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Factors Affecting Water Vapor Flow in the Soil

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ABSTRACT

Soil water vapor movement carries mass of water and its bulk quantity in motion carries a specific amount of energy. The moment when water molecules obtain enough energy to release themselves from liquid surface, it involves phase transition from liquid to gas phase. The energy required for its release or freedom must be sourced from some energy source in the soil. The energy can come from the soil, water, or gas phase in the immediate surrounding. As such, the water phase transition into gas phase absorbs energy and causes temperature variation. Since vapor carries a significant amount of energy, its evaporation can cause a significant amount of temperature in the soil. Hence, studying vapor movement in soil and the factors affecting vapor movement is an important subject of concern. The current study is looking into factors that affect soil water vapor movement in the soil. The equation from Philip and de Vries was examined in the current study. We found that soil water vapor movement was not limited to factors affecting vapor phase, the vapor phase was in continuous interaction with other factors that affecting liquid water movement and heat flux in the soil.

1. Introduction

Soil is an important body of interface place between groundwater and atmospheric. Naturally, the layer of soil on a daily or monthly basis does not go through a lot of changes in terms of its solid physical properties such as bulk density, porosity, and soil texture. The properties change largely depends on non-periodic events such as flood, forest fire, urbanization, or plantation activities. Other soil physical properties subjected to daily or monthly variation includes the soil moisture content and temperature variation. Soil moisture content can vary from nearly dry during drought season to almost saturated condition during raining season. While soil temperature appears at the lowest at nighttime, and then, it peaks when it is exposed to ninety-degree sunlight from soil surface, normally in the afternoon. The soil moisture content attracts considerable attention from laboratory investigation, field scientists and practitioners because it has immediate utilities to human activities

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such as farming and plantation management. Similarly, soil temperature comes in close interaction with soil moisture content. Changes in soil temperature can bring about variation in soil moisture content, and similarly, changes in soil moisture content affect soil temperature. In recent years, soil temperature variation has received a considerable attention due to its close relation with climate change [1]. Soil temperature affects plant growth. In places where soil texture like sand or peat, fire on soil surface containing organic matter can occur. Hence, the soil temperature and soil moisture content are important soil physical parameters to observe and model.

In addition to mass and heat transport as described by liquid water movement and heat conduction, respectively, water vapor is another physical parameter of importance in soil. Mass transport in soil is not limited to liquid water movement, water vapor flux also contributes to mass transport in soil. Furthermore, water vapor carries a certain amount of heat as it moves. The heat storage contributed by vapor comes from the balance between vapor inflow and outflow from storage (bulk soil), besides the heat it obtains or releases during evaporation and condensation on soil surface, respectively. Furthermore, previous work by researcher has reported the importance of water vapor as a source of water for seed germination [2].

When the liquid water vaporizes, a significant amount of heat energy is utilized to mobilize water [3] from the surface of liquid to the vapor phase. Thus, evaporation is an essential mechanism to mobilize heat energy from liquid phase apart from vapor movement itself in transporting heat [4] in the soil. Outflux of vapor from the deeper soil body to the soil surface, or the evaporation of water at the soil surface, remained the most important mechanism to reduce soil temperature after exposure to heat from sunlight [5]. Thus, it helps in sustaining a relatively constant daily periodic soil temperature by soil capturing heat in the daytime and soil releasing heat in the nighttime [6].

Modeling mass transport in soil was often limited to liquid water movement. Water movement in unsaturated soil was governed by Richards' equation [7]. The equation was reported to have a mass balance problem when a matric suction based form was used [8]. To overcome the mass balance problem, a mix-based [9], a water-content [10] and matric suction-based [11] equation was used instead. The equation was considered inadequate when a large part of the mass transport was influenced by soil temperature variation in the soil [12]. Hence, heat transport equation based on Fourier's law [13] was being introduced alongside the mass transport equation to capture some of the mass fluxes caused by the daily temperature fluctuation in the soil. The heat equation gave rise to the movement of energy by conduction and the transfer of heat due to liquid water movement [14]. However, a problem occurred when the equation was used to model mass transport at the soil surface, when soil moisture can be evaporated or even heat can be transported by vapor in the soil body [15], [16]. Hence, the water vapor flux equation was introduced into the mass transport equation alongside the heat transfer equation. The set of equation was described in the Philip and de Vries [17]. Here after taken as PdV.

