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Exploration of No-Slip and Slip of Unsteady Squeezing Flow Fluid Through a Derivatives Series Algorithm

Abeer Majeed Jasim[1,*]

[1] Department of Mathematics, College of Science, University of Basrah, Basrah, Iraq

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ABSTRACT

In this work, an unsteady incompressible viscous magnetic-hydrodynamic squeezing of flow fluid is investigated. By the help of similar variables and appropriate transformations which play an important role to convert non-linear partial differential system into the non-linear ordinary differential equation. Also, the reduced boundary value of problem is resolved analytically by employing a derivatives series algorithm (DSA). The important key for this construction is necessary for the derivatives that appear as a coefficient in the power series. The impacts of conspicuous physical emerging parameters on the velocity distribution are described using sketched and interpreted at boundaries in cases of slip and no slip.

1. Introduction

Most physical phenomenon can describe the flow of fluid that is squeezed between two parallel objects. In many hydrodynamic machines and tools one of this phenomenon can been observed. In the nineteenth century, the modeling of the flow fluid was investigated due to its endless applications in various fields, therefore it has gained a lot of attention. The first studies in these flows were introduced by Stefan [1] who found an asymptotic solution and worked on Newtonian fluid. There are different applications of these flow [2-4] can be described in Figure 1.

Exceptionally, magneto-hydrodynamics refer to the study of fluids in the electromagnetic field. The use of magneto-hydrodynamic fluid as a lubricant is highly increased because of any unexpected change in the viscosity of this lubricant can be avoided under certain extreme conditions. Various authors have studied the impact of magnetic field on fluid flow [5-19]. In [20] by the presence of a magnetic field squeezing flow between two disks together was examined and between the rotating disks was investigated in [21,22]. A material which has filled fluid pores is known as a porous medium. The Low of Reynolds number results in the viscous force being very robust when compared with inertial forces. In such cases, in porous media inertial forces are at times neglected this flow system

* *Corresponding author.*
E-mail address: abeer.jassem@uobasrah.edu.iq

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is known creeping flow or stokes [23-25]. In addition, the properties of a porous medium can be shown in Figure 2.

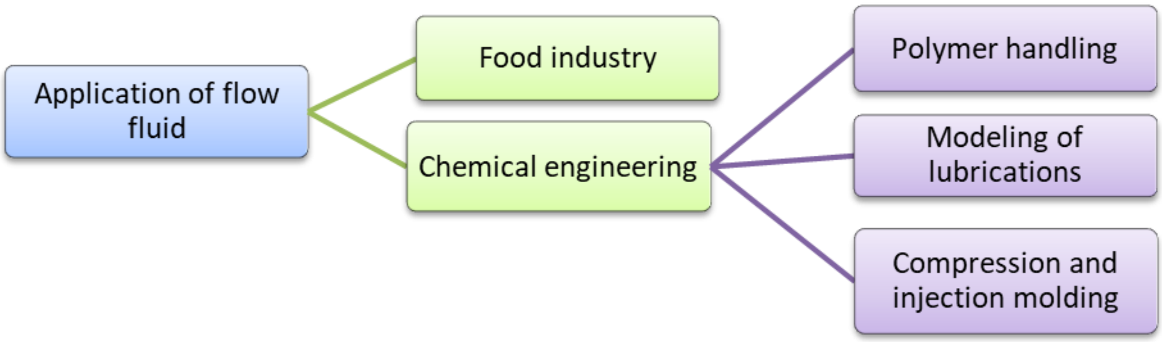

Fig. 1. The applications of flow fluid

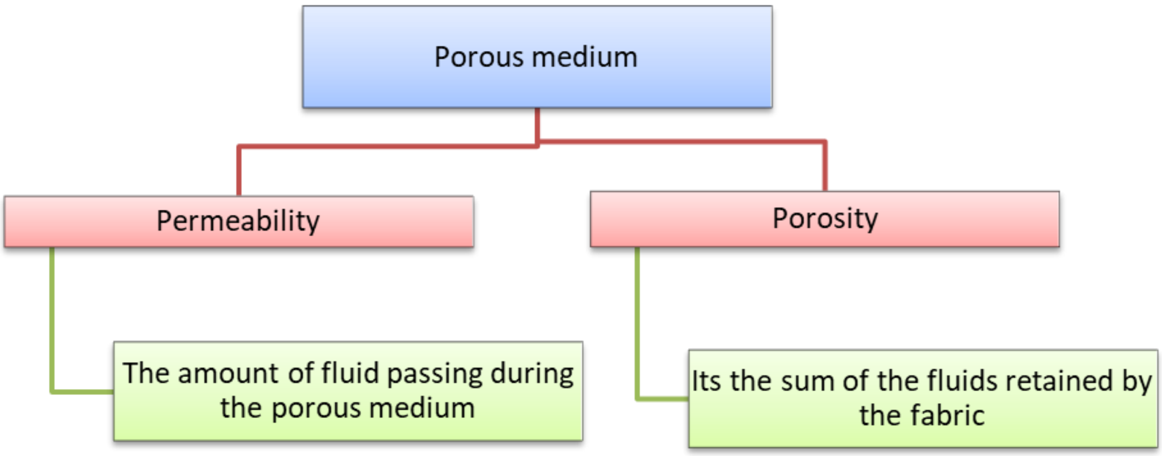

Fig. 2. The properties of porous medium

In a porous medium [26], most of the observations about the difficulty of calculating slip are based on theoretical analysis and numerical methods. Experimentally, through indirect approaches the slip has been confirmed. In the fluid solid interface, the slip effect is constant under boundary condition. The boundary condition offers different implementation in technology and science for example material processing, fluid transportation and rheometic measurements. In the microscopic level, the boundary condition for fluid is appeared as a non-slip condition, meanwhile, the instantaneous velocity will be zero at boundaries Nevertheless, no instantaneous slip conditions due to the shear stress have been investigated by the implication of unexpected increase in flow rates or decrease in the viscosity [27-34]. The slip condition is identified as an apparent slip in fluid mechanics. In the slip and no-slip at boundaries, the semi-analytical solutions of squeezing an unsteady flow fluid with porous impacts and magneto-hydrodynamic are discussed in this article. This issue is solved via a DSA in order to find analytical approximate solution. The proposed approach is designed to produce a new and efficient hybrid algorithm. The main feature of this approach is used to accelerate of convergence with less computational process. The results are represented by tables and graphs were compared with least squares homotopy perturbation method (LSHPM), homotopy perturbation method (HPM), and Runge-Kutta-Fehlberg (RKF5). The remaining sections are organized in the following: section 2 is shown to explain the statement of problem. The basic concept of the proposed approach is given in section 3. The application of this approach is illustrated in section 4. The results of the tables and figures are presented in section 5 and section 6, respectively. In addition, the conclusion is written in section 7.

2. The Statement of Problem

A porous medium between two infinite parallel plates and an unsteady rectilinear incompressible viscous magneto-hydrodynamic squeezing flow fluid are investigated. At any time $\check{t}$, always the distance between the plates is $2\zeta(\check{t})$. In the channel the central-Axis is $\check{x}$-Axis, and the perpendicular Axis to the channel is $\check{y}$-axis. The acting of the uniform magnetic field $= \eta(0, \eta_0, 0\ )$ along $\check{y}$-Axis. The properties of the magnetic field [5] can be outlined as follows

i. A very small magnetic Reynolds number causes the induced magnetic field to be small so it is considered [35-39].
ii. To the flow and the fluid direction of the magnetic field is perpendicular.
iii. Finally, $\eta_0 = \mathrm{H}_0\delta_0$, where δ_0 is the magnetic permeability.

