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Solitary Wave Solutions for Forced Nonlinear Korteweg-de Vries Equation by Using Approximate Analytical Method

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ABSTRACT

The Multistep Modified Reduced Differential Transform Method (MMRDTM) is proposed in this paper. It is implemented to solve the forced nonlinear Korteweg-de Vries (fnKdV) equations. There are several advantages of the proposed method. Using the suggested approach, a high-speed converging series can be approximated analytically. Furthermore, the number of calculated terms is reduced significantly. The nonlinear term in fnKdV equations is substituted with corresponding Adomian polynomials before applying the multi-step technique. As a result, we provided simpler and more effective ways to solve fnKdV equations. On top of that, the solutions can be approximated more accurately over a longer period of time. To show the MMRDTM's capability and accuracy, we consider several of the fnKdV examples to illustrate the proposed method's potential in analytical approximation. Then, the features of the solutions are represented in tabular and graphical forms. In conclusion, the proposed method delivers highly accurate and precise solutions for these types of equations.

1. Introduction

In a wide range of physics phenomena, the Korteweg-de Vries (KdV) equation is very useful in modelling the interaction and evolution of nonlinear waves equations. Russell's [1] experiments in 1844 led to the start of the work on the KdV equation. It was found by using one-dimensional evolution equation that describes how a long-surface gravity wave with a small amplitude moves through a shallow water channel.

Keskin and Oturanç [2] presented a general framework of the Reduced Differential Transform Method (RDTM) to solve the generalized equations of KdV equation. Comparison of RDTM with other established techniques demonstrating the current method's effectiveness. Abdou and Yildirim [3] came up with a way to use RDTM to find numerical solutions to time-fractional nonlinear evolution equations with initial conditions. The numerical findings showed the significant characteristics,

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efficacy and reliability of the suggested technique and graphically showed the impacts of distinct values.

Besides that, Abazari and Abazari [4] solved the generalised Hirota–Satsuma coupled KdV equation with Differential Transform Method (DTM) and RDTM. They demonstrated that the RDTM is not only incredibly effective but also very convenient and accurate for solving nonlinear equation systems. Furthermore, Ebenezer *et al.,* [5] proposed fractional RDTM to obtain analytic solution of time-fractional KdV equation. By using numerical examples, the obtained results showed that these techniques are accurate, reliable, and efficient. On the other hand, Triki *et al.,* [6] used the sine-cosine method and the Kudryashov generalised method to look into the potential KdV equations. The KdV equation with linear damping force is taken into consideration by Ali *et al.,* [7] for a large-scale problem like the tsunami.

On the other hand, the fractional RDTM was modified by Ray [8] and used to solve fractional KdV equations. In that paper, the nonlinear term was substituted with related Adomian polynomials which proposed by Kataria and Vellaisamy [9]. Therefore, the solutions to the nonlinear problem can be found more easily by using less calculation steps. El-Zahar [10] further came up with an adaptive multistep DTM to solve a singular pertubation initial-value problems. Fast-convergent series are generated by the method; therefore, the solution converges over a wide time range.

Recently, nonlinear KdV equations was solved by Hussin *et al.,* [11] using their proposed Multistep Modified Reduced Differential Transform Method (MMRDTM). Based on their finding, high-accuracy approximate solutions of the nonlinear KdV were achieved. Besides that, by using the same method, Hussin *et al.,* [12] solved the nonlinear Schrodinger equations with power-law nonlinearity. Moreover, Hussin *et al.,* [13] also solved the for the nonlinear Dispersive $K(m,n)$ equations using the MMRDTM. The findings indicated that the MMRDTM is a legitimate and efficient approach for obtaining an approximate analytic solution to nonlinear KdV equations with compact support.

The modification made by Ray [8] and El-Zahar [10] have inspired this study to come up a new application of the MMRDTM for solving the forced nonlinear Korteweg-de Vries (fnKdV) equations. This method is able to provide an analytical approximation in a fast-convergent sequence with a decreased number of calculated terms.

Basically, the shallow groundwater model defines the governing equations for the basic hydrodynamic model of tsunami generation explained in [14,15]. This evolution is based on the most basic theory of water waves that adequately describes the behaviour of the real ocean, as well as the system of partial differential equations discussed in the studies by Guyenne and Grilli [16], Layton and Panne [17] and Pelinobvs *et al.,* [18]. Although many researchers doubt the application of soliton theory to tsunami modeling by Constantin and Johnson [19], the approach to the solitary wave model nevertheless gives an idea of the impressive scales involved in deep water waves in a study by Li and Sattinger [20]. As a result, a numerical study of this equation is being carried out to investigate the solitary wave dynamics in shallow water.

