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Parameter Estimation Methods for Bivariate Copula in Financial Application

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ABSTRACT

In the field of finance, understanding the dependencies between financial variables is crucial for effective risk management and decision-making. Copulas are powerful tools used to model these dependencies, enabling more accurate financial predictions and assessments. However, accurate estimation of copula parameters remains a significant challenge, impacting the reliability of the dependency structures modeled. This study addresses this problem by evaluating the effectiveness of four parameter estimation methods: maximum likelihood estimation, pseudo maximum likelihood, inversion of the rank correlation coefficient based on Kendall's tau and Spearman's rho. The objective is to determine which method provides the most accurate estimates for financial applications. The methodology involves a comparative analysis using exchange rate data for the Malaysian Ringgit against the U.S. Dollar and the Euro, covering the period from 1999 to 2023, sourced from Bank Negara Malaysia. Various copula families, including Gumbel, Clayton, Frank, Gaussian, and Student's t, are employed to model the dependencies between these exchange rates. The study involves transforming the marginals of the sample data into uniform variates, estimating copula parameters, and evaluating the models' performance using the Root Mean Square Error (RMSE) as a measure of goodness-of-fit. The principal results indicate that the Frank copula, exhibits the lowest RMSE, demonstrating superior performance in capturing the dependency structure of the data. This finding highlights the suitability of the Frank copula for financial risk management and decision-making. In conclusion, this analysis enhances the understanding of copula parameter estimation in finance, providing insights into the effectiveness of different methods and their practical applications in modeling financial dependencies.

1. Introduction

Copulas, they are essentially functions that couple multivariate distribution functions to their one-dimensional margins. It is a powerful tool to model and understand the dependencies between random variables. Sklar theorem (1959) states that, any multivariate joint distribution can be

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expressed in terms of its marginals and a copula that represents the dependency structure. The models have become very popular within the scientific community, making it an essential tool for many researchers and practitioners and this is due to the idea of dividing the specification of multivariate models into two parts, which are the marginal distributions and the dependence structure (copula).

Bivariate copula models are a very useful tool in the field of finance, particularly for risk analysis and risk management. These models are important in capturing and representing complex dependencies between financial variables, which are essential for accurate modelling and decision-making. Since copula enables the separation of marginal behaviors from the dependency structure, it provides a more nuanced understanding of correlations and tail dependencies in data [1].

In practical finance applications such as asset pricing, portfolio optimization, and risk management, the precise estimation of copula parameters is crucial. These parameters directly influence the effectiveness of risk measures like Value at Risk (VaR) and Expected Shortfall (ES), which are highly sensitive to the dependency structures among financial assets [2]. Accurate parameter estimation can significantly enhance the reliability of these measures, thereby improving financial decision-making processes.

This study focuses on two parameter estimation methods for Archimedean copulas MLE which is a parametric estimation method and PML which is a semi parametric estimation method. MLE is a widely used parametric method that maximizes the likelihood function under the assumption that the data samples are derived from a population that fits the model parameters perfectly. This method is known for its asymptotic efficiency and consistency under regular conditions, making it a preferred choice for parameter estimation in many statistical applications, including copulas [3].

On the other hand, PML offers a more flexible approach by relaxing some of the stringent assumptions required by traditional MLE. This method can be particularly advantageous in situations where the true distribution is unknown or difficult to specify. PML is robust in handling deviations from typical assumptions, making it suitable for complex financial models characterized by asymmetries or heavy tails [4,5].

The primary objective of this study is to evaluate and compare the performance of MLE and PML in estimating copula parameters for financial data. We assess the GOF of the models using the RMSE as the evaluation metric. By identifying the most effective estimation method, this research seeks to provide valuable insights for financial decision-making.

The findings of this study are expected to contribute to the existing knowledge on copula models in finance, particularly in the context of parameter estimation methods. By offering a comparative analysis of MLE and PML, we aim to highlight each method and thereby guide practitioners in selecting the most appropriate approach for their specific modelling needs.

2. Literature Review

2.1 Archimedean Copula

Archimedean copulas are characterized by their simple mathematical structure and ease of use [6]. Archimedean copulas can model asymmetric dependencies and are particularly useful when dealing with non-linear relationships between variables. Common types of Archimedean copulas include Clayton, Gumbel, and Frank copulas and each of them captures different types of dependency structures [7].

