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Exploring Mathematics through Game-Based Learning Mastermind Game: Colour Code Breaker

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ABSTRACT

Mastermind, a classic code-breaking game for two players, features various versions with differing numbers of code pegs (ranging from 4 to 6) and rules concerning repeated colours and blank pegs. This paper explores a simplified variant of Mastermind, where solutions are restricted to code pegs with unique colours and no blank pegs. By focusing on a subset of two code pegs, we model each attempt as a linear system. Through three attempts that satisfy the criteria of a linear system, we determine the initial colour solution for each small group. Further analysis allows for the calculation of the correct colours and their positions within each group using probability measurements. This approach provides a systematic method for solving the simplified Mastermind game and demonstrates the application of linear systems and probabilistic techniques in code-breaking. The findings offer insights into the effectiveness of these mathematical methods in deriving accurate solutions and enhancing problem-solving strategies within the game.

1. Introduction

Exploratory learning is an educational approach where students actively seek out new information and knowledge instead of passively absorbing it. This method fosters curiosity, experimentation, and a readiness to delve into the unknown. Typically, students engage in novel activities like problem-solving before receiving instruction on the underlying concepts and procedures. Research has shown that exploratory learning can enhance learning outcomes in in-person courses without necessitating a complete overhaul of the course structure [1]. In exploratory learning, rather than following a strict sequence of training materials, the learner independently explores a system, often driven by the pursuit of a specific real or simulated task [2]. Active engagement and critical reflection plays crucial parts in educational settings [3-4]. Hence, the researchers introduce a four-dimensional framework to assist educators in assessing potential

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educational games within the topics or subject matter. However, this new exploratory learning does not diminish educator's roles, but instead, it helps educators to create more engaging study environment. In study conducted by Getchell *et al*., [5], the researchers emphasize on the exploratory learning where the researchers use computer game technologies to create engaging and interactive resources which support the explorative learning.

One games that can implement the exploratory learning is through Classic Mastermind or often called Mastermind games [6]. Mastermind, originally a code-breaking game, has application in various field including mathematics and psychology [7]. Martinsson and Su [8] stated that Mastermind is a famous code-breaking board game for two players. That one player or the code maker makes a hidden codeword that consists of a sequence of four colours. Meanwhile, the goal of the second player known as the codebreaker must guess this codeword. After each guess, the codemaker provides several black and white pegs indicating how close the guess is to the real codeword. The game is over when the codebreaker has made a guess identical to the hidden string. The Mastermind game is also known by various names and variants such as Bulls and Cows (a paper-pencil version) or Jotto (a letter-based version) [9]. Originally, the Mastermind was a two-player game with numbers where one player comes up with a four-digit numeric code while the other tries to interpret the code. To achieve this goal, the codebreaker gets feedback indicating the number of digits in their right positions and those that were part of the code, but in different positions [10]. Strom and Barolo [7] uses Mastermind game to enhance logical and understanding skills on the scientific concepts of students. In their studies, the researchers simulate and experimental process where one player creates a secret code, while the others attempts to deduce it using logic and reasoning. Their research is slightly different with our research, where in this research, we encourage the players (students) to use all knowledge regarding mathematics which is probability and linear system of equations to solve the Mastermind game.

Peyman *et al*., [19] studied the query complexity based on the permutation method for the guessing game Mastermind. In addition, El Ouali and Sauerland [11] also showed that de-terministic algorithms for the identification of a secret code in Black-Peg AB-Mastermind can be modified and applied to Yes-No AB-Mastermind. The latter is a new variant of AB-Mastermind which is harder to play for the codebreaker since a less informative error measure is provided. The Yes-No measure only returns the information on whether a query and the secret code coincide in any position, while the Black-Peg measure is the number of positions in which both codes coincide [12]. Merelo *et al*., [13] uses a different technique, where the researchers treat the Mastermind game as an optimization problem and introduced a new function for evolutionary algorithms that address the challenge of achieving the results as well as minimizing the time taken to solve the game. In later year, Merelo *et al*., [14] then focused on improving the efficiency of solving the Mastermind puzzle by examining the effect of different parameters towards the speed and effectiveness of the code-breaking process. The researchers firstly tested small and consistent set sizes using two scoring methods where by analyzing two methods which are entropy and most parts, they conclude that these methods influence the speed in finding the solutions of the code breaker game. Meanwhile, Monte-Carlo method and Sarsa (λ) algorithms is used by Lu *et al*., [15] in order to obtain the secret code of the opponent. The researchers use these two mathematical methods in order to solve the code breaker game.

