Semarak Engineering Journal

Journal homepage:
https://semarakilmu.com.my/journals/index.php/sem_eng/index
ISSN: 3036-0145

Advancing Heat Transfer: Exploring Nanofluids and Regression analysis on Lower Stagnation Point of a Horizontal Circular Cylinder for Brinkman-Viscoelastic fluid

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ARTICLE INFO

Article history:
Received 1 September 2024
Received in revised form 7 October 2024
Accepted 10 November 2024
Available online 15 December 2024

Keywords:
Hybrid nanofluids; heat transfer; Brinkman -Viscoelastic model; Carboxymethyl Cellulose (CMC); regression model

ABSTRACT

Nanofluids and hybrid nanofluids are increasingly employed in research, products, and technologies to enhance heat transfer efficiency. Recent investigations have focused on the convective heat transfer of viscoelastic nanofluids flowing through porous media, utilizing the Brinkman-Viscoelastic nanofluid model. In this study, the volume fraction of nanoparticles is used to characterize the nanofluids, while the heat transfer performance is quantified by the Nusselt number. The primary objective is to develop a regression model that evaluates the influence of nanoparticle volume fraction on the Nusselt number using simple linear regression analysis. Copper (Cu) nanoparticles and Carboxymethyl Cellulose (CMC) serve as the nanoparticle and base fluid, respectively. The governing equations for Brinkman-Viscoelastic nanofluid are simplified through non-dimensional and non-similarity transformations to enable analytical treatment. These simplified equations are numerically solved using the Runge-Kutta-Fehlberg method, and the results are used to construct and validate the regression model. This study provides insights into the relationship between nanoparticle concentration and thermal performance, contributing to advancements in heat transfer applications.

1. Introduction

Nanofluids are specialized fluids composed of a base fluid with suspended nanoparticles, often in the size of smaller than 100 nm [1]. Examples of base fluids are water and oil. Nanoparticles on the other hand can be composed of several components, including metals such as copper, oxides, or

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https://doi.org/10.37934/sej.7.1.110

carbon-based substances [2]. Integrating these nanoparticles markedly improves the thermal conductivity and heat transfer properties of the base fluid compared to conventional fluids.

In the context of convective boundary layers, nanofluids are crucial due to their superior thermal properties. The parameter that denotes the physical characteristics of nanoparticles in a nanofluid is the nanoparticle volume fraction, which is theoretically defined as the ratio of the volume of nanoparticles to the total volume of the fluid mixture [3]. Studies show that nanofluids exhibit greater Nusselt numbers than conventional fluids [1]. The Nusselt number (Nu) is a dimensionless quantity utilised in the study of convective heat transfer. It is the ratio of convective heat transfer to conductive heat transfer in a fluid [4]. The Nusselt number assists engineers and scientists in evaluating and predicting the effectiveness of heat transfer in diverse systems, including heat exchangers, and cooling systems in both natural and forced convection situations.

Given that the physical properties of nanofluids are defined by nanoparticle volume fraction, numerous researchers have examined the effect of this fraction on the Nusselt number utilising the Tiwari & Das model, as it incorporates nanoparticle volume fraction to analyse nanofluid behaviour [5]. Mahat *et al.,* [6] investigated the natural convection of a viscoelastic nanofluid past a horizontal circular cylinder using the Tiwari & Das model and they identified that nanoparticle volume fraction increases the Nusselt number. Zokri *et al.,* [7] studied the impact of mixed convection on the lower stagnation point flow of Jeffrey nanofluid from a horizontal circular cylinder also using the Tiwari & Das model. They used Carboxymethyl cellulose (CMC) water as the base fluid while copper as the nanoparticles in their study. The results indicate that both the velocity and temperature profiles increase with higher nanoparticle volume fractions. The investigation of nanofluids and convective heat transfer utilizing the Tiwari & Das model is progressing and evolving with various geometries, boundary conditions and fluid enhancements, as demonstrated in the references [8-12].

