{i+1}^j - 2\bar{f}_i^j + \bar{f}_{i-1}^j) - \frac{bc}{2\Delta\eta} f_i^j (f_{i+1}^j - f_{i-1}^j) (\bar{f}_{i+1}^j - \bar{f}_{i-1}^j) \\ & + \frac{(1+\zeta)}{\Delta\tau} f_i^j - (2i\Omega + M + c - Gr^*\theta^*)f_i^j \end{aligned} \quad (15)$$

and

$$\frac{1}{\Delta\tau} \theta_i^{j+1} = \frac{1}{\Delta\tau} \theta_i^j + \frac{1}{Pr(\Delta\eta)^2} (\theta_{i+1}^j - 2\theta_i^j + \theta_{i-1}^j) \quad (16)$$

The initial and boundary conditions are $f_1^j = A(j - 1)\Delta\tau$, $\theta_1^n = 1$ for $j = 1, 2, 3, \dots, f_L^n = 0$, $\theta_L^n = 0$ for every $n = 1, 2, 3, \dots, L = L_1, L_2, L_3$ and $f_i^1 = 0$ for $i = 1, 2, 3, \dots, N + 1$. The equation can be written in matrix form, with dimension $N - 2$

$$[A_f][f] = [b_f] \tag{17}$$

and

$$[A_\theta][\theta] = [b_\theta] \tag{18}$$

To satisfy semi-infinite boundary condition in Eq. (12), this research investigated the fluid flow problem at three lengths which are $L = L_1, L_2, L_3 = 6, 12, 18$. In this paper, the step sizes are $\Delta\eta = 1/4$ and $\Delta\tau = 1/5$ with time iteration is 6. Since the equations involve complex functions such that in Eq. (8) and is very difficult to calculate manually, MATLAB programming is coded to find the output. Table 1 presents the velocity profile obtained from FDM with $a, b, c, \zeta, \Omega, M, Gr, Pr, t = 1$ keep fixed. They show that the velocities start different either increase or decrease at $\eta = 4$. Table 2 presents the temperature distribution where they start different at $\eta = 4$. These prove that the velocity and temperature are different for different length. In order to obtain the accurate results for each node, these three lengths will be blended together in nonlinear least square in the next section.

The data from Table 1 and Table 2 are plotted to see the convergence results. Figure 1 shows the curvature of results which they are converges to the solution at each length, $L = L_1, L_2, L_3 = 6, 12, 18$.

Table 1
 The velocity profile at different lengths by FDM

eta(η)	u (real part)		eta(η)	v (imaginary part)		eta(η)
	$L = 6$	$L = 12$		$L = 6$	$L = 12$	
0	1	1	0	1	1	0
2	0.0268	0.0268	2	0.0268	0.0268	2
4	0.0008932	0.0008939	4	0.0008932	0.0008939	4
6	0	2.74×10^{-5}	6	0	2.74×10^{-5}	6
8		7.88×10^{-7}	8		7.88×10^{-7}	8
10		2.157×10^{-8}	10		2.157×10^{-8}	10
12		0	12		0	12
14			14			14
16			16			16
18			18			18

Table 2
 The temperature distribution at different lengths by FDM

θ			
eta(η)	$L = 6$	eta(η)	$L = 6$
0	1	0	1
2	0.01208	2	0.01208
4	0.000146	4	0.000146
6	0	6	0
8		8	
10		10	
12		12	