In many cases, the source of the tsunami moves at varying speeds and directions. The fnKdV equation has been shown to describe the resonant mechanism of tsunami wave generation by atmospheric disturbances moving at near critical speed in the ocean in the study by Shen [21]. In a series of articles, Shen [22,23] outlined that there are two supercritical solitary waves and one subcritical downstream cnoidal wave. Solitary waves emerge at the variable topographic effects on the evolution of the internal undular bores of depression [24]. Hoe *et al.,* [25] also investigated the effect of rapidly varying topography by using mathematical model of the variable-coefficient extended KdV. Hayytov *et al.,* [26] investigate computational performance of multistep schemes in solving hyperbolic model based on one-dimensional linear wave equation. Francesca *et al.,* [27] developed new scheme to solve non-linear wave equation for acoustical modelling.

The dynamical system may be simplified considering the resonant character of tsunami generation. It is shown that it is possible to derive simpler nonlinear, dispersive wave models, which can be used for our potential initial numerical investigation, by making further simplifying assumptions. If waves are believed to travel in only the positive x-direction then it can be shown that the waves have a steady form at a first approximation. For the detailed derivations of the fnKdV equations, see in the study by Yaacob *et al.,* [28].

2. Methodology

2.1 The Development of Multistep Modified Reduced Differential Transform Method

The fnKdV equation to be considered in this paper is as follows

$$\frac{\partial u}{\partial t}+\alpha u\frac{\partial u}{\partial x}+\beta\frac{\partial^3 u}{\partial x^3}=\frac{\partial f}{\partial x} \tag{1}$$

Eq. (1) represents a different approach to describing the governing equations for the basic hydrodynamic model of tsunami generation caused, for example, by atmospheric disturbances [14,28]. The term $\partial f/\partial x$ is called forcing term. The appropriate evolution equation, which approximates the Boussinesq equation asymptotically, leads to this Eq. (1) of fnKdV. It can be assumed that the forcing term in the fnKdV can be derived from atmospheric disturbances. Different forms of this equation have been investigated widely and numerical results indicate that the solution contains a set of solitary waves [18,29]. Obviously, the behaviour of tsunami waves on the open ocean is much more complex than the solitary wave model; however, this fnKdV equation was used in Yaacob *et al.,* [28] as a simple mathematical model that could describe the modelling of tsunami waves.

Now simulate the solitary wave evolution with the force in motion of the KdV equation. In order to demonstrate the roles of solitons in the forced dynamics, it has been assumed in the scheme that the forcing term being added is sufficiently weak in Pelinovsky *et al.,* [18], so that it remains in shape during the phase of interaction.

By applying the MMRDTM to Eq. (1) and utilising its fundamental properties, we obtain

$$U_{k+1,i}(x)=\left(\frac{1}{k+1}\right)\left(-\alpha\sum_{r=0}^{k}U_{k-r,i}(x)\frac{\partial}{\partial x}U_{r,i}(x)-\beta\frac{\partial^3}{\partial x^3}U_k(x)+\frac{\partial f}{\partial x}\right) \tag{2}$$

with transformed initial condition

$$U_0(x)=f(x) \tag{3}$$

Now, write the nonlinear term

$$N(u,t)=\sum_{n=0}^{\infty}A_n(U_0(x),U_1(x),\ldots,U_n(x))t^n, \tag{4}$$

where A_n is the appropriate Adomian's polynomials. A recent novel method which was proposed by Kataria and Vellaisamy [9] for calculating the Adomian polynomials such as

$$\begin{aligned}&A_0=N\big(U_0(x)\big),\\&A_n(U_0(x),U_1(x),\ldots,U_n(x))=\frac{1}{2\pi}\int_{-\pi}^{\pi}N\left(\sum_{k=0}^{n}U_k(x)e^{ikx}\right)e^{-in\lambda}\,d\lambda,\quad n\geq 1\end{aligned} \tag{5}$$