In addition, the moving of the plate symmetrically from the central axis in the channel leads to form the problem that have the modeling as follows

$$\frac{\partial \check{u}_1}{\partial \check{x}} + \frac{\partial \check{u}_2}{\partial \check{y}} = 0 \tag{1}$$

$$\check{\rho}\left(\frac{\partial \check{u}_1}{\partial \check{t}} + \check{u}_1\frac{\partial \check{u}_1}{\partial \check{x}} + \check{u}_2\frac{\partial \check{u}_1}{\partial \check{y}}\right) = -\frac{\partial \check{p}}{\partial \check{x}} + \gamma\left(\frac{\partial^2 \check{u}_1}{\partial \check{x}^2} + \frac{\partial^2 \check{u}_1}{\partial \check{y}^2}\right) - \sigma\eta_0^2\check{u}_1 - \frac{\delta}{\kappa}\check{u}_1 \tag{2}$$

$$\check{\rho}\ \left(\frac{\partial \check{u}_2}{\partial \check{t}} + \check{u}_1\frac{\partial \check{u}_2}{\partial \check{x}} + \check{u}_2\frac{\partial \check{u}_2}{\partial \check{y}}\right) = -\frac{\partial \check{p}}{\partial \check{y}} + \gamma\left(\frac{\partial^2 \check{u}_2}{\partial \check{x}^2} + \frac{\partial^2 \check{u}_2}{\partial \check{y}^2}\right) - \frac{\delta}{\kappa}\check{u}_2 \tag{3}$$

3

The components of the velocity are $\check{u}_1$ along $\check{x}$-Axis and $\check{u}_2$ along $\check{y}$-Axis. $\check{\rho}$, γ, δ, κ and σ are assigned to the density, kinematic viscosity, dynamic viscosity, constant permeability, and electric conductivity respectively. The generalized pressure $\check{h}$ with the vorticity function ϖ and can be define as

$$\varpi = \frac{\partial \check{u}_2}{\partial \check{x}} - \frac{\partial \check{u}_1}{\partial \check{y}} \tag{4}$$

$$\check{h} = \frac{\check{\rho}}{2}[\check{u}_1^2 + \check{u}_2^2] + \check{p} \tag{5}$$

Substitution of Eq. (4) and Eq. (5) in Eq. (1) – Eq. (3), the equations of mass and momentum transformed become as

$$\frac{\partial \check{u}_1}{\partial \check{x}} + \frac{\partial \check{u}_2}{\partial \check{y}} = 0 \tag{6}$$

$$\frac{\partial \check{h}}{\partial \check{x}}+\check{\rho}\left(\frac{\partial \check{u}_1}{\partial \check{t}}-\check{u}_2\varpi\right)=-\gamma\frac{\partial \varpi}{\partial \check{y}}-\sigma\eta_0^2\check{u}_1-\frac{\delta}{\kappa}\check{u}_1 \tag{7}$$

$$\frac{\partial \check{h}}{\partial \check{y}}+\check{\rho}\left(\frac{\partial \check{u}_2}{\partial \check{t}}-\check{u}_1\varpi\right)=-\gamma\frac{\partial \varpi}{\partial \check{x}}-\frac{\delta}{\kappa}\check{u}_2 \tag{8}$$

By using Eq. (7) and Eq. (8) through the elimination of gradient pressure leads to equation as follows

$$\check{\rho}\left(\frac{\partial \varpi}{\partial \check{t}}+\check{u}_1\frac{\partial \varpi}{\partial \check{x}}-\check{u}_2\frac{\partial \varpi}{\partial \check{y}}\right)=\gamma\Delta^2\varpi-\sigma\eta_0^2\frac{\partial \check{u}_1}{\partial \check{y}}-\frac{\delta}{\kappa}\varpi \tag{9}$$

The boundary conditions of the stated problem are given in following forms

$$\check{u}_1(\check{x},\check{y},\check{t})=0,\ \ \check{u}_2(\check{x},\check{y},\check{t})=\check{u}_{2w}(t),\ \ at\ \ \check{y}=\zeta \tag{10}$$

$$\check{u}_2(\check{x},\check{y},\check{t})=0,\ \ \frac{\partial \check{u}_1(\check{x},\check{y},\check{t}\ )}{\partial\alpha}=0,\ \ at\ \ \check{y}=0 \tag{11}$$

Here, $\check{u}_{2w}(t)=\frac{d\zeta}{d\check{t}}$ is the velocity of plates and assuming that $\alpha=\frac{\check{y}}{\zeta(\check{t})}$ is dimensionless variable. The mathematical form of the Eq. (6) and the Eq. (9) is

$$\frac{\partial \check{u}_1}{\partial \check{x}}+\frac{\partial \check{u}_2}{\zeta(\check{t})\partial\alpha}=0 \tag{12}$$

$$\check{\rho}\left(\frac{\partial \varpi}{\partial \check{t}}+\check{u}_1\frac{\partial \varpi}{\partial \check{x}}-\check{u}_2\frac{\partial \varpi}{\zeta(\check{t})\partial\alpha}\right)=\gamma\Delta^2\varpi-\sigma\eta_0^2\frac{\partial \check{u}_1}{\zeta(\check{t})\partial\alpha}-\frac{\delta}{\kappa}\varpi \tag{13}$$

The boundary conditions on $\check{u}_1(\check{x},\alpha,\check{t})$ and $\check{u}_2(\check{x},\alpha,\check{t})$ are

$$\check{u}_1(\check{x},\alpha,\check{t})=0,\ \ \check{u}_2(\check{x},\alpha,\check{t})=\check{u}_{2w}(t)\ ,\qquad at\ \ \alpha=1 \tag{14}$$

$$\check{u}_2(\check{x},\alpha,\check{t})=0,\qquad \frac{\partial \check{u}_1(\check{x},\alpha,\check{t}\ )}{\partial\alpha}=0,\qquad at\ \ \alpha=0 \tag{15}$$

The definition of the velocity components are

$$\check{u}_1=\frac{\kappa-\check{x}}{\zeta(\check{t})}\check{u}_{2w}\mathrm{H}'(\alpha),\quad \check{u}_2=\check{u}_{2w}(\check{t})\mathrm{H}(\alpha) \tag{16}$$

Offset of Eq. (14) in Eq. (12) and Eq. (13), yields the following

$$\frac{d^4\mathrm{H}}{d\alpha^4}-\frac{\zeta\check{u}_{2w}}{\gamma}\left[\mathrm{H}\,\frac{d^3\mathrm{H}}{d\alpha^3}-\frac{d\mathrm{H}}{d\alpha}\,\frac{d^2\mathrm{H}}{d\alpha^2}-\alpha\,\frac{d^3\mathrm{H}}{d\alpha^3}-2\frac{d^2\mathrm{H}}{d\alpha^2}\right]-\frac{\zeta^2}{\gamma\check{u}_{2w}}\frac{d\check{u}_{2w}}{dt}\frac{d^2\mathrm{H}}{d\alpha^2}-Mg\frac{d^2H}{d\alpha^2}-Mp\,\frac{d^2H}{d\alpha^2}=0 \tag{17}$$

Mg is the MHD parameter and Mp is porous parameter. The boundary condition can be defined by the Eq. (14) and Eq. (15)

$$\mathrm{H}(1) = 1, \quad \frac{d\mathrm{H}(1)}{d\alpha} = 0, \quad \mathrm{H}(0) = 0, \quad \frac{d^2\mathrm{H}(0)}{d\alpha^2} = 0 \tag{18}$$

Subsequently, similar solution, led to define the following

$$\frac{\zeta \breve{u}_{2w}}{\gamma} = Rn, \quad \frac{\zeta^2}{\gamma \breve{u}_{2w}} \frac{d\breve{u}_{2w}}{dt} = Rn\, Qn \tag{19}$$

Rn and Qn can be introduced as the functions for $\breve{t}$ and these functions are fixs for the similar solutions. By integrating $\frac{\zeta \breve{u}_{2w}}{\gamma} = Rn$, thus we yield

$$\zeta(\breve{t}) = (2\gamma Rn\, \breve{t} + \zeta_0^2)^{1/2} \tag{20}$$

Eq. (13) and Eq. (14) given that $Qn = -1$. The final from of the resulting equation with Rn represents Reynolds number as

$$\frac{d^4\mathrm{H}}{d\alpha^4} - Rn \left[\mathrm{H}\frac{d^3\mathrm{H}}{d\alpha^3} - \frac{d\mathrm{H}}{d\alpha}\frac{d^2\mathrm{H}}{d\alpha^2} - \alpha \frac{d^3\mathrm{H}}{d\alpha^3} - 3\frac{d^2\mathrm{H}}{d\alpha^2}\right] - \mathrm{Mg}\frac{d^2\mathrm{H}}{d\alpha^2} - Mp\frac{d^2\mathrm{H}}{d\alpha^2} = 0 \tag{21}$$

Two cases for the boundary conditions are provided as the following: follows

iv. The boundary condition has no slip parameter

$$\mathrm{H}(1) = 1, \quad \frac{d\mathrm{H}(1)}{d\alpha} = 0, \quad \mathrm{H}(0) = 0, \quad \frac{d^2\mathrm{H}(0)}{d\alpha^2} = 0 \tag{22}$$

v. The boundary condition has slip parameter

$$\mathrm{H}(1) = 1, \quad \frac{d\mathrm{H}(1)}{d\alpha} = \Lambda \frac{d^2\mathrm{H}(1)}{d\alpha^2}, \quad \mathrm{H}(0) = 0, \quad \frac{d^2\mathrm{H}(0)}{d\alpha^2} = 0 \tag{23}$$