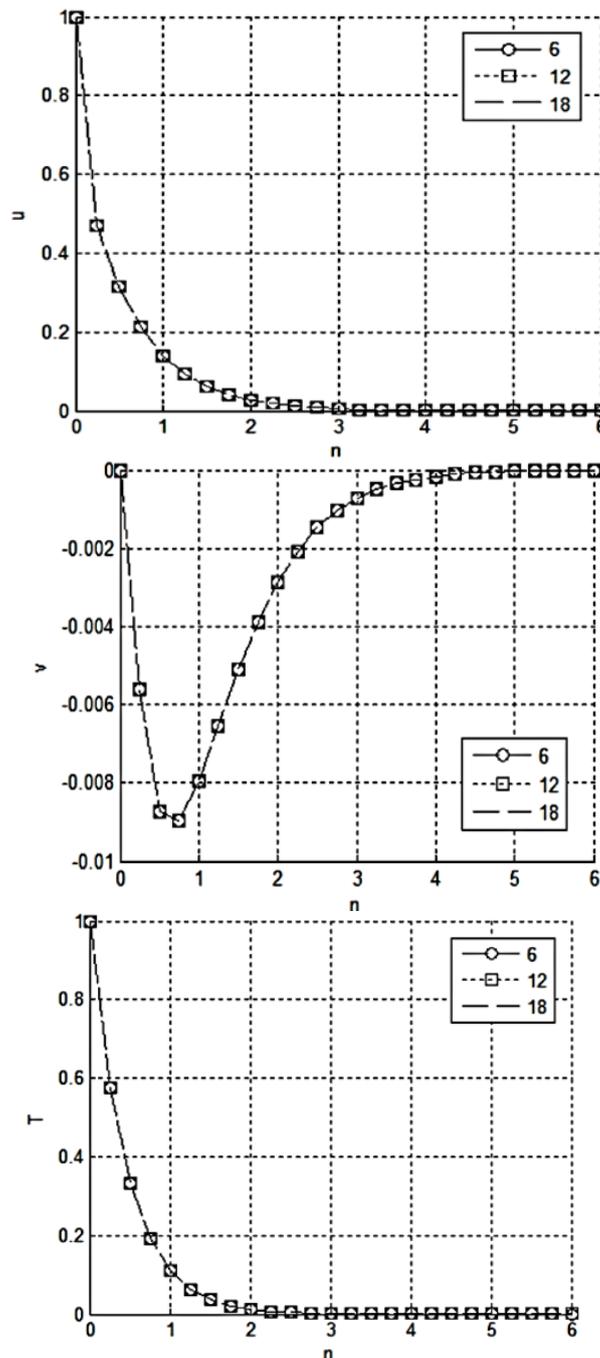


Fig. 1. FDM at $L = L_1, L_2, L_3 = 6, 12, 18$ with $a, b, c, \zeta, \Omega, M, Gr, Pr, t = 1$ fixed

4.2 Asymptotic Interpolation Method

The nonlinear least square follow a special function of $y(L_i) = a_1 + a_2 e^{-a_3^2 L_i}$ [25,26]. MATLAB is used to achieve the best fit of data. Table 3 until Table 5 show the results up to the $L = 6$. Using minimisation techniques, coefficient $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ are found from three nonlinear equations as follow

$$\sum_{i=1}^3 -2(y_i - a_1 - a_2 e^{-a_3^2 x_i}) = 0$$

$$\sum_{i=1}^3 -2e^{-a_3^2 x_i} (y_i - a_1 - a_2 e^{-a_3^2 x_i}) = 0 \tag{19}$$

$$\sum_{i=1}^3 2(y_i - a_1 - a_2 e^{-a_3^2 x_i}) \cdot 2a_3 x_i (a_2 e^{-a_3^2 x_i}) = 0$$

The data obtained by the hybrid method shown in Table 3 , Table 4 and Table 5 operating with three different lengths $L = L_1, L_2, L_3 = 6, 12, 18$ to define the infinite length. y_1, y_2 and y_3 are the results at different lengths and ε_i is prediction errors, $\varepsilon_i = y_i - y(L) = y_i - (a_1 + a_2 e^{-a_3^2 x_i})$. The results obtained at each node are similar with a_1 .

Table 3
 The real part of velocity profile after asymptotic interpolation method

η	L	y_1	y_2	y_3	$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$			ε_i	u
0	L_1	1	1	1	1.0000			9.728×10^{-8}	1
	L_2		1	1	0.8504				
	L_3			1	1.1483				
2	L_1	0.0268	0.0268	0.0268	0.0268	0.8589	1.1491	9.7297×10^{-8}	0.0268
	L_2	0.0407	0.0268	0.0268					
	L_3	0.0407	0.0407	0.0268					
4	L_1	0.0009	0.0009	0.0009	0.0009	0.8590	1.1492	9.7529×10^{-8}	0.0009
	L_2	0.0014	0.0009	0.0009					
	L_3	0.0014	0.0014	0.0009					
6	L_1	0.0000	0.0000	0.0000	0.0000	0.8561	1.1521	1.0589×10^{-7}	0.0000
	L_2	0.0000	0.0000	0.0000					
	L_3	0.0000	0.0000	0.0000					

Table 4
 The imaginary part of velocity profile after asymptotic interpolation method

η	L	y_1	y_2	y_3	$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$			ε_i	u
0	L_1	0	0	0	0.0000			9.729×10^{-8}	0.0000
	L_2		0	0	0.8591				
	L_3			0	1.1491				
2	L_1	-0.0029	-0.0029	-0.0029	-0.0029			9.7288×10^{-8}	-0.0029
	L_2	-0.0039	-0.0029	-0.0029	0.8592				
	L_3	-0.0039	-0.0039	-0.0029	1.1491				
4	L_1	-0.0002	-0.0002	-0.0002	-0.0002			9.7225×10^{-8}	-0.0002
	L_2	-0.0002	-0.0002	-0.0002	0.8592				
	L_3	-0.0002	-0.0002	-0.0002	1.1490				
6	L_1	0.0000	0.0000	0.0000	-0.0000			9.5355×10^{-8}	-0.0000
	L_2	0.0000	0.0000	0.0000	0.8599				
	L_3	0.0000	0.0000	0.0000	1.1484				