The $U_k(x)$ values can be computed by substituting Eq. (3) into Eq. (2) and performing iterative calculations on the resultant data. Then, the n-terms approximation solution is given by the set of values $\{U_k(x)\}_{k=0}^{n}$ of the inverse transformation as follows:

$$u(x,t) = \sum_{k=0}^{K} U_k(x)t^k , \qquad t \in [0,T]$$

Suppose that we use the nodes $t_i = ih$ to divide the interval $[0,T]$ into M subintervals $[t_{i-1}, t_i]$, where $i = 1,2,\ldots,M$, and each step is $h = \frac{T}{M}$. The MMRDTM's fundamental concepts are as follows. Firstly, modified RDTM (MRDTM) is applied to the initial value problem of interval $[0,t_1]$. Then, use the initial conditions $u(x,0) = f_0(x)$, and $u_1(x,0) = f_1(x)$ to get the following approximate result,

$$u_1(x,t) = \sum_{k=0}^{K} U_{k,1}(x)t^k, \qquad t \in [0,t_1]$$

At each subinterval $[t_{i-1}, t_i]$, the initial conditions $u_i(x,t_{i-1}) = u_{i-1}(x,t_{i-1}), (\partial/\partial t)u_i(x,t_{i-1})$ are used for $i \geq 2$ and the implementation of multistep RDTM to the initial value problem on $[t_{i-1}, t_i]$, where t_{i-1} replaces t_0. Next multistep scheme for repeating process $u(x,0) = f_0(x), u_1(x,0) = a$. In order to generate a series of approximations $u_i(x,t), i = 1,2,\ldots,M$, for the solution $u(x,t)$, the following steps must be performed repeatedly:

$$u_i(x,t) = \sum_{k=0}^{K} U_{k,i}(x)(t-t_{i-1})^k, \qquad t \in [t_{i-1}, t_i]$$

In fact, the MMRDTM yields the following solutions:

$$u(x,t) = \begin{cases} u_1(x,t), & t \in [0,t_1] \\ u_2(x,t), & t \in [t_1,t_2] \\ \vdots & \vdots \\ u_M(x,t), & t \in [t_{M-1},t_M] \end{cases}$$

Obviously, the algorithm of MMRDTM is simple and offers improved computing performance across the board regardless of the value of h. It is crucial to note that after the step size $h = T$, the MMRDTM is reduced to the MRDTM.

3. Results

3.1 Example 1

We consider the fnKdV by using different forcing terms in this paper. Forcing Term, $\partial f/\partial x = x^2$. Consider the fnKdV equation with the quadratic term x^2 as the forcing term. Let $\alpha = -6$ and $\beta = 1$, then Eq. (1) is simplified as introduced by David *et al.,* [31]:

$$\frac{\partial u}{\partial t} - 6u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = x^2 \qquad (6)$$

with initial condition

$$u_0(x,t) = \frac{-2e^x}{(1+e^x)^2}$$

and exact solution

$u(x,t) = \frac{-2e^{x-t}}{(1+e^{x-t})^2}$

Using basic properties of MMRDTM and then applying MMRDTM to Eq. (6), will obtain

$$U_{k+1,i}(x) = \left(\frac{1}{k+1}\right)\left(6\sum_{r=0}^{k} U_{k-r,i}(x)\frac{\partial}{\partial x}U_{r,i}(x) - \frac{\partial^3}{\partial x^3}U_k(x) + \delta(k-2)\right) \quad (7)$$

with transformed initial condition

$$U_0(x) = \frac{-2e^{x}}{(1+e^{x})^2}. \quad (8)$$

Now write first four examples of the nonlinear term as

$A_0 = U_0^2(x),$
$A_1 = 2U_0(x)U_1(x),$
$A_2 = 2U_0(x)U_2(x) + U_1^2(x),$
$A_3 = 2U_0(x)U_3(x) + 2U_1(x)U_2(x).$

These nonlinear terms, we use general form of formula A_n Adomian polynomials as Eq. (5). We calculate these Adomian polynomials formula by using Maple 2021. The $U_k(x)$ values can be computed by substituting Eq. (8) into Eq. (7) and performing iterative calculations on the resultant data. Then, the n-terms approximation solution is given by the set of values $\{U_k(x)\}_{k=0}^{n}$ of the inverse transformation as follows:

$u(x,t) = \sum_{k=0}^{K} U_k(x)t^k, \qquad t \in [0,T]$

Figure 1(a) shows graph of the exact solution, Figure 1(b) shows graph of approximate solution MMRDTM for $t \in [0,2]$ and $x \in [-5,5]$ while Figure 1(c) shows graph of approximate solution MRDTM for $t \in [0,2]$ and $x \in [-5,5]$. Observe that, the graphs of the solutions which obtained by the MMRDTM look similar to the graphs of the corresponding exact solutions. That means, the MMRDTM yielded solutions close to the exact solution of fnKdV equations. The performance error analyses obtained by MMRDTM are summarized in Table 1 for $x = 1$.

