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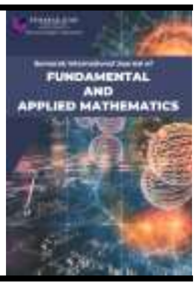

Convective and Permeable Surface Boundary Conditions on Stagnation-Point Flow over a Shrinking Sheet

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ABSTRACT

This paper aims to analyze the behavior of the stagnation-point flow and heat transfer over a convective and permeable shrinking sheet. The governing partial differential equations are converted into ordinary differential equations by similarity transformations before being solved numerically using the bvp4c function built-in MATLAB software. Results found that dual solutions exist for the shrinking parameter. Effect of suction/injection parameter on the skin friction and heat transfer coefficients as well as the velocity and temperature profiles are presented in tables and graphs. The analysis indicates that the skin friction coefficient and the local Nusselt number as well as the velocity and temperature were influenced by suction/injection parameter.

1. Introduction

The boundary layer flow over a shrinking sheet is a fluid dynamic problem that arises when a surface is contracting, pulling the fluid near it toward the surface. This scenario is commonly observed in industrial processes like material cooling, polymer processing, and in the study of heat transfer and fluid dynamics. Wang [1] was the first who pointed out the flow over a shrinking sheet while he was working on the flow of a liquid film over a stretching sheet. Later, Miklavčič and Wang [2] obtained the viscous flow induced by a shrinking sheet in the presence of suction. This research concludes that the solution is not unique at the certain rate of suction parameter, and the shrinking sheet offers a nonlinear fluid phenomenon. Tan *et al.*, [3] have investigated the mathematical modelling of boundary layer flow over a time-dependent shrinking sheet with permeable surface. They have concluded that the velocity of the flow increases as the suction parameter increases and the decreasing shrinking parameter. In addition, they have found that the triple solutions exist with two branches are linearly stable, while the third branch is linearly unstable and physically not realizable.

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Recently, the study of boundary layer flow over a shrinking sheet with the effects of radiation is examined by Amran and Ali [4]. They found that dual solutions exist for shrinking sheet. Other than viscous fluid, Othman *et al.,* [5] have studied the boundary layer flow towards stationary point flow over shrinking surface in nanofluid.

The analysis of stagnation-point flow in fluids is crucial for various engineering and industrial applications such as cooling, nuclear reactors, electronic and many hydrodynamics processes. The boundary layer of stagnation point flow is a critical concept in fluid mechanics, describing the thin layer of fluid near a solid surface where the velocity transitions from zero at the stagnation point to the free-stream velocity as the distance from the surface increases. The problem of stagnation flow towards a shrinking sheet was studied by Wang [6], and he concluded that the flow over a shrinking sheet is likely to exist; either an adequate suction on the boundary is imposed, or a stagnation flow is considered. Later, Bhattacharyya *et al.,* [7] analyzed the effects of partial slip on the steady boundary layer stagnation-point flow of an incompressible fluid and heat transfer towards a shrinking sheet. Lok *et al*., [8] studied the steady axisymmetric stagnation point flow of a viscous and incompressible fluid over a shrinking circular cylinder with mass transfer in the presence of suction. Next, the stagnation point flow over a permeable shrinking sheet with slip effects and suction case were discussed by Fauzi *et al.*, [9]. Their study summarized that the velocity slip and suction delay the boundary layer separation whereas the temperature slip does not affect the boundary layer separation. The stagnation point flow for the case of stretching/shrinking cylinder has been considered by Mat *et al.,* [10]. The study has found that dual solutions exist for the case of a shrinking cylinder, and the surface of the cylinder has increased the velocity of the flow and the heat transfer rate. Besides, the numerical solution for the MHD boundary layer flow and heat transfer past a shrinking case with suction is investigated by Jhankal and Kumar [11]. It was found that the velocity of the flow increases with the suction effect. Besides that, the investigation into boundary layer stagnation point flow also attracted many researchers in different fluid types and the impact of parameters on the flow as mentioned in the papers of Samat *et al*., [12], Yashkun *et al*., [13] and Japili *et al*., [14]

Building on the work of Aman *et al.*, [15], we extend their study by investigating steady, stagnation-point flow over a stretching/shrinking sheet in a viscous, incompressible fluid, incorporating the suction/injection and velocity slip effects. We analyze and discuss the impact of key parameters, specifically the suction/injection parameter and slip effects on the skin friction coefficient and the heat transfer rate at the surface.

