Spheroidal First Order Polarization Tensor (SFOPT) Toolkit

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ARTICLE INFO	ABSTRACT
  	The polarization tensor (PT) has been utilized in numerous applications involving electric and electromagnetic fields, such as metal detection, landmine detection, and electrical imaging. In these applications, the PT is implemented in the process of identifying object, where the object can be represented in the form of the first order PT. Thus, it is crucial to have an application that provides easy access to calculate the first order PT for the object. However, the existing application in the literature has some limitations, focusing solely on computing the first order PT for a prolate spheroid with semi axes $a > b = c$ and an oblate spheroid with semi axes $a < b = c$. Therefore, the purpose of this study is to invent a graphical user interface (GUI) for the Spheroidal First Order Polarization Tensor (SFOPT) Toolkit which facilitates efficient computations and visualizations related to spheroids. The SFOPT Toolkit will be developed by using App Designer in MATLAB. The SFOPT interface integrates four essential functions: computation of the first order PT for a spheroid, classification of spheroidal types, three-dimensional visualization of spheroids, and determination of semi axes from the computed first order PT. Through illustrative examples, we demonstrate the effectiveness and versatility of the SFOPT framework, offering insights into its practical utility and potential applications in diverse fields. The reliability of the toolkit is also presented, and the findings shows the error of computations are small. The toolkit is a user-friendly application since the users can easily access it by downloading the application instead of running the coding themselves. Moreover, this toolkit can be a reference for researchers to compute the first order PT for a spheroid and determine the semi axes (size) of the spheroid from the given first order PT.

1. Introduction

The concept of the Polarization Tensor (PT) has been explored in numerous applications, particularly in electric and electromagnetic fields. These applications include biomedical engineering, particularly to enhance electrical imaging [1] and to locate the wireless capsule endoscopes [2]. Furthermore, PT is also utilized in metal detection for security purposes in locating and discriminating

the threat and non-threat objects [3-6]. Moreover, it is utilized in landmine detection for landmine clearance [7,8] and detecting small metallic objects buried in mineralized soil [9].

In these applications, PT plays a crucial role in the identification process of objects since it contains important information about the object's material, size, shape, and orientation, as mentioned by Ammari and Kang [1]. Therefore, researchers have made numerous attempts to derive the formulas of the PT for some objects [10,11]. Conversely, some researchers focus on implementing different approaches to numerically compute the PT since it cannot be computed analytically [12,13]. Meanwhile, some researchers focus on exploring the properties of the PT theoretically [14,15] and validating the theoretical findings by presenting some numerical examples [16].

The simplest form of the PT is called as the first order PT, where the formula of the first order PT is given by Ammari and Kang [1], particularly when the object is assumed to be an ellipsoid. This formula was later modified by Yunos and Khairuddin [17], where they incorporated depolarization factors into the first order PT. While it's acknowledged that the PT for many objects can be represented by the first order PT of an ellipsoid, as stated by Khairuddin *et al.*, [18], there is also a notable interest in exploring the first order PT for spheroids.

A spheroid is one of the types of ellipsoids and can be divided into two categories, which are prolate spheroid and oblate spheroid. The spheroid is represented as Eq. (1) in the Cartesian coordinate system with the semi axes denoted as a, b, and c:

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1. \tag{1}$$

The spheroid can be further classified as a prolate or an oblate spheroid, depending on its semi axes value. It will be classified as a prolate spheroid if the semi axes satisfy $a > b = c$, $b > a = c$, and $c > a = b$. Conversely, the spheroid will be classified as an oblate spheroid when the semi axes satisfy $a < b = c$, $b < a = c$, and $c < a = b$. Figure 1 illustrates a prolate spheroid with semi axes $c > a = b$ whereas Figure 2 demonstrates the shape of an oblate spheroid with semi axes $c < a = b$.
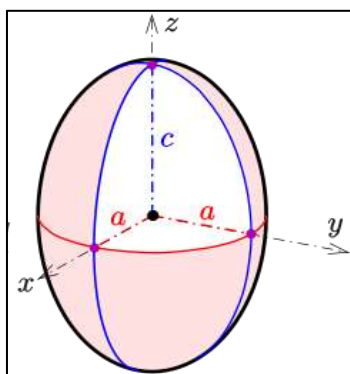
Fig. 1. Example of a prolate spheroid with $c > a = b$
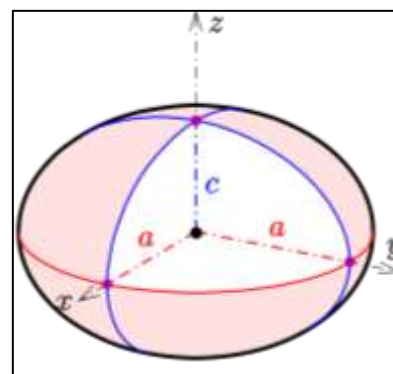
Fig. 2. Example of an oblate spheroid with $c < a = b$

The first order PT for a spheroid can be computed analytically using the formula proposed by Yunos and Khairuddin [17]. However, in order to determine the size of the spheroid, the calculations need to be computed semi-analytically as discussed by Khairuddin *et al.*, [19]. Eventually, it requires the researcher to type in the code based on the respective formula. Thus, an application named the 'Spheroidal First Order Polarization Tensor Calculator' was previously developed. It has two functions

which are to calculate the first order PT for a spheroid and to determine the size of a spheroid by inserting specific information about the spheroid.

