Terminal Synergetic Control for Plate Heat Exchanger

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ABSTRACT

Terminal synergetic control (TSC) is proposed as a control strategy for the temperature management of a plate heat exchanger. The controller is designed by incorporating a selected macro variable with a time-varying sliding surface. The primary objective is to maintain precise control over the outlet temperature of the cold water. To assess the convergence characteristics of the newly proposed TSC approach, the simulation results achieved using TSC featuring a time-varying macro variable are compared to those obtained from the conventional synergetic control (SC) method. With an appropriate macro variable, the simulation results indicate a notable improvement in the convergence rate provided by our designed TSC method, compared to the conventional one. The desirable property of control input, the chattering-free condition, achieved by both TSC and SC approaches emphasizes the advantage of the synergetic control-based techniques over the conventional sliding mode controller. In conclusion, synergetic control-based techniques offer superior potential solutions for nonlinear feedback control problems.

Keywords:
Heat exchanger; synergetic control; terminal synergetic control; nonlinear feedback control; time-varying macro variable

1. Introduction

Heat exchangers find extensive utility across diverse industrial processes, e.g., oil and gas industries, food processing, and HVAC systems. A heat exchanger's efficiency is a vital aspect that influences the overall performance of the system. One of the ways to improve the efficiency of heat exchangers is by using advanced control strategies for the effective regulation of the exit temperature of the fluid. As presented in previous studies, both linear and nonlinear feedback control techniques have been applied to heat exchanger units [1–11]. Due to the fundamental nonlinear dynamics of heat exchangers, the sliding mode control (SMC) technique is commonly employed for temperature control in such systems and other applications [1-4,7,12]. SMC has shown itself to be

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successful in controlling dynamic systems with uncertainties and disturbances. However, the primary issue with the conventional SMC approach is that it introduces chattering into the control input signal. Alternatively, synergetic control (SC) is introduced in this work. The SC control theory was initially proposed by Kolesnikov [13,14]. It is an effective and efficient method for controlling various engineering systems such as power systems, robotics, thermal systems, and epidemic systems [15-27]. The desirable properties of SC techniques are global stability, robustness to bounded disturbance, chattering-free characteristics, and parameter insensitivity with properly selected macro variables [16-18,21,22].

Terminal synergetic control (TSC) is the enhanced control method in terms of improving convergence while maintaining the beneficial characteristics of the SC approach [21,28-31]. In the TSC method, both macro variables which are a function of an error, and/or the corresponding dynamic evolutions are selected so that the finite time convergence of the macro variables and errors are satisfied [21,28,29,32-37]. The TSC method has been implemented in various applications in previous literature [21,28-37].

In this paper, we design the TSC approach with a selected macro variable based on the time-varying sliding surface. The control objective is to regulate the cold fluid temperature of the plate heat exchanger at its exit. The following are the highlights of this study

- There has been no previous research that presents the development of the TSC approach specifically for plate heat exchanger systems, to the author’s knowledge.
- As mentioned in Santi et al., [17,18], sliding surfaces can be utilized as a macro variable in the SC approach. Also, its convergence rate is affected by the selection of the macro variable [27]. To improve the convergence rate of the control system, the concept of the time-varying sliding mode is applied in the selecting procedure of the macro variable.
- In our controller design procedure, the control input of the system (flow rate of hot water) is restricted to nonnegative values for a practical implementation. The auxiliary system serves to accommodate the effect of nonnegative restriction.
- Our developed TSC method significantly improves in convergence rate over the conventional SC method.
- The chatter-free properties of control input are guaranteed by both control techniques (TSC and SC).

2. Dynamic Model of the Plate Heat Exchanger

The simplified schematic of the plate heat exchanger is presented in Figure 1. $T_{co}$ and $T_{ho}$ are the outlet temperatures of the cold and hot water, respectively. $T_{ci}$ and $T_{hi}$ are the inlet temperature of the cold and hot water, respectively.
The mathematical model of the plate heat exchanger is based on the conservation of energy principle [4,11]. Two differential equations can be constructed to describe the model as shown in Eq. (1)

\[
\begin{align*}
\dot{T}_{co}(t) &= -K_1(T_{co}(t) - T_{ho}(t)) + \frac{U_c}{V_c} (T_{ci} - T_{co}(t)) \\
\dot{T}_{ho}(t) &= -K_2(T_{ho}(t) - T_{co}(t)) + \frac{1}{V_h} (T_{hi} - T_{ho}(t)) u(t),
\end{align*}
\]