Table 5
 The temperature distribution after asymptotic interpolation method

η	L	y_1	y_2	y_3	$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$	ε_i	θ
0	L_1	1	1	1	1.0000	9.728×10^{-8}	1
	L_2		1	1	0.8504		
	L_3			1	1.1483		
2	L_1	0.0121	0.0121	0.0121	0.0121 0.8590 1.1491	9.729×10^{-8}	0.0121
	L_2	0.0120	0.0121	0.0121			
	L_3	0.0120	0.0120	0.0121			
4	L_1	0.0001	0.0001	0.0001	0.0001 0.8591 1.1491	9.7297×10^{-8}	0.0001
	L_2	0.0003	0.0001	0.0001			
	L_3	0.0003	0.0003	0.0001			
6	L_1	0.0000	0.0000	0.0000	0.0000 0.8589 1.1493	9.7826×10^{-8}	0.0000
	L_2	0.0000	0.0000	0.0000			
	L_3	0.0000	0.0000	0.0000			

Table 6 shows the comparison results from FDM and hybrid method of finite difference - asymptotic interpolation with all parameters set to $a, b, c, \zeta, \Omega, M, Gr, Pr, t = 1$. The output from FDM clearly show that the terminal node received zero value but not in hybrid method. The results of the hybrid method show that as length increases, the velocity profile and temperature increase and approaching zero which is satisfied the boundary condition of $f(\eta, \tau) \rightarrow 0$ as $\eta \rightarrow \infty$. This findings present that the hybrid method is better compared to FDM.

Table 6
 Comparison of velocity profile by FDM and hybrid method

$\eta(\eta)$	u		v		θ	
	FDM	Hybrid Method	FDM	Hybrid Method	FDM	Hybrid Method
0	1	1	0	0	1	1
1	0.1406	0.1406	-0.007967	-0.007966	0.1099	0.1099
2	0.0268	0.0268	-0.002858	-0.002858	0.01208	0.01208
3	0.004955	0.004956	-7.319×10^{-4}	-7.316×10^{-4}	0.001328	0.001329
4	0.0008932	0.0008943	-1.624×10^{-4}	-1.622×10^{-4}	0.000146	0.0001464
5	0.0001532	0.0001583	-3.206×10^{-5}	-3.288×10^{-5}	1.586×10^{-5}	1.643×10^{-5}
6	0	2.779×10^{-5}	0	-6.065×10^{-6}	0	-2.147×10^{-6}

4.3 Validation With HAM

The validation of the hybrid method is carried out by comparing the result with Nazari [29] without heat transfer. Figure 2(a) depicts the real and imaginary parts of the velocity profile obtained from HAM. Meanwhile Figure 2(b) depicts the velocity profile from the present study, which has been obtained using a hybrid method. This graph shows that the velocity profiles for both methods increase as time increases.