3. The Basic Steps of the DSA

The assuming solution for the coefficient of power series is an important base to construct the approximate analytical solution formula. So, these coefficients can be computed differential method. We summarize the details of a derivatives of the series scheme in the following steps [22-25]

Considering the ordinary differential equation, we can write

$$G\left(\mathrm{H}(\alpha), \frac{d\mathrm{H}(\alpha)}{d\alpha}, \frac{d^2\mathrm{H}(\alpha)}{d\alpha^2}, \frac{d^3\mathrm{H}(\alpha)}{d\alpha^3}, \ldots, \frac{d^{(n-1)}\mathrm{H}(\alpha)}{d\alpha^{(n-1)}}, \frac{d^{(n)}\mathrm{H}(\alpha)}{d\alpha^{(n)}}\right) \tag{24}$$

Rewriting of the Eq. (24) becomes

$$\frac{d^{(n)}\mathrm{H}(\alpha)}{d\xi^{(n)}} = G\left(\mathrm{H}(\alpha), \frac{d\mathrm{H}(\alpha)}{d\alpha}, \frac{d^2\mathrm{H}(\alpha)}{d\alpha^2}, \frac{d^3\mathrm{H}(\alpha)}{d\alpha^3}, \dots, \frac{d^{(n-1)}\mathrm{H}(\alpha)}{d\alpha^{(n-1)}}\right) \tag{25}$$

G is a function for $\mathrm{H}(\alpha)$ and its derivatives, $\mathrm{H}(\alpha)$ is an unknown function, and α is independent variable. Integration Eq. (25) with respect to α n-times on interval $[0, \alpha]$, become the following

$$\mathrm{H}(\alpha) = \sum_{j=1}^{n} \frac{\alpha^{j-1}}{(j-1)!}\mathrm{H}^{(j-1)}(0) + L^{-1}K[\mathrm{H}(\alpha)] \tag{26}$$

where

$$K[\mathrm{H}(\alpha)] = G\left(\mathrm{H}(\alpha), \frac{d\mathrm{H}(\alpha)}{d\alpha}, \frac{d^2\mathrm{H}(\alpha)}{d\alpha^2}, \frac{d^3\mathrm{H}(\alpha)}{d\alpha^3}, \dots, \frac{d^{(n-1)}\mathrm{H}(\alpha)}{d\alpha^{(n-1)}}\right), \; L^{-1}(.) = \int_0^{\alpha} \dots \int_0^{\alpha} (.)(d\alpha)^n$$

Assuming that

$$[K(\alpha)] = \sum_{m=1}^{\infty} \frac{d^{(m-1)}K[\mathrm{H}_0(\alpha)]}{d\alpha^{(m-1)}} \tag{27}$$

Rewriting the Eq. (26)

$$K[\mathrm{H}(\alpha)] = K[\mathrm{H}_0(\alpha)] + K[\mathrm{H}_0(\alpha)] + K''[\mathrm{H}_0(\alpha)] + K'''[\mathrm{H}_0(\alpha)] + \cdots \tag{28}$$

Offset of Eq. (28) in Eq. (26), we acquire

$$\mathrm{H}(\alpha) = \mathrm{H}_0(\alpha) + \mathrm{H}_1(\alpha) + \mathrm{H}_2(\alpha) + \mathrm{H}_3(\alpha) + \cdots \tag{29}$$

where

$$\mathrm{H}(0) = \sum_{j=1}^{n} \frac{\alpha^{j-1}}{(j-1)!}\mathrm{H}^{(j-1)}(0), \;\; \mathrm{H}_1(\alpha) = L^{-1}K[\mathrm{H}_0(\alpha)], \;\; \mathrm{H}_2(\alpha) = L^{-1}K[\mathrm{H}_0(\alpha)],$$

$$\mathrm{H}_3(\alpha) = L^{-1}K'''[\mathrm{H}_0(\alpha)], \qquad \mathrm{H}_4(\alpha) = L^{-1}K''''[\mathrm{H}_0(\alpha)], \dots. \tag{30}$$

The derivative of K with respect to α which is considered as a crucial part of the NA. By starting to calculate $K[\mathrm{H}(\alpha)]$, $K'[\mathrm{H}(\alpha)]$, $K''[\mathrm{H}(\alpha)]$, …, we can write

$$K[\mathrm{H}(\alpha)] = G\left(\mathrm{H}(\alpha), \frac{d\mathrm{H}(\alpha)}{d\alpha}, \frac{d^2\mathrm{H}(\alpha)}{d\alpha^2}, \frac{d^3\mathrm{H}(\alpha)}{d\alpha^3}, \dots, \frac{d^{(n-1)}\mathrm{H}(\alpha)}{d\alpha^{(n-1)}}\right) \tag{31}$$

$$K'[\mathrm{H}(\alpha)] = \sum_{t_1=1}^{n} K_{\mathrm{H}^{(t_1-1)}}.(\mathrm{H}_\alpha)^{(t_1-1)} \tag{32}$$

$$K''[\mathrm{H}(\alpha)] = \sum_{t_2=1}^{n}\sum_{t_1=1}^{n} K_{\mathrm{H}^{(t_1-1)}\mathrm{H}^{(t_2-1)}}.(\mathrm{H}_\alpha)^{(t_1-1)}.(\mathrm{H}_\alpha)^{(t_2-1)} + \sum_{t_1=1}^{n} K_{H^{(t_1-1)}}.(H_{\alpha\alpha})^{(t_1-1)} \tag{33}$$

$$K''[g(\xi)] = \sum_{t_2=1}^{n}\sum_{t_1=1}^{n} K_{\mathrm{H}^{(t_1-1)}\mathrm{H}^{(t_2-1)}}.(\mathrm{H}_\alpha)^{(t_1-1)}.(\mathrm{H}_{\alpha\alpha})^{(t_2-1)}, + \sum_{t_1=1}^{n} K_{\mathrm{H}^{(t_1-1)}}.(\mathrm{H}_{\alpha\alpha\alpha})^{(t_1-1)} + \sum_{t_3=1}^{n}\sum_{t_2=1}^{n}\sum_{t_1=1}^{n} K_{H^{(t_1-1)}H^{(t_2-1)}H^{(t_3-1)}} \ (H_\alpha)^{(t_1-1)}.(H_\alpha)^{(t_2-1)}.(H_\alpha)^{(t_3-1)} \tag{34}$$

$\vdots$

The presumption of the solution H and the operator K are analytic functions, so the mixed of the derivatives are equivalence. We note that the derivative function of H is unknown, and we can propose the following hypothesis

$$\mathrm{H}_\alpha = \mathrm{H}_1 = L^{-1}K[\mathrm{H}_0(\alpha)], \quad \mathrm{H}_{\alpha\alpha} = \mathrm{H}_2 = L^{-1}K'[\mathrm{H}_0(\alpha)],$$
$$\mathrm{H}_{\alpha\alpha\alpha} = \mathrm{H}_3 = L^{-1}K''^{[\mathrm{H}_0(\alpha)]}, \qquad \mathrm{H}_{\alpha\alpha\alpha\alpha} = \mathrm{H}_4 = L^{-1}K'''^{[\mathrm{H}_0(\alpha)]}, \dots, \tag{35}$$