The PdV equation comes with a comprehensive theoretical support, but it was not free from limitation needing continuous improvement. One aspect of the PdV equation frequently becomes the subject of investigation, for instance, vapor enhancement factor on the temperature gradient term driving vapor movement in the soil [18]. The term itself was more like a multiplier or correction factor than any theoretical equation with adequate physical representation [19]. As a result, it frequently attracts considerable interest from many researchers to investigate and identify the underlying mechanism involved in representing the vapor enhancement factor. In this study, we re-examined the governing equation of mass and heat transport as layout by PdV, with emphasis on parameters affecting the changes of water vapor movement. The examination is important to clearly distinguish between parameters' increment or decrement which either enhancing vapor movement

or limiting the vapor movement. Hence, the finding of this study can be used to better understand the changes soil properties on water vapor movement.

2. Review of the Governing Equation

The review of the equation was conducted on the mass and heat transport as in the equation given by Philip and de Vries. Initially, mass transport was examined. Then, the heat transfer equation was examined. The investigation was based on the condition that when changing a parameter, the changes on the vapor flux were identified. The partial differential equation from heat and mass were supported by the subsidiary's equations, for example, van Genuchten equation that relates matric suction with soil moisture content, and also the unsaturated hydraulic conductivity with soil moisture content. To simplify the scope of the investigation, the current study limits the identification on whether the change in parameter was either increasing or decreasing the water vapor flux in the soil. The method used was commonly called one-at-a-time sensitivity analysis. However, in the current study, numerical quantification of changing the input parameter to study the effect of output result on water vapor flow was not conducted, but rather focused on a qualitative study. It was considered sufficient because the current study aim was to form a complete picture of relationships between the physical parameters with that of water vapor movement.

The mass transport of water vapor mass flux [20] is as follows,

$$\frac{q_v}{\rho_L} = - \frac{D\Omega\theta_a}{\rho_L} \frac{\partial\rho_v}{\partial z} \quad (1)$$

The right-side term implies water vapor diffusion due to water vapor density difference in space. The diffusion coefficient of water vapor is given by,

$$D = 2.29 \times 10^{-5} \left(\frac{T}{273.15} \right)^{1.75} \quad (2)$$

The tortuosity factor in the porous soil relates to soil porosity and soil moisture content and it is given by,

$$\Omega = (\phi - \theta_L)^{2/3} \quad (3)$$

The volumetric air content has similar relation as in tortuosity but without the power over the relation and it is given by,

$$\theta_a = \phi - \theta_L \quad (4)$$

Equation 1 of the $\frac{\partial\rho_v}{\partial z}$ is referring to the change of water vapor density in space that can be expanded by chain rule, and it is given by,

$$\frac{\partial\rho_v}{\partial z} = \left(\frac{\partial\rho_v}{\partial\rho_s} \frac{\partial\rho_s}{\partial T} + \frac{\partial\rho_v}{\partial h_m} \frac{\partial h_m}{\partial T} \right) \frac{\partial T}{\partial z} + \frac{\partial\rho_v}{\partial\psi_m} \frac{\partial\psi_m}{\partial z} \quad (5)$$

The term on the left-hand side refers to the variation of water vapor density with space, which is contributed by the change of temperature and matric suction in space.

The thermal vapor diffusivity is given by,

$$D_{Tv} = \eta \frac{D\Omega\theta_a}{\rho_L} \left(\frac{\partial\rho_v}{\partial\rho_s} \frac{\partial\rho_s}{\partial T} + \frac{\partial\rho_v}{\partial h_m} \frac{\partial h_m}{\partial T} \right) \quad (6)$$

And, isothermal water vapor diffusivity is given by,

$$D_{mv} = \frac{D\Omega\theta_a}{\rho_L} \frac{\partial\rho_v}{\partial\psi_m} \quad (7)$$

The change in water vapor density with saturated water vapor density is given by,

$$\frac{\partial \rho_v}{\partial \rho_s} = h_m \quad (8)$$

The relative humidity is given by [17],

$$h_m = \exp\left(\frac{gM\psi_m}{RT}\right) \quad (9)$$

The soil matric suction is influenced by soil moisture content and soil temperature as,

$$\psi_m = \psi_e \left(\frac{\theta_L}{\theta_s}\right)^{-b} \exp(-\beta(T - T_o)) \quad (10)$$