Meanwhile, entropy mastermind is a code-breaking game based on the classic game Mastermind. In entropy mastermind, a secret code is generated from a probability distribution by random drawing and replacement [16]. The entropy mastermind game is a probabilistic version of the classic code-breaking game, involving inductive, deductive, and scientific reasoning. Whether help in the form of a hint was available and manipulated within subjects. Results showed that participants tended to ask for help late in the gameplay, often when they already had all the necessary information needed to crack the code [17]. The application of the entropy mastermind games has been explained by Özel *et al.*, [18]. Some of the schoolchildren in the range of ages between eight to 10 played a version of entropy mastermind with jars and coloured marbles in which a hidden code to be interpreted was generated at random from an urn with a known, visually presented probability distribution of marble colours. In this study, they managed to describe the novel game-based mathematics intervention for fostering children's intuitions about entropy and probabilities using the entropy mastermind. Prabhu and Woodruff [19] further explored the secrets in Mastermind games by proposing set H of n hidden points that the codebreaker must learn all the points in H while minimizing the number of queries they make.

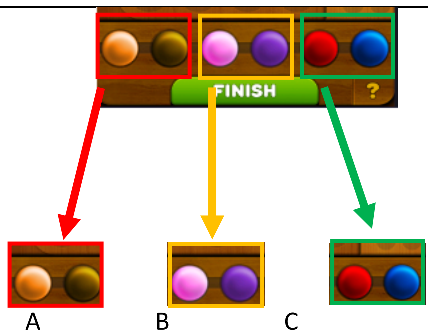
In summary, Mastermind was an interactive game for children, in which one player must predict the correct sequence of coloured pins arranged by the opponent through a limited series of attempts. The opponent provides feedback for each guess, indicating the number of correct coloured pins and the number of correct coloured pins in the correct position. In the deductive version, a series of guesses and the associated feedback are predetermined so that the code can be unambiguously inferred from the given premises. Hence, this paper aims to combine the principle of exploratory learning, which is highlighted through Mastermind game. The exploration of Mastermind games through the implementation of mathematical concepts which is linear systems and probability aims to provide details understanding on the mathematical concepts in games as well as nurturing problem solving skills for the learners.

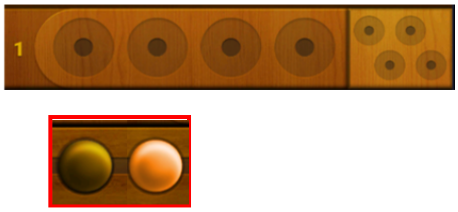
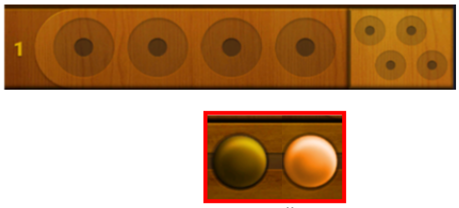
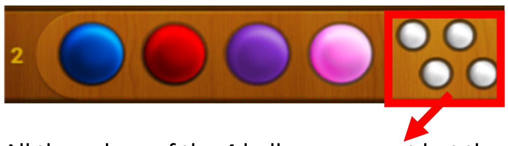
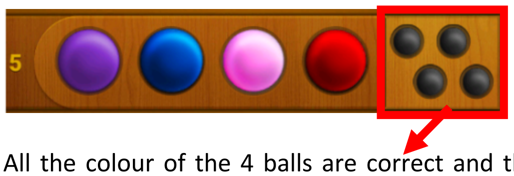
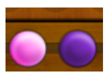
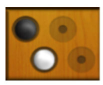
2. The Mathematics behind Colour Code Breaker