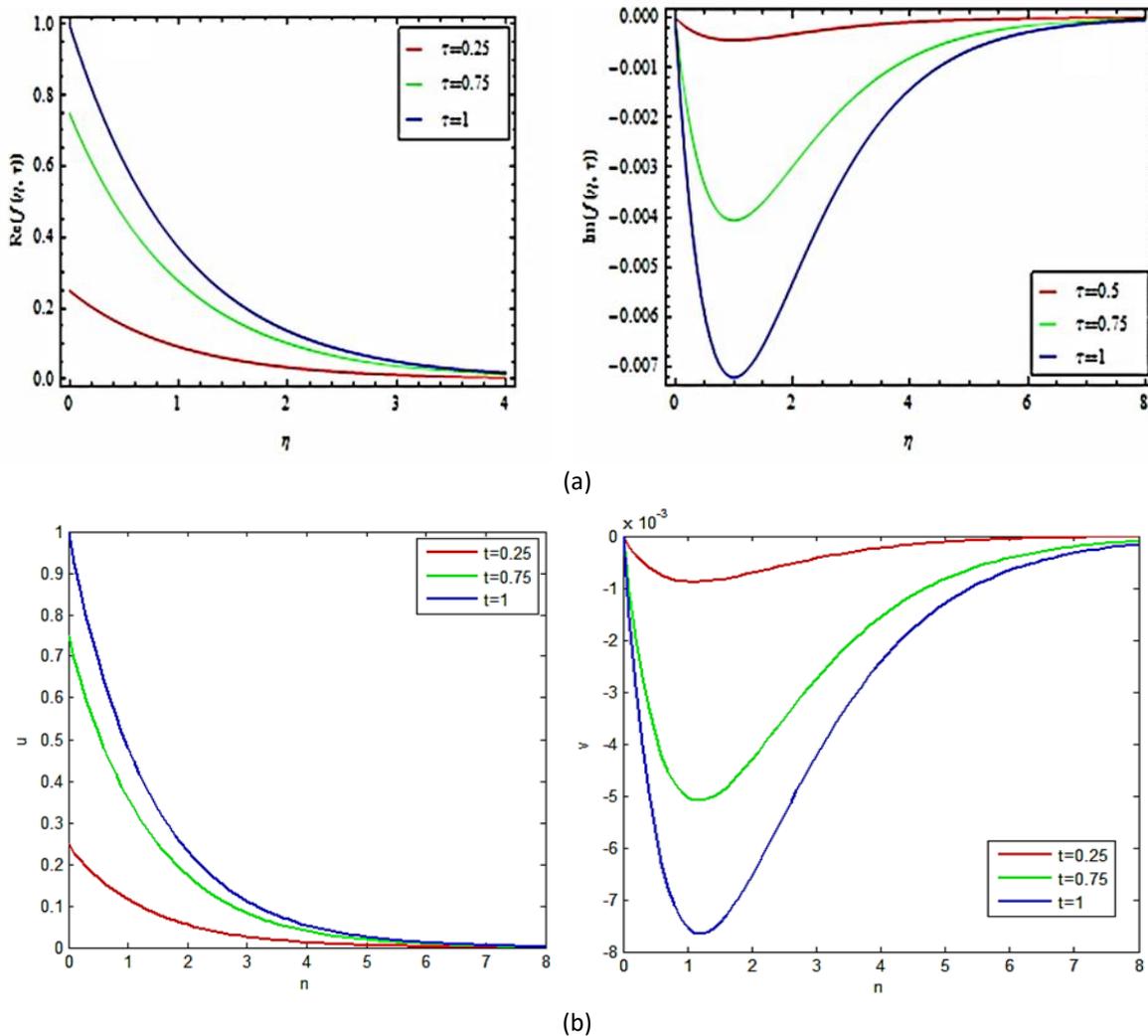
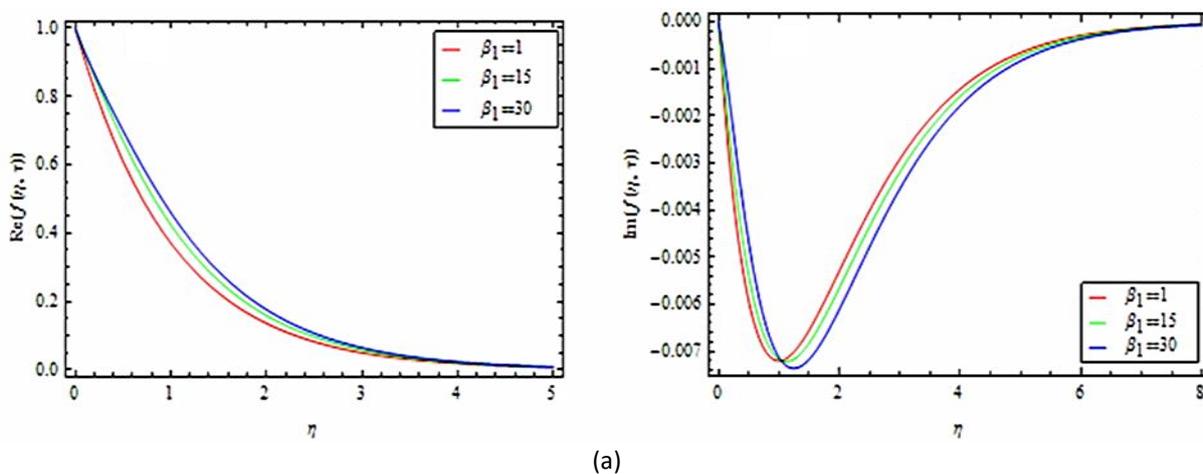


Fig. 2. Comparison of velocity profile for different value of time, τ with others parameter fixed (a) HAM (Nazari [29]) (b)Hybrid method (present work)

The velocity profile from HAM is also shown in Figure 3(a), and the current results from the hybrid method are shown in Figure 3(b). The figures seem to have the same curvature pattern. With an increasing in the third-grade parameter, the velocity profile increases.