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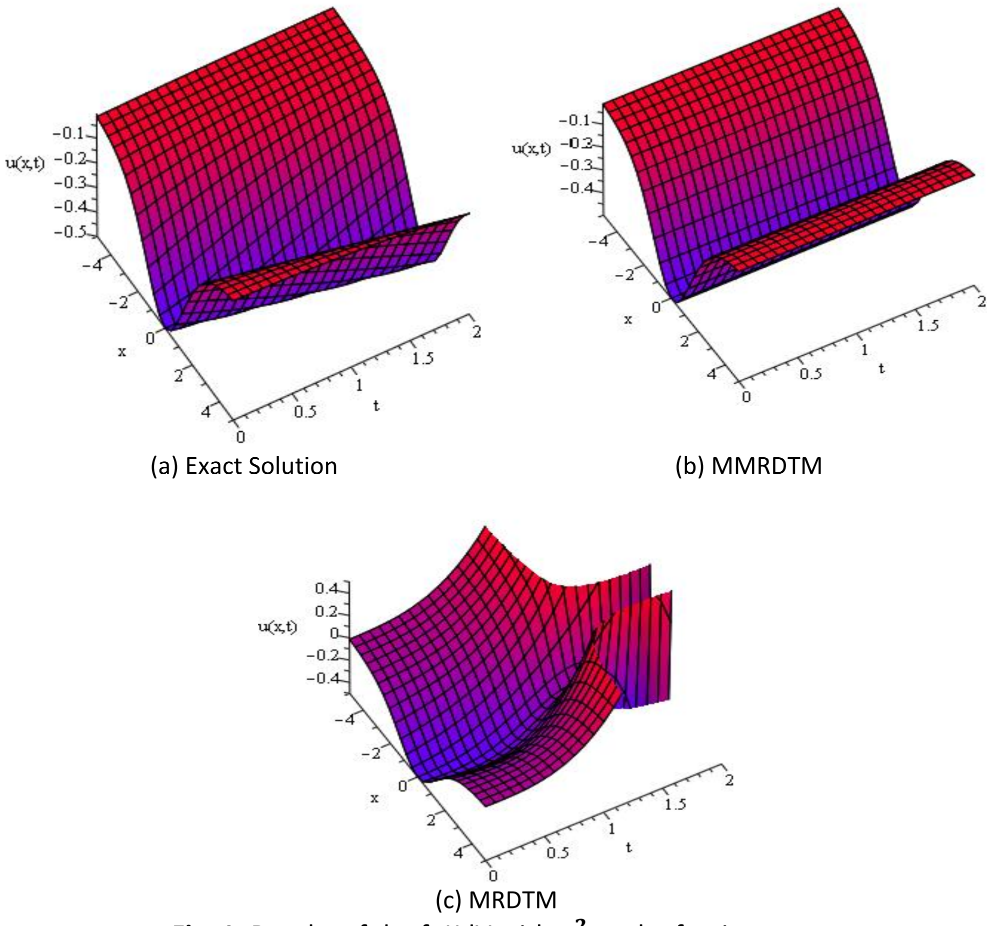
Table 1
Comparison error of MMRDTM and MRDTM for Example 1

t	Exact Solution	Absolute Error MMRDTM	Absolute Error MRDTM
0.1	-0.4110006146	4.000×10^{-10}	3.420×10^{-4}
0.2	-0.4278193930	4.300×10^{-9}	2.797×10^{-3}
0.3	-0.4434257466	7.110×10^{-8}	9.605×10^{-3}
0.4	-0.4575684810	5.680×10^{-7}	2.304×10^{-2}
0.5	-0.4700074244	2.893×10^{-6}	4.527×10^{-2}
0.6	-0.4805214914	1.102×10^{-5}	7.814×10^{-2}
0.7	-0.4889166234	3.431×10^{-5}	1.229×10^{-1}
0.8	-0.4950331454	9.197×10^{-5}	1.800×10^{-1}
0.9	-0.4987520804	2.198×10^{-4}	2.487×10^{-1}
1.0	-0.5000000000	4.794×10^{-4}	3.266×10^{-1}
1.1	-0.4987520804	9.703×10^{-4}	4.094×10^{-1}
1.2	-0.4950331454	1.846×10^{-3}	4.902×10^{-1}
1.3	-0.4889166232	3.330×10^{-3}	5.593×10^{-1}
1.4	-0.4805214914	5.742×10^{-3}	6.030×10^{-1}
1.5	-0.4700074242	9.517×10^{-3}	6.037×10^{-1}
1.6	-0.4575684810	1.524×10^{-2}	5.391×10^{-1}
1.7	-0.4434257464	2.366×10^{-2}	3.807×10^{-1}
1.8	-0.4278193930	3.575×10^{-2}	9.416×10^{-2}
1.9	-0.4110006146	5.270×10^{-2}	3.622×10^{-1}
2.0	-0.3932238666	7.599×10^{-2}	1.038