2.1 The Definition of Symbols

In the colour code breaker game, grasping the specific definitions and meanings of the symbols used is essential. These symbols are vital for categorizing colour codes and solving the puzzles. To navigate and strategize effectively, it is crucial to understand what each symbol represents and how it influences the overall solution. This foundational knowledge is fundamental to mastering the game and achieving accurate results. Table 1 below provides a detailed explanation of these symbols before we delve into the methodology of the colour code game.

Table 1
Definitions of symbols

Symbol	Description	Example
A	The first group comprises of the first and second balls out of six balls. Note that the location and sequence of the colour cannot be switched.	
B	The second group comprises of the third and fourth balls out of six balls. Note that the location and sequence of the colour cannot be switched.	
C	The third group comprises of the fifth and sixth balls out of six balls. Note that the location and sequence of the colour cannot be switched.	** Note that the location and colour of the balls in each compartment cannot be changed.

A1, A2, B1, B2, C1, C2	The symbol A1 means the first ball in the first group. The symbol A2 means the second ball in the first group. The symbol B1 means the first ball in the second group and etc.	 A1 A2 B1 B2
'	The position of the two balls in a specific group is switched.	 A'
"	The position of the two balls in a specific group is switched and location of the group is put at the back.	 A"
W	The white ball on the right means the colour of the ball is correct but the position is wrong.	 All the colour of the 4 balls are correct but they are put in the wrong position.
B	The black ball on the right means both the colour and position of the ball are correct.	 All the colour of the 4 balls are correct and their position are correct! Bingo!
+	The combination of two groups.	 Equation that can be formed from the above trial is:  +  =  Which implies A + B = 1b 1w

To effectively address and solve the complexities associated with mastering the colour code game, it is crucial to first identify and thoroughly understand the concept of colour grouping. This involves analyzing and categorizing colours based on their relationships and positions within a given framework. As noted in the definition, for the expression, $A+B \neq B+A$ it indicates that A must be positioned in the first column, while B needs to be placed in the second column.

The initial step in tackling this challenge involves making a foundational assumption regarding the colour code game. Specifically, we should operate under the assumption that the solution can be approached by utilizing a strategic framework, which incorporates keywords related to location and positional information. By doing so, we can systematically break down the problem and apply relevant principles to identify patterns and ultimately decipher the colour code accurately. Our first assumption is:

$$\begin{aligned} &A + B \rightarrow \text{Trial 1} \\ &C' + B' \rightarrow \text{Trial 2} \\ &C + A'' \rightarrow \text{Trial 3} \end{aligned} \tag{1}$$

In this context, *A, B,* and *C* represent the initial positions of the colours before the game begins. For instance, if the original arrangement of colours for the balls is depicted in Figure 1, then *A* denotes the yellow and gold balls, *B* signifies the pink and purple balls, and *C* indicates the red and blue balls. Specifically, *A* is located in the left column, while both *B* and *C* are positioned in the right column.