The viscoelasticity of the fluid and the porosity of the medium into where the fluid flows can further enhance convective heat transfer, rather than solely relying on nanofluid. The Brinkman-viscoelastic model is employed to analyze the convective heat transfer of viscoelastic fluids traversing a porous media. This model integrates the Brinkman model and the viscoelastic model, initially employed by Kanafiah *et al.*, [13]. However, this model did not consider the presence of nanofluid, despite its prominence. This paper will integrate the Brinkman-viscoelastic model with the Tiwari & Das model to investigate the mixed convection of a viscoelastic fluid containing nanoparticles flowing over a horizontal circular cylinder embedded in a porous material.

The primary aim is to examine the effect of nanoparticle volume fraction on the Nusselt number. A linear regression analysis will be performed to elucidate the correlation between nanoparticle volume fraction and Nusselt number. The Brinkman-viscoelastic nanofluid model will be simplified into more manageable equations before being solved with the Runge-Kutta Fehlberg Method. Data for regression will be gathered from the solutions, subsequently allowing for the calculation of the regression model.

2. Methodology

2.1 Governing Equations

Suppose that a Brinkman viscoelastic fluid is suspended with nanoparticles flowing towards a horizontal circular embedded in a porous medium, and mixed convection is examined. Figure 1 depicts the problem's physical model along with the coordinate system. Let the horizontal circular cylinder that is embedded in a porous medium has radius a. T_w and T_∞ are used to represent the constant temperature of the surface of the cylinder and the constant ambient respectively. The free stream velocity denoted by $\frac{1}{2}U_\infty$ is directed in the vertical upward direction relative to the cylinder, while the symbol g represents the acceleration of gravity. Given that the study is centered at the lower stagnation point, the variables $\overline{x}$ and $\overline{y}$ are assigned specific values. In particular, $\overline{x}$ is set to 0, while $\overline{y}$ is chosen to be perpendicular to the surface of the cylinder. The three governing equations for Brinkman-viscoelastic nanofluid flowing over a horizontal circular cylinder embedded with porous medium are as follows [9,14,15,16]

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{u}}{\partial \overline{y}} = 0 \tag{1}$$

$$\begin{aligned}\frac{\mu_{nf}}{K}\overline{u} = \frac{\mu_{nf}}{\phi}\frac{\partial^2 \overline{u}}{\partial \overline{y}^2} + \overline{u}_e\frac{\partial \overline{u}_e}{\partial \overline{x}} + k_0\left[\overline{u}\frac{\partial^3 \overline{u}}{\partial \overline{x}\partial \overline{y}^2} + \overline{v}\frac{\partial^3 \overline{u}}{\partial \overline{y}^3} - \frac{\partial \overline{u}}{\partial \overline{y}}\frac{\partial^2 \overline{u}}{\partial \overline{x}\partial \overline{y}} + \frac{\partial \overline{u}}{\partial \overline{x}}\frac{\partial^2 \overline{u}}{\partial \overline{y}^2}\right] \\ +\left[\varphi\rho_s\beta_s + (1-\varphi)\rho_f\beta_f\right]g\left(T - T_\infty\right)\sin\left(\overline{x}/a\right)\end{aligned} \tag{2}$$

$$\left(\rho C_p\right)_{nf}\left(\overline{u}\frac{\partial T}{\partial \overline{x}} + \overline{v}\frac{\partial T}{\partial \overline{y}}\right) = k_{nf}\left(\frac{\partial^2 T}{\partial \overline{x}^2} + \frac{\partial^2 T}{\partial \overline{y}^2}\right) \tag{3}$$

subject to the Constant Wall Temperature (CWT) boundary condition

$$\begin{aligned}&\overline{u} = 0,\;\; \overline{v} = 0,\;\; T = T_w \text{ on } \overline{y} = 0 \\ &\overline{u} \to \overline{u}_e,\; T \to T_\infty \text{ as } \overline{y} \to 0\end{aligned} \tag{4}$$