Wherefore, Eq. (31)-Eq. (34) determined by:

$$K[\mathrm{H}_0(\alpha)] = G\left(\mathrm{H}_0(\alpha), \frac{d\mathrm{H}_0(\xi)}{d\alpha}, \frac{d^2\mathrm{H}_0(\xi)}{d\alpha^2}, \frac{d^3\mathrm{H}_0(\xi)}{d\alpha^3}, \dots, \frac{d^{(n-1)}\mathrm{H}_0(\xi)}{d\alpha^{(n-1)}}\right)$$

$$K'[\mathrm{H}_0(\alpha)] = \sum_{t_1=1}^{n} K_{\mathrm{H}_0{}^{(t_1-1)}}.(\mathrm{H}_1)^{(t_1-1)} \tag{36}$$

$$K''[\mathrm{H}_0(\alpha)] = \sum_{t_2=1}^{n}\sum_{t_1=1}^{n} K_{\mathrm{H}_0{}^{(t_1-1)}\mathrm{H}_0{}^{(t_2-1)}}.(\mathrm{H}_1)^{(t_1-1)}.(\mathrm{H}_1)^{(t_2-1)} + \sum_{t_1=1}^{n} K_{H_0{}^{(t_1-1)}}.(H_2)^{(t_1-1)} \tag{37}$$

$$K''[\mathrm{H}_0(\alpha)] = 3.\sum_{t_2=1}^{n}\sum_{t_1=1}^{n} K_{\mathrm{H}_0{}^{(t_1-1)}\mathrm{H}_0{}^{(t_2-1)}}.(\mathrm{H}_1)^{(t_1-1)}.(\mathrm{H}_2)^{(t_2-1)}, + \sum_{t_1=1}^{n} K_{\mathrm{H}_0{}^{(t_1-1)}}.(\mathrm{H}_3)^{(t_1-1)} + \sum_{t_3=1}^{n}\sum_{t_2=1}^{n}\sum_{t_1=1}^{n} K_{H_0{}^{(t_1-1)}H_0{}^{(t_2-1)}H_0{}^{(t_3-1)}} \ (H_1)^{(t_1-1)}.(H_1)^{(t_2-1)}.(H_1)^{(t_3-1)} \tag{38}$$

$\vdots$

Plugging of Eq. (35)- Eq. (38) in Eq. (29), through employing the above steps can obtained the required analytical solution for the Eq. (24).

4. The Implementation of the Derivatives Series Algorithm for Squeezing Flow Fluid

The DSA is implemented to solve the ordinary differential system of equations. 9 -11 in order which described in the previous section to find the analytical approximate solution $\mathrm{H}(\alpha)$ as it can be acquired from the required information as follows:

From the first step, and by integrating the Eq. (9) 4 times with respect to α on $[0, \alpha]$, we will have

$$\mathrm{H}(\alpha) = J_{11} + J_{12}\alpha + J_{13}\frac{\alpha^2}{2!} + J_{14}\frac{\alpha^3}{3!} + Rn\left[\mathrm{H}\frac{d^3\mathrm{H}}{d\alpha^3} - \frac{d\mathrm{H}}{d\alpha}\frac{d^2\mathrm{H}}{d\alpha^2} - \alpha\frac{d^3\mathrm{H}}{d\alpha^3} - 3\frac{d^2\mathrm{H}}{d\alpha^2}\right] + \mathrm{Mg}\frac{d^2\mathrm{H}}{d\alpha^2} + Mp\frac{d^2\mathrm{H}}{d\alpha^2} \quad (39)$$

Rewrite the equations as follows

$$\mathrm{H}(\alpha) = J_{11} + J_{12}\alpha + J_{13}\frac{\alpha^2}{2!} + J_{14}\frac{\alpha^3}{3!} + L^{-1}K[\mathrm{H}(\alpha)]$$

Which

$$J_{11} = \mathrm{H}(0),\; J_{12} = \mathrm{H}'(0),\; J_{13} = \mathrm{H}''^{(0)},\; J_{14} = \mathrm{H}'''^{(0)}, \qquad and\; L^{-1} = \int_0^\alpha\int_0^\alpha\int_0^\alpha\int_0^\alpha(.)(d\alpha)^4$$

$$K[\mathrm{H}(\alpha)] = Rn\left[\mathrm{H}\frac{d^3\mathrm{H}}{d\alpha^3} - \frac{d\mathrm{H}}{d\alpha}\frac{d^2\mathrm{H}}{d\alpha^2} - \alpha\frac{d^3\mathrm{H}}{d\alpha^3} - 3\frac{d^2\mathrm{H}}{d\alpha^2}\right] + \mathrm{Mg}\frac{d^2\mathrm{H}}{d\alpha^2} + Mp\frac{d^2\mathrm{H}}{d\alpha^2} \quad (40)$$

From the boundary conditions Eq. (10) and Eq. (11) the equation becomes

$$\mathrm{H}(\alpha) = J_{12}\alpha + J_{14}\frac{\alpha^3}{3!} + L^{-1}K[\mathrm{H}(\alpha)] \quad (41)$$

where

$$\mathrm{H}_0 = J_{12}\alpha + J_{14}\frac{\alpha^3}{3!}, \qquad H_1 = L^{-1}K[H_0(\alpha)], \qquad H_2 = L^{-1}K'[H_0(\alpha)], \ldots \quad (42)$$

From the above step, we can write

$$\mathrm{K}[\mathrm{H}(\alpha)] = Rn\left[\mathrm{H}\frac{d^3\mathrm{H}}{d\alpha^3} - \frac{d\mathrm{H}}{d\alpha}\frac{d^2\mathrm{H}}{d\alpha^2} - \alpha\frac{d^3\mathrm{H}}{d\alpha^3} - 3\frac{d^2\mathrm{H}}{d\alpha^2}\right] + \mathrm{Mg}\frac{d^2\mathrm{H}}{d\alpha^2} + Mp\frac{d^2\mathrm{H}}{d\alpha^2} \quad (43)$$

$$K'[\mathrm{H}(\alpha)] = \sum_{t_1=1}^{4} K_{\mathrm{H}^{(t_1-1)}}.(\mathrm{H}_\alpha)^{(t_1-1)} \quad (44)$$

$\vdots$

The following hypothesis can be suggested as

$$\mathrm{H}_{\alpha} = \mathrm{H}_1 = L^{-1}K[\mathrm{H}_0(\alpha)], \qquad \mathrm{H}_{\alpha\alpha} = \mathrm{H}_2 = L^{-1}K'[\mathrm{H}_0(\alpha)] \tag{45}$$

Now, we need to extract the first derivatives of K as follows

$$\begin{aligned}
&k_{\mathrm{H}_0} = \mathrm{Rn}\,\mathrm{H}_0''', k_{\mathrm{H}_0\mathrm{H}_0} = k_{\mathrm{H}_0\mathrm{H}_0'} = 0, k_{\mathrm{H}_0\mathrm{H}_0'''} = Rn,\\
&k_{\mathrm{H}_0\mathrm{H}_0\mathrm{H}_0} = k_{\mathrm{H}_0\mathrm{H}_0\mathrm{H}_0'} = k_{\mathrm{H}_0\mathrm{H}_0'\mathrm{H}_0'} = k_{1\mathrm{H}_0\mathrm{H}_0'\mathrm{H}_0'} = k_{1\mathrm{H}_0\mathrm{H}_0''\mathrm{H}_0''} = k_{1\mathrm{H}_0\mathrm{H}_0'''\mathrm{H}_0'''} = 0,\\
&k_{\mathrm{H}_0'} = -\mathrm{Rn}\,\mathrm{H}_0'', k_{\mathrm{H}_0'\mathrm{H}_0} = k_{\mathrm{H}_0'\mathrm{H}_0'''} = 0, k_{\mathrm{H}_0'\mathrm{H}_0''} = -Rn,\\
&k_{\mathrm{H}_0\mathrm{H}_0\mathrm{H}_0'} = k_{\mathrm{H}_0\mathrm{H}_0'\mathrm{H}_0'} = k_{\mathrm{H}_0'\mathrm{H}_0'\mathrm{H}_0'} = k_{\mathrm{H}_0'\mathrm{H}_0'''\mathrm{H}_0'} = k_{\mathrm{H}_0'\mathrm{H}_0''\mathrm{H}_0''} = k_{\mathrm{H}_0'\mathrm{H}_0'''\mathrm{H}_0'''} = 0,\\
&k_{\mathrm{H}_0''} = Rn\,\mathrm{H}_0' - 3Rn + Mg + Mp, \qquad k_{\mathrm{H}_0''\mathrm{H}_0'''} = 0, k_{\mathrm{H}_0''\mathrm{H}_0'} = Rn,\\
&k_{1g_0''g_0g_0'} = k_{1g_0''g_0'g_0'} = k_{1g_0''g_0'g_0'} = k_{1g_0''g_0'''g_0'} = k_{1g_0''g_0''g_0''} = k_{1g_0''g_0'''g_0'''} = 0,\\
&k_{\mathrm{H}_0'''} = \mathrm{Rn}(\mathrm{H}_0 - \alpha),\ k_{\mathrm{H}_0'''\mathrm{H}_0''} = 0, k_{\mathrm{H}_0'''\mathrm{H}_0} = Rn,\\
&k_{\mathrm{H}_0'''\mathrm{H}_0\mathrm{H}_0'} = k_{1\mathrm{H}_0'''\mathrm{H}_0'\mathrm{H}_0'} = k_{\mathrm{H}_0'''\mathrm{H}_0'\mathrm{H}_0'} = k_{\mathrm{H}_0'''\mathrm{H}_0'''\mathrm{H}_0'} = k_{\mathrm{H}_0'''\mathrm{H}_0''\mathrm{H}_0''} = k_{\mathrm{H}_0'''\mathrm{H}_0'''\mathrm{H}_0'''} = 0,
\end{aligned} \tag{46}$$