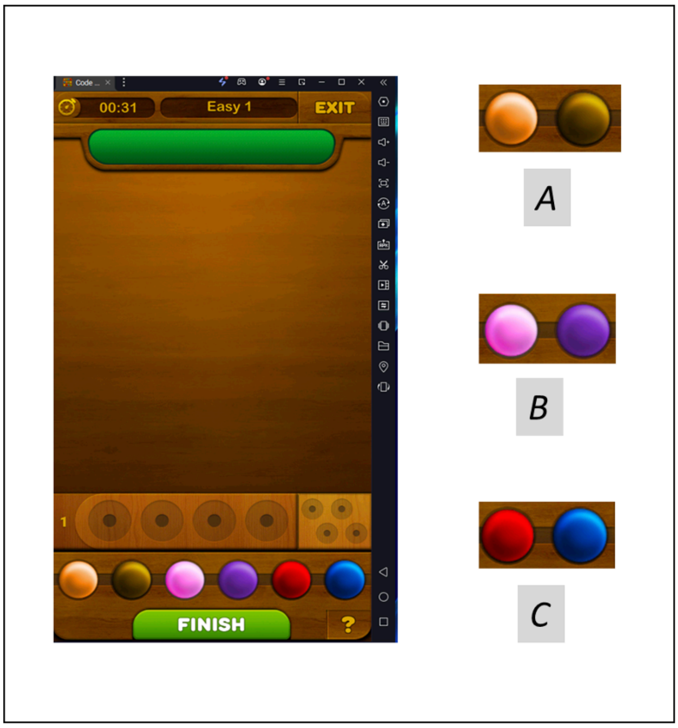


Fig. 1. The interface of the code breaker mastermind game

Referring to the initial assumption outlined in equation (1), *A′* represents the first shift applied to the first two colours, altering their positions. Figure 2 illustrates the changes made to each category of balls as a result of this shifting process.

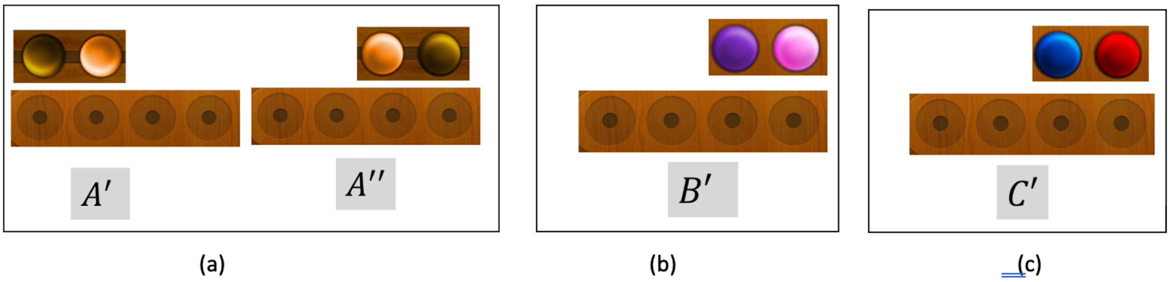


Fig. 2. Shifting of the colour of the ball

Initially, by referring to the first assumption made as in equation (1), it is observed that, the solution will appear at the right column of the game which is either white or black dot. One white dot indicates that, in the solutions, among the four balls that is placed, there is one colour of the balls is correct with wrong position. While for black dot, it is indicating that, the colour and the position of the balls is correctly placed. Figure 3 shows the output of the colour for the balls as we followed the assumption made as in equation (1) where from Figure 3, from the first and third output, one black dot and one white dot shows that, the arrangement of four balls for the first row is in correct for one unknown colour while falsely position for one colour among those four balls. For the second output, it is observed that four white dots appear where it indicates that, all four colour is correct however it is wrong in its position.

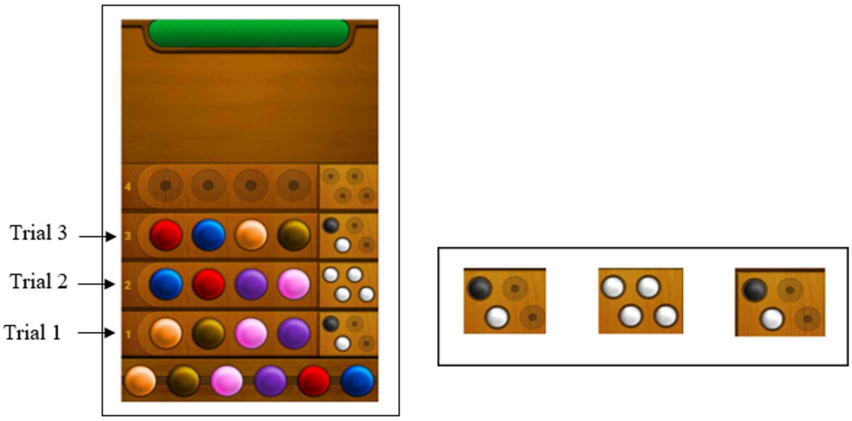


Fig. 3. The arrangement of the colour and the output of the code breaker game (for Trial 1, Trial 2 and Trial 3)