2.1.1 Clayton copula

Clayton copula is a member of the Archimedean family of copulas, widely used in finance for modelling dependencies between variables, particularly in scenarios involving asymmetric tail dependence. The Clayton copula is known for its ability to capture lower tail dependence, which means it effectively models situations where extreme low values in one variable are associated with extreme low values in another variable [8,9]. This property makes it particularly useful in credit risk modelling, where joint occurrences of extreme losses need to be accurately represented.

A study by Al-babtain *et al.,* [10] explored Clayton copula for real data applications. Their research demonstrated the flexibility and potential of Clayton copulas in modelling skewed and symmetric data sets, emphasizing their importance in capturing dependencies in financial data. By utilizing Clayton copulas, the study highlighted improved accuracy in parameter estimation and the ability to model lower tail dependencies [10,11].

2.1.2 Gumbel copula

Introduced by Gumbel in 1960, the Gumbel copula is particularly effective for modeling upper tail dependencies, making it highly valuable in financial scenarios where extreme events like market rallies occur. Unlike the Clayton copula, which captures lower tail dependence, the Gumbel copula excels with extreme high values.

A study by Tinungki *et al.,* [12] during the COVID-19 pandemic showed its effectiveness in estimating VaR for telecommunication stocks in Indonesia, improving risk assessment accuracy. Dewick and Liu [13] also highlighted their use in predicting financial contagion and understanding economic dependencies. In 2024, Ahmad *et al.,* [14] found the Gumbel copula best modelled drought patterns in Baluchistan, aiding water resource management and mitigation strategies.

2.1.3 Frank copula

Introduced by Frank in 1979, the Frank copula is an Archimedean copula recognized for its capability to symmetrically model both positive and negative dependencies. Unlike the Clayton and Gumbel copulas, which focus on lower and upper tail dependencies respectively, the Frank copula offers a balanced approach, making it suitable for various financial applications where dependencies are not necessarily extreme.

The uses of Bayesian mixture copula estimation demonstrate that Frank copulas effectively capture complex dependencies in financial markets [9]. This enhances the accuracy of risk assessments and portfolio optimization [13]. Additionally, Frank copula's symmetric dependence structure improves the modelling of relationships between various risk factors, enhancing the accuracy of risk assessments and the pricing of insurance products, particularly in valuing natural catastrophe risks [15].

2.2 Parameter Estimation Methods
2.2.1 Maximum Likelihood Estimation (MLE)

MLE is a parametric technique that focuses on determining the parameter values that maximize the likelihood function, based on the assumption that the data samples come from a population that perfectly matches the model parameters. MLE is known for its asymptotic efficiency and consistency under regular conditions, making it a preferred choice for parameter estimation in many statistical applications, including copulas [16].

A study by Zhang *et al.,* [3] examined MLE methods for copula models, highlighting challenges and innovations through Monte Carlo simulations and real data applications. The research emphasized MLE's effectiveness in portfolio risk management, particularly for complex, high-dimensional financial data, demonstrating the importance of optimization methods in enhancing MLE reliability.

2.2.2. Pseudo Maximum Likelihood (PML)

PML extends MLE by relaxing some of the stringent distributional assumptions, making it more adaptable to models where the true distribution is unknown or complex. This method transforms the sample data into uniform variates using the empirical cumulative distribution function (ECDF) before applying the likelihood function. PML is robust in handling deviations from typical assumptions, making it suitable for complex financial models characterized by asymmetries or heavy tails [16].

The limitations of traditional PML estimation for copula models often overestimates dependence, especially in small samples [4]. The study proposes modifications using the median or mode of order statistics. Through simulations, the modified PML estimators outperform traditional methods like Kendall’s tau and Spearman’s rho in terms of bias and mean square error. Applying these estimators to insurance data shows significant variations in dependence estimation between different products, demonstrating their practical relevance [4].

3. Methodology

3.1 Research Data

The data used in this study consists of exchange rates for the Malaysian Ringgit MYR against USD and EUR. The dataset spans from 1999 to October 2023 and was sourced from Bank Negara Malaysia. This extensive data set provides a comprehensive range of historical exchange rate observations, essential for robust analysis. The methodology used in this is research to compare the effectiveness of different parameter estimation methods in financial applications using copula models. This includes data collection, selection of copula families, parameter estimation methods, and the evaluation of model performance using RMSE.