(a) Exact Solution (b) MMRDTM

(c) MRDTM

Fig. 1. Results of the fnKdV with x^2 as the forcing term

3.2 Example 2

Forcing Term, $\partial f/\partial x = \sin(x)$. Consider the fnKdV equation with the trigonometric function $\sin(x)$ as the forcing term by David *et al.,* [31]

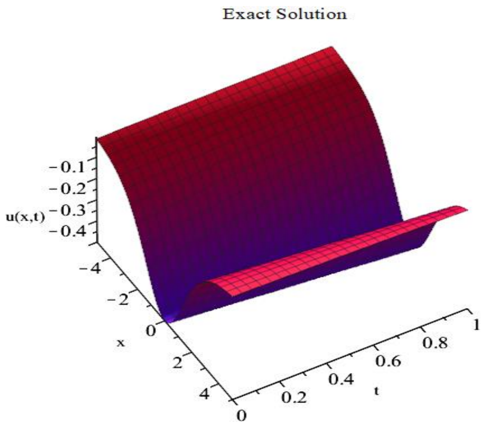
$$\frac{\partial u}{\partial t} - 6u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = \sin(x) \tag{9}$$

Using basic properties of MMRDTM and then applying MMRDTM to Eq. (9), we can obtain

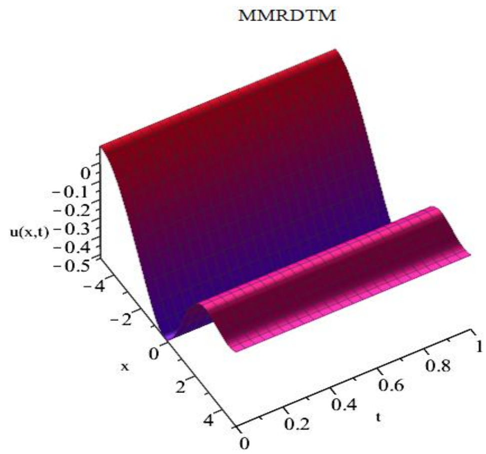
$$U_{k+1,i}(x) = \left(\frac{1}{(k+1)}\right)\left(6\sum_{r=0}^{k} U_{k-r,i}(x)\frac{\partial}{\partial x}U_{r,i}(x) - \frac{\partial^3}{\partial x^3}U_k(x) + \sin(x)\right),$$

with transformed initial condition Eq. (8).

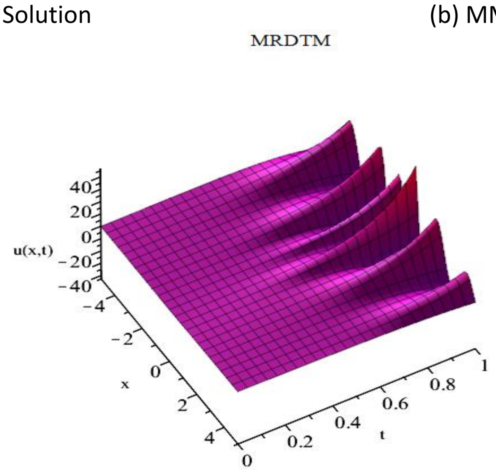
Figure 2(a) shows graph of the exact solution, Figure 2(b) shows graph of approximate solution MMRDTM for $t \in [0,1]$ and $x \in [-5,5]$ while Figure 2(c) shows graph of approximate solution MRDTM for $t \in [0,1]$ and $x \in [-5,5]$. The graph of MMRDTM is far better than MRDTM which depicts a good agreement with the exact solution. Therefore, it is shown that MMRDTM has high accuracy for solving this type of fnKdV equations.