where \( K_1 = \frac{UA}{C_p c c c} \) and \( K_2 = \frac{UA}{C_p h h h} \) . The parameters of the system Eq. (1) are summarized as follows [1,4,11]: \( U \) refers to the overall heat transfer coefficient. \( A \) is defined as the total heat transfer surface area that the fluid contacts. \( C_{c,c} \) and \( C_{c,h} \) represent the specific heat capacity of cold and hot water, respectively. \( \rho_c \) and \( \rho_h \) are the density of hot and cold water, respectively. \( V_h \) and \( V_c \) are the volume of the hot and cold sides, respectively. Cold water is flowing with a flow rate denoted as \( U_c \) while hot water is supplied at a flow rate marked as \( u(t) \) with the latter serving as the system’s control input.

The system’s state variable \( T_{co} \) and \( T_{ho} \), the model represented in Eq. (1) can be transformed algebraically into an input-output state-space representation as follows

\[
\begin{align*}
\dot{z}_1(t) &= T_{co}(t) \\
\dot{z}_2(t) &= T_{ho}(t) - T_{co}(t).
\end{align*}
\]

To control the outlet temperature of the cold water (\( T_{co} \)) to a set point value, the state equations illustrated in Eq. (2) with the corresponding input-output equation can be expressed in the following equations

\[
\begin{align*}
\dot{z}_1(t) &= K_1 z_2(t) + \frac{U_c}{V_c} (T_{ci} - z_1(t)) \\
\dot{z}_2(t) &= -K_2 z_2(t) + \frac{1}{V_h} (T_{hi} - z_1(t) - z_2(t)) u(t) \\
y(t) &= z_1(t).
\end{align*}
\]

For the practicality of the controller design, Eq. (3) can be further modified by defining the state variables as \( x_1(t) = z_1(t) - T_{cr} \) and \( x_2(t) = K_1 z_2(t) + \frac{U_c}{V_c} (T_{ci} - z_1(t)) \) where \( T_{cr} \) is the set point temperature. The model can therefore be rewritten as the following state equation

Let \( U_o \sqsubseteq \{ (x_1(t), x_2(t)) \ | \ g_o \not= g(x_1(t) + g_2 x_2(t), \forall t \geq 0 \} \sqsubseteq R^2 \) and,

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f(x) + g(x) u,
\end{align*}
\]
where $x \in [x_1, x_2] \in U_0 \text{, } f(x) = f_0 - f_1x_1(t) - f_2x_2(t) \text{, } g(x) = g_0 - g_1x_1(t) - g_2x_2(t)$. It is worth to note that $g(x) \neq 0 \ \forall t \in [0, \infty)$ since $x \in U_0$.

The constants variables of Eq. (4) are given by $f_0 = \frac{K_2U_c}{V_c}(T_{ci} - T_{cr})$, $f_1 = \frac{K_1U_c}{V_c}$, $f_2 = K_2 + \frac{U_c}{V_c}$, $g_0 = \frac{K_1}{V_b}T_{hi} - \left(1 + \frac{U_c}{K_1V_c}T_{ci} + \frac{U_c}{K_2V_c}T_{ci}\right)$, $g_1 = \frac{K_1}{V_b}\left(1 + \frac{U_c}{K_1V_c}\right)$, and $g_2 = \frac{1}{V_b}$.

The state equations Eq. (4) are further used throughout the controller design process in the following section.

3. Control of Heat Exchanger

3.1 Control Objective

To control the dynamic of the heat exchanger, the outlet temperature of the cold water $T_{co}(t)$ needs to be regulated to the set point temperature $T_{cr}$ by the designed controller. An error corresponding to the control objective can then be expressed as Eq. (5)

$$e_1 = x_1 - x_{ir},$$

where $x_{ir}$ is the reference signal corresponding to the desired output cold water $T_{cr}$.

3.2 Controller Design

According to previous studies, the controller design procedure for regulating the output temperature of the cold water is summarized and conducted as follows [13-21,24,28].

First, the macro variable is selected in terms of the error based on the control objective. In this paper, the macro variable is selected based on the sliding surface of the sliding mode control [38]

$$\psi_1 = \dot{e}_1 + c_1(t)e_1 + c_2(t),$$

where the coefficient $c_1(t)$ and the function $c_2(t)$ in Eq. (6) are such that

$$c_1(t) = \frac{a_1 \exp(-k_1 t)}{1 + \exp(-k_1 t)},$$

and

$$c_2(t) = a_2 \exp(-k_2 t),$$

where the exponent terms $k_1$ and $k_2$ are positive real numbers. The coefficients $a_1$ and $a_2$ are selected based on an initial condition as outlined in a study by Wang et al., [38]. This selection of coefficients is aimed at manipulating the macro variable surface to cross the initial condition, thereby enhancing the convergence time of the surface.