However, there are some limitations to this application. It can only compute the first order PT for a prolate spheroid with semi axes $a > b = c$ or an oblate spheroid with $a < b = c$, as the algorithm is derived from Yunos and Khairuddin [17]. Additionally, the computation of the semi axes of the spheroid is also restricted to some cases only where the given first order must align with the study from Khairuddin *et al.*, [19].

Hence, the objective of this study is to improve the existing application by extending its capabilities to accommodate all types of prolate and oblate spheroids, rebranding it as the 'Spheroidal First Order Polarization Tensor (SFOPT) Toolkit'. This advanced toolkit is dedicated to performing computations of the first order PT for all types of spheroids. The toolkit incorporates four essential functions: calculating the first order PT for diverse spheroid types, classifying the specific type of spheroid, visualizing the spheroid through a three-dimensional graph and determining the spheroid based on the given first order PT.

2. Mathematical Formulation of the First Order Polarization Tensor

This section will review the formula of the first order PT since the toolkit requires some information to be inserted. The formula of the first order PT for spheroid is actually simplified from the explicit formula of the first order PT for ellipsoid given by Yunos and Khairuddin [17] where the formula is in terms of the depolarization factors.

If the object, B is an ellipsoid represented by Eq. (1), the first order PT with a conductivity k denoted by $M(k,B)$ is given by

$$M(k,B) = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix}, \qquad (2)$$

where $M_i = \frac{(k-1)|B|}{(1-d_i)+kd_i}$ for $i = 1,2,3$ with $|B|$ represented as the volume of B and d_i is the depolarization factors defined by Milton [20] by

$$d_1 = \frac{abc}{2} \int_0^\infty \frac{dy}{(y+a^2)^{3/2}\sqrt{(y+b^2)(y+c^2)}}, \qquad (3)$$

$$d_2 = \frac{abc}{2} \int_0^\infty \frac{dy}{(y+b^2)^{3/2}\sqrt{(y+a^2)(y+c^2)}}, \qquad (4)$$

$$d_3 = \frac{abc}{2} \int_0^\infty \frac{dy}{(y+c^2)^{3/2}\sqrt{(y+a^2)(y+b^2)}}, \qquad (5)$$

However, according to Stoner [21] and Mohamad Yunos *et al.*, [22], the depolarization factors in Eq. (3) can be further simplified and stated as Proposition 1.

Proposition 1 Let a, b and c be the semi principal axes of an ellipsoid and d_1 is one of the depolarization factors for the ellipsoid. Thus,

    i.   If $a > b = c$, then $d_1 = \frac{1-\psi^2}{\psi^2}\left(\frac{1}{2\psi}ln\left(\frac{1+\psi}{1-\psi}\right) - 1\right)$, where $\psi = \sqrt{1 - \left(\frac{b}{a}\right)^2}$.

ii.   If $a < b = c$, then $d_1 = \frac{1}{\varphi^2}\left(1 + \frac{\sqrt{1-\varphi^2}}{\varphi}\sin^{-1}\varphi\right)$, where $\varphi = \sqrt{1 - \left(\frac{a}{b}\right)^2}$.

In addition, another property of depolarization factors is given in Proposition 2.

Proposition 2 The depolarization factors d_1, d_2 and d_3 satisfy $d_1 + d_2 + d_3 = 1$.

Therefore, as the spheroid comprises two identical axes, and considering both Proposition 1 and Proposition 2, Yunos and Khairuddin [17] suggested that Eq. (2) can be reduced to
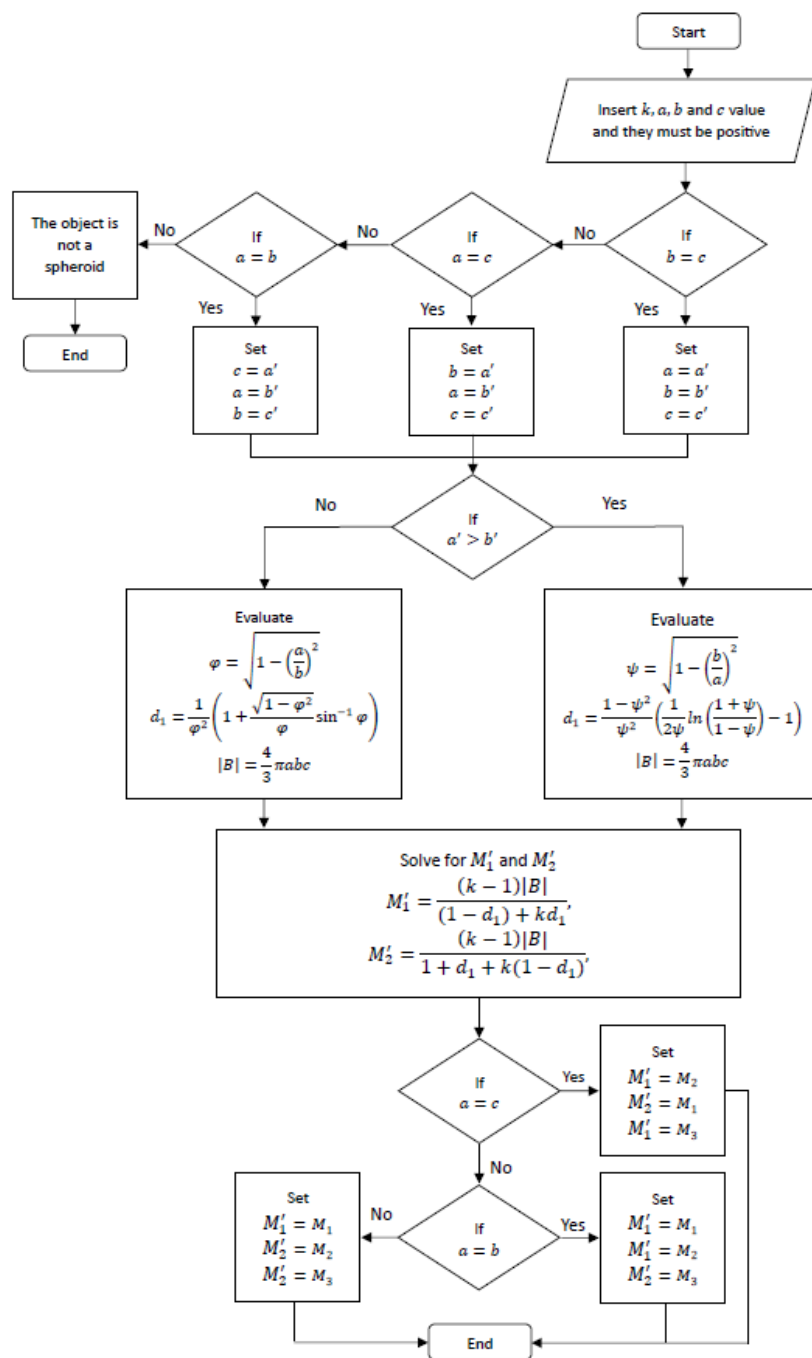
$$M(k,B) = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_2 \end{bmatrix}, \tag{6}$$