3.2 Copula

3.2.1 Clayton copula

The Clayton copula, introduced by Clayton (1978), is particularly effective in modelling lower tail dependence, which is useful in financial contexts where extreme low values of one variable are associated with extreme low values of another variable. The Clayton copula is defined by the following CDF Eq. (1):

$$C\ (u, v; \theta) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta} \quad (1)$$

where copula parameter, θ controls the strength of dependence.

3.2.2 Gumbel Copula

The Gumbel copula is renowned for its capacity to capture upper tail dependence, making it particularly appropriate for financial contexts where extreme high values in one variable are linked with extreme high values in another. The Gumbel copula is defined by the following CDF Eq. (2):

$$C\,(u, v; \theta) = \exp - [(-\ln u)^{\theta} + (-\ln v)^{\theta}]^{(1/\theta)} \tag{2}$$

where θ ≥ 1.

3.2.3 Frank Copula

The Frank copula is distinctive in its capability to symmetrically model both positive and negative dependencies. This characteristic makes it well-suited for various financial applications where the dependencies between variables are moderate rather than extreme. The Frank copula is defined by the following CDF Eq. (3):

$$(u, v; \theta) = -\frac{1}{\theta} ln \left| 1 + \frac{(exp\{-\theta u\} - 1)\,(exp\{-\theta v\} - 1)}{exp\{-\theta\} - 1} \right| \tag{3}$$

where θ ≠ 1.

3.3 Parameter Estimation Methods
3.3.1 Maximum Likelihood Estimation (MLE)

MLE, which is used to estimate copula parameters, involves the following steps:

i. Construct the likelihood function for copula Eq. (4):

$$L(\alpha, \beta, \theta) = \prod_{i=1}^{n} c[\,F_x(x_i; \alpha), F_y(y_i; \beta); \theta]\,.\,f_x(x_i; \alpha)\,.\,f_y(y_i; \beta) \tag{4}$$

ii. Find the log-likelihood function of Eq. (4) by taking the natural logarithm of the equation.
iii. Differentiate the log-likelihood function with respect to each parameter α, β, and θ. Then, set the derivatives to zero as such Eq. (5), Eq. (6), and Eq. (7):

$$\frac{\partial[ln\,L(\alpha,\beta,\theta)]}{\partial\alpha} = 0 \tag{5}$$

$$\frac{\partial[ln\,L(\alpha,\beta,\theta)]}{\partial\beta} = 0 \tag{6}$$

$$\frac{\partial[ln\,L(\alpha,\beta,\theta)]}{\partial\theta} = 0 \tag{7}$$

iv. Obtain the ML estimators denoted by, $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\theta}$ by solving the equations above simultaneously.

3.3.2 Pseudo Maximum Likelihood (PML)

PML is a semi parametric method that relaxes some of the assumptions of MLE, making it more adaptable to cases where the true distribution is unknown. The steps involved in PML are:

i. Transform the sample data into uniform varieties using empirical distribution function as Eq. (8) and Eq. (9):

$$u = \hat{F}_n(x) = \frac{1}{n+1}\sum_{j=1} I_{[x_j \leq x_i]} \tag{8}$$

$$v = \hat{F}_n(y) = \frac{1}{n+1}\sum_{j=1} I_{[y_j \leq y_i]} \tag{9}$$

ii. Maximize the copula log-likelihood model $l(\alpha, \beta, \theta)$to estimate the dependence parameter, θ as follows in Eq. (10):

$$\hat{\theta} = \arg max \sum_{i=1}^{n} \ln c\ [\hat{F}_n(x_i), \hat{F}_n(y_i); \theta] = \arg max \sum_{i=1}^{n} \ln c\ [u_i, v_i; \theta] \tag{10}$$

3.4 Root Mean Squared Error (RMSE)

The GOF of copula models is commonly assessed using RMSE which measures the difference between the observed values and the values predicted by the model, providing a quantitative measure of model accuracy. It is defined as Eq. (11)

$$RMSE = \sqrt{\frac{1}{n}\Sigma_{\mathrm{i}=1}^{n}(p_i - o_i)^2} \tag{11}$$

where number of observations, *n*, predicted value for the *i*-th observation, p_i, and the actual observed value, o_i.

4. Results

4.1 Data Analysis

The dataset consists of monthly exchange rates for the MYR against USD and EUR from January 1999 to October 2023, sourced from Bank Negara Malaysia. Table 1 summarizes the descriptive statistics for these exchange rates, showing significant variations between the two currency pairs.