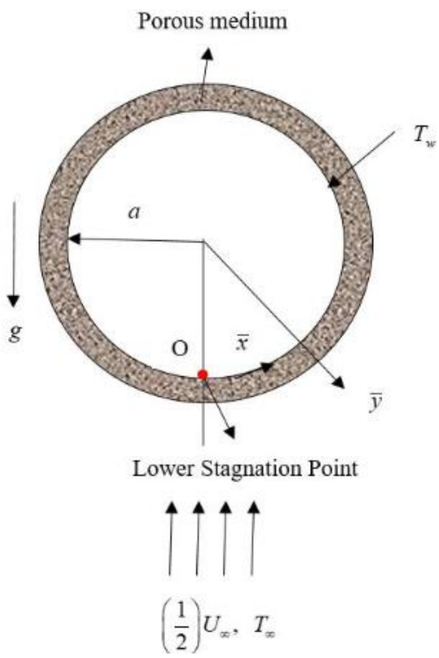

Fig. 1. The problem's coordinate system and model

Nanoparticle volume fraction in the equations is denoted as φ. The other parameters that represents the characteristics of nanofluids in the governing are μ_{nf}, $\left(\rho C_p\right)_{nf}$ and α_{nf} that represents dynamic viscosity, heat capacitance and thermal diffusivity of nanofluid respectively [16]. Eq. (1)-(3) represent the continuity, momentum, and energy equations of the study, respectively. Here T is the fluid temperature, $\bar{p}$ is the fluid pressure, μ_f is the viscosity of the fluid fraction, β_s and β_f are the thermal expansion coefficients of the solid and of the fluid, respectively, ρ_s is the density of the solid fraction and ρ_f is the density of the fluid fraction. The relations between nanofluid and conventional fluid are given by [17]

$$\begin{gathered}\mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}},\alpha_{nf}=\frac{k_{nf}}{\left(\rho C_p\right)_{nf}},\left(\rho C_p\right)_{nf}=(1-\varphi)\left(\rho C_p\right)_f+\varphi\left(\rho C_p\right)_s \\ \frac{k_{nf}}{k_f}=\frac{\left(k_s+2k_f\right)-2\varphi\left(k_f-k_s\right)}{\left(k_s+2k_f\right)+\varphi\left(k_f-k_s\right)}\end{gathered} \tag{5}$$

The variables in the equations are defined as follows: k_{nf} represents the nanofluid's effective thermal conductivity, k_f represents the fluid's thermal conductivity, k_s represents the solid's thermal conductivity, $\left(\rho C_p\right)_{nf}$ represents the nanofluid's heat capacity, $\left(\rho C_p\right)_f$ represents the fluid's heat capacity and $\left(\rho C_p\right)_s$ represents the solid's heat capacity [17]. Carboxymethyl Cellulose (CMC) is a viscoelastic fluid, and a study by al-Rashed assessed its application as the base fluid in a microchannel heat sink (MCHS), revealing that it possesses a perfect heat transfer ratio. Copper is a nanoparticle with excellent thermal conductivity, along with others such as Arabic Gum (Ag) and Aluminium (Al) [18]. The base fluid utilised in this study is Carboxymethyl Cellulose (CMC) water, and the nanoparticle employed is Copper (Cu).

In order to reduce the partial differential equations into simpler ordinary differential equations so that the equations can be solved, non-dimensional variables and similarity transformation variables are applied. The non-dimensional variables employed in this study are defined as [13]

$$\begin{gathered}x=\bar{x}/a, y=Pe^{1/2}\left(\bar{y}/a\right), u=\bar{u}/U_\infty,\ v=Pe^{1/2}\left(\bar{v}/U_\infty\right), \\ \theta=\left(T-T_\infty\right)/\left(T_w-T_\infty\right), u_e\left(\bar{x}\right)=\bar{u}_e\left(\bar{x}\right)/U_\infty, p=\left(\bar{p}-p_\infty\right)/\left(\rho_{nf}U_\infty^2\right)\end{gathered} \tag{6}$$

while the similarity transformation variables are given as [13]

$$\psi=xf\left(x,y\right),\ \ \theta=\theta\left(x,y\right), \tag{7}$$

with ψ represents the function of stream while θ represents the fluid's temperature and both are defined as $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$. After applying Eq. (6) and (7), Eq. (1)-(4) are simplified into Eq. (8)-(10).