Once all the outputs from the games have been obtained, they are substituted into the first assumption. Consequently, equation (1) will be adjusted to focus solely on colour matching.

$$\begin{aligned} A + B &= 1W1B \\ C' + B' &= 4W \\ C + A'' &= 1B1W \end{aligned} \tag{2}$$

Consequently, equation (1) will be adjusted to focus solely on colour. We consider only colour matching and disregard the positions, resulting in Equation (2) being transformed into Equation (3).

$$
\begin{aligned}
&A + B = 2 \\
&C' + B' = 4 \\
&C + A'' = 2
\end{aligned}
\tag{3}
$$

2 in this first row and third of the linear system indicate one black and one white dots while 4 in the second row indicate four white dots. The solution of the colour of the game can be easily obtain by solving the linear system in equation (3). By transforming the equation into matrix system of equation, equation (3) then yield to equation (4) as stated below.

$$
\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}
\begin{bmatrix} A \\ B \\ C \end{bmatrix}
=
\begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}
\tag{3}
$$

The inverse matrix method is used to solve the matrix system that is obtained as in equation (4) which yield to equation (5) as stated below.

$$
\begin{aligned}
\begin{bmatrix} A \\ B \\ C \end{bmatrix}
&=
\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}^{-1}
\begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix} \\
&=
\begin{bmatrix} 0.5 & -0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 \end{bmatrix}
\begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}
\end{aligned}
\tag{3}
$$

By multiplying the matrix in the right hand side of equation (5), it will yield to the following solution of the games as shown in equation (6).

$$
\begin{bmatrix} A \\ B \\ C \end{bmatrix}
=
\begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}
\tag{3}
$$

The solution reveals that the colour scheme for the colour code breaker game comprises two colours from the B category and two colours from the C category—specifically pink, purple, red, and blue. However, the correct positions of these colours are still inaccurately determined. To identify the correct positioning and colours of the balls, we use a simple probability approach. Given that A=0, we can derive the results from Equation (2).

$$[0 \quad B] = 2 = 1Black1White$$

$$[C' \quad 0] = 2 = 2White$$
$$[0 \quad B'] = 2 = 2White \tag{3}$$

$$[C \quad 0] = 2 = 1Black1White$$

Therefore, the analysis and interpretation of the results derived from Equation (7) can be comprehensively summarized in the table provided below. This summary facilitates a clearer understanding of the implications of Equation (7) and aids in drawing meaningful conclusions from the data.

Table 2
Definitions of symbols

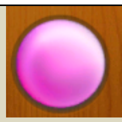
No. of Possibility	Column 1		Column 2	
	Colour 1	Colour 2	Colour 3	Colour 4
2	 $B2$		 $B1$	
2	 $B1$			 $B2$
2	 $C1$		 $C2$	
2		 $C2$	 $C1$	

Table 3 provides a comprehensive listing of the possible solutions, detailing the correct positions and colours of the balls. This table outlines the various configurations that satisfy the criteria for the correct arrangement, offering a clear view of the potential outcomes. By examining this table, one can evaluate the different possibilities and understand the range of solutions that align with the given parameters.

Table 3
The possibility of the solution of the balls (extra trials)

Trial	Column 1		Column 2	
	Colour 1	Colour 2	Colour 3	Colour 4
4	$B2$	$C2$	$B1$	$C1$
5	$C1$	$B2$	$B1$	$C2$
6	$C1$	$B1$	$C2$	$B2$
7	$B1$	$C2$	$C1$	$B2$

The probability of success in this scenario ranges from a minimum of 1 to a maximum of 4, given the four possible outcomes. On average, to achieve a win, one would need to undertake approximately 2.5 extra trials in addition to the initial 3 basic trials. This means that while the minimum number of trials required is 1, the maximum could extend up to 4, and typically, achieving a successful outcome will involve a total of around 5.5 trials.