Table 1
Descriptive statistics of exchange rates

Variable	MYR against EUR	MYR against USD
Minimum	3.2447	2.4769
Maximum	5.1258	4.4746
Mean	4.4315	3.7320
Standard Deviation	0.4633	0.4546

These statistics suggest a relatively volatile currency exchange environment for the MYR, with more pronounced fluctuations against the EUR. The scatter plot in Figure 1 illustrates the correlation between the exchange rates of MYR against USD and EUR, revealing a weak but positive linear relationship.

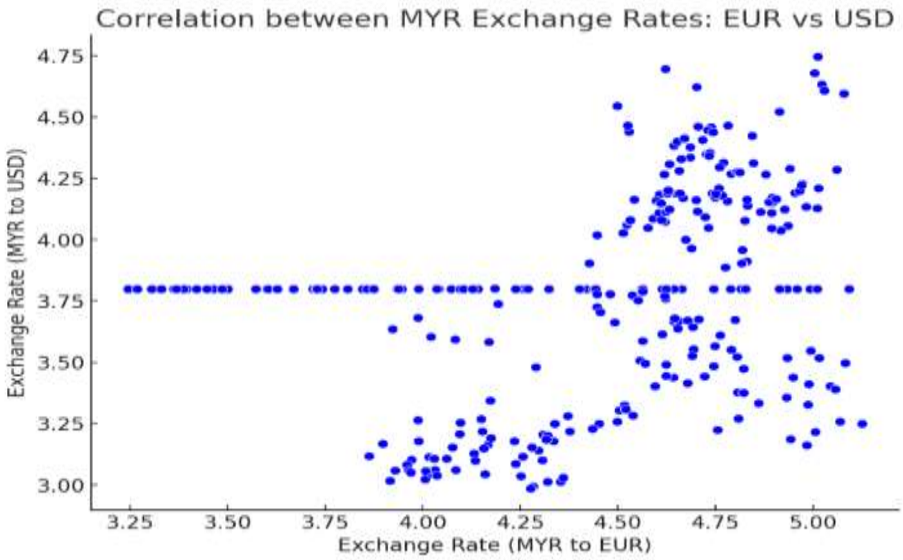


Fig. 1. Scatter Plot of MYR/USD vs. MYR/EUR Exchange Rates

The time series plot in Figure 2 provides insights into the variability of the MYR exchange rates against the EUR and USD over time. The USD rate shows periods of increase and stability, whereas the EUR rate exhibits a more consistent upward trend, especially in the later years.