Where

$$M_1 = \frac{(k-1)|B|}{(1-d_1)+kd_1}, \tag{7}$$

$$M_2 = \frac{(k-1)|B|}{1+d_1+k(1-d_1)}. \tag{8}$$

Hence, the $M(k,B)$ for spheroid with semi axes $a > b = c$ and $a < b = c$ can be calculated by using Eq. (6), Eq. (7) and Eq. (8) with appropriate d_1 from Proposition 1. However, the $M(k,B)$ for other type of prolate and oblate spheroid, it can be obtained by implementing rotation towards the spheroid. Figure 3 illustrates the algorithm of computing the $M(k,B)$ for spheroid at a fixed k.

Start

Insert k, a, b and c value and they must be positive

The object is not a spheroid

No — If $a = b$

No — If $a = c$

No — If $b = c$

End

Yes

Set
$c = a'$
$a = b'$
$b = c'$

Yes

Set
$b = a'$
$a = b'$
$c = c'$

Yes

Set
$a = a'$
$b = b'$
$c = c'$

No — If $a' > b'$ — Yes

Evaluate

$$\varphi = \sqrt{1 - \left(\frac{a}{b}\right)^2}$$

$$d_1 = \frac{1}{\varphi^2}\left(1 + \frac{\sqrt{1-\varphi^2}}{\varphi}\sin^{-1}\varphi\right)$$

$$|B| = \frac{4}{3}\pi abc$$

Evaluate

$$\psi = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

$$d_1 = \frac{1-\psi^2}{\psi^2}\left(\frac{1}{2\psi}ln\left(\frac{1+\psi}{1-\psi}\right) - 1\right)$$

$$|B| = \frac{4}{3}\pi abc$$

Solve for M_1' and M_2'

$$M_1' = \frac{(k-1)|B|}{(1-d_1) + kd_1},$$

$$M_2' = \frac{(k-1)|B|}{1 + d_1 + k(1-d_1)},$$

If $a = c$ — Yes

Set
$M_1' = M_2$
$M_2' = M_1$
$M_1' = M_3$

No

Set
$M_1' = M_1$
$M_2' = M_2$
$M_2' = M_3$

No — If $a = b$ — Yes

Set
$M_1' = M_1$
$M_1' = M_2$
$M_2' = M_3$

End

Fig. 3. The flowchart of calculating the first order PT of the spheroid with a fixed conductivity