Fig. 4. The solution of the colour code breaker game round 1

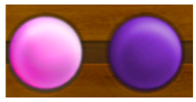
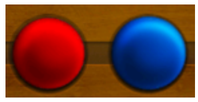
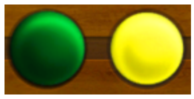
With the foundational strategies and educational contexts established, we now turn our attention to the results and examples from Game Round 1. This upcoming section will provide concrete illustrations of how the possibility and deduction approaches are applied in practice. By examining these examples, we will gain valuable insights into the effectiveness of each approach and how they contribute to solving colour code breaker games. The detailed analysis of results will further elucidate the practical implications of these strategies, offering a comprehensive understanding of their impact and utility in both educational and game settings.

3. Results and Examples

3.1 Examples of Game Round 1 using Probability Approach

To better understand the dynamics of the colour code breaker game, we will examine specific examples from Game Round 1. These examples illustrate the practical application of the game's rules and strategies, providing insights into how the colour codes are interpreted and solved. By reviewing these cases, players can gain a clearer grasp of the methodologies involved and enhance their approach to succeeding in the game.

Table 4
Definitions of symbols

Steps	Instructions to Solve
1. Given: 	Divide the six balls with different colours into 3 compartments: Group A:  Group B:  Group C:  Consider  as A1,  as A2 etc.
2. Given: 	Form the system of equations: $A + B = 3W$ $C' + B' = 1B\ 1W$ $C + A = 3W$ From the above equations, form the associated matrix of: $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$ Solving the above matrix to obtain: $\begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ From above, we can know that the correct combination should consists of both $A1$ and $A2$, regardless of the position. Besides that, since $B = 1$, then we should have either $B1$ or $B2$. Furthermore, from $C = 1$, we know that the correct answer should consist of either $C1$ or $C2$. Note that we can only determine the existence of the balls with specific colour, but we do not know the correct position.

3. Make a guess and given:

Forming the conditions:

$$[A \quad 0] = 2\ (2\ White)$$
$$[0 \quad B] = 1\ (1\ White)$$
$$[C' \quad B'] = 2\ (1\ Black\ 1\ White)$$
$$[C \quad 0] = 1\ (1\ White)$$
$$[0 \quad A] = 2\ (2\ White)$$

Determine the most helpful conditions. Combining the conditions of $[A \quad 0] = 2\ (2\ White)$ and $[0 \quad A] = 2\ (2\ White)$ allows us to come out with the possibility of four cases below:

A2	A1			=2W

		A2	A1

	A1	A2	

A2			A1

Other conditions are unable to provide sufficient information. However, from the condition $[0 \quad B] = 1\ (1\ White)$, we know that if B1 exists, then B2 cannot exist. Same goes to the condition $[C \quad 0] = 1\ (1\ White)$ which implies if C1 exists, then C2 cannot exists.

Due to the lacking of information, we need to make a guessing and analyse the further results obtained.

From the results obtained from our guess of

A1	A2	C1	B1

, it can be seen that 2 White and 2 Black dots are obtained, which means all the colours are correct, however, there are two balls are not placed in the correct position. Thus, we do not need to change the colours of the ball further. By referring to four possible cases of A1 and A2 mentioned in the previous step, we can conclude that the position of the A1 and A2 should be reversed. Finally, the correct answer should be

A2	A1	C1	B1

3.2 Examples of Game Round 1 using Deduction Approach

There is another approach that can be applied to solve the colour code breaker which is the deduction approach. The deduction approach is a technique that involves analyzing the feedback from each guess to infer the correct colour sequence. This approach is a logical method used to derive a conclusion or solve a problem by reasoning from known facts or general principles. It involves making inferences based on evidence, rules, or given data to narrow down possibilities and reach a specific answer. The deduction approach begins with the stating of the hypothesis. The player begins with a guess based on the available information such as the combination of colours in Mastermind. Then, the player will analyze the feedback gained for the guess made in the previous step. Here, the number of black and white dots indicates how many colours are correct and whether they are in the right position. For instance, if one black dot appears means one colour is in the correct position.