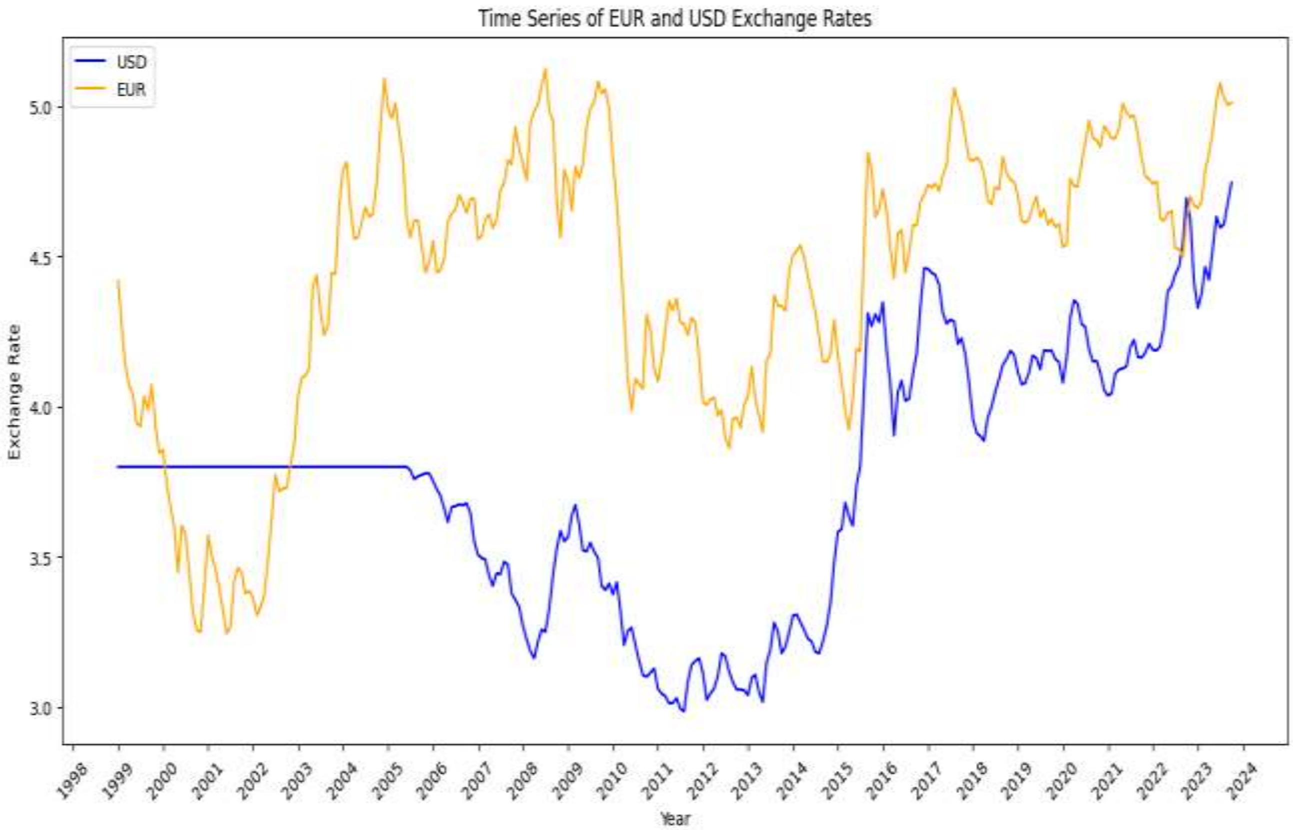


Fig. 2. Time series plot of MYR/USD vs. MYR/EUR exchange rates

Then, the correlation between the exchange rates of MYR against EUR and MYR against USD was assessed using Pearson's correlation coefficient and Kendall's tau correlation. These measures help understand the linear relationship and the strength of dependence between the currency pairs. The results are summarized in Table 2.

Table 2
Correlation coefficients between MYR/EUR and MYR/USD exchange rates

Correlation Coefficient	Coefficient Value	p-value
Pearson's coefficient	0.3100	0.0000
Kendall's tau	0.2520	0.0000

The Pearson's correlation coefficient of 0.3100 indicates a low to moderate positive correlation, meaning that as the exchange rate of one currency pair increases, the other also tends to increase, albeit not strongly. Kendall's tau correlation coefficient of 0.2520 also indicates a positive correlation, but with a weaker strength compared to Pearson's coefficient. Both correlations have a highly significant p-value of 0.0000, indicating that the observed correlations are statistically significant and unlikely to be due to random chance. This suggests a meaningful relationship between the MYR/EUR and MYR/USD exchange rates as identified by both correlation methods which could be modelled with a copula.

4.2 Specifying Marginal Distributions

This section evaluates three distributions for analysing the exchange rates of the MYR against the EUR and the USD: the normal distribution, the Generalized Extreme Value (GEV) distribution (see Figure 4), and the log logistic distribution (see Figure 3). These distributions were chosen based on their characteristics and potential to model the unique behaviours observed in the exchange rate data.

The normal distribution is chosen for its characteristic of data variation around the mean, particularly seen in USD exchange rates. Then, the GEV distribution is chosen for effective in modelling extreme values, suitable for capturing tail behaviours in financial datasets. Finally, the log logistic distribution is selected to investigate its performance in modelling data with heavier tails and a skewed nature, particularly observed in EUR exchange rates.

The goodness-of-fit for the chosen distributions was assessed using the Kolmogorov-Smirnov (KS) test, which determines the best fit for the observed data to a specified distribution. Lower KS values indicate a better model fit. The results are presented in Table 3.

Table 3
Goodness-of-Fit Test using the Kolmogorov-Smirnov test

Distribution	MYR against EUR	MYR against USD
Normal Distribution	0.06476	0.16597
GEV Distribution	0.03608	0.16597
Log Logistic Distribution	0.02349	0.16597

The results indicate that the log logistic distribution provides the best fit for the MYR against EUR, while the GEV distribution demonstrates the best fit for the MYR against USD exchange rates. This suggests that the log logistic distribution effectively captures the heavier tails in EUR data, and the GEV distribution is more suitable for modelling the extreme values in USD data. This step of fitting the marginals into parametric distribution is crucial since it is an underlying assumption for the MLE method.