$$\frac{f'}{(1-\varphi)^{2.5}} = \frac{\Gamma f'''}{(1-\varphi)^{2.5}} + k_1\left[2ff''' - ff^{(4)} - (f'')^2\right] + \left[(1-\varphi) + \varphi\left(\rho_s\beta_s / \rho_f\beta_f\right)\right]\lambda\theta\frac{\sin x}{x}$$
$$+\frac{\sin x}{x} + k_1 x\left[f'\frac{\partial f'''}{\partial x} - \frac{\partial f}{\partial x}f^{(4)} + \frac{\partial f'}{\partial x}f''' - f''\frac{\partial f''}{\partial x}\right] \tag{8}$$

$$\frac{k_{nf}}{k_f}\left[\frac{(\rho C_p)_f}{(1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s}\right]\theta'' + f\theta' = x\left[f'\frac{\partial\theta}{\partial x} - \frac{\partial f}{\partial x}\theta'\right] \tag{9}$$

$$\begin{aligned}&f(x,0)=0, \quad f'(x,0)=0, \quad \theta(x,0)=1 \quad \text{at } y=0,\\ &f'(x,\infty)\to\frac{\sin x}{x}, \quad f''(x,\infty)\to 0, \ \theta(x,\infty)\to 0 \quad \text{as } y\to\infty.\end{aligned} \tag{10}$$

When $x=0$ which is at the lower stagnation point, Eq. (8)-(10) are written to a simpler form of differential equations as shown below

$$\frac{f'}{(1-\varphi)^{2.5}} - \frac{\Gamma f'''}{(1-\varphi)^{2.5}} - k_1\left[2ff''' - ff^{(4)} - (f'')^2\right] - \left[(1-\varphi) + \varphi\left(\rho_s\beta_s / \rho_f\beta_f\right)\right]\lambda\theta - 1 = 0 \tag{11}$$

$$\frac{k_{nf}}{k_f}\left[\frac{(\rho C_p)_f}{(1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s}\right]\theta'' + f\theta' = 0 \tag{12}$$

with the boundary condition

$$\begin{aligned}&f(0)=0, \quad f'(0)=0, \quad \theta(0)=1\\ &f'(\infty)\to 1, \quad f''(\infty)\to 0, \quad \theta(\infty)\to 0\end{aligned} \tag{13}$$

The Nusselt number Nu_x is a dimensionless number that represents the convective heat transfer. It is defined as follows

$$Nu_x = \frac{aq_w}{k_f(T_w - T_\infty)} \tag{14}$$

where $q_w = -k_{nf}\dfrac{\partial T}{\partial y}$ [17]. After conducting non-dimensional and non-similarity transformations using Eq. (6) and (7), the Nusselt number Eq. (14) turns into

$$Nu_x = -\frac{k_{nf}}{k}Pe^{1/2}\frac{\partial\theta}{\partial y} \tag{15}$$

where Pe is Peclect number. The simplified Eq. (11)-(13) and Eq. (15) are then numerically solved using the Runge Kutta Fehlberg method, which has its built-in program in the Maple software. The solution will thereafter serve as the data for the regression analysis.

2.2 Regression Analysis

Regression models characterize the association between variables by fitting a line to the empirical data. Linear regression models utilize a straight line to estimate the variation of a dependent variable in response to changes in an independent variable [19]. The equation for a simple linear regression is

$$y = m_0 + m_1 x + e \tag{16}$$

In this context, x represents the independent variable, while y denotes the dependent variable. The constant of the regression line is denoted by m_0, m_1 signifies the slope of the regression line, and e represents the error of the regression. Linear regression identifies the optimal line that best fits the data by determining the regression coefficient m_1 that minimizes the error e [20]. In this study, the variable x represents the nanoparticle volume fraction, φ while y denotes the reduced Nusselt number, $Nu_x Pe^{-1/2}$.