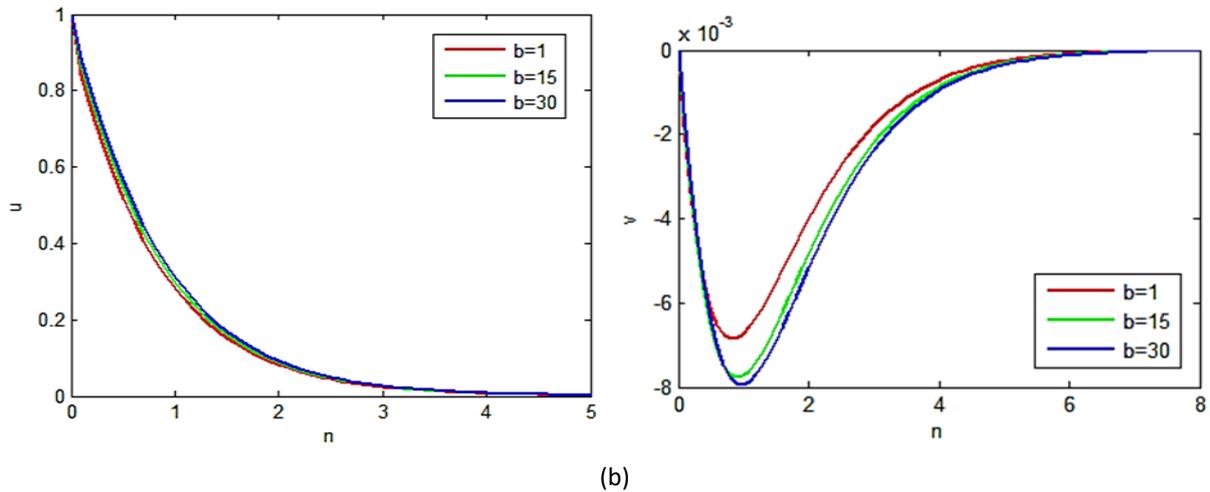


Fig. 3. Comparison of velocity profile for different value of third grade fluid, b with others parameter fixed (a) HAM (Nazari [29]) (b)Hybrid method (present work)

5. Results and Findings

MATLAB Software with Intel(R) Core (TM) i7 processor was used to illustrate the findings. Different values of parameter were tested. Figure 4 depicts the velocity distribution at the real and imaginary parts for various porosity parameter values. The results show that as c increases from 1 to 15, the velocity decreases. This result is similar with Zeeshan [30], who discovered that increasing porosity reduces fluid velocity. This is related to permeability, where the fluid can quickly move through the medium with large pores. According to Billen [31], the loose medium has greater permeability than the consolidated medium.

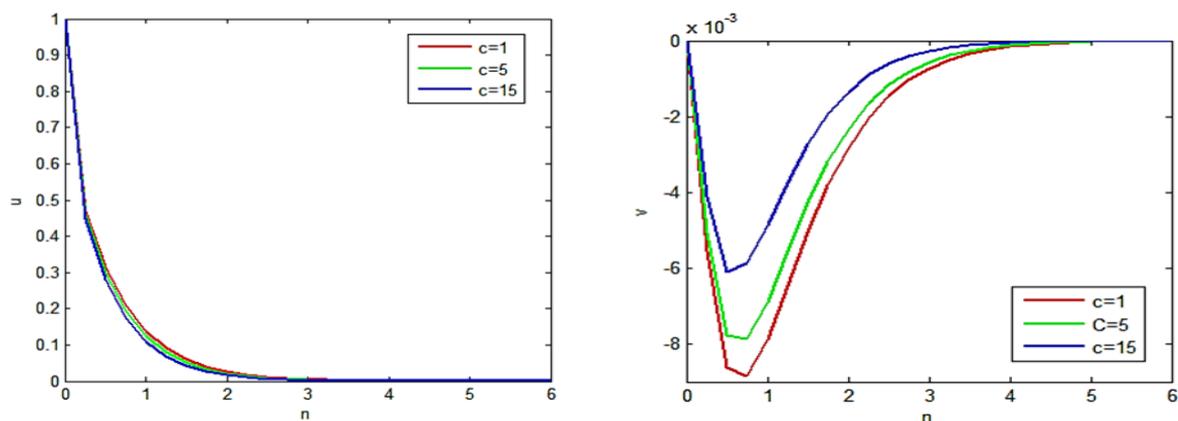


Fig. 4. Velocity profile for different value of porous parameter, c with others parameter fixed at $a, b, \zeta, \Omega, M, Gr, Pr, t = 1$

In this fluid issue, the value of the Grashof number is chosen to be $Gr > 0$, which corresponds to fluid medium cooling, as highlighted by Nayak [8]. Figure 5 depicts the results for various Grashof number values. A large number of Gr decreases kinematic viscosity and thus increases the velocity profile's boundary layer. Anita [12] investigated the impact of different values of Gr and reported to a similar conclusion. It should be noted that for $Pr = 1$, both heat and momentum dissolve over the fluid in about the same ratio.