(a) Exact Solution



(b) MMRDTM



(c) MRDTM

Fig. 2. Results of the fnKdV with $sin(x)$ as the forcing term

3.2 Example 3

Forcing Term, $\partial f/\partial x = e^x$. Consider the fnKdV equation with the exponential term e^x as the forcing term by David *et al.,* [31],

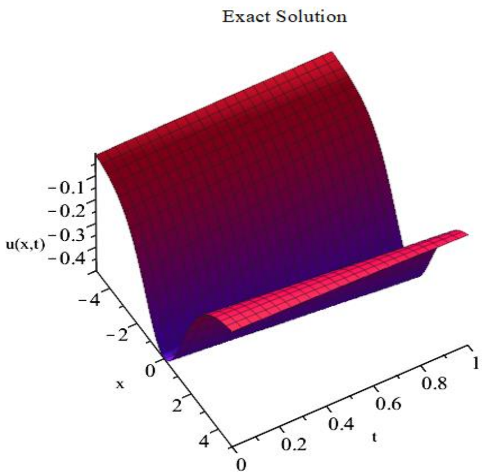
$$\frac{\partial u}{\partial t} - 6u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = e^x. \tag{10}$$

Using basic properties of MMRDTM and then applying MMRDTM to Eq. (10), we can obtain

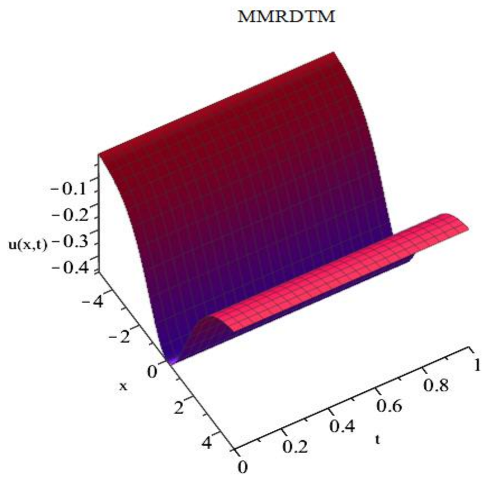
$$U_{k+1,i}(x) = \left(\frac{1}{(k+1)}\right)\left(6\sum_{r=0}^{k} U_{k-r,i}(x)\frac{\partial}{\partial x}U_{r,i}(x) - \frac{\partial^3}{\partial x^3}U_k(x) + e^x\right),$$

with transformed initial condition Eq. (8).

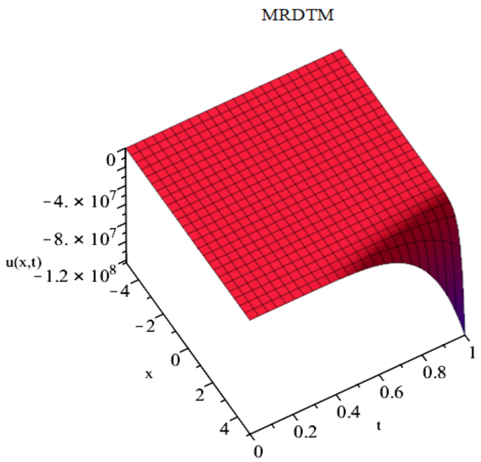
Figure 3(a) shows graph of the exact solution, Figure 3(b) shows graph of approximate solution MMRDTM for $t \in [0,1]$ and $x \in [-5,5]$ while Figure 3(c) shows graph of approximate solution MRDTM for $t \in [0,1]$ and $x \in [-5,5]$. Graph of MMRDTM shows a good agreement with the exact solution. Therefore, it is shown that MMRDTM has high accuracy for solving this type of fnKdV equations.



(a) Exact Solution



(b) MMRDTM



(c) MRDTM

Fig. 3. Results of the fnKdV with e^x as the forcing term

4. Conclusions

In this study, a new approximate analytical method known as MMRDTM is developed and implemented to obtain solitary wave solutions for forced nonlinear KDV equation. The technique was modified to replace the nonlinear term with its Adomian polynomials in this novel technique. Then multistep approach was adopted to get accuracy solution. According to the findings and the graphical representation of the results, the estimated fnKdV solutions were attained with high precision. Finally, we can state that the analytic approximation solution obtained to this type of equation using the MMRDTM outperforms the MRDTM in terms of effectiveness, consistency, and accuracy. The Maple 2021 software was used to perform all computations in this paper.

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