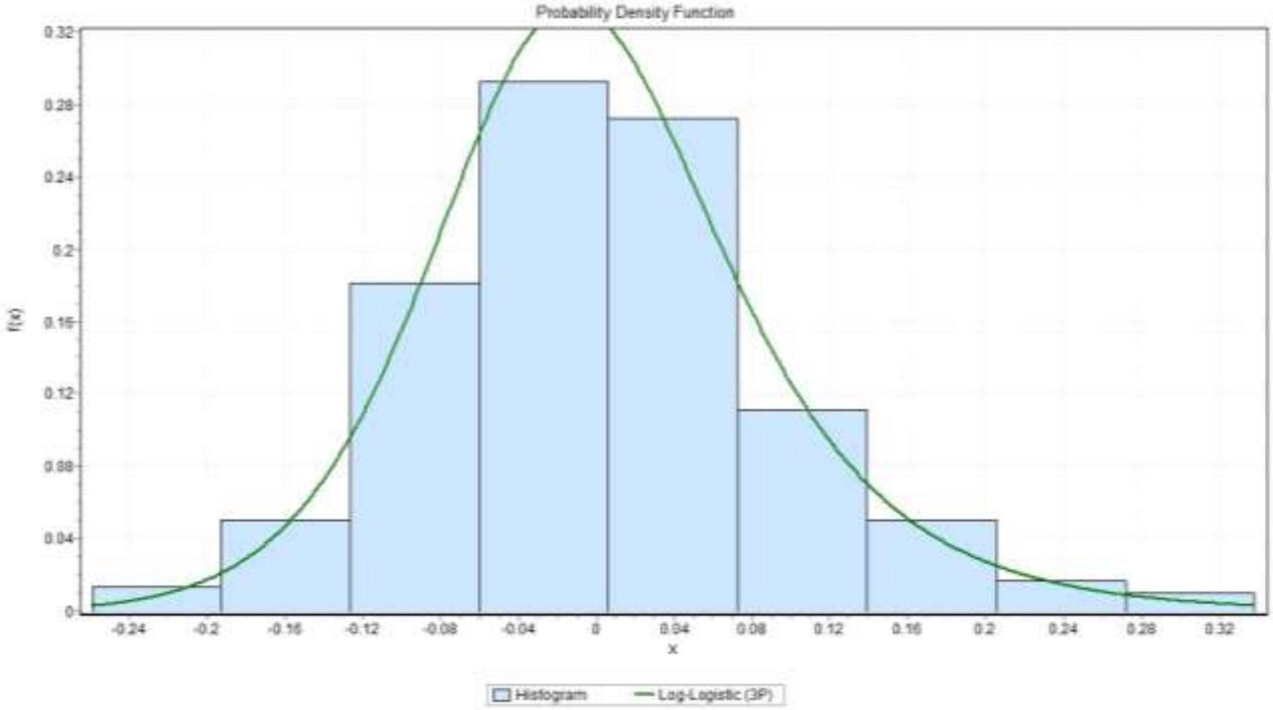


Fig. 3. PDF of the fitted Log Logistic distribution for MYR/EUR

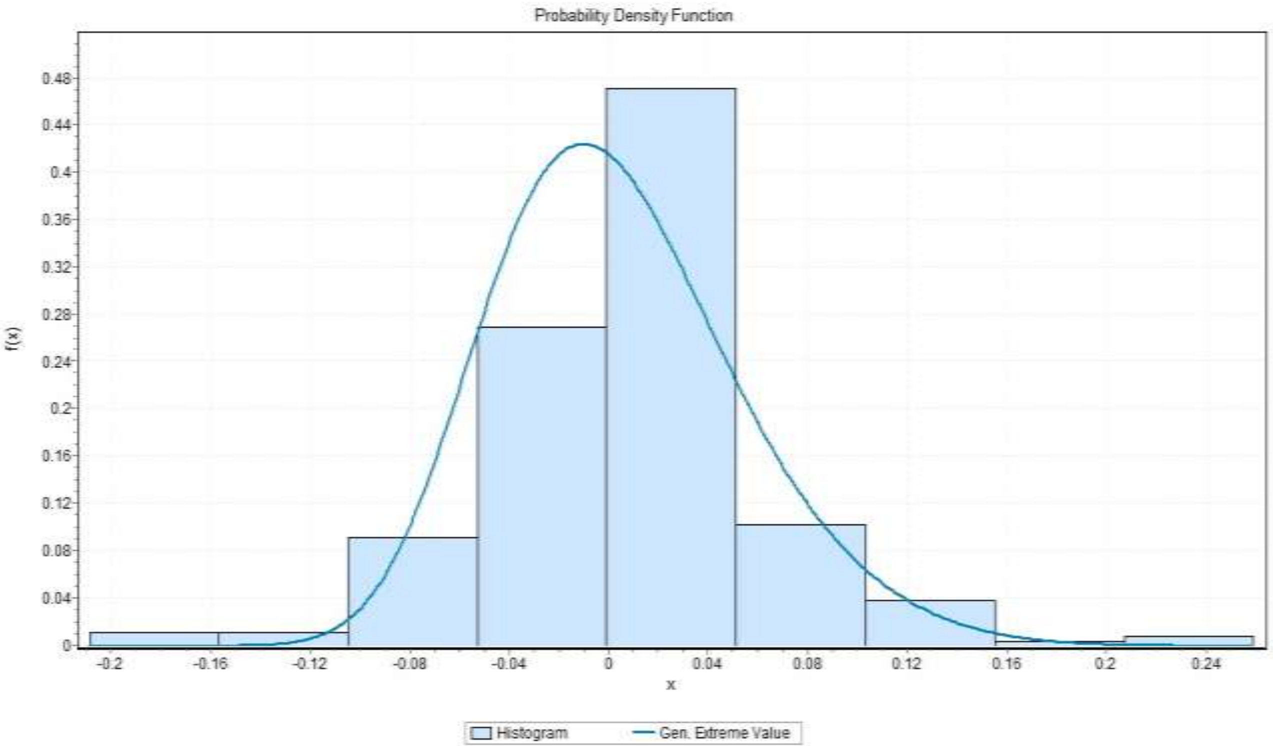


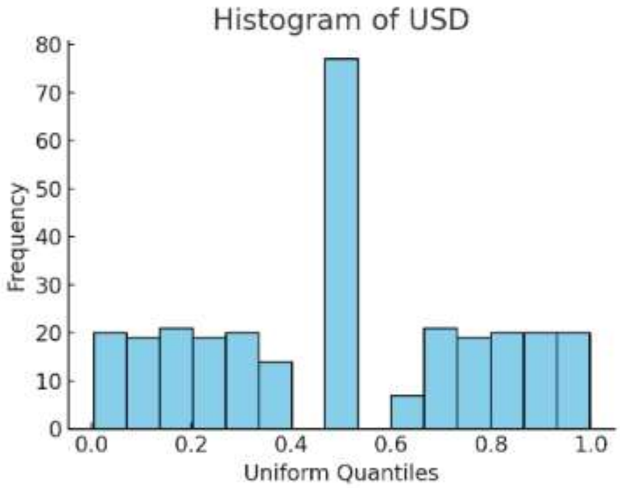
Fig. 4. PDF of the fitted GEV distribution for MYR/USD

For methods that do not require parametric marginal distributions, namely PML, the ECDF is used to transform data. This nonparametric method allows for the estimation of marginal distributions using actual observations, which is critical for accurately capturing the underlying data distributions and dependencies.