To conduct linear regression, a dataset for the two variables is required. The data are derived from the numerical solution of simplified governing equations utilizing the Runge Kutta Fehlberg approach. To verify the presence of a linear relationship in the data, it is advisable to create a scatter plot with φ shown on the horizontal axis and $Nu_x Pe^{-1/2}$ on the vertical axis. The correlation coefficient is computed to assess the strength of the linear link between the two variables. Upon confirmation of the strength, the regression line can be established utilizing the least squares method with the assistance of *Excel* software. Upon obtaining the regression line, the r^2-value indicating the model's performance will be assessed. The assessment will involve determining the probable error of the correlation coefficient, which serves as an indicator of its accuracy and reliability. This can be determined using the following formula [21]

$$P.E(r) = 0.6745\frac{1-r^2}{\sqrt{n}} \tag{17}$$

where r^2 is the correlation coefficient and n is the number of data. The correlation coefficient is deemed reliable only when the value of r is approximately six times greater than the probable error [19]. Figure 2 below illustrates the process for the regression analysis.

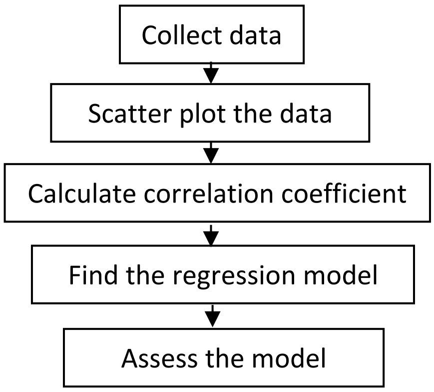

Fig. 2. Flow of linear regression

3. Results

Prior to doing the regression analysis, the value of $-\theta'(0)$ derived from the Runge Kutta Fehlberg technique is validated. The findings are juxtaposed with those of Kanafiah *et al.*, [13] and Nazar *et al.,* [14], as illustrated in Table 1. The table demonstrates substantial concordance in the results, so affirming the accuracy of the proposed calculation. As the solutions have been validated as accurate, fifty values of the reduced Nusselt number are extracted for fifty corresponding nanoparticle volume fractions. The numbers presented in Table 2 serve as the data for the regression analysis. Figure 3 illustrates the scatter plot of the data, indicating that an increase in nanoparticle volume fraction elevates the reduced Nusselt number, demonstrating a substantial linear correlation.

Table 1
Comparison for values of $-\theta'(0)$ with $\Gamma = 0.1$, $k_1 = 0$, $\varphi \to 0$ and several value of $\lambda = 1, 2, 3$.

	$-\theta'(0)$		
λ	[14]	[13]	Present
1	0.7791	0.7790	0.7790
2	0.8706	0.8705	0.8705
3	09460	0.9458	0.9458

Table 2
The value of reduced Nusselt number $Nu_x Pe^{-1/2}$ and nanoparticle volume fraction φ

φ	$Nu_x Pe^{-1/2}$	φ	$Nu_x Pe^{-1/2}$	φ	$Nu_x Pe^{-1/2}$	φ	$Nu_x Pe^{-1/2}$	φ	$Nu_x Pe^{-1/2}$
0	0.662348261	0.02	0.679898735	0.04	0.697283705	0.06	0.714517689	0.08	0.731614624
0.002	0.664111186	0.022	0.681644435	0.042	0.699013663	0.062	0.716233315	0.082	0.733317302
0.004	0.665872319	0.024	0.683388495	0.044	0.700742126	0.064	.717947595	0.084	0.735018759
0.006	0.667631676	0.026	0.685130929	0.046	0.702469108	0.066	0.719660532	0.086	0.736719008
0.008	0.669389273	0.028	0.686871754	0.048	0.704194624	0.068	0.721372139	0.088	0.73841806
0.01	0.671145124	0.03	0.688610982	0.05	0.705918686	0.07	0.723082431	0.09	0.740115931
0.012	0.672899246	0.032	0.69034863	0.052	0.707641309	0.072	0.72479142	0.092	0.741812632
0.014	0.674651653	0.034	0.69208472	0.054	0.7093622507	0.074	0.726499121	0.094	0.743508176
0.016	0.676402361	0.036	0.693819249	0.056	0.711082294	0.076	0.728205546	0.096	0.745202576
0.018	0.678151382	0.038	0.69555224	0.058	0.712800683	0.078	0.729910709	0.098	0.746895847

Regression analysis is performed via the *Data Analysis* tool in *Microsoft Excel*. The value of multiple R (r), correlation coefficient (r^2) and the probable error ($P.E(r)$) are presented in Table 3.