The variation in water vapor density with relative humidity depends on the following relation,

$$\frac{\partial \rho_v}{\partial h_m} = \rho_s \quad (11)$$

The saturated vapor density is given by [21],

$$\rho_s = \frac{\exp\left(19.819 - \frac{4975.9}{T}\right)}{1000} \quad (12)$$

The water vapor density change also has relation to matric suction in the soil, which is given by,

$$\frac{\partial \rho_v}{\partial \psi_m} = \rho_s \frac{\partial h_m}{\partial \psi_m} \quad (13)$$

The change in relative humidity with soil matric suction has the following relation,

$$\frac{\partial h_m}{\partial \psi_m} = \frac{gM}{RT} \exp\left(\frac{gM\psi_m}{RT}\right) \quad (14)$$

The mass transport of liquid mass flux [22] is as follows,

$$\frac{q_L}{\rho_L} = -K \frac{\partial \psi_m}{\partial T} \frac{\partial T}{\partial z} - K \frac{\partial \psi_m}{\partial z} - K \bar{i} \quad (15)$$

The right-side first term represents liquid water diffusion flux due to temperature gradient. The second term on the right indicates liquid water diffusion flux by matric suction gradient, and the last term represents the effect of gravitational pull on water flux vertically.

The thermal liquid water diffusivity is given by,

$$D_{TL} = -K \frac{\partial \psi_m}{\partial T} \quad (16)$$

The unsaturated hydraulic conductivity [23] is as follows,

$$K = K_s \left(\frac{\theta_L}{\theta_s}\right)^{2b+3} \frac{\mu(T_o)}{\mu(T)} \quad (17)$$

The change of soil matric suction with temperature is as follows. Modified from Milly [24],

$$\frac{\partial \psi_m}{\partial T} = -\psi_e \left(\frac{\theta_L}{\theta_s}\right)^{-b} \beta \exp(-\beta(T - T_o)) \quad (18)$$

The heat transfer equation [14] is governed by,

$$q_h = -\lambda_* \frac{\partial T}{\partial z} + L_o q_v + c_v (T - T_o) q_v + c_L (T - T_o) q_L \quad (19)$$

The right-side of equation first term refers to heat transfer by conduction, while the second term refers to heat transfer by latent of vaporization as the water vapor moves. The third term refers to

sensible heat transfer as water vapor being transported in space, and similarly, the fourth term is the liquid water sensible heat transfer as the liquid water transfer in the soil.

Equation 18 can be expanded into the following form. Modified from Nassar and Horton [25],

$$q_h = -\left\{\lambda + c_v(T - T_o)\rho_L D_{Tv} + c_L(T - T_o)\rho_L D_{TL}\right\} \frac{\partial T}{\partial z} - \left\{\left[L(T) + c_L(T - T_o)\right]\rho_L D_{mv} + c_L(T - T_o)\rho_L K\right\} \frac{\partial \Psi_m}{\partial z} - c_L(T - T_o)\rho_L K \bar{i} \quad (20)$$

The right-side first term refers to the accumulation of conduction, sensible heat by water vapor, and sensible heat by liquid water, under the influence of temperature gradient. Whereas the second term caused by soil matric suction gradient is a combined action of latent heat of vaporization and sensible heat by isothermal water vapor diffusion and also the liquid water sensible heat transfer. The final term refers to the sensible heat transfer because of gravity.

3. Results and Discussion

Mass transport in soil is a combined effect of both liquid water and water vapor mass transfer. Simultaneously, mass transport also drives heat transfer in the soil, apart from heat conduction in the soil. In addition, temperature variation in the soil drives mass transport. Hence, the feedback loop between mass and heat transfer needing both mechanisms involved to be discussed even when reviewing water vapor mass movement alone. The current review begins by reviewing the mass transport equation of water vapor, then followed by liquid water mass transfer equation. Finally, the heat transfer equation was reviewed.