Substitution of Eq. (43) – Eq. (46) in Eq. (41), we obtain

$$\mathrm{H}_0 = J_{12}\alpha + \frac{1}{6}J_{14}\alpha^3 \tag{47}$$

$$\mathrm{H}_1 = \frac{1}{5}\left[-\frac{1}{6}\,Rn\,J_{14} + \frac{1}{24}(Mg + Mp)\,J_{14}\right]\alpha^5 - \frac{1}{2520}Rn\,{J_{14}}^2\,\alpha^7 \tag{48}$$

$$\begin{aligned}
\mathrm{H}_2 = \Big(&-\frac{1}{630}J_{12}\,J_{14}\,Rn^2 + \frac{1}{2520}J_{12}\,J_{14}Rn\,Mg + \frac{1}{2520}J_{12}\,J_{14}Rn\,Mp + \frac{1}{210}J_{14}Rn^2\\
&- \frac{1}{504}J_{14}Rn\,Mg - \frac{1}{504}J_{14}Rn\,Mp + \frac{1}{5040}J_{14}Mg^2 + \frac{1}{2520}J_{14}Mg\,Mp\\
&+ \frac{1}{2520}J_{14}Mg\,Mp + \frac{1}{5040}J_{14}Mp^2\Big)\alpha^7\\
&+ \left(\frac{1}{11340}{J_{14}}^2Rn^2 - \frac{1}{45360}J_{12}{J_{14}}^2Rn^2 - \frac{1}{60480}{J_{14}}^2Rn\,Mg\right.\\
&\left.- \frac{1}{60480}{J_{14}}^2Rn\,Mp\right)\alpha^9 - \frac{1}{2494800}Rn^2{J_{14}}^3\alpha^{11}
\end{aligned} \tag{49}$$

Substitution of Equations .47-49 in Eq. (41), we get the following

$$\begin{aligned}
\mathrm{H}(\alpha) = J_{12}\alpha &+ \frac{1}{6}J_{14}\alpha^3 + \frac{1}{5}\left[-\frac{1}{6}\,Rn\,J_{14} + \frac{1}{24}(Mg + Mp)\,J_{14}\right]\alpha^5\\
&+ \left[-\frac{1}{2520}Rn\,{J_{14}}^2 - \frac{1}{630}J_{12}\,J_{14}\,Rn^2 + \frac{1}{2520}J_{12}\,J_{14}Rn\,Mg + \frac{1}{2520}J_{12}\,J_{14}Rn\,Mp\right.\\
&+ \frac{1}{210}J_{14}Rn^2 - \frac{1}{504}J_{14}Rn\,Mg - \frac{1}{504}J_{14}Rn\,Mp + \frac{1}{5040}J_{14}Mg^2 + \frac{1}{2520}J_{14}Mg\,Mp\\
&\left.+ \frac{1}{2520}J_{14}Mg\,Mp + \frac{1}{5040}J_{14}Mp^2\right]\alpha^7 + \cdots
\end{aligned} \tag{50}$$

5. The Discussion of Tables

Table 1 shows the convergence values of $\mathrm{H}'(0)$ and $\mathrm{H}''(0)$. The schemed tables are listed to see the influences of emerging physical parameters Reynolds number Rn, magneto-hydrodynamic parameter Mg, porous parameter Mp, slip parameter Λ on the axial velocity $\mathrm{H}(\alpha)$, and the radial velocity $\mathrm{H}'(\alpha)$. The comparisons of the solutions by the new approach with Fehlberg RK, LSHPM, and HP are investigated as can be seen in Tables 2-5. these tables show that the solutions are fully compatible. From fixed point theorems [15-18], the convergence condition can be addressed as follows

$$\epsilon^k = \begin{cases} \dfrac{\|\mathrm{H}_{k+1}\|}{\|\mathrm{H}_1\|}, & \|\mathrm{H}_1\| \neq 0 \\ 0, & \|\mathrm{H}_1\| = 0 \end{cases} \qquad for \quad k = 1,2,.... \tag{51}$$

As shown in Table 6, the application of the convergence condition leads to find values ϵ^k. This table proved that the values for ϵ^k was between 0 and 1, therefore the solutions of the new approach is converged.

Table 1
The convergence values of $\mathrm{H}'(0)$ and $\mathrm{H}'''(0)$ for $Rn = 0.5, Mg = 1$

$\Lambda = 0, Mp = 0.7$		$\Lambda = 0.9, Mp = 0.7$		$\Lambda = 0.9, Mp = 0.8$		
Approximation	$\mathrm{H}'(0)$	$\mathrm{H}'''(0)$	$\mathrm{H}'(0)$	$\mathrm{H}'''(0)$	$\mathrm{H}'(0)$	$\mathrm{H}'''(0)$
Order 1	1.5039341	-3.058345	0.6847	1.9248	0.6902	1.8816
Order 2	1.5040116	3.059038	0.6856	1.9188	0.6907	1.8781
Order 3	1.5040116	0.6856	0.6856	1.9188	-0.6907	1.8781

Table 2
The compared solutions between Fehlberg KR, HPM, LSHPM, and the present solutions

$\Lambda = 0,$	$Rn = 0{,}5,$	$Mg = 1,$	$Mp = 0.7$	
α	Fehlberg RK5	HPM[5]	LSHPM[5]	Present Solutions
0.0	0.000000	0.000000	0.000000	0.000000
0.1	0.149891	0.149891	0.149891	0.149891
0.2	0.296726	0.296726	0.296726	0.296726
0.3	0.347456	0.347456	0.347456	0.437456
0.4	0.569050	0.569050	0.569050	0.569050
0.5	0.688501	0.688500	0.688501	0.688501
0.6	0.792825	0.792825	0.792825	0.792825
0.7	0.879069	0.879068	0.879068	0.879068
0.8	0.944296	0.944296	0.944296	0.944296
0.9	0.985583	0.985583	0.985583	0.985583
1.0	1.000000	1.000000	1.000000	1.000000

Table 3
The compared solutions between Fehlberg KR, HPM, LSHPM, and present solutions

$\Lambda = 0,$	$Rn = 1,$	$Mg = 1,$	$Mp = 0.3$	
α	Fehlberg RK[5]	HPM[5]	LSHPM[5]	Present Solutions
0.0	0.000000	0.000000	0.000000	0.000000
0.1	0.156416	0.156262	0.156416	0.156420
0.2	0.308960	0.308676	0.308960	0.308968
0.3	0.453866	0.4538494	0.453866	0.453877
0.4	0.587568	0.587161	0.587568	0.587582
0.5	0.706796	0.706409	0.706796	0.706811
0.6	0.808646	0.808327	0.808646	0.808661
0.7	0.890646	0.890426	0.890626	0.890658
0.8	0.950797	0.950682	0.950797	0.950803
0.9	0.987588	0.987555	0.987588	0.987579
1.0	1.000000	1.000000	1.000000	1.000000

Table 4
The compared solutions between Fehlberg KR, HPM, LSHPM, and present solution

$\Lambda = 0.9$	$Rn = 0{,}5,$	$Mg = 1,$	$Mp = 0.8$	
α	Fehlberg RK[5]	HPM[5]	LSHPM[5]	Present Solutions
0.0	0.000000	0.000000	0.0000000	0.000000
0.1	0.069388	0.069393	0.069388	0.069388
0.2	0.140654	0.140663	0.140654	0.140654
0.3	0.215670	0.215683	0.215670	0.215670
0.4	0.296303	0.296317	0.296303	0.296302
0.5	0.384403	0.384419	0.384403	0.384402
0.6	0.481805	0.481821	0.481805	0.481804
0.7	0.590316	0.590331	0.590316	0.590316
0.8	0.711710	0.711721	0.711710	0.711710
0.9	0.847715	0.847721	0.847715	0.847715
1.0	1.000000	1.000000	1.000000	1.000000

Table 5
The compared solutions between Fehlberg KR, HPM, LSHPM, and present solution