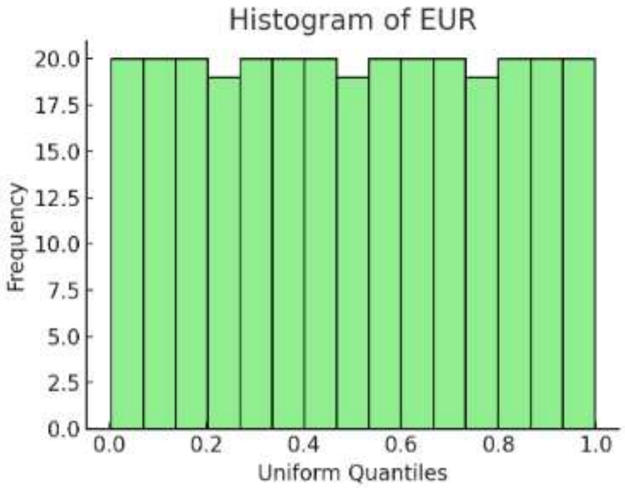
According to Sklar's theorem, the joint distribution of two variables can be uniquely represented by a copula function that incorporates their individual marginal CDFs along with a parameter that binds these marginal together. Transforming using the ECDF scales the marginal (see Figure 5) to follow a uniform distribution from 0 to 1. The ECDF of a data point y_i from the sample Y can be calculated as such Eq. (12):