2. Methodology

Consider a two-dimensional flow near stagnation-point on a convective and permeable shrinking sheet. It is assumed that the forms $u_e(x) = ax$ and $u_w(x) = bx$ are of the free stream and the stretching/shrinking velocities along the x-axis, where a and b is a positive constant. It is also assumed that the mass flux velocity, v_w with $v_w < 0$ for suction and $v_w > 0$ for injection. Below these boundary layer approximations, the governing equations is derived as follows (Aman *et al.,* [15])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where (u,v) are the component of velocity in the (x,y) axes, T is the fluid temperature in the boundary layer, ν is the kinematic viscosity, α is the thermal diffusivity. The flow field is defined by the following boundary conditions

$$\begin{aligned} &u=u_w+u_{slip},\quad v=v_w \quad \text{at} \quad y=0 \\ &u\to u_e \quad \text{as} \quad y\to\infty \end{aligned} \tag{4}$$

where $u_{slip}=L\dfrac{\partial u}{\partial y}$ is the slip velocity factor.

The bottom surface of the sheet is heated through convection from a hot fluid of temperature, T_f which provides a heat transfer coefficient, h_f. Under this assumption, referring Aman *et al.,* [15], the boundary conditions for the thermal field can be written as

$$\begin{aligned} &-k\frac{\partial T}{\partial y}=h_f\left(T_f-T_w\right) \quad \text{at} \quad y=0 \\ &T\to T_\infty \quad \text{as} \quad y\to\infty \end{aligned} \tag{5}$$

where k is the thermal conductivity and T_w is the uniform temperature over the top surface of the sheet. Here we have $T_f>T_w>T_\infty$.

Following Aman *et al.,* [13], the following similarity variables are used:

$$\eta=\left(\frac{u_e}{\nu x}\right)^{1/2}y,\;\; \psi=\left(\nu x u_e\right)^{1/2}f(\eta),\;\; \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty} \tag{6}$$

where η is the similarity variable, $f(\eta)$ is the dimensionless stream function and $\theta(\eta)$ is the dimensionless temperature, ψ is the stream function defined as usual $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$ which Eq. (1) is identically satisfied. By applying Eq. (6), we get

$$u=axf'(\eta) \text{ and } v=-(\nu a)^{1/2}f(\eta) \tag{7}$$

where primes denote differentiation with respect to η. Substituting Eq. (6) and Eq. (7) into Eq. (2) and Eq. (3), we obtain the following nonlinear ordinary differential equations:

$$f'''+ff''+1-f'^2=0 \tag{8}$$

$$\frac{1}{\Pr}\theta''+f\theta'=0 \tag{9}$$

subject to the boundary conditions

$$f(0)=\gamma, \quad f'(0)=\varepsilon+\delta f''(0), \quad \theta(0)=-\beta\left[1-\theta(0)\right]$$
$$f'(\eta)\to 1, \quad \theta(\eta)\to 0 \quad \text{as} \quad \eta\to\infty \tag{10}$$

where $\gamma=-v_w/(av)^{1/2}$ is the suction/injection parameter where $\gamma>0$ and $\gamma<0$ indicate the suction effect and the injection effect, respectively, $\varepsilon=b/a$ is the stretching/shrinking parameter, with $\varepsilon>0$ correspond to stretching and $\varepsilon<0$ for shrinking, $\delta=L(a/\nu)^{1/2}$ is the velocity slip parameter and $\beta=(\nu/a)^{1/2}\,h/k$ is the convective heat transfer parameter.

The primary quantities of interest in this study are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as (see Aman *et al.,* [15])

$$C_f=\frac{\tau_w}{\rho u_e^2/2}, \quad Nu_x=\frac{xq_w}{k\left(T_f-T_\infty\right)}. \tag{11}$$

Here, τ_w is the skin friction at the wall and q_w is the wall heat transfer which are given by

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{12}$$

with μ and k are the dynamic viscosity and the thermal conductivity, respectively. Substituting the Eq. (6) and Eq. (12) in Eq. (11), we get

$$\frac{1}{2}C_f\,\mathrm{Re}_x^{1/2}=f''(0), \quad Nu_x/\mathrm{Re}_x^{1/2}=-\theta'(0) \tag{13}$$

where $\mathrm{Re}_x=u_e x/\nu$ is the local Reynolds number.