Next, to calculate the semi axes of the spheroid, the algorithm is taken from Khairuddin *et al.*, [19] and demonstrated as Figure 4.
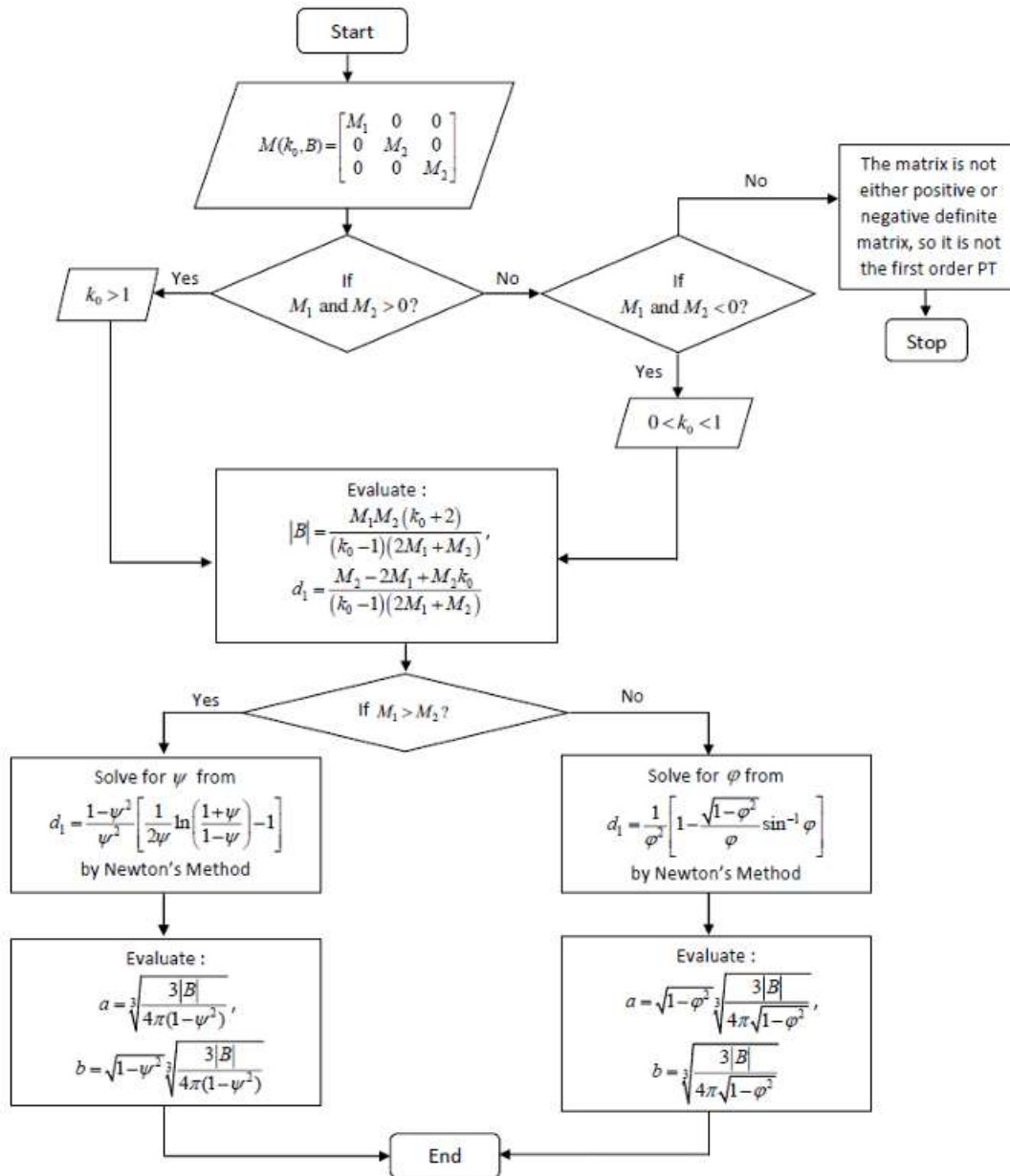
Fig. 4. The flowchart of calculating the semi axes of the spheroid with a fixed conductivity by Khairuddin *et al.,* [19]

Figure 4 explains the flow of calculating the semi axes of a spheroid from the given $M(k, B)$ but the given $M(k, B)$ is restricted in the form of Eq. (6) only. Thus, Muminov *et al.,* [23] have extended the findings to the case of obtaining a prolate spheroid with semi axes $b > a = c$ and $c > a = b$ together with an oblate spheroid with semi axes $b < a = c$ and $c < a = b$ by implementing rotation toward the given $M(k, B)$ and the obtained semi axes.

Hence, SFOPT Toolkit will implement the algorithm as demonstrated in Figure 3 to compute $M(k, B)$ for spheroid. On the other hand, the flow chart in Figure 4 is employed, and the technique of rotating the given $M(k, B)$ from Muminov *et al.,* [23] is also utilized to compute the semi axes of the spheroid. This applies specifically to cases where the semi axes are in the form of $b > a = c, c > a = b, b < a = c$ and $c < a = b$.

3. Methodology

3.1 Generalization of graphical user interface (GUI)

MATLAB stands out as a preeminent mathematical software widely embraced for its capabilities in data analysis, data visualization, algorithm development, and numerical calculations. It comes equipped with a variety of functions specifically designed for working with matrices. This not only makes programming more straightforward but also helps overcome different computational difficulties. To make the computation of the first order PT for spheroids faster, this study created a graphical user interface (GUI) using MATLAB App Designer. The layout of the toolkit was created using the set of interactive user interface (UI) components available in MATLAB App Designer (version R2023a). The operational setting involves an external environment, and the application's code is scripted using an external editor seamlessly integrated with a layout view. The interface page includes two views, which are Design and Code views. These view pages can be accessed through a button on the design page. In the design view, users can structure the application layout, while in the code view, coding is required for each tool to specify the type and characteristics of the data to be incorporated into the code.