Considering the physical characteristics of the heat exchanger system, the control input (fluid volumetric flow rate) should be of a non-negative value. Thus, the constraint of the control input
needs to be considered. To handle this non-negative constraint, we implement the concept of the auxiliary system as presented in the study by Qi et al. [39]. Consequently, the error is modified as follows

\begin{equation}
    e_i = x_i - x_i^r - z_i,
\end{equation}

where the variable \( z_i \) is the state variable of the auxiliary system. The auxiliary system is defined as Eq. (10) [39]

\begin{align}
    \dot{z}_1 &= -c_{z_1} z_1 + z_2 \\
    \dot{z}_2 &= -c_{z_2} z_2 + g(x) \Delta u,
\end{align}

where \( \Delta u \square u - v \) and \( v \) is a nominal control.

Then, the dynamic evolution of the macro variable is defined as Eq. (11) [14,17,18,21,24,28]

\begin{equation}
    T_i \psi_i^{p/q} + \psi_i = 0,
\end{equation}

where \( T_i \) is the control parameter. The parameter \( T_i \) affect the rate of convergence of the macro variable to the manifold in Eq. (11). The exponent \( p \) and \( q \) are positive odd numbers and defined as \( 1 < p / q < 2 \). In the light of Eq. (5), Eq. (9), and Eq. (10), Eq. (11) becomes

\begin{equation}
    T_i \left( \frac{\psi_i^{p/q}}{T_i} \right) = f(x) + g(x) u + \left[ c_1 \dot{e}_1 + c_1 e_1 + \dot{c}_2 \right] - \ddot{x}_i - \left( -c_{z_1} \left[ -c_{z_1} z_1 + z_2 \right] - c_{z_2} z_2 + g(x) \Delta u \right),
\end{equation}

Finally, solve for the nominal control, \( v(t) \), Eq. (12) is then given by

\begin{equation}
    \left( \frac{-\psi_i}{T_i} \right)^{q/p} = f(x) + g(x) u + \left[ c_1 \dot{e}_1 + c_1 e_1 + \dot{c}_2 \right] - \ddot{x}_i - \left( -c_{z_1} \left[ -c_{z_1} z_1 + z_2 \right] - c_{z_2} z_2 + g(x) \Delta u \right).
\end{equation}

Then,

\begin{align}
    &\left( \frac{-\psi_i}{T_i} \right)^{q/p} = f(x) + g(x) u + \left[ c_1 \dot{e}_1 + c_1 e_1 + \dot{c}_2 \right] - \ddot{x}_i - \left( -c_{z_1} \left[ -c_{z_1} z_1 + z_2 \right] - c_{z_2} z_2 \right) - g(x) \Delta u \\
    &\left( \frac{-\psi_i}{T_i} \right)^{q/p} = f(x) + g(x) u + \left[ c_1 \dot{e}_1 + c_1 e_1 + \dot{c}_2 \right] - \ddot{x}_i + c_{z_1} \left[ -c_{z_1} z_1 + z_2 \right] + c_{z_2} z_2 - g(x) \Delta u \\
    &\left( \frac{-\psi_i}{T_i} \right)^{q/p} = f(x) + g(x) (u - \Delta u) + \left[ c_1 \dot{e}_1 + c_1 e_1 + \dot{c}_2 \right] - \ddot{x}_i + c_{z_1} \left[ -c_{z_1} z_1 + z_2 \right] + c_{z_2} z_2
\end{align}

\begin{equation}
    g(x) (u - \Delta u) = \left( \frac{-\psi_i}{T_i} \right)^{q/p} - f(x) + \dot{x}_i - \left( -c_{z_1} \left[ -c_{z_1} z_1 + z_2 \right] + c_{z_2} z_2 \right) - \left[ c_1 \dot{e}_1 + c_1 e_1 + \dot{c}_2 \right].
\end{equation}
Recall that $v = u - \Delta u$, thus, the nominal control $v$ can be determined as

$$v = g(x)^{-1} \left( \begin{array}{c} -\gamma_1 \\ \frac{T}{r} \end{array} \right) + f(x) + \dot{x}_v - c_{z_1} - c_{z_2} - c_{\dot{z}_1} + c_{\dot{z}_2}.$$