3. Results

The ordinary differential equations (8) and (9), subject to the boundary conditions (10) have been solved numerically by means of boundary value problem solver bvp4c, a built-in MATLAB function for some values of governing parameters, namely the suction/injection parameter γ and the stretching/shrinking parameter ε while the velocity slip parameter δ, convective heat transfer parameter β, and the Prandtl number Pr were fixed at 1 for the sake of brevity.

Table 1 shows the comparison of critical values of the stretching/shrinking parameter, ε_c with those of previous studies (Wang [6], Bachok *et al.,* [16,17]) when the effect of suction/injection and convective boundary condition are neglected. The comparison shows an excellent agreement thus give confidence to the numerical results to be reported further. The values of the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ included in Table 2 for future reference for stretching and shrinking cases.

Table 1
Comparison values of the critical value ε_c when $\Pr = \beta = 1$ and $\gamma = \delta = 0$

Author(s)	ε_c
Wang [6]	1.24657
Bachok *et al.,* [16]	1.24657
Bachok *et al.,* [17]	1.24657
Present results	1.246581

Table 2
Values of the skin friction coefficient $f''(0)$ and $-\theta'(0)$ for different values of γ and ε when $\Pr = \delta = \beta = 1$

ε	γ	$f''(0)$	$-\theta'(0)$
	-0.1	-0.623798733	0.446728865
2	0	-0.63091738	0.464884815
	0.1	-0.63795954	0.482427254
	-0.1	0	0.423741313
1	0	0	0.443790768
	0.1	0	0.463121111
-1.5	-0.1	1.303123956	0.31255109
		(0.312672977)	(0.0040096)
	0	1.345538501	0.345742757
		(0.285463955)	(0.004481296)
	0.1	1.384932718	0.376757478
		(0.259284704)	(0.005207834)
-2	-0.1	1.409578419	0.249006229
		(0.841200239)	(0.063961598)
	0	1.496368723	0.296870693
		(0.767041873)	(0.05748092)
	0.1	1.567087213	0.337972811
		(0.707272549)	(0.054953665)

( ) second solution

Variations of the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ with stretching/shrinking parameter ε and suction/injection parameter γ are shown in Figures 1 and 2. It is seen that there are region of dual solutions for $\varepsilon_c < \varepsilon \le -1$, unique solutions for $\varepsilon > -1$ and no solutions for $\varepsilon < \varepsilon_c$ where ε_c is the critical value of ε. In Figures 1 and 2, the solid lines represent the first solution, while the dashed lines denote the second solution. Based on numerical computations, the critical values of ε_c 0.1 are $\varepsilon_c = \text{-}2.19192, \text{-}2.33013$ and $\text{-}2.4789$ for $\gamma = \text{-}0.1, 0$ and, as presented in Figures 1 and 2. As reported by previous studies, for example, Merkin [18], Weidman *et al.,* [19], and Harris *et al.,* [20], the first solution is stable and physically reliable, and the second solution is not. We expect this finding holds for the present numerical solutions.

Figure 1 indicates that the skin friction coefficient increases as γ increase. This could be because suction effect enhances surface shear stress, slows down the fluid flow, and consequently increase the surface velocity gradient, as consistent with the trend shown in Figure 3. Form Figure 1, it is evident that the values of $|\varepsilon_c|$ for which dual solutions exist increase as γ. This indicates that suction broadens the range of dual solutions for the similarity equations (8)-(10).

Figure 2 portrays variations of the local Nusselt number which represents the heat transfer rate with ε for some suction/injection parameter γ. As the suction/injection parameter increases, the local Nusselt number also increase. This phenomenon occurs because of suction effect has reduced the thermal boundary layer thickness and in turn decrease the temperature gradient at the surface, as consistent with temperature profile in Figure 4.