$\Lambda = 0.9$	$Rn = 0.3,$	$Mg = 0.9,$	$Mp = 0.9$	
α	Fehlberg RK[5]	HPM[5]	LSHPM[5]	Present Solutions
0.0	0.000000	0.000000	0.000000	0.000000
0.1	0.073128	0.073117	0.073128	0.073128
0.2	0.147841	0.147820	0.147841	0.147841
0.3	0.225732	0.225703	0.225732	0.225732
0.4	0.308414	0.308379	0.309414	0.308413
0.5	0.397527	0.397490	0.397527	0.397526
0.6	0.494748	0.494712	0.494748	0.494748
0.7	0.601801	0.601770	0.601801	0.601800
0.8	0.720459	0.720436	0.720459	0.720459
0.9	0.852559	0.852546	0.852559	0.852559
1.0	1.000000	1.000000	1.000000	1.000000

Table 6
The values of $\epsilon^{k}_{\|.\|_2}$ for convergence tests

	No slip at boundaries	Slip at boundaries
k	$Rn = 0.5,\ Mg = 1, Mp = 1,\ \Lambda = 0.9$	
1	0.01387130116	0.008366782491
2	0.00019861339	0.000072298680
3	0.00037886151	0.000227907622
⋮	⋮	⋮
k	$Rn = 1,\ Mg = 1,\ \ Mp = 0.7,\ \ \Lambda = 0.7$	
1	0.02856764588	0.09732209781
2	0.00056762514	0.00791379491
3	0.00249023541	0.00848829477
⋮	⋮	⋮

6. The Discussion of Figures

In this section, the study of influence of different physical parameters for the axial velocity $\mathrm{H}(\alpha)$ and radial velocity profile $\mathrm{H}'(\alpha)$ by using the graphical curves is introduced. The results of approximate solutions that pass through a porous medium for an unsteady squeezing of MHD flow fluid at the boundaries in the cases of slip and no slip employing NHM are also discussed. For these cases, we can appoint the following.

6.1 The Boundary Conditions with No Slip Parameter

Figure 3 - 8 indicate the effect of Rn, Mg and Mp on the axial velocity and radial velocity. In fluid, Rn can be defined as the relationship between inertial forces and viscous forces as well as it has the potential scale for the effective measurement appeared in fluid behavior with a larger scale. As a result from Figure 3 and 4, it seems that the increasing of Rn leads to increase the axial velocity while decreasing the radial velocity near the central axis with increasing this velocity near wall when increment Rn. Mg plays a vital role in the resistance contribution that generated by Lorentz force of the magnetic pressure field. As a result from Figure 5 and 6, it can be seen that the increase of Mg pointed to decrease the axial velocity with increasing in the radial velocity occurred near the central axis and decreasing near wall of the channel. Whereas, the behavior of Mp increment is similar to Mg as shown in Figure 7 and 8.