From water vapor equation, Eq. (1) shows water vapor movement depends on water vapor diffusion coefficient, tortuosity factor, volumetric air content, and water vapor density difference in space. Refer to Figure 1. The liquid water density was assumed insignificant, and it has been excluded from the accounting the mass transfer of liquid water in the soil, as it appears on both sides of the Eq. (1). The diffusion coefficient captured the random motion of water vapor flow rate in the soil, and it appears to increase with increasing soil temperature that increases the soil water vapor flux. Soil tortuosity refers to the easiness of the pathway the soil water vapor must go through to reach the flow destination. Hence, the more restrictive the pathway for the water vapor flow, the lower the soil tortuosity. The tortuosity factor only increases when either soil has a high porosity or the volumetric water content decreases, which translates into increasing soil water vapor movement. Similar situations occur to volumetric air content, it increases with either soil with high porosity or decreasing volumetric water content towards drying condition.

A high-water vapor density gradient appears to drive higher water vapor movement, as in Eq. (1). The water vapor density gradient can be expanded, as in Eq. (5), and it increases with increasing soil temperature gradient and soil matric suction gradient. The soil temperature gradient increment leads to decreasing the derivative of soil relative humidity with respect to soil temperature, as well as increasing the derivative of soil water vapor density with respect to the soil relative humidity. Simultaneously, the equal increment of soil temperature gradient also causes increment in the derivative of saturated water vapor density with respect to soil temperature, and the increment in the derivative of water vapor density with respect to the saturated water vapor density. As for the soil matric suction gradient, its increment causes the derivative of soil water vapor density with respect to the soil matric suction to increase.

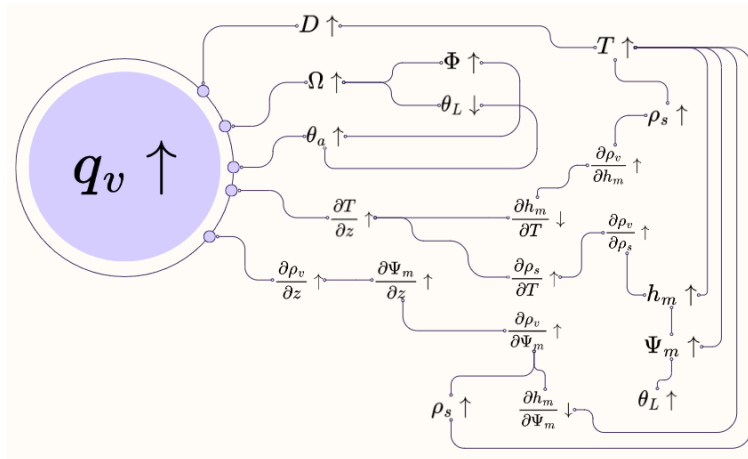


Fig. 1. The schematic diagram shows the relation of water vapor flux as a function of different parameters in the soil. The vertical arrow indicates the increasing value of the parameter in relation to other parameters either increasing (arrow upward) or decreasing (arrow downward)

The terms at the forefront of the temperature gradient, as in Eq. (5), are clustered to form thermal water vapor diffusivity, which increases when the soil temperature gradient increases. In the same way, the term before the soil matric suction gradient increases is clustered to form the isothermal water vapor diffusivity that increases as the soil matric suction gradient increases. The derivative of water vapor density with respect to the saturated water vapor density is equivalent to soil relative humidity, as in Eq. (8), where it is a function of soil matric suction and soil temperature, Eq. (9). The soil relative humidity is apparently increased when either the soil matric suction or soil temperature increases. The soil matric suction, as in Eq. (10), increases with increasing soil moisture content and soil temperature. Also, increasing the value of the derivative of soil water vapor density with respect to soil relative humidity is equivalent to the increment of saturated water vapor density, which corresponds to the increasing soil temperature, Eq. (12).

The derivative of soil water vapor density with respect to soil matric suction, as in Eq. (13), consists of the multiply of saturated water vapor density and the derivative of soil relative humidity with respect to soil matric suction. Both terms increase as the soil temperature increases.