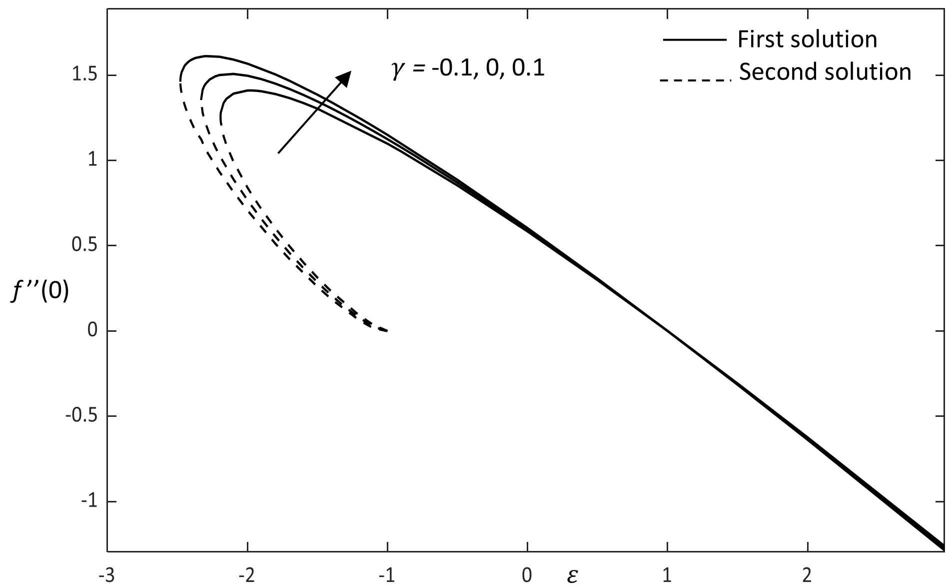

Fig. 1. Skin friction coefficient $f''(0)$ against ε for different values of γ when $\Pr = \delta = \beta = 1$

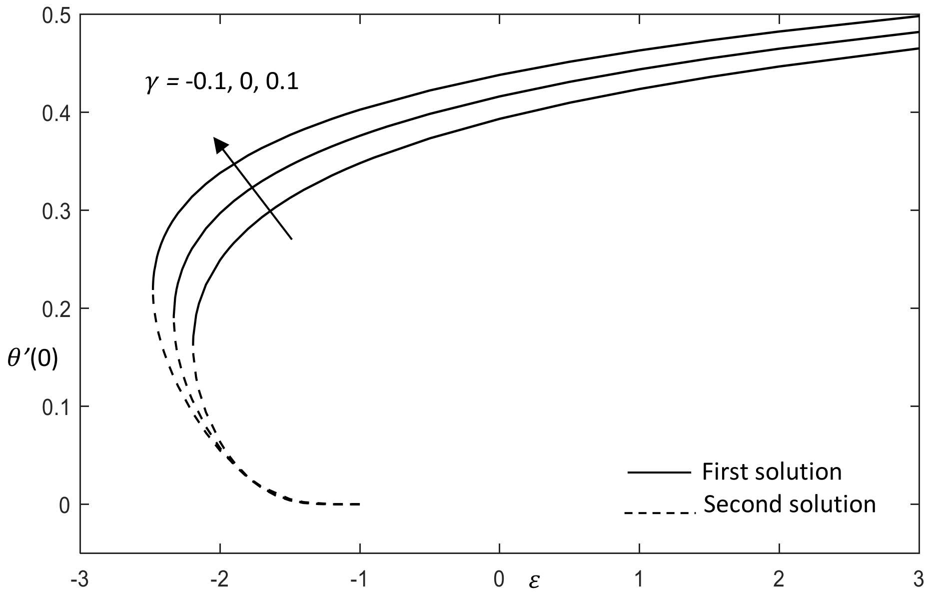

Fig. 2. Nusselt number $-\theta'(0)$ against ε for different values of γ when $\Pr = \delta = \beta = 1$

Figures 3 and 4 present the sample of velocity and temperature profiles for different values of the suction/injection parameter γ. It is clearly shown that all of these profile approach the far field boundary conditions (10) asymptotically, thus support the validity of the present results beside supporting the duality nature of solutions as presented in Figures 1 and 2. Figure 3 presents the velocity profiles $f'(\eta)$ for some values of the suction/injection parameter γ. For a stable solution, it is apparent that the velocity increases as the value of suction/injection parameter γ increase. This leads to reduction in the momentum boundary layer thickness, leading to a rise in flow velocity near the surface.