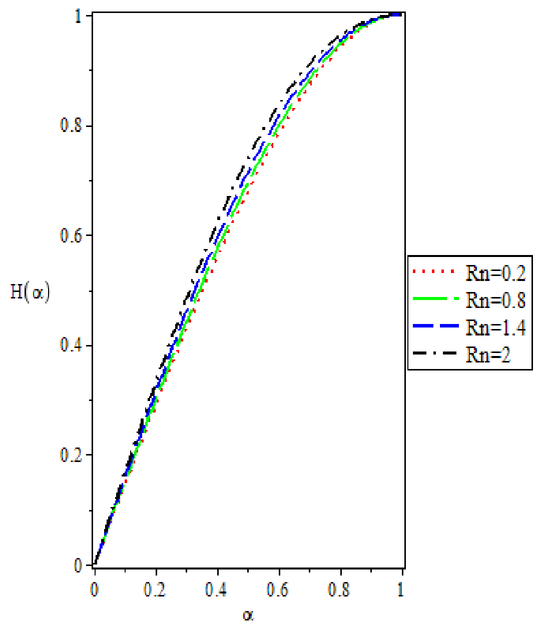

Fig. 3. The impact of Rn on $\mathrm{H}(\alpha)$ for $Mg = Mp = 1$

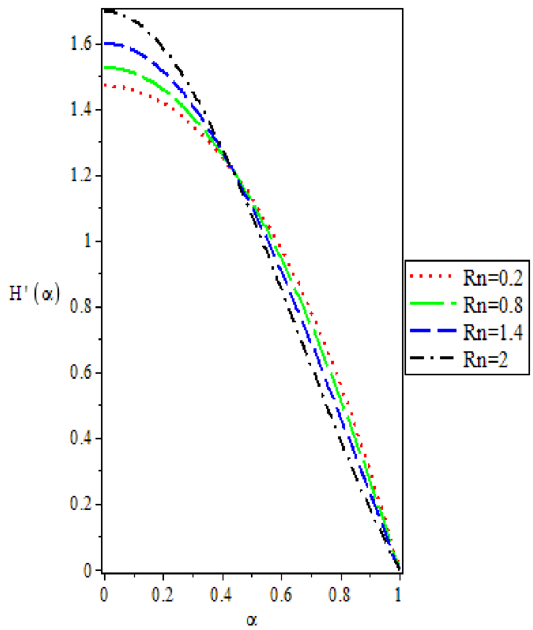

Fig. 4. The impact of Rn on $\mathrm{H}'(\alpha)$ for $Mg = Mp = 1$

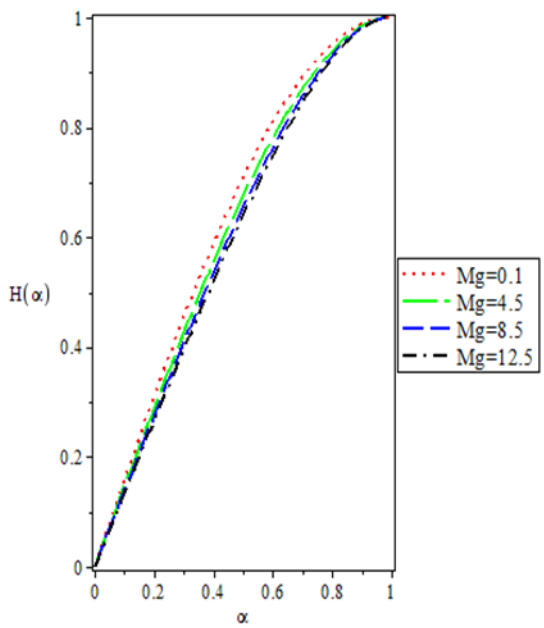

Fig. 5. The impact of Mg on $\mathrm{H}(\alpha)$ for $Rn = Mp = 1$

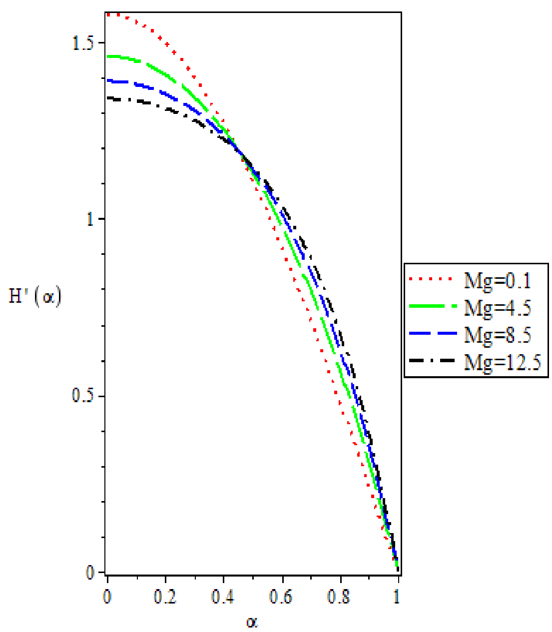

Fig. 6. The impact of Mg on $\mathrm{H}'(\alpha)$ for $Rn = Mp = 1$

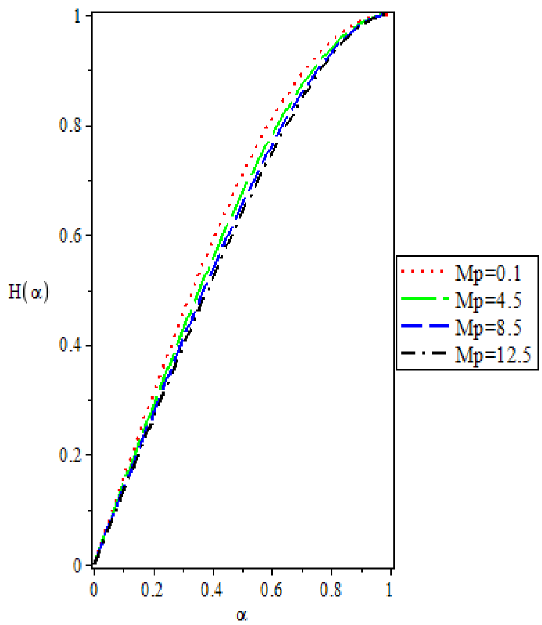

Fig. 7. The behavior of Mp on $\mathrm{H}(\alpha)$ for $Rn = Mg = 1$

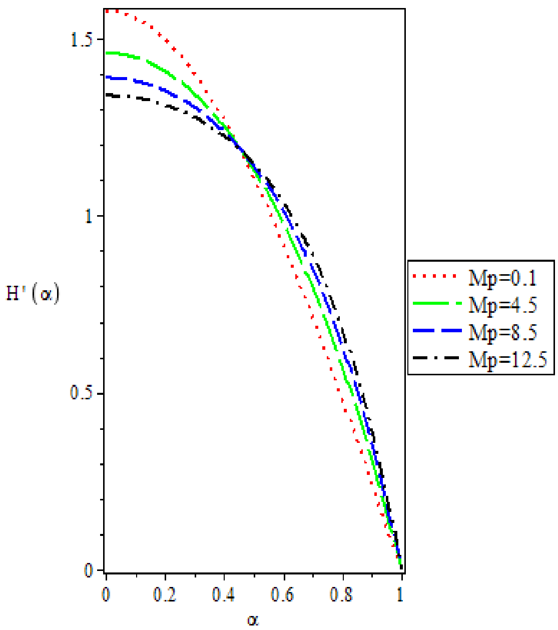

Fig. 8. The behavior of Mp on $\mathrm{H}'(\alpha)$ for $Rn = Mg = 1$

6.2 The Boundary Conditions with Slip Parameter

From Figure 9–16, it can be seen the effect of Rn, Mg, Mp, and Λ on the axial velocity and radial velocity. Figure 9-14 prove that the effect of Rn, Mg, and Mp for the slip condition is opposite in the no-slip condition. Moreover, the behavior of the curves Λ on the axial velocity increases when the slip parameter is visited. While, we observed the decreasing of the radial velocity near the center axis with increasing near the wall for channel.

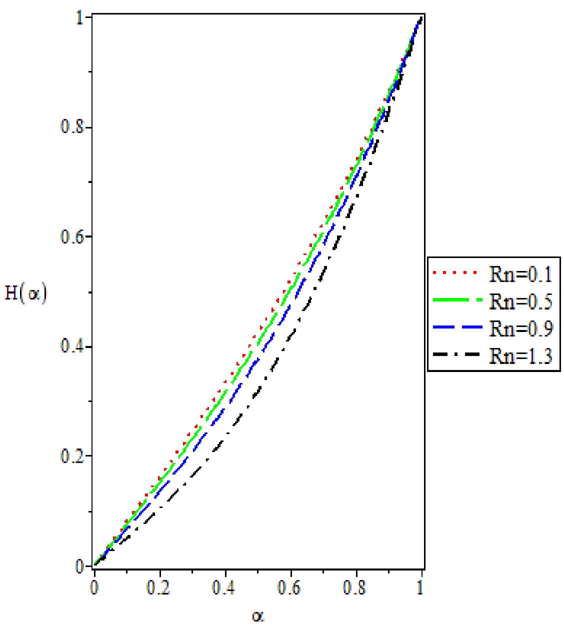

Fig. 9. The impact of Rn on $\mathrm{H}(\alpha)$ for $Mg = Mp = 1, \Lambda = 1$

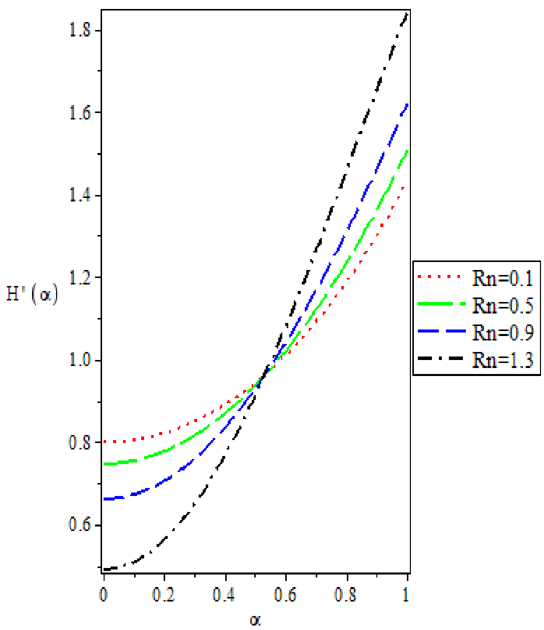

Fig. 10. The impact of Rn on $\mathrm{H}'(\alpha)$ for $Mg = Mp = 1, \Lambda = 1$

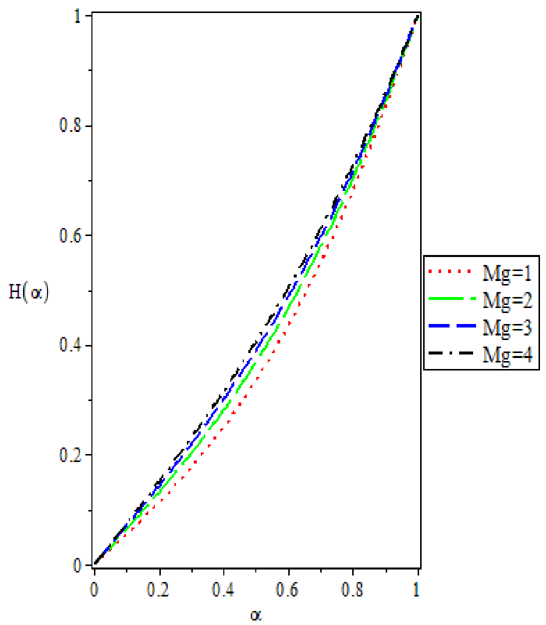

Fig. 11. The impact of Mg on $\mathrm{H}(\alpha)$ for $Rn = \Lambda = 1, Mp = 0.1$

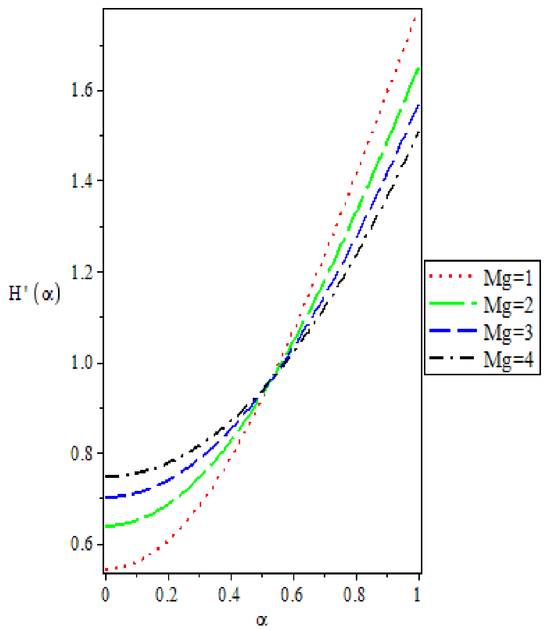

Fig. 12. The impact of Mg on $\mathrm{H}'(\alpha)$ for $Rn = \Lambda = 1, Mp = 0.1$

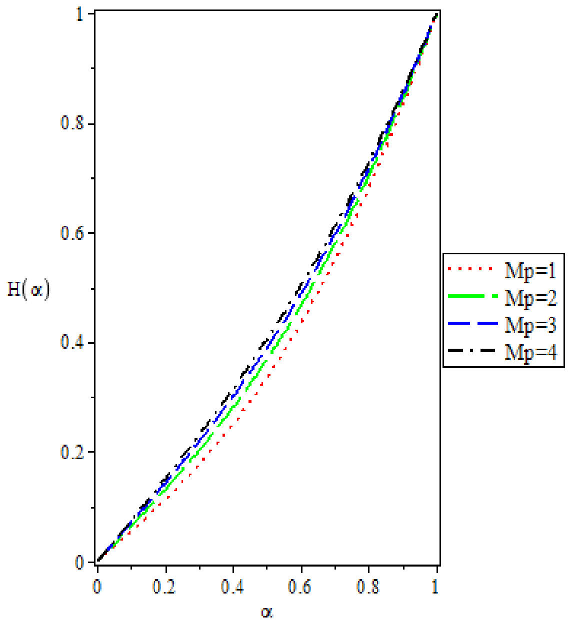

Fig. 13. The behavior of Mp on $\mathrm{H}(\alpha)$ for $Rn = \Lambda = 1, Mg = 0.1$