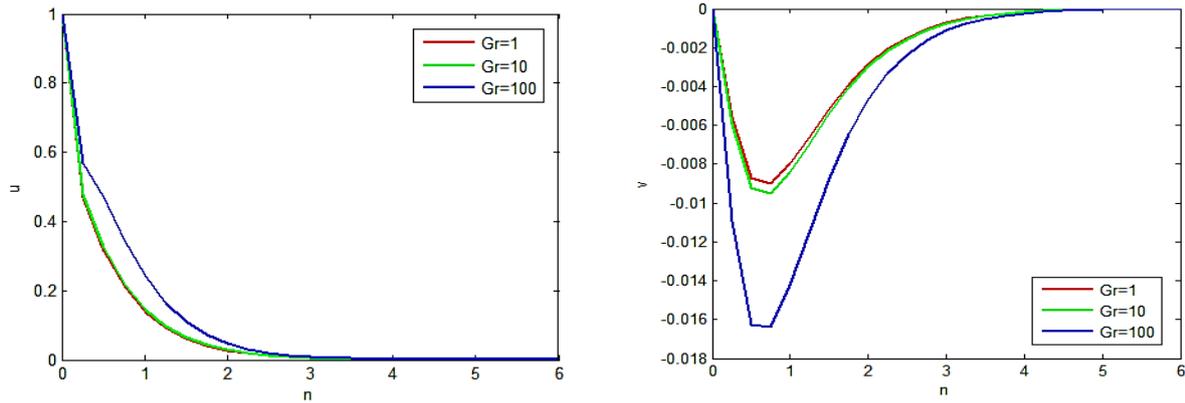


Fig. 5. Velocity profile for different value of Grashoff number, Gr with others parameter fixed at $a, b, c, \zeta, \Omega, M, Pr, t = 1$

Figure 6 depicts the relationship between the Grashof number, Gr and the Prandtl numbers, Pr . Four Gr and Pr couple sets are investigated. $Pr < 1$ shows that thermal diffusivity is the dominant factor. As a result, for every value of Gr the temperature distribution increased. In this case, heat conduction happens. The thickness of the thermal boundary layer is greater than that of the velocity boundary layer. A liquid metal, such as mercury, is a typical example of this fluid. Meanwhile, $Pr > 1$ indicates that momentum diffusivity is dominant and essential to the convection phase. As a result, the thermal boundary layer becomes thinner. However, it is also shown that for $Gr > Pr$, the boundary layer is thicker than for $Gr < Pr$. Furthermore, it is observed that as Pr increases, the temperature decreases [32,33].

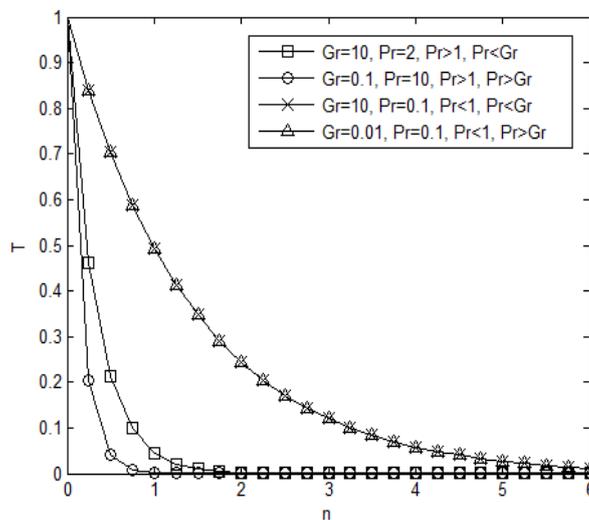


Fig. 6. Effect of Gr and Pr towards the temperature distribution

Figure 7 depicts the effect of a magnetic field on the velocity of a fluid. Physically, as the magnetic parameter increases from 5 to 15 the Lorentz force emerges and slows the fluid motion [34]. The fluid's velocity decreases because of this condition. The effect of different values of second- and third-grade parameters on the velocity profile of fluid flow is investigated. Figure 8 shows that increasing the non-Newtonian parameter α leads to increase in velocity. This result is consistent with the results of previous research by Siddiqui [35]. Figure 9 also shows that increasing the parameter b cause

velocity to increase. Figure 10 also presents the relationship between time and velocity profile are directly proportional where velocity increase when time increase.