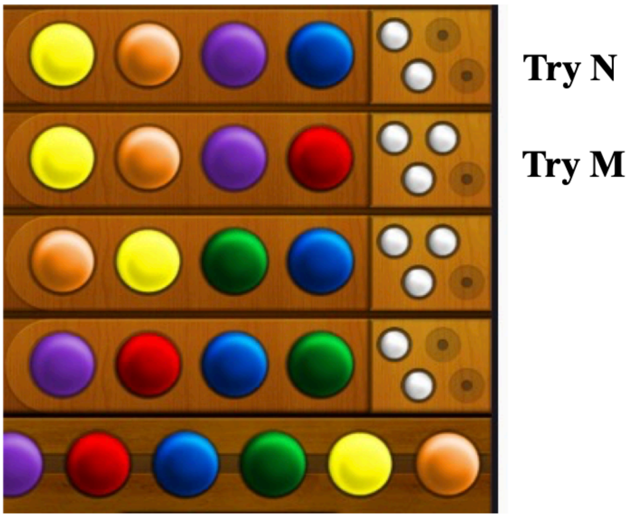


Fig. 5. The solution of the colour code breaker game round 1 using deduction approach

Let say, we obtained the third trial and the forth trial as in equation (7) and equation (8).

Try **M** $\quad [C1 \quad C2 \quad C3 \quad C4] = 3White$ (7)

Try **N** $\quad [C1 \quad C2 \quad A1 \quad A2] = 3White$ (8)

where
$$Try\ M - TryN = [C1, C2, A1, A2] - [C1, C2, A1, B1] = [*,*,*, A2] - [*,*,*, B1] = 3w - 2w = 1w$$

Hence, we get $A2 - B1 = 1White \rightarrow A2 = 1White$, please note that $B1$ is impossible to be $White$, which will induce the solution of $A2 = 2White$. After this step, the player can eliminate the possibilities by eliminating those colours that are not correct from the guess. Use the information gained to make a more informed guess. This involves tweaking your guess to test new possibilities while eliminating previously disproven ones. These guesses based on the feedback are continued until the correct solution is obtained. In short, a deduction approach is about using what you know such as facts, rules, and feedback to systematically uncover the solution.

4. Conclusions

In conclusion, the colour code breaker game can be effectively approached through two primary strategies which are the possibility approach and the deduction approach. The possibility approach involves exploring all potential outcomes to identify the correct solution. This method emphasizes the importance of experimentation, trial, and error, allowing players to gain in-sights through a process of discovery and adjustment. Conversely, the deduction approach re-lies on logical reasoning and systematic elimination to solve the puzzles. It involves analyzing the given clues and progressively narrowing down the possibilities to arrive at the correct answer.

These approaches are deeply intertwined with principles of exploratory and STEM education. The possibility approach resonates with exploratory education, which values hands-on experimentation and encourages learners to engage with problems in an open-ended manner. This method fosters a learning environment where students are encouraged to explore various solutions and learn from their experiences.

On the other hand, the deduction approach aligns closely with STEM education, which emphasizes structured problem-solving, critical thinking, and analytical skills. STEM education aims to develop students' abilities to apply logical reasoning and systematic approaches to complex problems, mirroring the deductive strategies used in the colour code breaker game. By integrating both approaches, players not only enhance their problem-solving skills but also gain a deeper understanding of the cognitive processes involved in both exploratory and STEM-oriented learning. These strategies collectively contribute to a well-rounded educational experience, bridging theoretical knowledge with practical application, and preparing individuals for a range of intellectual challenges.

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