The first step of creating Spheroidal First Order Polarization Tensor Toolkit is to design the GUI and layout at the design view. When designing the layout, visual components are incorporated using the drag-and-drop approach. The layout was designed according to the objectives of this toolkit. After that, the code to run the function was programmed in the code view. The codes involve the computation of the $M(k, B)$ for spheroid where the algorithm is illustrated as Figure 3, whereas the algorithm of determining the semi axes of the spheroid from the given $M(k, B)$ was referred to Figure 4 and the findings in Muminov *et al.*, [23]. On the other hand, the classification of the type of spheroid was added and the visualization of the spheroid was given as well. Figure 5 presents the whole process of creating SFOPT Toolkit application.
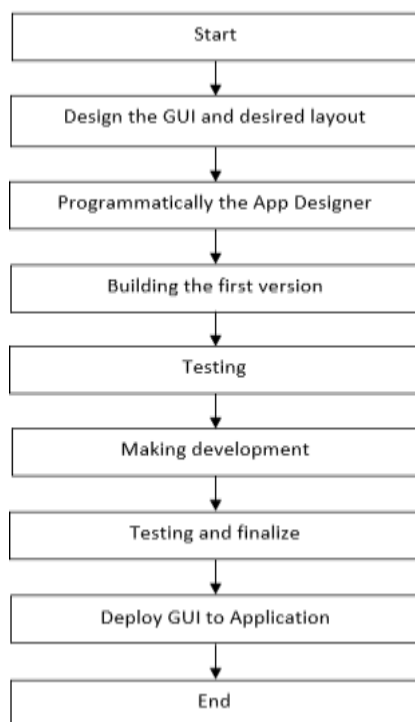
Fig. 5. The process of creating Spheroidal First Order Polarization Tensor (SFOPT) Toolkit application

3.2 Input Information

According to the algorithm given in Figure 3, it is possible to compute the $M(k, B)$ of the spheroid when the conductivity of the spheroid denoted as k and the semi axes represented as a, b, and c are known. Hence, the input information that need to be inserted by the users are k, a, b, and c. Users are required to insert numeric values for k, a, b, and c. For instance, if $k = 10^3$, the users should insert 10000 for k. The same rule applies to a, b, and c values. It's essential to note that the unit for a, b, and c in this toolkit should be in meters.

In addition, the inputs require to classify the type of spheroid and demonstrate the spheroid in three dimensions are semi axes a, b, and c. Next, in order to compute the spheroid from the given $M(k, B)$, the required information is the conductivity of the spheroid, k, and the given $M(k, B)$, which are the diagonal values of the $M(k, B)$, denoted as M_1, M_2, and M_3.

4. Results

In this section, we present the graphical user interface (GUI) for the SFOPT Toolkit, designed to facilitate efficient computations and visualizations related to spheroids.

4.1 Layout of Spheroidal First Order Polarization Tensor Toolkit

The GUI generally consists of four functions: (1) to calculate the first order PT of a spheroid; (2) to classify the type of spheroid; (3) to visualise the spheroid in three-dimension graph; and (4) to determine the semi axes of the spheroid from the given first order PT. The code that forms the underlying of GUI is named as 'SFOPT Toolkit'. This code is created to produce the graphical controls of the GUI at specific locations and includes various callbacks. These callbacks essentially contain all the other functions associated with the GUI controls.

Figure 6 demonstrates the overall layout of Spheroidal First Order Polarization Tensor (SFOPT) Toolkit in MATLAB App Designer. The layout started from the descriptions about the toolkit at the top. Then, each function was separated using the panel for better interface and easy to understand by users. The detail for each function will be explained in the next subsection.
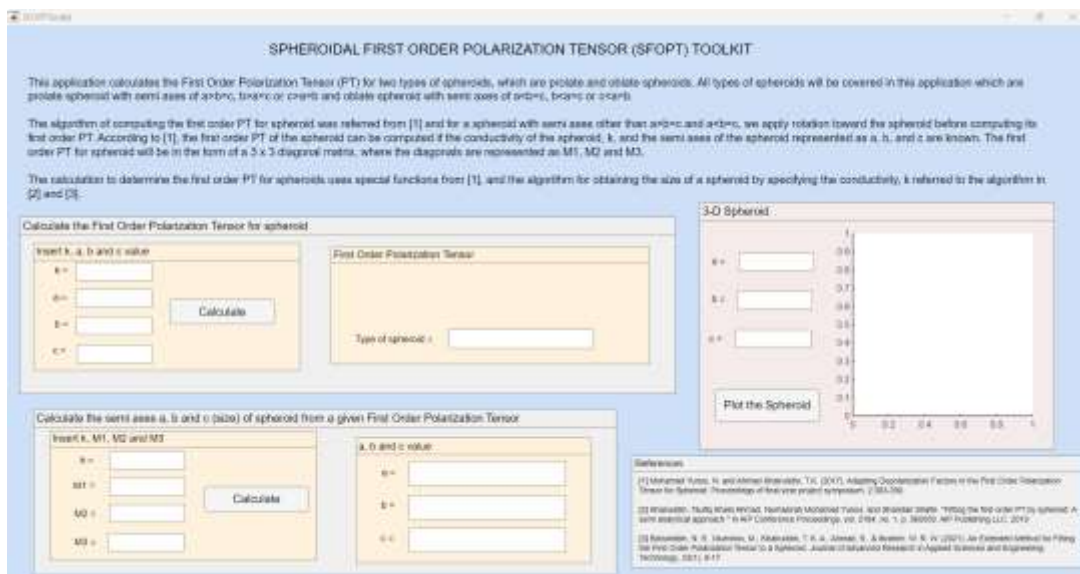

Fig. 6. The overall layout of Spheroidal First Order Polarization Tensor (SFOPT) Toolkit in the MATLAB App Designer