While mass transport is a combination of both liquid water and water vapor, a snapshot moment of time on water vapor flux does not seem to directly be affecting the liquid water movement. The liquid water mass flux appears to be independent from that of water vapor mass movement, though they are similarly driven by soil matric suction gradient and soil temperature gradient, as in Eq. (15). When both mass transport equations are included in the partial differential equation, as in PdV, to account for mass change with time, a moment of time that changes in mass movement in liquid water, it will change mass flux in water vapor, and vice versa. Thus, increasing the liquid water movement is coupled with increasing water vapor movement. Therefore, factors affecting liquid water movement will also affect water vapor movement. In addition to that, liquid water mass flux is affected by gravitational force that as for now is not considered affecting water vapor movement.

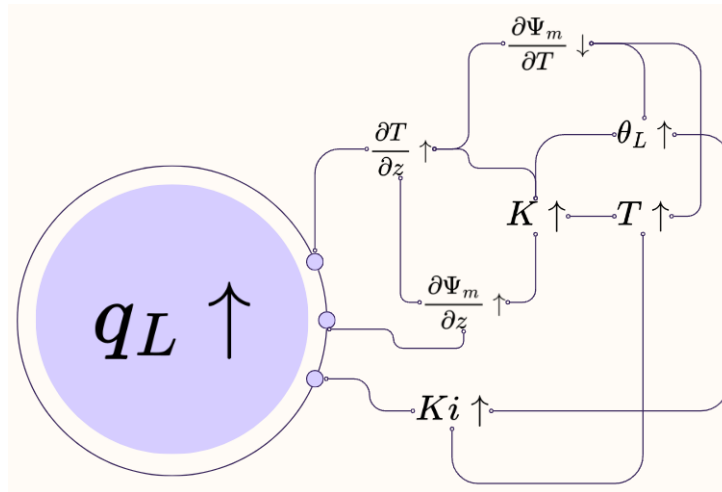


Fig. 2. The schematic diagram shows the relation of liquid water flux as a function of different parameters in the soil. The vertical arrow indicates the increasing value of the parameter in relation to other parameters either increasing (arrow upward) or decreasing (arrow downward)

The liquid water mass movement is affected by soil temperature gradient, soil matric suction gradient, and gravity, as in Eq. (15). Refer to Figure 2. While gravity is a constant force, increasing the soil temperature gradient does give rise to the decreasing value on the derivative of soil matric suction with respect to the soil temperature (Eq. (18)), which multiply with unsaturated hydraulic conductivity, as in Eq. (16). Multiplication forms the term variable known as the thermal liquid water diffusivity that increases as the soil temperature, soil moisture content, and the soil temperature gradient increases. The unsaturated hydraulic conductivity increases with increasing soil moisture content and increasing soil temperature, Eq. (17). When soil matric suction gradient increased, the unsaturated hydraulic conductivity also increased. Hence, the soil matric suction gradient and soil temperature gradient increment give rise to liquid water movement. Similarly, increasing soil moisture content or soil temperature also increases the liquid water movement by gravity.

Generally, the heat transfer in the soil is affected by soil temperature gradient and soil matric suction gradient. Refer to Figure 3. Also, gravitational pull will speed up the heat transfer vertically downward movement. The soil temperature gradient consists of three terms, which are the apparent thermal conductivity heat flux under temperature gradient, the water vapor thermal diffusivity sensible heat flux, and the liquid water thermal diffusivity sensible heat flux. When the soil temperature gradient increases, all the three terms increase. Under the soil matric suction gradient, there are also three terms, and they are water vapor isothermal diffusivity-latent heat flux, water vapor isothermal diffusivity sensible heat flux, and liquid water isothermal diffusivity sensible heat flux. The first two terms combined appear to reduce as the soil matric suction gradient increases. However, the last term increases as the soil matric suction gradient increases. The overall combination of the three terms appeared to increase as the soil matric suction gradient increased. Finally, the gravity driven heat flux increases when the soil moisture content or the soil temperature increases. The ability to quantify soil heat flux has significant importance because it raises the soil temperature apart from moving heat energy in the soil forward or deeper into the soil.