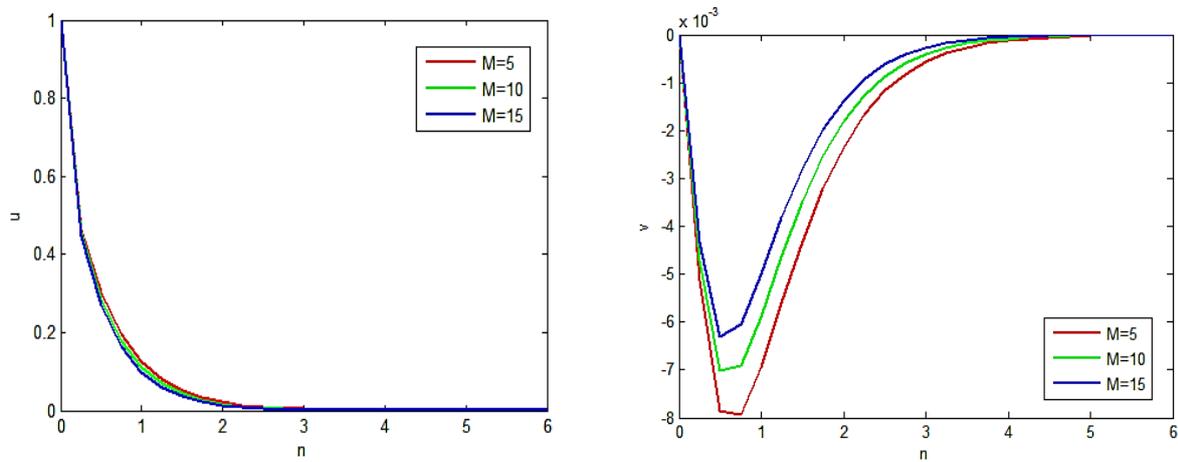


Fig. 7. Velocity profile for different value of magnetic parameter, M with others parameter fixed at $a, b, c, \zeta, \Omega, Gr, Pr, t = 1$

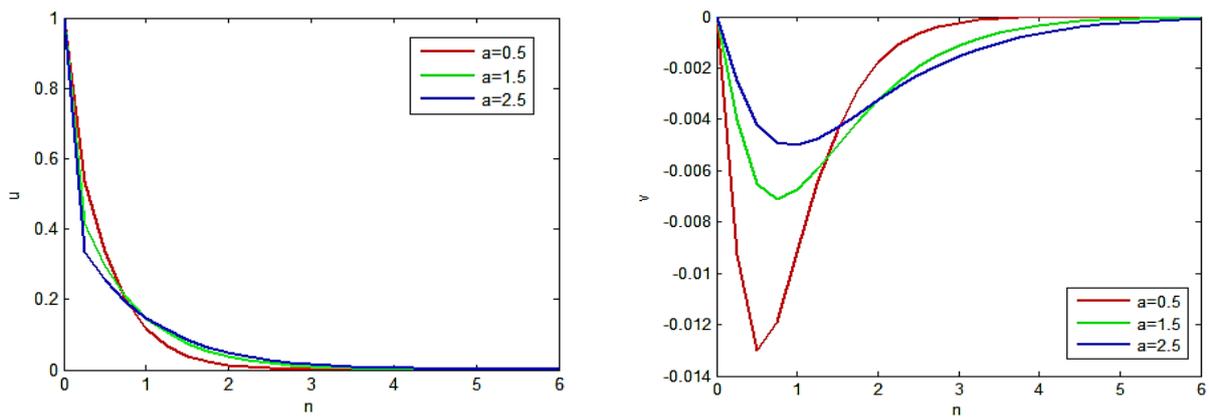


Fig. 8. Velocity profile for different value of second-grade parameter, a with others parameter fixed at $b, c, \zeta, M, \Omega, Gr, Pr, t = 1$

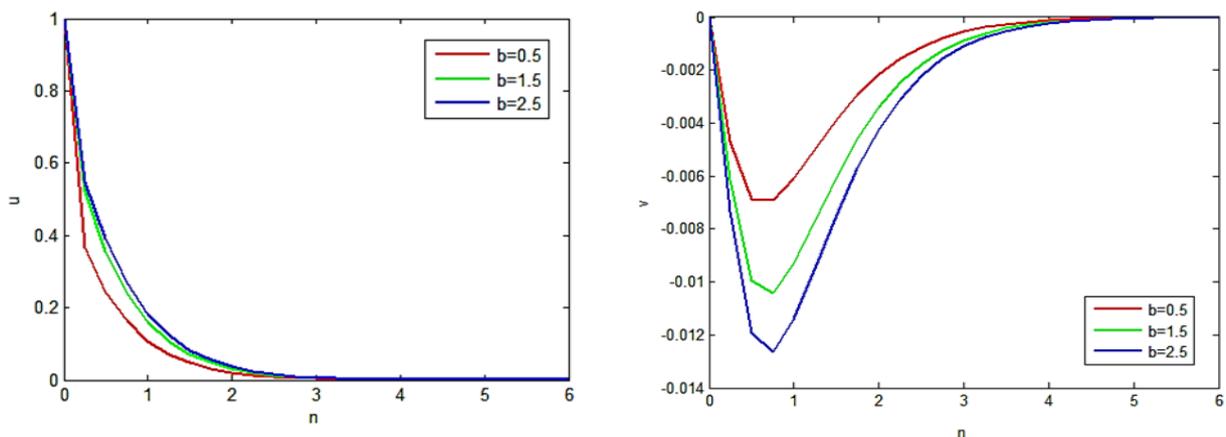


Fig. 9. Velocity profile for different value of third-grade parameter, b with others parameter fixed at $a, c, \zeta, M, \Omega, Gr, Pr, t = 1$