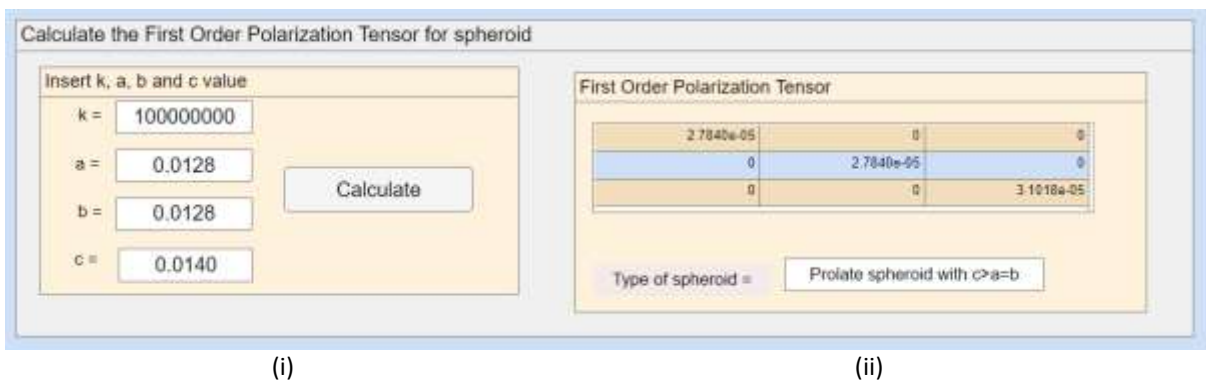## 4.2 Calculate the first order PT for a spheroid

The first function of this toolkit is to compute the first order PT for a spheroid. The underlying code is implemented based on the algorithm in Figure 3. Figure 7 illustrates the interface for calculating the first order PT for a spheroid before execution. As depicted in Figure 7, there are two panels with the headings 'Insert $k, a, b,$ and c value' and 'First Order Polarization Tensor.' To compute the first order PT, the users need to insert the conductivity (k) and the semi axes values ($a, b,$ and c) in the provided edit box. Subsequently, the users should click the 'Calculate' push button. This button executes the coded calculation when clicked.

(i)   (ii)
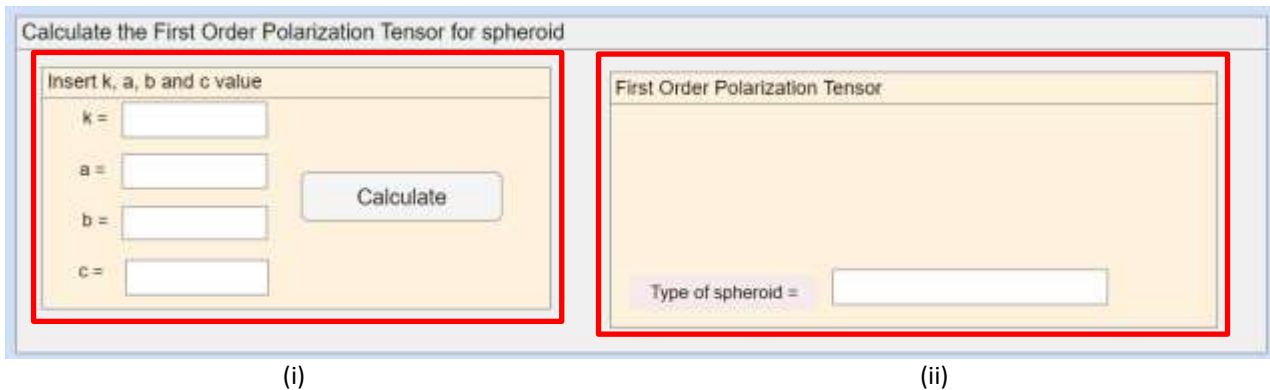
Fig. 7. The layout of calculating the first order PT for a spheroid before execution is as follows: (i) Users are required to insert the necessary information. (ii) The output is displayed after all the required information in (i) has been filled in

After clicking the push button, the output comprising the first order PT of the spheroid will promptly appear in the panel titled 'First Order Polarization Tensor.' Additionally, the spheroid will be classified as either a prolate or oblate spheroid based on the inserted values.

In order to assess the reliability of this toolkit, the study conducted by Khairuddin *et al.*, [24] was used as an example. Let's consider the values $k = 100000000, a = 0.0128, b = 0.0128,$ and $c = 0.014$. Figure 8 illustrates the interface for calculating the first order PT for a spheroid.

(i)   (ii)

Fig. 8. The layout of calculating the first order PT for a spheroid after execution is as follows: (i) The values of $k = 100000000, a = 0.0128, b = 0.0128,$ and $c = 0.014$ are the input. (ii) The resulting first order PT is presented in the form of a 3×3 matrix in a table. Additionally, the type of spheroid will be classified

Upon clicking the 'Calculate' button, the output of the computation of the first order PT is displayed, as shown in Figure 8 (ii). It's important to note that the first order PT is presented in the form of a 3×3 matrix, thus, the way of presenting the result is stated as Eq. (9) as follows,

$$M = 10^{-5} \begin{bmatrix} 2.7840 & 0 & 0 \\ 0 & 2.7840 & 0 \\ 0 & 0 & 3.1018 \end{bmatrix}. \tag{9}$$