The correlation coefficient of 0.999969698 signifies that 99% of the entire variance in $Nu_x Pe^{-1/2}$ can be anticipated based on nanoparticle volume fraction φ. Further, from the values of m_0 and m_1 give the simple linear regression model as stated below

$$Nu_x Pe^{-1/2} = 0.662656435 + 0.862123356\varphi \tag{18}$$

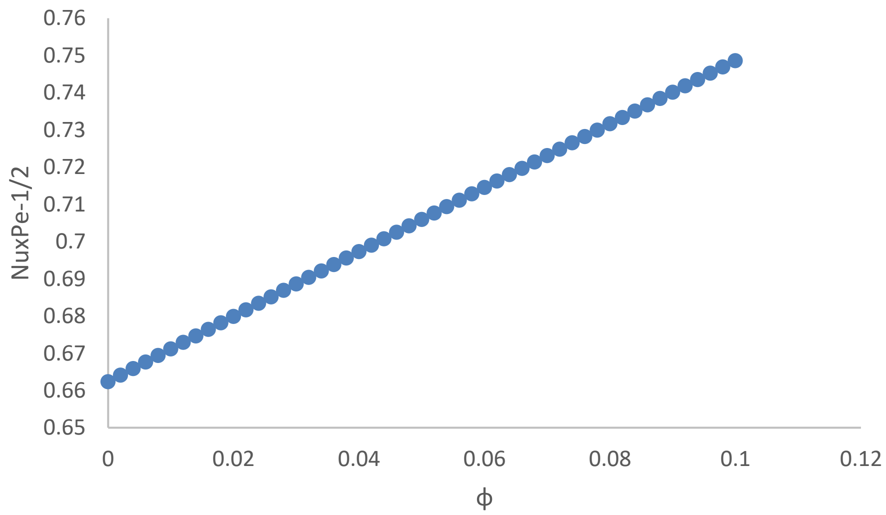

Fig. 3. $Nu_x Pe^{-1/2}$ vs φ

Table 3
Regression analysis results

r	r^2	m_0	m_1	$P.E(r)$	$r / P.E(r)$
0.99998	0.99997	0.662656435	0.862123356	0.00000143	698795.2408

Table 4 presents the estimated values of $Nu_x Pe^{-1/2}$ for various values of φ, revealing a consistent growing trend in the estimated values of $Nu_x Pe^{-1/2}$ as the nanoparticle volume fraction φ is augmented, accompanied by a minor residual. A robust linear correlation between the two variables is often assured when the value of $r / P.E(r)$ exceeds 6.

Table 4
The estimated and actual value of $Nu_x Pe^{-1/2}$

φ	Estimated $Nu_x Pe^{-1/2}$	Actual $Nu_x Pe^{-1/2}$	e
0	0.662656435	0.662348261	-0.00031
0.008	0.669553422	0.669389273	-0.00016
0.016	0.676450409	0.676402361	-0.000048
0.028	0.686795889	0.686871754	0.000076
0.036	0.693692876	0.693819249	0.000126
0.048	0.704038357	0.704194624	0.000156
0.056	0.710935343	0.711082294	0.000147
0.068	0.721280824	0.721372139	0.000091
0.076	0.728177811	0.728205546	0.000028
0.088	0.738523291	0.73841806	-0.00011

4. Conclusions

In conclusion, numerical analysis employing the Runge Kutta Fehlberg method is utilized to examine the impact of nanoparticle volume fraction on the reduced Nusselt number, revealing that an increase in nanoparticle volume fraction elevates the reduced Nusselt number. The regression analysis reinforces the findings, indicating that the correlation coefficient approaches 100%, demonstrating that the variance in the reduced Nusselt number could be explained by the nanoparticle volume fraction and the regression model is expressed as follows

$$Nu_x Pe^{-1/2} = 0.662656435 + 0.862123356\varphi$$

Acknowledgement
The authors sincerely acknowledge Universiti Malaysia Pahang Al-Sultan Abdullah for the financial support provided under research grant PGRS2303145. Appreciation is also extended to Universiti Teknologi MARA (UiTM) Kelantan and Terengganu Branch and the Federal College of Education, Nigeria, for their invaluable guidance and support.

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