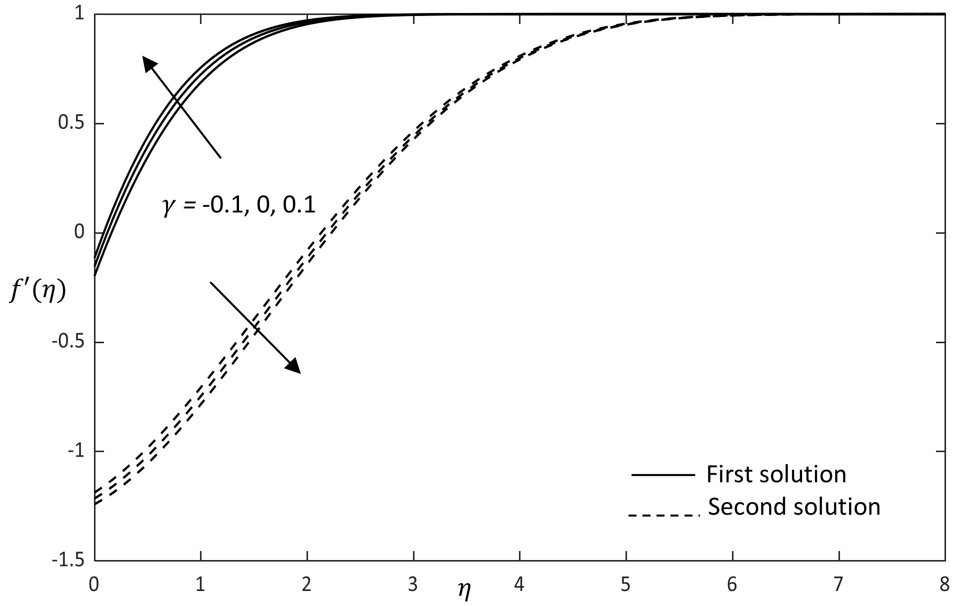

Fig. 3. Velocity profiles $f'(\eta)$ for different values of γ when $\mathrm{Pr} = \delta = \beta = 1$ and $\varepsilon = -1.5$ (shrinking case)

Figure 4 shows the temperature profiles $\theta(\eta)$ for some values of the suction/injection parameter γ. It is found that as γ increase, the fluid temperature within the boundary layer decrease and as consequences reducing the temperature gradient at the surface. As a results, the heat transfer rate at the surface $-\theta'(0)$ increases with an increase in suction/injection parameter γ, as verified by the data in Table 2.

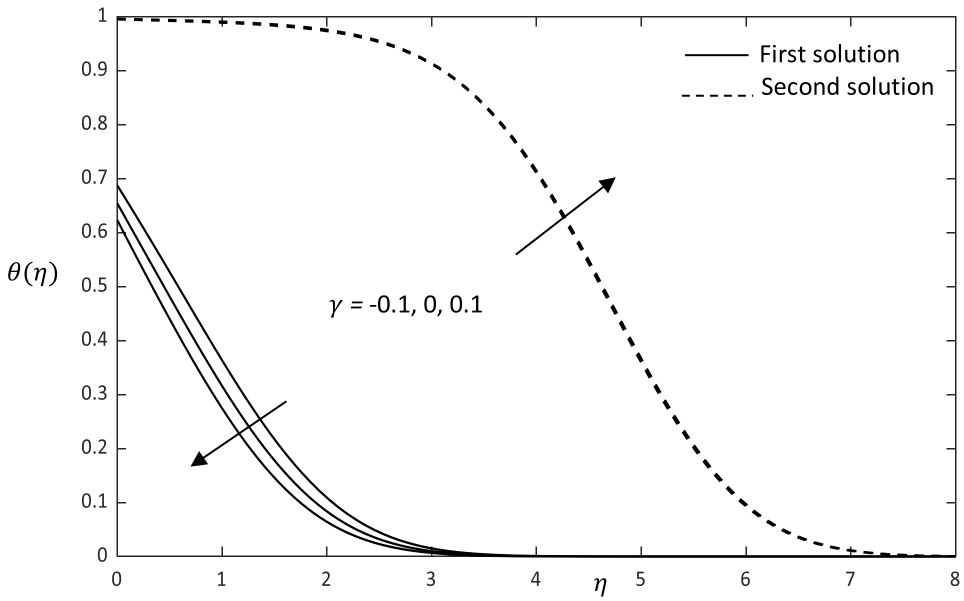

Fig. 4. Temperature profiles $\theta(\eta)$ for different values of γ when $\mathrm{Pr} = \delta = \beta = 1$ and $\varepsilon = -1.5$ (shrinking case)

4. Conclusions

In this paper, we considered the numerical solution of stagnation-point flow and heat transfer over a convective and permeable shrinking sheet and solved numerically using bvp4c function built in MATLAB software. The analysis shows that the skin friction coefficient and the local Nusselt number as well as the velocity and temperature were influenced by suction/injection parameter. It is observed that as the suction/injection parameter increases, the skin friction coefficient and the local Nusselt number also increase. Numerical results showed that dual solutions were found to exist for the shrinking sheet. Both velocity and temperature profiles obtained satisfied the far field boundary conditions asymptotically, supporting the validity of the present numerical results and the existence of the dual solutions.

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