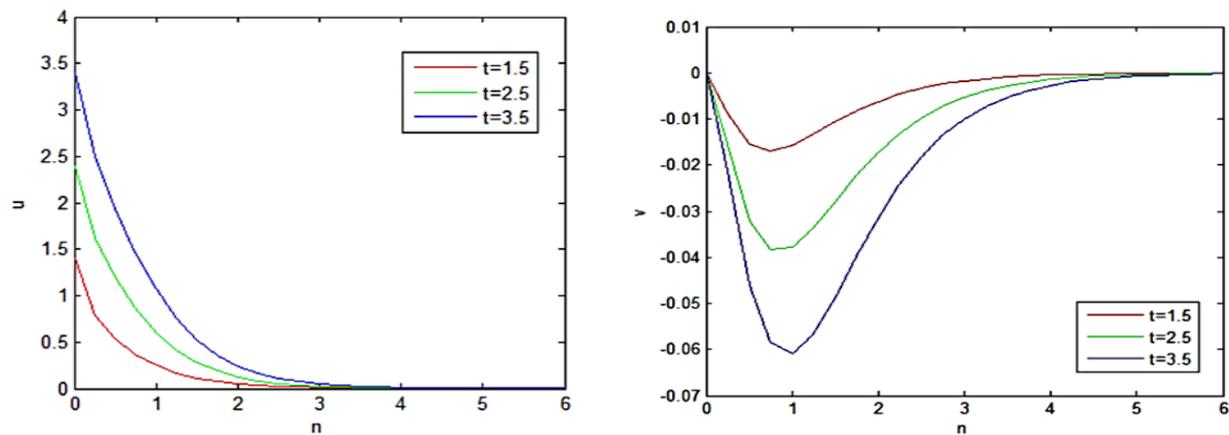


Fig. 10. Velocity profile for different value of third-grade parameter, t with others parameter fixed at $a, b, c, \zeta, M, \Omega, Gr, Pr = 1$

6. Discussions

The third-grade fluid flows in a rotating frame through a magnetic field and a porous medium have been analysed. In order to address the fluid flow problem, a hybrid method of finite difference – asymptotic interpolation is adopted. To fulfil the infinite size of the domain, this hybrid method used three different lengths in FDM as shown in Figure 1, Table 1 and Table 2, which had been later combined with an asymptotic function $y = a_1 + a_2 e^{-a_3^2 L}$ [25,26]. The best fit is achieved by using a non-linear least square curve fit. The hybrid process solution is a_1 [25,26]. Table 3 until Table 5 indicate the hybrid solution and how the outcomes met the condition in Eq. (16). There is no exact solution to this problem that has been published. The issue has been compared with semi-analytical HAM in the absence of heat transfer, as shown in Figure 2 and Figure 3.

Heat transfer is assigned to the fluid flow problem in the following section. The lengths of the problem $L = L_1, L_2, L_3 = 6, 12, 18$ are chosen to present the limiting case $\eta \rightarrow \infty$. Table 6 shows that the outputs of the hybrid method are larger than the outputs of the finite difference method. Various values of the porosity parameter, Grashof number, Prandtl number, MHD parameter, time, second- and third-grade parameters are tested to see how they influence the velocity and temperature distribution. The outcomes are depicted in Figure 4 until Figure 10. The fluid velocity decrease as the porosity parameter increased. Increasing the Grashof number, on the other hand, will boost the buoyancy force of the flow and leads to increase the velocity profile [33]. Figure 4 describes the relationship between Prandtl and Grashof numbers. Temperature goes up for a low Prandtl number ($Pr < 1$). The fluid flow in a magnetic field. As a result, the higher the MHD, the slower the velocity. The results are consistent with previous studies.

7. Conclusions

The problem of unsteady constant acceleration MHD third-grade fluid flow in a rotating frame through porous medium with a presence of heat transfer is conducted. A new hybrid method of finite difference method and asymptotic interpolation method is adopted to deal with the semi-infinite size of the fluid flow problem. The procedure of method is shown in separate sections with all the parameters fixed; $a, b, c, \zeta, \Omega, M, Gr, Pr, t = 1$. The advantage of the hybrid method over FDM is presented. It reveals that the results from the hybrid method are greater and fulfil the boundary condition compared to the FDM that presents zero result at infinite length. The result is validated

with HAM. Various parameters with different value are tested and the findings show that there is relation between parameters with velocity and temperature distribution.

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