Next, Eq. (9) is further verified by calculating the matrix norm to find the difference between the computed first order PT and the given first order PT from Khairuddin *et al.*, [24]. For this purpose, let M be the given first order PT and $\widehat{M}$ denotes the computed first order PT for the spheroid, then,

$$M - \widehat{M} = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix} \tag{10}$$

is calculated to compute the error, e to be the matrix norm,

$$e = \sqrt{m_{11}^2 + m_{22}^2 + m_{33}^2}. \tag{11}$$

Thus, by considering the M from Khairuddin *et al.*, [24] and $\widehat{M}$ from Eq. (9), utilizing Eq. (10) and Eq. (11), it is found that,

$$e = 1.6201 \times 10^{-7}. \tag{12}$$

It can be described that this toolkit is reliable for computation of the first order PT for spheroid since the e calculated in Eq. (12) is small and equal to 0 at 6 decimal places which indicates that the computed first order PT is almost similar to the given first order PT from Khairuddin *et al.*, [24].

In addition, based on $a = 0.0128$, $b = 0.0128$, and $c = 0.014$, the toolkit has classified the spheroid as a prolate spheroid with $c > a = b$.
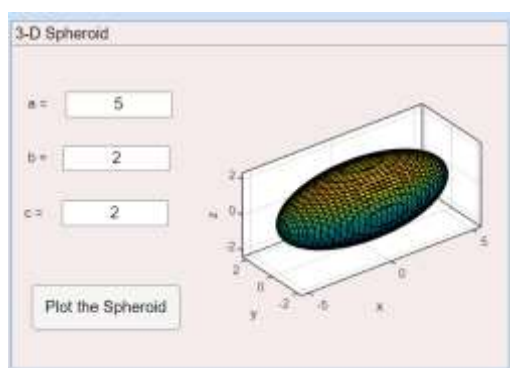
4.3 Plot Three-Dimensional Spheroid

The second function of this toolkit is to visualize the spheroid in three dimensions. Figure 9 illustrates the interface for plotting the spheroid in three dimensions. The required information to plot the spheroid includes the semi axes values (a, b, and c). Once a, b, and c value is inserted, the users need to click the 'Plot the Spheroid' button to visualize the spheroid. The spheroid will appear at the axes.
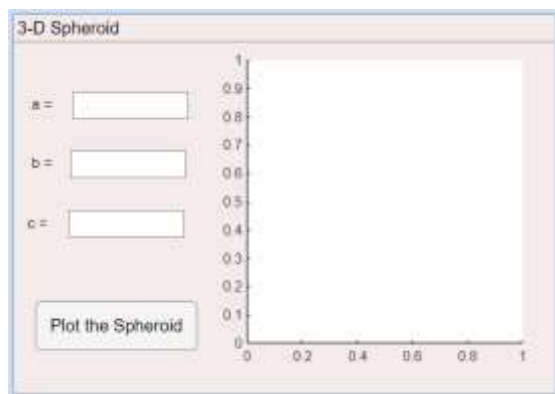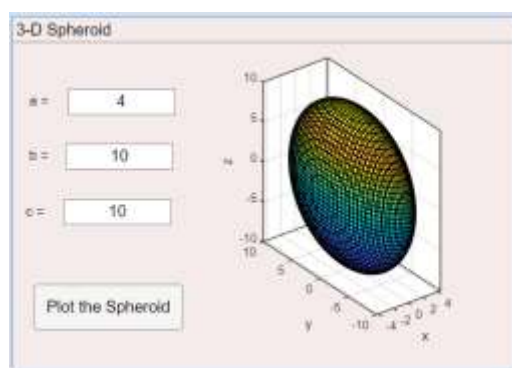
Fig. 9. The layout of plotting the spheroid in three-dimension

Two examples, one for each prolate and oblate spheroid, are provided to demonstrate the plotting function. Figure 10 illustrates a prolate spheroid in three dimensions with values $a = 5$, $b = 2$ and $c = 2$. Meanwhile, Figure 11 presents an oblate spheroid in three dimensions with values $a = 4$, $b = 10$ and $c = 10$. This function assists the users in visualizing the shape of the spheroid without the need to write the code.

**Fig. 10.** The layout of plotting the spheroid in three-dimension when $a = 5$, $b = 2$ and $c = 2$

Fig. 11. The layout of plotting the spheroid in three-dimension when $a = 4$, $b = 10$ and $c = 10$

4.4 Compute the Spheroid from Its First Order PT

The last function of this toolkit is to compute the size a spheroid based on the given first order PT. Here, the semi axes represent the size of the spheroid. Thus, SFOPT Toolkit will provide a faster way of obtaining the semi axes of the spheroid by following the algorithm in Figure 4 and as explained by Muminov *et al.*, [23]. For these purposes, the diagonals of the given first order PT must be either $M_1 = M_2$, $M_1 = M_3$ or $M_2 = M_3$. Figure 12 presents the interface of calculating the semi axes of a spheroid.

As shown in Figure 12, there are two panels with the headings 'Insert k, M_1, M_2, and M_3' and 'a, b, and c value'. Here, the users need to insert the value of k, M_1, M_2, and M_3. After clicking the push button, the output comprising the semi axes of the spheroid will promptly appear in the panel titled 'a, b, and c value'.