(15)

### 3.3 Proof of Stability

Define the Lyapunov function in terms of the macro variables as Eq. (16) \[17,18,21,24,28\]

$$V = \frac{1}{2} \psi_1^2.$$  

(16)

Then, the derivative of the Lyapunov function is calculated as Eq. (17)

$$\dot{V} = \psi_1 \dot{\psi}_1.$$  

(17)

From Eq. (5) and Eq. (6), the Lyapunov function in Eq. (17) becomes

$$\dot{V} = \psi_1 \left[ c_{\dot{z}_1} + c_{\dot{z}_2} + c_{\dot{z}_1} + c_{\dot{z}_2} \right]$$

$$= \psi_1 \left[ f(x) + g(x)u - \dot{x}_v - \left( -c_{z_1} - c_{z_2} - g(x) \Delta u \right) + c_{\dot{z}_1} + c_{\dot{z}_2} \right]$$

$$= \psi_1 \left[ f(x) + g(x)(u - \Delta u) - \dot{x}_v + c_{z_1} - c_{z_2} + c_{\dot{z}_1} + c_{\dot{z}_2} \right]$$

$$= \psi_1 \left[ f(x) + g(x) + c_{\dot{z}_1} + c_{\dot{z}_2} \right]$$

(18)

Substituting $v$ into Eq. (18) yields

$$\dot{V} = \psi_1 \left( -\frac{1}{T} \psi_1 \right)^{\frac{q}{p}} \left( -\frac{1}{T} \right)^{\frac{q}{p}} \psi_1^{\frac{p+q}{p}} V^{\frac{p+q}{2p}} = -\lambda V^{\frac{p+q}{2p}},$$

(19)

where $\lambda = (T) \frac{q}{2p}$ and, $\lambda > 0$, $0 < \frac{p+q}{2p} < 1$ by construction.

By lemma 1, we can conclude that the macro variables will converge to zero in a finite time $t_s$. Further description of lemma 1 as below \[28,40,41\]

Consider the system in Eq. (20):

$$\dot{x} = f(x)$$

(20)

where $x$ denotes a state vector and $x \in \mathbb{R}^n$. If there exists a positive-definite and continuous Lyapunov function with the following inequality
\[ \dot{V}(t) \leq -\lambda V^\sigma(t), \forall t \geq t_0, V(t_0) \geq 0, \]  
where \( \lambda \) is a positive constant and \( \sigma \) is a constant exponent with \( 0 < \sigma < 1 \). Then, the following inequality

\[ V^{1-\sigma} \leq V^{1-\sigma}(t_0) - \lambda(1-\sigma)(t-t_0), \quad t_0 < t < t_s \]  
holds for given an initial time \( t_0 \), and

\[ V(t) = 0, \forall t \geq t_s, \]  
where \( t_s \) is determined as

\[ t_s = t_0 + \frac{V^{1-\sigma}(t_0)}{\lambda(1-\sigma)}. \]  

On the manifold \( \psi_1 = 0 \), the convergence of the error \( e_i(t) \) to zero according time varying differential equation

\[ \dot{\psi}_i = -\left( a_i \exp(-k_it) \right) e_i - a_2 \exp(-k_2t) \]  
together with appropriate value of the value the \( a_1, a_2, k_1, \) and \( k_2 \), e.g. \( a_1 > 0, a_2 > 0, 0 < k_1 \leq k_2 \). However, it is worth noting that, by this condition, it will limit the initial condition to be crossed by the specified macro variable in Eq. (6). In conclusion, this implies that the heat exchanger’s cold-water temperature can be properly controlled by the designed controller.