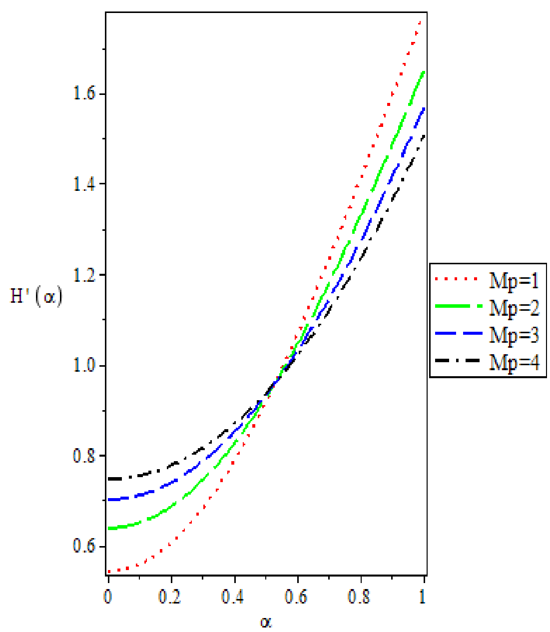

Fig. 14. The behavior of Mp on $\mathrm{H}'(\alpha)$ for $Rn = \Lambda = 1, Mg = 0.1$

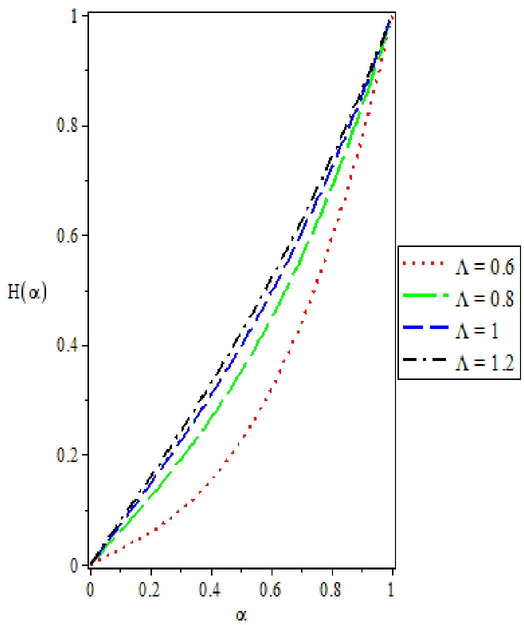

Fig. 15. The behavior of Λ on $\mathrm{H}(\alpha)$ for $Rn = Mp = 0.5, Mg = 1$

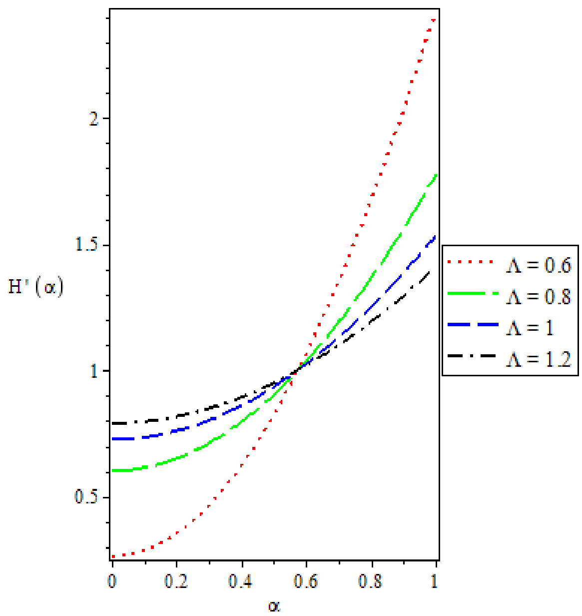

Fig. 16. The behavior of Λ on $\mathrm{H}'(\alpha)$ for $Rn = Mp = 0.5, Mg = 1$

7. Conclusion

In this work, the cases of no slip and slip at boundary condition of unsteady squeezing flow during a porous medium among two parallel plates infinite are analyzed by using a derivatives series algorithm. The obtained results proved that the effectiveness of this approach for obtaining the solution with a clear manner. Also, these results demonstrated that this approach is less computational cost with more consistent algorithm in terms of accuracy as compared to homotpy analysis scheme. This approach applied in various fields of engineering and science. The behavior of the curves axial velocity can be grouped as Figure 17 and 18.

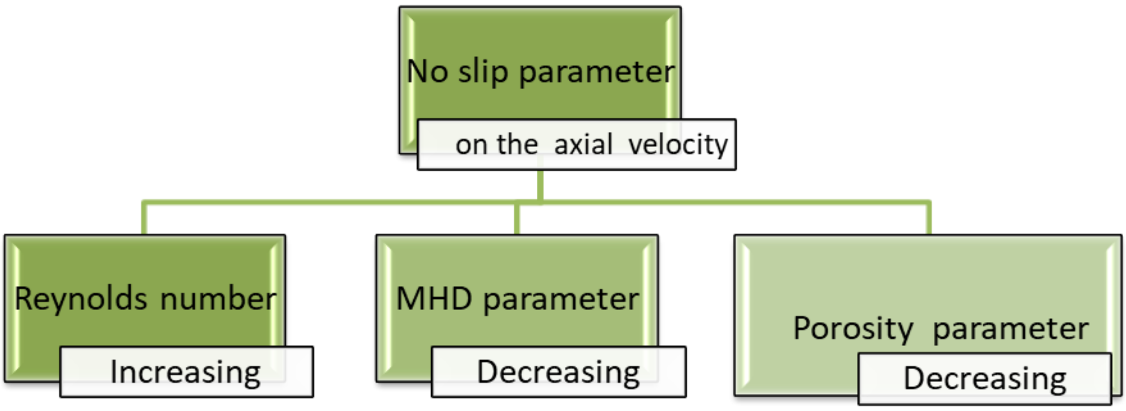

Fig. 17. The behavior of physical parameter in no slip case

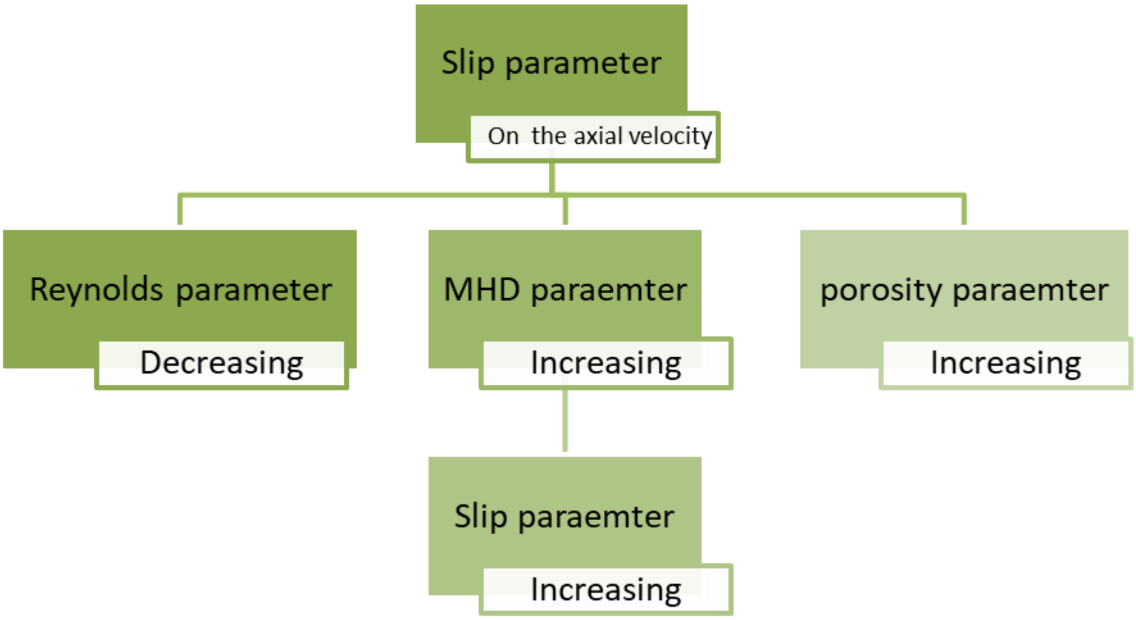

Fig. 18. The behavior of physical parameter in the slip case

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