(i)   (ii)
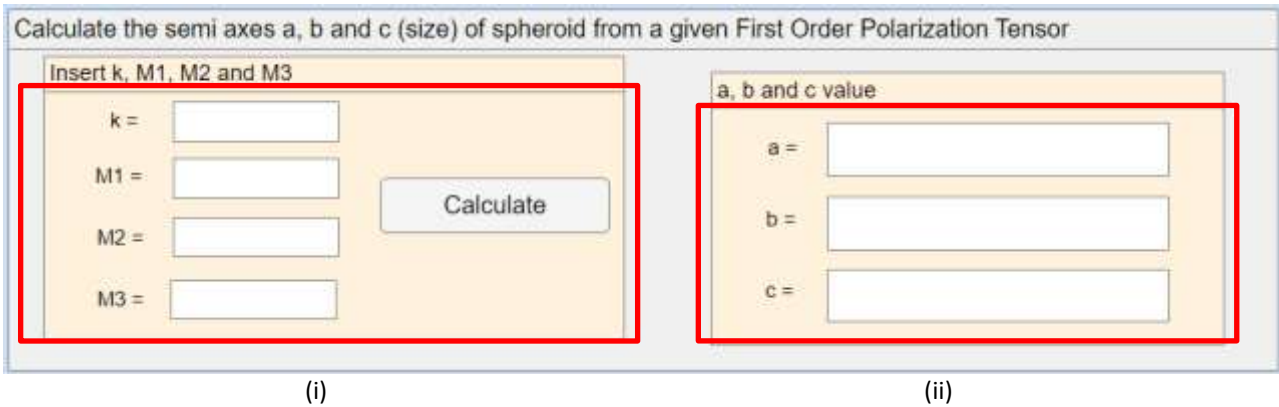
Fig. 12. The layout of calculating the semi axes of a spheroid from the given first order PT is as follows: (i) Users need to insert some information such as the conductivity of the spheroid, k, as well as the diagonals of the given first order PT represented by M_1, M_2, and M_3. (ii) The output is displayed after all the required information in (i) has been filled in

In order to assess the reliability of this toolkit, the study conducted by Muminov *et al.*, [23] was used as an example. Given $k = 500$, $M_1 = 12.85$, $M_2 = 12.85$, and $M_{33} = 8.51$. Figure 13 demonstrates the interface of calculating the semi axes of the spheroid.

(i)   (ii)
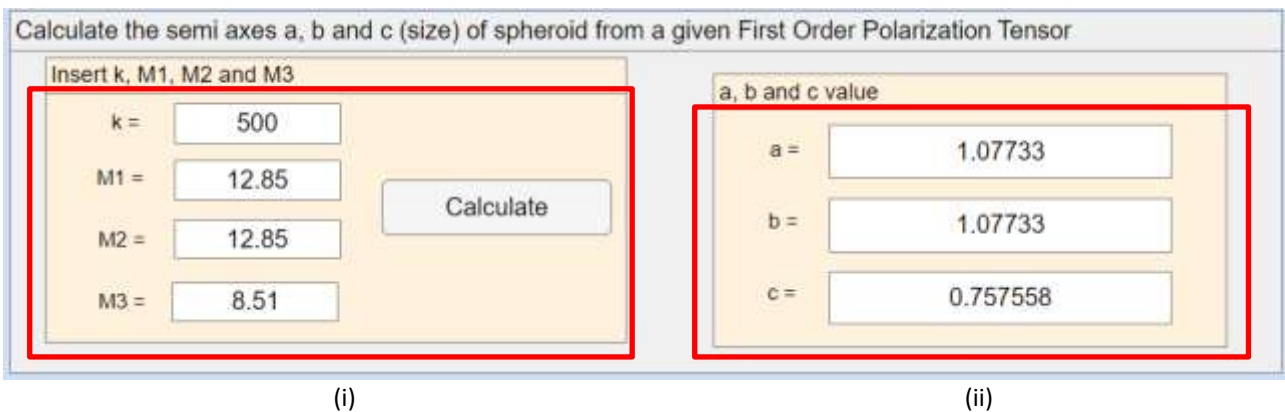
Fig. 13. The layout of calculating the semi axes of a spheroid from the given first order PT is as follows: (i) Users need to insert some information, such as the conductivity of the spheroid, k, as well as the diagonals of the given first order PT represented by M_1, M_2, and M_3. (ii) The output is displayed after all the required information in (i) has been filled in

Upon clicking the 'Calculate' button, the result of the semi axes of the spheroid will promptly appear in (ii). The obtained value for a, b, and c was then checked first in the process of testing this toolkit by computing the first order PT using Eq. (2) for the obtained value for a, b, and c together with k. After that, Eq. (10) and Eq. (11) are used to calculate the e.

Thus, from the calculation, it is found that,

$$e = 1.4142 \times 10^{-4}. \tag{13}$$

From Eq. (13), the e is small and if it is rounded to three decimal places, the e will be 0. Hence, this indicates that this toolkit is a reliable application of obtaining the semi axes of the spheroid.

5. Conclusions

In conclusion, this research focuses on providing a quicker way of computing the first order PT for spheroid, classifying the type of spheroid, obtaining the semi axes of the spheroid and visualising the spheroid in three dimensions. The toolkit is a user-friendly application accessible to researchers in related fields. Additionally, this application will assist researchers in saving time by eliminating the need to write code themselves.

Acknowledgement

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