$$\hat{F}(y_i) = \frac{1 + rank\ of\ y_i\ among\ Y}{n+1} \tag{12}$$

This transformation standardizes the exchange rate data to a uniform distribution, which is then used to fit the selected copula. This step is crucial for evaluating the performance of both parametric (MLE) and semi parametric (PML) methods in capturing dependencies in financial data.



(a)



(b)

Fig. 5. ECDF of the marginals (a) MYR/USD (b) MYR/EUR

4.3. Results of Estimated Dependence Parameter

This section presents the results from the estimation of dependence parameters using various copula families across two methods: MLE and PML. Each method was applied to three copula families, which are Gumbel, Clayton, and Frank, to assess their effectiveness in modelling the dependencies between the exchange rates of the Malaysian Ringgit against major currencies.

Table 4
Estimated dependence parameter, θ of the copula obtained from MLE and PML

Copula Family	MLE	PML
Gumbel	1.0898	1.1350
Clayton	0.1795	0.1790
Frank	0.7454	0.7536

Table 4 shows the estimated dependence parameters for each copula family using both MLE and PML methods. The results indicate relatively consistent estimates across both methods for each copula family, with some variability depending on the specific method used. The variability in the estimated parameters highlights the importance of selecting the appropriate method and copula family based on the specific characteristics of the financial data being analyzed since the choice of copula and estimation technique can significantly impact the interpretation of dependencies.

4.4. Goodness of Fit and Model Selection

To evaluate the performance of different copula models and estimation methods, the RMSE was used as the statistical GOF measure in this study where a lower RMSE value indicates a better fit of the model to the data, suggesting more accurate parameter estimates.

Table 5
RMSE values for model evaluation

Copula Family	MLE	PML
Gumbel	0.0939	0.0843
Clayton	0.0727	0.0401
Frank	0.0476	0.0061

The results in Table 5 indicate that the Frank copula model consistently shows the lowest RMSE across both MLE and PML methods, demonstrating its effectiveness in capturing complex dependencies in financial markets. PML, in particular, shows the lowest RMSE values, indicating it as the most accurate method for parameter estimation in this study.

The low RMSE value of 0.0061 for the Frank copula under PML suggests that this method provides the most accurate fit to the data, capturing the dependencies with great precision. This indicates that PML is highly effective in modelling the complex relationships between the exchange rates, making it a preferred method for such analyses.

The MLE method also performs well with the Frank copula, with an RMSE of 0.0476. While this value is higher than that of PML, it still reflects a reliable estimation method. The relatively close RMSE values between MLE and PML for the Frank copula suggest that both methods are robust, although PML has a slight advantage in precision.

The performance of PML with the Frank copula demonstrates its capability in modelling the dependency structure between the EUR and USD exchange rates. This copula is particularly effective in capturing moderate tail dependence characteristics observed between these currencies. In financial terms, using the Frank copula can help in accurately estimating and managing risks associated with currency fluctuations. This accuracy is beneficial in forecasting and hedging against potential market movements based on the relationship between different currencies.

5. Conclusions

This study evaluated the effectiveness of parameter estimation methods, MLE and PML for Archimedean copulas in modelling dependencies between financial variables. Using exchange rate data for the MYR against USD and EUR, the research focused on the Clayton, Gumbel, and Frank copulas. The findings indicate that the Frank copula consistently showed the lowest RMSE values across both methods, with PML emerging as the more accurate estimation method. Specifically, the RMSE for the Frank copula was 0.0476 using MLE and 0.0061 using PML, highlighting PML's better precision in parameter estimation. This highlights the importance of selecting suitable copula models and estimation methods for accurate financial risk management. The Frank copula, estimated via PML, proved particularly effective in capturing dependencies in exchange rate data. These insights can help financial analysts to better assess risks and strategize based on the relationships between different currencies.

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