4. Simulation

This section encompasses the simulation of the controlled heat exchanger system, which was carried out to assess the viability and practicality of the newly proposed controller. The design parameters of the system are from Almutairi and Zribi [4] as follows: \( T_{ci} = 20^\circ C \), \( T_{hi} = 80^\circ C \), \( U = 300 \text{ W/m}^2 \cdot ^\circ C \), \( A = 0.0672 \text{ m}^2 \), \( C_{p,c} = C_{p,h} = 4180 \text{ J/kg} \cdot ^\circ C \), \( \rho_c = \rho_h = 1000 \text{ kg/m}^3 \), \( V_c = V_h = 0.000537 \text{ m}^3 \), and \( U_c = 150 \text{ cm}^3 / \text{min} \) The control input, the hot water flow rate, is also restricted to nonnegative values with \( u_{\text{max}} = 3000 \text{ cm}^3 / \text{min} \). The parameters for the designed TSC method are set as follows: \( T_i = 50 \), \( p = 7 \), \( q = 5 \), \( k_1 = 0.0001 \), \( k_2 = 0.6 \), and \( a_i = 3 \). Then, the corresponding coefficient \( a_2 \) is obtained as \( a_2 = 29.46 \). To assess the convergence characteristics of the proposed TSC method, the simulation results achieved using the TSC method with time-varying macro variable are compared with the results obtained from the conventional SC method. In the conventional SC method, the macro variable is specified as \( \psi_1 = \dot{\psi}_i + a_i\psi_1 \) and the corresponding dynamic evolution is \( T_i\dot{\psi}_i + \psi_1 = 0 \) [17,18,21].

Figure 2 shows the outlet temperature of the cold side. Both the TSC and SC control approaches effectively achieve the desired outcome of driving the outlet temperature to the designated set point of \( 40^\circ C \). The proposed TSC method shows a faster convergence rate (approximately 200 seconds faster) compared to the conventional one. Similarly, as presented in Figure 3, the TSC approach converges to the steady state temperature faster than the SC method for the output temperature of
the hot side. Figure 4 illustrates the flow rate of the hot water which is the system’s control input. The control input under the TSC approach performs impulsive behaviour (spike) with a lower peak compared to the conventional SC approach. This smoother input and chattering-free condition are the preferable characteristics for the practical implementation of water flow rate manipulation.

Furthermore, a simulation was conducted on the heat exchanger, subject to the bounded disturbances occurring between 500 seconds and 700 seconds. This simulation was performed using the TSC method’s designed controller, with the disturbance \( d(t) \) defined as \( d(t) = A_{\text{dis}} \sin \omega_{\text{dis}} t \), where \( A_{\text{dis}} = 0.025 \) and \( \omega_{\text{dis}} = 0.025\pi \). This disturbance occurs in the second equation of (4). The simulation results were compared with those obtained using the SMC approach with the sliding surface of \( s_1 = \dot{s}_1 + a_{\text{smc}} e_1 \), where \( a_{\text{smc}} = 0.05 \). This SMC approach was derived based on the reaching law of \( \dot{s}_1 = -k_{\text{sw}} \text{sign}(s) \), where the switching gain is selected as \( k_{\text{sw}} = 0.8 \) [42-44]. The controller parameters are selected to ensure similar convergence rates for both the TSC and SMC methods. The time-domain responses of the cold and hot water in the heat exchanger system, considering the effect of the disturbance, are depicted in Figure 5 and Figure 6, respectively. The corresponding control inputs are presented in Figure 7. As evidenced in Figure 5, both the TSC and SMC methods effectively control the outlet cold water temperature to the desired setpoint. Additionally, Figure 6 demonstrates the temperature of the hot water converges to its corresponding level under both controllers. As shown in Figure 7, the control input under the TSC method remains free from chattering as expected, whereas the control input of the SMC method exhibits chattering. Both parts of the simulation confirm that the TSC method’s designed controller achieves the desired characteristics of improved convergence rates and chattering-free control input.

![Fig. 2. Time responses of the outlet temperature of cold water under the TSC and the conventional SC approach](image)

\[ T_{\text{co}}: \text{TSC} \]
\[ T_{\text{co}}: \text{SC} \]
\[ T_{\text{co}_{\text{ref}}} \]
**Fig. 3.** Time responses of the outlet temperature of hot water under the TSC and the conventional SC approach

**Fig. 4.** Control inputs, the volumetric flow rate of hot water, under the TSC and the conventional SC approach
**Fig. 5.** Time responses of the outlet temperature of cold water under the TSC and the conventional SMC approach

**Fig. 6.** Time responses of the outlet temperature of hot water under the TSC and the conventional SMC approach
5. Conclusions

In this paper, to control the outlet temperature of the heat exchanger, we develop the TSC method with a selected macro variable based on the time-varying sliding surface. The proposed TSC method's stability is thoroughly investigated within the framework of the control system. The investigation demonstrates that the proposed TSC method ensures stability for the controlled heat exchanger system. The simulation results indicate the notable improvement in the convergence rate provided by our designed TSC method, compared to the conventional SMC method. In addition, the chattering-free condition of control input raises the value of the TSC technique, compared to the conventional SMC method. Consequently, applying the TSC technique offers a feasible and attractive method for controlling the temperature of heat exchanger systems.

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