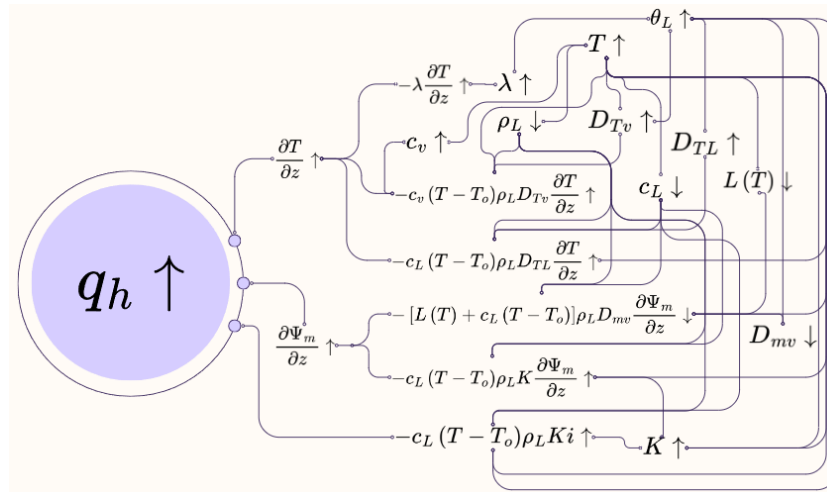


Fig. 3. The schematic diagram shows the relation of soil heat flux as a function of different parameters in the soil. The vertical arrow indicates the increasing value of the parameter in relation to other parameters either increasing (arrow upward) or decreasing (arrow downward)

3. Conclusion

Soil physics begins with the understanding of water movement in the soil. It was extended to include heat transfer in the soil. Refinement of water and heat flow requires the inclusion of water vapor movement in the soil, especially at the soil surface where water vapor remains the only means of mass loss to the atmosphere, apart from evapotranspiration from trees and plants. Modeling soil water, vapor and heat transfer in soil have many areas of importance, especially current environment is experiencing rapid changing climate has increased the concern on water resources availability, forest fire, food security, including the climate change and global warming. Hence, a better understanding of the physical parameters with clear feedback loops with a clear indication on the chain of reaction, when a single parameter is changed, has useful reference to engineer and scientist. The current study has clearly revealed the interconnection between parameters begins with parameters which increase water vapor movement. This is followed by liquid water movement, and heat transfer. This study reviews the mechanisms used in the PdV model, and also open up the opportunity to identify areas of improvement for mechanism interaction that is yet available in the current model.

Nomenclature

q_L	liquid water flux (kg/m ² /s)
q_v	water vapor flux (kg/m ² /s)
q_h	heat energy transfer (J/m ² /s)
ρ_L	density of liquid water (kg/m ³)
ρ_v	water vapor density (kg/m ³)
ρ_s	saturated water vapor density (kg/m ³)
D	diffusion coefficient (m ² /s)
Ω	tortuosity (dimensionless)

θ_a	volumetric air content (m^3/m^3)
θ_s	saturated volumetric water content (m^3/m^3)
θ_L	volumetric water content (m^3/m^3)
D_{TL}	thermal diffusivity of liquid water ($\text{m}^2/\text{s/K}$)
K	unsaturated hydraulic conductivity (K , m/s) as a function of soil water content
K_s	saturated hydraulic conductivity (m/s)
$\mu(T_o)$	dynamic viscosity (kg/m/s) of water at reference temperature T_o
$\mu(T)$	dynamic viscosity (kg/m/s) of water at temperature T
\bar{i}	unit vector (dimensionless)
dT	temperature difference (K)
dz	spatial difference (m)
$d\psi_m$	soil matric suction difference (-m)
D_{Tv}	thermal water vapor diffusivity ($\text{m}^2/\text{s/K}$)
D_{mv}	isothermal water vapor diffusivity ($\text{m}^2/\text{s/K}$)
η	vapor enhancement factor (dimensionless)
T	soil temperature (K)
T_o	reference temperature (K)
ϕ	soil porosity (m^3/m^3)
M	molecular weight of water (kg/mol)
h_m	relative humidity in the soil (dimensionless)
g	gravity (m/s^2)
R	gas constant (J/K/mol)
λ	apparent thermal conductivity (J/m/s/K)
L_o	latent heat of vaporization (J/kg) at reference temperature T_o
$L(T)$	latent heat of vaporization (J/kg) at reference temperature T
c_L	liquid water specific heat (J/kg/K)
c_v	water vapor specific heat at constant pressure (J/kg/K)

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